Towards a low dissipation FE scheme for scale resolving turbulent compressible flows

Lucas Gasparino F. da Silva[*][†], Oriol Lehmkuhl[*], Daniel Mira Martinez[*]
[*]Barcelona Supercomputing Center, Barcelona, Spain
[†]Universitat Politècnica de Catalunya, Barcelona, Spain
E-mail: {lucas.gasparino, oriol.lehmkuhl, daniel.mira}@bsc.es

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I. EXTENDED ABSTRACT

This work focuses on developing a highly accurate Finite Element numerical scheme for Large-Eddy simulation of turbulent compressible flows, where low numerical dissipation is required. The proposition is to develop a low-dissipation stabilizing term into an algorithm, implement it on a readily-available CFD code and test results to assess performance and suitability.

A. Discretisation basics

In general, numerical solution of partial differential equations like the Navier-Stokes set governing fluid flow involves some form of space-time discretisation procedure, where the physical domain is divided into a set of points called grid, or a set of connected subdomains referred to as *elements* or *volumes*. If the problem is transient, then the continuous time must also be subdivided and discretised to allow for a local temporal solution. In finite elements, the simplest form of discretisation procedure is the Continuous Galerkin method, which yields a central-like, 2nd order discrete solution to the problem.

For convection dominated problems (those containing a transport-like term), direct application of a standard FE technique often results in oscillatory, or fully unstable, solutions, and therefore some form of stabilization is required, generally referred to as artificial diffusion, as the added terms have this exact behavior. Common stabilized methods are the Streamline Upwind Petrov Galerkin method (SUPG) and its variations, which achieve a stable solution by effectively reducing discretization order, ensuring the scheme remains monotonicity preserving.

Compared to a standard Galerkin discretization procedure, classical stabilized methods such as SUPG and nth Taylor-Galerkin are known to introduce strong diffusion terms into the discrete Navier-Stokes equations, compatible with their low-order nature. Although this has minor influence over RANS and URANS models, Large-Eddy and Direct-Numerical simulations (LES and DNS) are heavily affected by it, and one is forced to move towards high-order central schemes, which have narrow stability ranges. In the compressible range, the hyperbolic nature of the problem exacerbates this stability issues, and if shock waves form, or any other form of discontinuity exists, the method fails completely. As Finite Volume schemes solve this issue by means of Riemann solvers on the shock and use of flux limiters that reduce the scheme's order locally, these are quite complex to implement in Finite Elements, and thus a different approach is necessary.

B. Entropy Viscosity and uniqueness of solutions

Research on stabilization of general hyperbolic problems has led to the development of a low-dissipation nonlinear term named *Entropy Viscosity*, which, when used in a Finite Elements context, behaves in a similar way to a FV flux-limiter, i.e., ensuring that 1st order solutions are employed only at high gradient regions, such as a shock or interface.

This method is based on the answer of a purely mathematical question: to find a unique solution for a general hyperbolic equation. As the strong form of this problem is not approachable, a *viscous solution* is sought that satisfies a weak form of the reformulated conservation law. The true solution is then found by assuming the limiting case where viscosity vanishes. Such viscous solution is referred to as the *entropy solution* of the hyperbolic problem, as it must satisfy *Kružkov's entropy condition*.

The mathematical theory described above lends itself well to a Finite Element discretisation procedure, as Kružkov's entropy condition is already stated in an ideal weak form. Moreover, it was found that the theory developed for 1D scalar cases can be directly extended to multi-dimensional hyperbolic (and quasi-hyperbolic) systems like the Euler and Navier Stokes equations, even if a nonlinear source term exists. This indicates a possible candidate for a low-dissipation algorithm theory.

C. Research steps

The first part of this research project has focused on adapting the generic concept developed on hyperbolic equations to the 1D Euler system describing inviscid compressible flows, as well as studying in practice how well it worked when compared to other approaches. Following previous work already developed by Guermond *et. al.* led to a unique implementation in the FE context, which has shown remarkably good performance overall. It is worth mentioning that this method lends itself well to explicit temporal discretization, and was only explored in this context. One interesting, and quite useful, property of the scheme is its capability for handling nonlinear source terms, such as the ones appearing in combustion problems.

D. Conclusion

The initial success of this study encourages extending the method to 2D and 3D applications of viscous flows, where turbulence is allowed to develop in the latter case. Applications to be tested range from thermally affected incompressible laminar flows to LES in subsonic regime and, should these work as expected, chemically reacting and transonic cases with weak shock formation. This later part is to be coupled with the in-house code Alya, a Finite-Elements multiphysics software. Should this research prove successful, the result will be a big step forward to its CFD capabilities in compressible turbulent ranges.

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Lucas Gasparino received his BSc degree in Mechanical Engineering from North Universidade Paulista (UNIP), Sao Paulo, Brazil, in January 2016. He completed his MSc degree in Numerical Methods and Finite Elements in Engineering Mechanics from Swansea University, UK in September 2017. Since 2018, he has been with the Computational Fluid Dynamics group of Barcelona Supercomputing Center (BSC) as a PhD student in the CASE department. His experiences include Finite Element modeling and analysis of structures and structural components under static and dynamic conditions, as well as computational analysis of complex flows.