of labour, as illustrated in section 5. An additional remark may further clarify the informational structure of the model. The optimal tax formulas derived in the paper assume knowledge of wage levels and elasticities, but only require *anonymous* information on the wage distribution at the *firm level*. Such information is not sufficient, however, to implement optimal lump-sum taxation since personalised lump-sum taxes can be levied only on the basis of the wage distribution at the *individual level*.

3. Optimal linear taxation of labour

From (2.5) it is apparent that an increase in source-based taxes translates into an identical increase in the producer interest rate, while leaving the consumer interest rate unaffected. As a result, source-based taxes cannot redistribute income by exploiting differences in consumers' saving behaviour as, for example in Haufler (1997) and Lopez et al. (1996). Nonetheless, the change in the producer interest rate modifies the demand for labour and induces a variation in equilibrium wage levels. Given constant returns to scale and no-joint production, the final effect on wages can be retrieved from the equilibrium conditions in production. The zero profit conditions for the two sectors are

$$c^1(w^1, r) = p^1 \tag{3.1}$$

$$c^2\left(w^2, r\right) = p^2 \tag{3.2}$$

where c^i represents the unit cost function. In the absence of commodity taxes, producer prices, p^i, are equal to the given world price levels, so that equilibrium wages are functions of the producer interest rate only. Implicit differentiation and application of Shephard's lemma entail that

$$\frac{\partial w^i}{\partial r} = -\frac{c_r^i}{c_{w^i}^i} = -\frac{K^i}{L^i} \equiv -\gamma^i \tag{3.3}$$

where K^i and L^i denote, respectively, the demand for capital and the demand for labour in sector i. In other words, the burden of source-based taxation is shifted completely onto the two immobile factors as an increase in the producer interest rate reduces wages in both sectors. Furthermore, in each sector the wage reduction equals the capital-labour ratio, γ^i. As a result, source-based taxes do not necessarily lead to a proportional fall in the wage level but they may well modify the distribution of income between agents endowed with different types of labour.

The issue analysed in the rest of this section is whether the government can exploit the *differential incidence* of a source-based tax to improve on the income distribution achieved with linear taxes on labour income. I tackle the question in two steps. In the next subsection I investigate whether the introduction of a source-based tax is welfare-improving given optimal differential taxation of labour and optimal residence-based capital income taxes. This is rather an artificial problem. If the government can directly observe the type of labour supplied by each individual, the first best allocation can be implemented through personalised lump-sum transfers. Nevertheless, the analysis of differential linear taxation of labour provides a useful benchmark for interpreting the optimal tax formulas derived in subsection (3.2) under the assumption that the government can observe directly neither wages nor the labour supply. Finally, in subsection (3.3) I analyse the optimality of differential origin-based commodity taxation.

3.1. Differential labour taxation

When each type of labour can be taxed at a different rate the optimal taxation problem faced by the government is

$$\max_{t_L^1, t_L^2, t_S, b} \quad W\left(nV\left(\omega^1, b\right), (1-n)\,V\left(\omega^2, b\right)\right) \tag{3.4}$$

subject to

$$R + b - t_L^1 n w^1 l^1 - t_L^2 (1-n)\, w^2 l^2 - t_S \left(n\gamma^1 l^1 + (1-n)\,\gamma^2 l^2\right) \leq 0 \tag{3.5}$$

where R is an exogenous budget requirement, l^i is the individual supply of labour of type i. Using subscripts to denote partial derivatives, the first order conditions for b, t_L^1 and t_L^2 and t_S are respectively

$$\begin{aligned} &n\left\{\left(\alpha^1 - 1\right) + \left[t_L^1 w^1 l_b^1 + t_S \gamma^1 l_b^1\right]\right\} + \\ &(1-n)\left\{\left(\alpha^2 - 1\right) + \left[t_L^2 w^2 l_b^2 + t_S \gamma^2 l_b^2\right]\right\} = 0 \end{aligned} \tag{3.6}$$

$$w^1 n\left\{\left(1-\alpha^1\right) l^1 - \left[t_L^1 w^1 l_w^1 + t_S \gamma^1 l_w^1\right]\right\} = 0 \tag{3.7}$$

$$w^2 (1-n)\left\{\left(1-\alpha^2\right) l^2 - \left[t_L^2 w^2 l_w^2 + t_S \gamma^2 l_w^2\right]\right\} = 0 \tag{3.8}$$

$$\begin{aligned} &\left(1-t_L^1\right)\gamma^1 n\left\{\left(1-\alpha^1\right) l^1 - \left[t_L^1 w^1 l_w^1 + t_S \gamma^1 l_w^1\right]\right\} + \\ &\left(1-t_L^2\right)\gamma^2 (1-n)\left\{\left(1-\alpha^2\right) l^2 - \left[t_L^2 w^2 l_w^2 + t_S \gamma^2 l_w^2\right]\right\} + \\ &t_S\left(\gamma_r^1 n l^1 + \gamma_r^2 (1-n)\, l^2\right) = 0 \end{aligned} \tag{3.9}$$

where α^i is the social marginal valuation of income accruing to consumers of type i measured in terms of government revenue.[1]

Expression (3.9) shows that a marginal increase in the source-based tax produces three different effects on welfare.

First, it reduces the net wage of skilled workers by an amount equal to $\left(1-t_L^1\right)\gamma^1$. The expression inside the curly brackets gives the social evaluation of this reduction. The first term represents the direct effects on both the government budget constraint (l^1) and the consumer's utility ($-\alpha^1 l^1$). The second term identifies the indirect effects on the government of changes in the labour supply.

Second, it reduces the net wage of the unskilled by an amount equal to $\left(1-t_L^2\right)\gamma^2$. As for wage 1, the expression inside the curly brackets represents the direct and indirect effects on social welfare.

Third, it raises the producer interest rate and brings about a variation in the domestic capital stock equal to $\gamma_r^1 n l^1 + \gamma_r^2 (1-n) l^2$. By taking the derivative of (3.3) one obtains

$$\gamma_r^i \equiv -\frac{\partial^2 w^i}{\partial (r)^2} = \frac{c_{rr}^i c_w^i - c_r^i c_{wr}^i}{\left(c_w^i\right)^2} - \frac{c_{rw}^i c_w^i - c_r^i c_{ww}^i}{\left(c_w^i\right)^2}\gamma^i \tag{3.10}$$

which shows that γ_r^i is always non-positive since c_{rr}^i and c_{ww}^i are non-positive whilst c_{rw}^i and c_{wr}^i are non-negative numbers. Hence by increasing its source-based tax the country experiences a capital outflow that decreases both revenue and welfare.

[1] If λ denotes the the multiplier associated to the government budget constraint (i.e. the social marginal value of revenue) α^i is defined as follows:

$$\alpha^i \equiv \frac{\partial W}{\partial V^i}\frac{\partial V^i}{\partial b} \Big/ \lambda.$$

Notice that condition (3.6) implies that the multiplier is different from zero.

It is apparent that the first two effects of the source-based tax are proportional to the effects produced by the two taxes on labour represented by the left hand sides of (3.7) and (3.8). When t_L^1 and t_L^2 are set to their optimal values, these effects vanish. The impact of the source-based tax on social welfare reduces to the revenue loss due to the capital outflow. In fact, substituting (3.7) and (3.8) into (3.9) gives

$$t_S\left(\gamma_r^1 n l^1 + \gamma_r^2 (1-n) l^2\right) = 0. \tag{3.11}$$

Equation (3.11) implies that the optimal source-based tax is equal to zero when $\gamma_r^i \neq 0$. There is just one particular case where this condition is not met: when a Leontief technology is adopted in both sectors. In this case (3.7), (3.8) and (3.9) are linearly dependent and the optimal source-based taxation is indeterminate. Summarizing,

Proposition 3.1. *The optimal source-based tax is zero when each type of labour can be taxed at a different rate.*

This result is not just a corollary of Diamond and Mirrlees' theorem on production efficiency as commodity taxes are not set at their optimal level. The proposition is an extension of the result obtained by Bucovetsky and Wilson (1991) in a framework with identical individuals. In the next section I show that the optimal source-based tax is not zero if the government is constrained to use a uniform tax on labour.

In order to interpret the formulas that are presented in the following, it is expedient to elaborate the first order conditions (3.6)-(3.9). Let β^i be the *net* social marginal valuation of income accruing to consumers of type i measured in terms of government revenue, i.e.:

$$\beta^i \equiv \alpha^i + \left(t_L^i w^i + t_S \gamma^i\right) l_b^i.$$

It is *net* as it takes into account the taxes paid on a unit transfer to individual i due to the income effects on the labour supply. Using Proposition 3.1, the Slutsky relationship and standard algebraic manipulations,[2] the first order conditions (3.6)-(3.9) give:

$$n\beta^1 + (1-n)\beta^2 = 1 \tag{3.12}$$

$$\frac{t_L^1}{1-t_L^1} = \frac{\left(1-\beta^1\right)}{\varepsilon_{lw}^{c1}} \tag{3.13}$$

$$\frac{t_L^2}{1-t_L^2} = \frac{\left(1-\beta^2\right)}{\varepsilon_{lw}^{c2}} \tag{3.14}$$

where ε_{lw}^{ci} is the elasticity of the compensated labour supply with respect to the wage. Condition (3.12) states that the lump-sum transfer equates the average net social marginal utility of income to 1. It implies that the two terms $\left(1-\beta^i\right)$ are either opposite in sign or both equal to 0. The optimal tax rates on labour satisfy a standard inverse elasticity rule adjusted for distributional considerations. When the government is indifferent with regard to the distribution of income, that is $\alpha^1 = \alpha^2 = 1$ in competitive equilibrium with uniform lump-sum taxation, there is no reason to resort to distortionary taxation. By contrast, if the government wishes to change the distribution of income that arises in the competitive equilibrium, it levies a tax on the wage of workers with the lower net social marginal valuation of income and pays a subsidy to workers

[2] See, for example, Atkinson and Stiglitz (1981) pp. 386-388.

with the higher net social marginal valuation of income. The tax and subsidy rates decrease with the elasticity of the labour supply on which they are levied or granted.

3.2. Uniform labour taxation

As previously remarked, problem (3.4) does not take account of the informational constraints faced by the government in a consistent manner. Personalised lump-sum transfers are deemed to be infeasible even if the government can directly observe the type of labour supplied by each individual. In the rest of this paper I resolve this inconsistency by assuming that the government cannot directly observe either the individual wage or the labour supply, but only labour income.[3] Hence, differential labour taxation is infeasible and the government is left with two alternatives: uniform linear taxation or non linear taxation of labour income. Uniform linear taxation is considered first, while the analysis of non-linear income taxation is postponed to section 4.

When labour income is taxed at a uniform rate the optimal tax problem becomes

$$\underset{t_L, t_S, b}{Max} \quad W\left(nV\left(\omega^1, b\right), (1-n)\, V\left(\omega^2, b\right)\right) \tag{3.15}$$

subject to

$$R + b - t_L\left(nw^1 l^1 + (1-n)\, w^2 l^2\right) - t_S\left(\gamma^1 n l^1 + \gamma^2 (1-n)\, l^2\right) \leq 0 \tag{3.16}$$

The first order conditions for b, t_L and t_S read respectively

[3] As remarked in section 2, in order to sustain the unfeasiblility of differential linear taxation when labour is sector specific, one must further assume both that the government cannot observe the sector where each individual works and that it cannot levy taxes on labour at the firm level.

$$
\begin{aligned}
&n\left\{\left(\alpha^1-1\right)+\left[t_L w^1 l_b^1+t_S \gamma^1 l_b^1\right]\right\}+ \\
&(1-n)\left\{\left(\alpha^2-1\right)+\left[t_L w^2 l_b^2+t_S \gamma^2 l_b^2\right]\right\}=0
\end{aligned} \tag{3.17}
$$

$$
\begin{aligned}
&w^1 n\left\{\left(1-\alpha^1\right) l^1-\left[t_L w^1 l_w^1+t_S \gamma^1 l_w^1\right]\right\}+ \\
&w^2(1-n)\left\{\left(1-\alpha^2\right) l^2-\left[t_L w^2 l_w^2+t_S \gamma^2 l_w^2\right]\right\}=0
\end{aligned} \tag{3.18}
$$

$$
\begin{aligned}
&(1-t_L)\gamma^1 n\left\{\left(1-\alpha^1\right) l^1-\left[t_L w^1 l_w^1+t_S \gamma^1 l_w^1\right]\right\}+ \\
&(1-t_L)\gamma^2(1-n)\left\{\left(1-\alpha^2\right) l^2-\left[t_L w^2 l_w^2+t_S \gamma^2 l_w^2\right]\right\}+ \\
&t_S\left(\gamma_r^1 n l^1+\gamma_r^2(1-n) l^2\right)=0
\end{aligned} \tag{3.19}
$$

The first two effects of the source-based tax, produced by the change in the two net wages, do not vanish although the labour tax has been set to its optimal level according to (3.18). In fact, by substituting (3.18) into (3.19) and rearranging, one obtains

$$
\begin{aligned}
&\frac{\varepsilon_{wr}^2-\varepsilon_{wr}^1}{r}\omega^2(1-n)\left\{\left(\alpha^2-1\right) l^2+\left[t_L w^2 l_w^2+t_S \gamma^2 l_w^2\right]\right\}+ \\
&t_S\left(\gamma_r^1 n l^1+\gamma_r^2(1-n) l^2\right)=0.
\end{aligned} \tag{3.20}
$$

where ε_{wr}^i represent both the elasticity of wage i with respect to the producer interest rate and the ratio between total interest and wages paid in sector i.

The first term in equation (3.20) describes the effect on social welfare of the income redistribution brought about by a marginal increase in the source-based tax. In order to interpret this expression it is expedient to decompose the final change in equilibrium wages using the elasticities ε^i_{wr}:

$$\frac{\partial w^1}{\partial r}\Big/ w^1 = \frac{\varepsilon^1_{wr}}{r} \tag{3.21}$$

$$\frac{\partial w^2}{\partial r}\Big/ w^2 = \frac{\varepsilon^1_{wr}}{r} + \frac{\varepsilon^2_{wr} - \varepsilon^1_{wr}}{r}. \tag{3.22}$$

The source-based tax reduces both wages by a percentage equal to $\varepsilon^1_{wr}\big/ r$. Then it brings about an additional variation in the wage of the unskilled that is equal to $\left(\left|\varepsilon^1_{wr}\right| - \left|\varepsilon^2_{wr}\right|\right)\big/ r$ as a percentage of the initial level. This additional change may represent an increase if $\left|\varepsilon^1_{wr}\right| < \left|\varepsilon^2_{wr}\right|$ or a further decrease if $\left|\varepsilon^1_{wr}\right| < \left|\varepsilon^2_{wr}\right|$. The proportional reduction in both wages does not affect social welfare as the labour tax is at its optimal level. Hence the first term in (3.20) contains exclusively the additional increase (decrease) in the wage of the unskilled that is measured by the multiplicative factor outside the curly brackets. The expression inside the curly brackets gives the social evaluation of such a change as explained when discussing (3.9).

As in (3.9), the second term in (3.20) represents the revenue loss due to capital outflow.

Expression (3.20) shows that source-based taxation is a substitute for differential labour taxation. In fact, there are just two circumstances in which the (3.20) implies that the optimal source-based tax is zero. The first is when the source-based tax produces the same proportional reduction in both wages (i.e. $\varepsilon^2_{wr} = \varepsilon^1_{wr}$). In this case the source-based tax is Pareto dominated by the uniform labour tax as the latter reduces wages but does not affect the return on the domestic capital stock. The second is when differential labour taxation is not socially desirable. In fact, the tax rates that solve problem (3.4) are also a solution of problem (3.15) if the optimal rate on skilled labour is equal to the optimal rate on unskilled labour. In such case each single term in (3.18) is equal to zero and (3.20) reduces to (3.11). Further, a solution of problem (3.15) solves the first order conditions of problem (3.4) when the expression in curly brackets in (3.20) is equal to zero. However, the analysis presented at the end of the last section has shown that uniform labour taxation cannot be a solution of the optimal taxation problem, except where the government, in the absence of distributional objectives, finances its expenditure with a poll-tax.

Summarizing,

Proposition 3.2. *When $\alpha^1 \neq \alpha^2$ in competitive equilibrium with uniform lump-sum taxation and the government can implement an optimal linear tax on labour income, the optimal source-based tax is different from zero if and only if $\varepsilon^2_{wr} \neq \varepsilon^1_{wr}$.*

It is useful to compare the results presented up to this point with the conclusions reached by Gerber and Hewitt (1987). They argue that for a small open economy it may be expedient to grant a source-based capital *subsidy* but never desirable to resort to source-based taxes. These results are based on two crucial assumptions. The first is that the government cannot directly transfer income to workers either through subsidies to labour or through a uniform lump-sum grant even though it can levy taxes at different rates on skilled and unskilled labour. The second is that the wages of skilled workers are proportional to the wages of the unskilled.

The first assumption is needed to avoid the outcome of Proposition 3.1: if the government can levy positive as well negative differential taxes on labour there is no reason to resort to source-based capital taxes or subsidies. The second assumption, is responsible for the inefficiency of a source-based capital tax. When wages are proportional to each other, $\varepsilon^2_{wr} = \varepsilon^1_{wr}$: a source-based tax does not redistribute labour income but uniformly reduces the wage level. The same outcome can be achieved with labour taxes, while avoiding the revenue loss due to capital flight. By contrast, a source-based capital subsidy turns out to be efficient because it is the only instrument that allows income to be transferred from the skilled to the unskilled.

The government can grant a source-based subsidy on capital and finance it with a tax on skilled labour. The source-based subsidy raises both wages but only the unskilled enjoy a higher income as the labour tax more than compensates for the increase in the wage of the skilled.

By allowing for differential incidence the model analysed in this paper provides a rationale for source-based subsidies to capital that does not depend on ad hoc restrictions on labour income taxation. Furthermore, expression (3.20) suggests that even a *positive* source-based tax can be efficient, depending on the elasticities ε^i_{wr} and the social evaluation of an increase in the wage of the unskilled. For example, when a marginal increase in the net wage of the unskilled is socially desirable (i.e. the expression in curly brackets in (3.20) is positive), the optimal source-based tax is positive when the burden is shifted onto the wage of the skilled more than proportionally (i.e. $\left|\varepsilon^1_{wr}\right| > \left|\varepsilon^2_{wr}\right|$). This conclusion can be strengthened by solving condition (3.18) for t_L and substituting into (3.19). Tedious but straightforward manipulations yield:

$$t_S = \frac{\left(\varepsilon^2_{wr} - \varepsilon^1_{wr}\right)(1-n)\,\omega^2 l^2 \left[\varepsilon^{c2}_{lw}\left(1-\beta^1\right) - \varepsilon^{c1}_{lw}\left(1-\beta^2\right)\right] n w^1 l^1}{\Delta} \tag{3.23}$$

where Δ denotes an expression which is always positive[4]. As explained in the previous section condition (3.17) implies that the two terms $\left(1-\beta^i\right)$ are opposite in sign when the government has redistributional objectives. Hence the expression in the square bracket in (3.23) is positive when the government wants to redistribute income towards the unskilled, that is when $\beta^2 > \beta^1$, while it is negative when the government aims to transfer income from the unskilled to the

[4] It is

$$\begin{aligned}\Delta \equiv &\left(\frac{\varepsilon^2_{wr}}{r} - \frac{\varepsilon^1_{wr}}{r}\right)^2 n l^1 w^1 \varepsilon^1_{lw}(1-n)\, l^2 w^2 \varepsilon^2_{lw} \\ &- \left(n w^1 l^1 \varepsilon^1_{lw} + (1-n)\, w^2 l^2 \varepsilon^2_{lw}\right)\left(\gamma^1_r n l^1 + \gamma^2_r (1-n)\, l^2\right).\end{aligned}$$

skilled, that is when $\beta^2 < \beta^1$. This leads immediately to the following result:

Proposition 3.3. *Under an optimal linear tax on labour income,*

$$\operatorname{sgn}(t_S) = \operatorname{sgn}\left(\left(\varepsilon_{wr}^2 - \varepsilon_{wr}^1\right) \times \left(\beta^2 - \beta^1\right)\right). \tag{3.24}$$

We can conclude that the government of a small open economy levies a *positive* source-based tax on capital income if a higher proportion of the tax burden is shifted onto the class of workers with the lower net social marginal utility of income.

3.3. Optimal origin-based taxes

The conclusions drawn for source-based taxation can be easily applied to *uniform* origin-based taxation. If world prices are given, origin-based taxes are shifted completely onto the immobile factors and a uniform *ad valorem* origin-based commodity tax can exactly replicate a source-based tax on capital income[5].

The preceding analysis does not answer the question whether the government should levy *differential* origin-based commodity taxes. Such taxation provides an additional tool for redis-

[5] To see this point, assume that t_L^* and t_S^* are optimal tax rates. Taxes $t'_L \equiv \left(1 - t_L^*\right)\left(1 + \frac{t_S^*}{r^*}\right) - 1$, $t'_S = 0$ and a uniform origin-based *ad valorem* tax $\tau'_O \equiv \frac{t_S^*}{r^*} \Big/ \left(1 + \frac{t_S^*}{r^*}\right)$ are consistent with the original consumer equilibrium prices and satisfy the zero profit conditions

$$c^1\left(\frac{\omega^1}{(1-t_L^*)}, r^* - t_S^*\right) = p^{*1} \Leftrightarrow c^1\left(\frac{\omega^1}{(1-t_L^*)\left(1+\frac{t_S^*}{r^*}\right)}, r^*\right) = \frac{p^{*1}}{\left(1+\frac{t_S^*}{r^*}\right)} = p^{*1}\left(1 - \tau'_O\right)$$
$$c^2\left(\frac{\omega^2}{(1-t_L^*)}, r^* - t_S^*\right) = p^{*2} \Leftrightarrow c^2\left(\frac{\omega^2}{(1-t_L^*)\left(1+\frac{t_S^*}{r^*}\right)}, r^*\right) = \frac{p^{*2}}{\left(1+\frac{t_S^*}{r^*}\right)} = p^{*2}\left(1 - \tau'_O\right)$$

tribution. The no-profit conditions

$$c^1(w^1, r^*) = p^{1*} - t_O^1 \tag{3.25}$$

$$c^2\left(w^2, r^*\right) = p^{2*} - t_O^2 \tag{3.26}$$

show that gross wages can be independently manipulated through t_O^1 and t_O^2. However, it is not apparent whether it is desirable to resort to this additional instrument since uniform origin-based commodity taxation and a linear labour income tax are sufficient to control the distribution of the two net wages.

The question can be resolved by analyzing the government maximisation problem

$$\underset{t_L, t_S, t_O, b}{Max} \quad W\left(nV\left(\omega^1, b\right), (1-n)\, V\left(\omega^2, b\right)\right) \tag{3.27}$$

subject to

$$R + b - t_L\left(nw^1 l^1 + (1-n)\, w^2 l^2\right) - t_O^1 \theta^1 n l^1 - t_O^2 \theta^2 (1-n)\, l^2 \leq 0 \tag{3.28}$$

where

$$\theta^i \equiv \frac{\partial w^i}{\partial p^i} = \frac{1}{c^i_{w^i}} = \frac{X^i}{L^i} \tag{3.29}$$

and X^i is domestic production of the good i. As for source-based taxes, origin-based taxes do not enter directly into social welfare as they do not affect commodity consumer prices.

The first order conditions for t_L, t_O^1 and t_O^2 read respectively

$$w^1 n \left\{ \left(1-\alpha^1\right) l^1 - \left[t_L w^1 l_w^1 + t_O^1 \theta^1 l_w^1\right] \right\} + \\ w^2 \left(1-n\right) \left\{ \left(1-\alpha^2\right) l^2 - \left[t_L w^2 l_w^2 + t_O^2 \theta^2 l_w^2\right] \right\} = 0 \tag{3.30}$$

$$-\left(1-t_L\right) \theta^1 \left\{ \left(\alpha^1 - 1\right) l^1 + \left[t_L w^1 l_w^1 + t_O^1 \theta^1 l_w^1\right] \right\} - t_O^1 \theta_p^1 l^1 = 0. \tag{3.31}$$

$$-\left(1-t_L\right) \theta^2 \left\{ \left(\alpha^2 - 1\right) l^2 + \left[t_L w^2 l_w^2 + t_O \theta^2 l_w^2\right] \right\} - t_O^2 \theta_p^2 l^2 = 0. \tag{3.32}$$

Conditions (3.31) and (3.32) state that the optimal commodity tax rates must balance the marginal variation in welfare due to income redistribution with the variation in revenues due to the change in domestic production. An increase in the origin-based tax in sector i, leads both to a decrease in the net wage of labour of type i, equal to $\left(1-t_L\right)\theta^2$, and to a decrease in per-capita production in the same sector, equal to $-\theta_p^i l^i$, as the derivative of (3.29) with respect to p^i,

$$\theta_p^i = -\frac{c_{ww}^i}{\left(c_w^i\right)^2}\theta$$

is always positive since $c_{ww}^i < 0$.

The comparison of condition (3.20) with conditions (3.31) and (3.32) suggests that the main difference between uniform and differential origin-based commodity taxation lies in their incidence on wages. Given that the two types of labour are sector specific, differential commodity taxation always allows one wage to be reduced with respect to the other, while the same objective can be achieved with uniform taxation only if $\varepsilon_{wr}^1 \neq \varepsilon_{wr}^2$. As a result, optimal origin-based commodity taxes are always different from zero when the expressions in curly brackets in (3.31) and (3.32) do not vanish, that is, when the net social marginal utility of income is different for skilled and unskilled workers. Summarising,

Proposition 3.4. *When $\alpha^1 \neq \alpha^2$ in competitive equilibrium with uniform lump-sum taxation and the government can implement an optimal linear tax on labour income, optimal origin-based commodity taxes are different from zero.*

A closer look at expressions (3.31) and (3.32) clarifies the rationale of differential origin-based commodity taxation. By substituting condition (3.30) into (3.31) and rearranging one obtains

$$(1-t_L)\,\theta^1 \frac{w^2\,(1-n)}{w^1 n}\left\{\left(\alpha^2-1\right)l^2+\left[t_L w^2 l_w^2+t_O \theta^2 l_w^2\right]\right\}-t_O^1\theta_p^1 l^1=0. \tag{3.33}$$

This expression shows that with an optimal linear income tax the redistribution of income brought about by the tax on good 1 has an effect on welfare that is proportional to the effect due to the redistribution of income produced by the tax on good 2. Why should the government resort to both taxes? The reason is that by mixing the two tax instruments the government reduces the revenue losses brought about by the redistribution of income. In order to achieve a unit increase in the net wage of the unskilled, the government has two options. The first, described by condition (3.32), is to reduce (increase) the tax (subsidy) on good 2 by an amount equal to $\left[(1-t_L)\,\theta^2\right]^{-1}$. As previously explained, this causes a revenue loss equal to $\left|\left[(1-t_L)\,\theta^2\right]^{-1} t_O^2\theta_p^2 l^2\right|$. The second, represented by condition (3.33), is to increase (reduce) the tax (subsidy) on good 1 by an amount equal to $\left[(1-t_L)\,\theta^1\right]^{-1}\left(w^1 n/\,w^2\,(1-n)\right)$. This in turn, reduces revenue by $\left|\left[(1-t_L)\,\theta^1\right]^{-1}\left(w^1 n/\,w^2\,(1-n)\right)t_O^1\theta_p^1 l^1\right|$. The desired increase in wage 2 is achieved efficiently when the marginal costs of the two tax instruments are equalised, that is, by substituting (3.32) into (3.33), when the following condition is satisfied

$$\frac{t_O^2 \theta_p^2 l^2}{(1-t_L)\,\theta^2} = -\frac{t_O^1 \theta_p^1 l^1}{(1-t_L)\,\theta^1} \frac{w^1 n}{w^2\,(1-n)}. \tag{3.34}$$

Using the fact that

$$p^i \theta^i = w^i + r\gamma^i \tag{3.35}$$

and rearranging, condition (3.34) can be rewritten as,

$$\frac{\tau_O^1}{\tau_O^2} = -\frac{(1-n)\,w^2 l^2}{n w^1 l^1} \frac{\eta_{lw}^2}{\eta_{lw}^1} \frac{\left(1+\left|\varepsilon_{wr}^2\right|\right)}{\left(1+\left|\varepsilon_{wr}^1\right|\right)} \tag{3.36}$$

where τ_O^i denotes the ad -valorem tax rate on good i and η_{lw}^i the elasticity of labour demand with respect to the wage. The striking feature of this expression is that the optimal ratio between the two commodity tax rates does not depend on the value judgements embedded in the social welfare functional. This is because the two taxes can achieve the same results in terms of income redistribution when coupled with a linear income tax. Another important implication of condition (3.36) is that the optimal tax rates have opposite signs. Hence, uniform ad -valorem commodity taxation cannot be a solution of the optimal taxation problem, apart from the trivial case where a government with no distributional objectives finances its expenditure exclusively through a uniform lump-sum tax. As to the level of the tax rates, condition (3.36) provides an inverse elasticity rule: the tax (subsidy) rate on good i decreases with total labour income, the elasticity of labour demand and the elasticity of the wage with respect to the interest rate in sector i.