View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Università del Salento: ESE - Salento University Publishing

6 Appendix

In this appendix we identify the range of technological and demand parameters for which an increase in public expenditure turns out to be expansionary.

As in the text, we start from the case in which public demand is more elastic than private demand ($\gamma > \rho$). Proposition 1 establishes that $\eta_{\widetilde{L}\widetilde{G}} > 0$ iff $\eta_{r\widetilde{G}} > (\alpha - 1)/\alpha$. This is always true in the presence of non increasing returns since $\eta_{r\widetilde{G}} > 0$. However, the above condition may also be verified under increasing returns provided that the reversal of the slope phenomenon occurs. In order to evaluate the relevance of the latter, we rewrite tha condition $\eta_{r\widetilde{G}} > (\alpha - 1)/\alpha$ with $\alpha > 1$ as

$$x^2 + \left(2\rho - 1 - \frac{\alpha}{\alpha - 1}\right)x + \rho(\rho - 1) < 0 \tag{A.1}$$

where $x = (\gamma - \rho)g$ and $g = \widetilde{G}/\widetilde{L}^{\alpha}$. Inequality (A.1) holds for $x \in]x_{\min}, x_{\max}[$, where $x_{\min}$ and $x_{\max}$ are the roots of the above second order polynomial. For these roots to be real, the following condition must be verified:

$$\left(2\rho - 1 - \frac{\alpha}{\alpha - 1}\right)^2 - 4\rho(\rho - 1) > 0$$

which implies

$$\rho < \frac{1}{4}\left[\frac{(2\alpha - 1)^2}{\alpha(\alpha - 1)}\right]. \tag{A.2}$$

Condition (A.2) imposes a constraint on the values of ρ and α, which is represented in Figure 2 for $1.01 < \alpha < 1.2$:

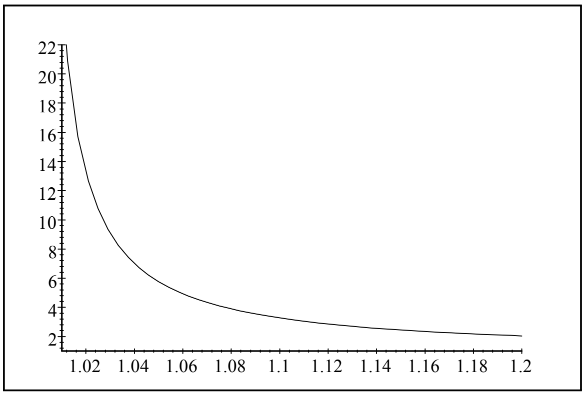

Figure 2

The higher the value of α, the smaller is the range of admissable values of ρ. For any couple of ρ and α satisfying (A.2), we may determine the corresponding interval $]x_{\min}, x_{\max}[$. For any x belonging to this interval, we obtain a relation between γ and g, consistent with the reversal of the slope:

$$\gamma = \frac{x}{g} + \rho.$$

For example, if $\alpha = 1.05$ and $\rho = 4$, $x_{\min} = 0.917$ and $x_{\max} = 13.083$. Choosing a value of $x = (\gamma - \rho)\, g$ close to $x_{\min}$, e.g. $x = 0.92$, we obtain the following relation between γ and g:

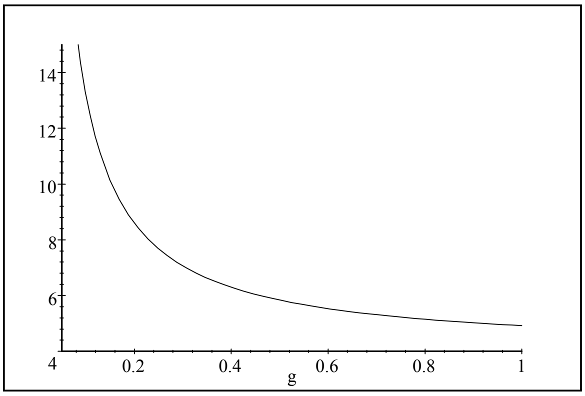

Figure 3

Figure 3 shows that a share of public expenditure on income equal to 20% requires an elasticity of public demand at least equal to 8.6, while $g = 0.3$ requires $\gamma \geq 7.07$. Had we chosen a lower value of ρ, e.g. $\rho = 3$, then $x_{\min} = 0.384$ and $x_{\max} = 15.616$. Then choosing $x = 0.39$, a value $g = 0.2$ requires $\gamma \geq 4.95$, while $g = 0.3$ requires $\gamma \geq 4.3$.

If returns are more increasing, say $\alpha = 1.1$ and, still consistently with (A.2), $\rho = 3$, $x_{\min} = 1.268$ and $x_{\max} = 4.732$. Choosing $x = 1.27$, we obtain that for $g = 0.2$ the elasticity γ must be greater than 9.35, while $g = 0.3$ requires $\gamma \geq 7.23$.

The discussion and the examples make it clear that once (A.2) is satisfied, the more increasing are returns for given ρ, the higher must be the value of γ for any given g. However, given returns, the lower the value of ρ, the lower the required value of γ.

Now we turn to the case of a public demand less elastic than private demand ($\gamma < \rho$). Proposition 2 states that in this case public expenditure is expansionary, $\eta_{\widetilde{L}\widetilde{G}} > 0$, iff $\eta_{r\widetilde{G}} < (\alpha - 1)/\alpha$. This is always verified for $\alpha \geq 1$, since now $\eta_{r\widetilde{G}} < 0$. However, Proposition 2 covers also situations of decreasing returns, provided that

$$z^2 - \left(2\rho - 1 - \frac{\alpha}{\alpha - 1}\right) z + \rho\,(\rho - 1) < 0 \tag{A.3}$$

where $z = (\rho - \gamma)\, g$. The roots of this second order polynomial are always real and they identify an interval $]z_{\min}, z_{\max}[$ within which inequality (A.3) holds and the reversal of the slope occurs. Obviously, the extreme values $z_{\min}$ and $z_{\max}$ depend on the technological and demand parameters α and ρ. Given ρ, we may therefore write $z_{\min} = z_{\min}(\alpha)$ and $z_{\max} = z_{\max}(\alpha)$.

Since γ must be greater than one, the following condition must hold:

$$z < (\rho - 1)\, g. \tag{A.4}$$

This implies that for any given ρ we cannot choose any $z \in \left]z_{\min}(\alpha), z_{\max}(\alpha)\right[$, but we are constrained to the values of z which satisfy both (A.3) and (A.4), with $g < 1$. For any α, let us choose one such value, $\underline{z}_{\min}(\alpha)$, arbitrarily close to $z_{\min}(\alpha)$. Then (A.4) allows us to identify a threshold value of g for any α:

$$g_{\min}(\alpha) = \frac{\underline{z}_{\min}(\alpha)}{(\rho - 1)}.$$

The function $g_{\min}(\alpha)$ is drawn in Figure 4, for $\rho = 4$ and $0.8 < \alpha < 1$

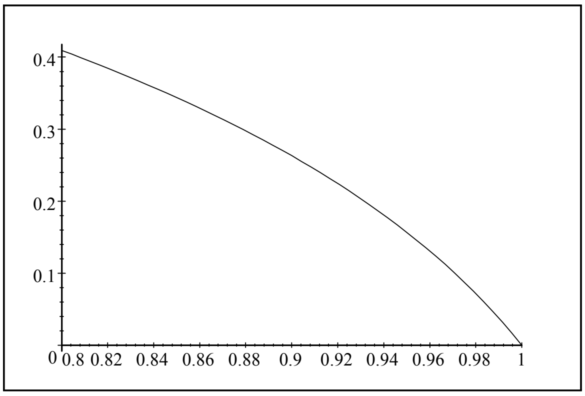

Figure 4

For all $1 > g \geq g_{\min}(\alpha)$, the reversal of the slope occurs, and the constraint on the demand elasticity parameters are verified. Therefore, for given ρ, and chosen $\underline{z}_{\min}(\alpha)$, we can use the definition of z and establish a relation between γ and $g \geq g_{\min}(\alpha)$, which ensures that public demand is expansionary:

$$\gamma = \rho - \frac{\underline{z}_{\min}(\alpha)}{g} \tag{A.5}$$

For example, if $\alpha = 0.95$ and $\rho = 4$, we have that $z_{\min} = 0.47$ and $z_{\max} = 25.53$. Therefore we may choose $\underline{z}_{\min} = 0.5$, so that $g_{\min} = 0.167$. The relation between γ and g is then represented in Figure 5:

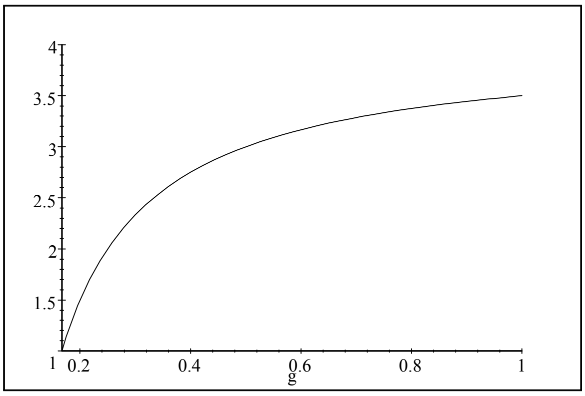

Figure 5

For all pairs (g, γ) lying below the curve, the reversal of the slope occurs. This implies that for $g = 0.2$, γ must be lower than 1.5. For $g = 0.3$, the maximum value of γ is 2.33. Had we chosen a higher value of ρ, e.g. $\rho = 5$, then $z_{\min} = 0.734$ and $z_{\max} = 27.266$. We may choose $\underline{z}_{\min} = 0.74$, so that $g_{\min} = 0.185$. Then for $g = 0.2$, γ must be lower than 1.3, while for $g = 0.3$, we have $\gamma \leq 2.54$.

If returns are more decreasing, e.g. $\alpha = 0.9$ and $\rho = 4$, the above procedure gives that $g_{\min} = 0.263$ Condition (A.5) implies $\gamma \leq 1.37$ for $g = 0.3$ and $\gamma \leq 2.03$ for $g = 0.4$.