4. Optimal non-linear taxation of labour income

In the preceding sections source- and origin-based taxes are presented as an indirect means of taxing labour at different rates when the labour tax is linear. However, the informational constraints do not bind the taxation on labour income to be linear: the government does not need to know individual wages and the labour supply in order to implement a nonlinear tax on labour income. Are source- and origin-based taxes still desirable when an optimal nonlinear income tax is levied?

The analysis developed up to this point suggests a negative answer. With a nonlinear income tax a different marginal tax rate can be set for each type of labour. Hence, source- and origin-based taxes cannot improve the distribution of net wages. However, I show in the following that an alternative rationale for source- and origin-based taxation does exist: by changing the distribution of gross wages, source- and origin-based taxes can relax the self-selection constraints that bind the non-linear tax.

The optimal taxation problem can be set up, as in Stiglitz (1986), in the following way. Let

$$U\left(x^{1i}, x^{2i}, s^{i}, l^{i}\right) \tag{4.1}$$

be the direct utility function for consumers of type i where x^{ji} is the demand for commodity j. A partially indirect utility function can be defined as follows,

$$V\left(l^{i}, m^{i}\right) \equiv \max_{x^{1},x^{2}} \left\{ U\left(x^{1}, x^{2}, l\right) \middle|\, p^{1*}x^{1} + p^{2*}x^{2} \leq m^{i} \right\} \tag{4.2}$$

where m^{i} represents after-tax labour income. Given that the labour supply is not observable, it is expedient to rewrite utility function (4.2) in terms of before-tax labour income $Y^i = w^i l^i$:

$$V^i \left(Y^i, m^i\right) \equiv V \left(\frac{Y^i}{w^i}, m^i\right) = V \left(l^i, m^i\right). \tag{4.3}$$

The tax paid by consumers of type i on labour income is given by $Y^i - m^i$. The assumption that $w^1 > w^2$ in both the pre-tax and the post-tax situation guarantees the fulfilment of the single crossing condition. Consequently, at most one of the two self-selection constraints is binding in the optimum. If we consider the case where the self-selection constraint is binding for skilled workers, the optimal source-based and income tax are given by the solution to the problem

$$\max_{Y^1, Y^2, m^1, m^2, t_S} \quad W \left(nV^1 \left(Y^1, m^1\right), (1-n) V^2 \left(Y^2, m^2\right)\right) \tag{4.4}$$

subject to the revenue and self-selection constraints

$$R - n \left(Y^1 - m^1\right) - (1-n) \left(Y^2 - m^2\right) - t_S \left(\gamma^1 n \frac{Y^1}{w^1} + \gamma^2 (1-n) \frac{Y^2}{w^2}\right) \leq 0 \tag{4.5}$$

$$V \left(\frac{Y^2}{w^1}, m^2\right) - V^1 \left(Y^1, m^1\right) \leq 0. \tag{4.6}$$

The first of the two constraints requires that revenue should be sufficient to finance the exogenous budget requirement R, while the second requires that skilled workers should not strictly prefer the allocation assigned to the unskilled. The first order conditions for Y^1, Y^2 and t_S read respectively

$$\frac{\alpha_1}{V_m^1} n V_l^1 \frac{1}{w^1} - \left[-n - t_S \gamma^1 n \frac{1}{w^1}\right] + \mu V_l^1 \frac{1}{w^1} = 0 \tag{4.7}$$

$$\frac{\alpha_2}{V_m^2}(1-n)V_l^2\frac{1}{w^2}-\left[-(1-n)-t_S\gamma^2(1-n)\frac{1}{w^2}\right]-\mu\left[V_l\left(\frac{Y^2}{w^1},m^2\right)\frac{1}{w^1}\right]=0 \tag{4.8}$$

$$\begin{aligned}
&\left\{\tfrac{\alpha_1}{V_m^1}nV_l^1\tfrac{1}{w^1}-\left[-n-t_S\gamma^1 n\tfrac{1}{w^1}\right]\right\}\gamma^1 l^1+\\
&\left\{\tfrac{\alpha_2}{V_m^2}(1-n)V_l^2\tfrac{1}{w^2}-\left[-(1-n)-t_S\gamma^2(1-n)\tfrac{1}{w^2}\right]\right\}\gamma^2 l^2+\\
&t_S\left(\gamma_r^1 nl^1+\gamma_r^2(1-n)l^2\right)-\mu\left[V_l\left(\tfrac{Y^2}{w^1},m^2\right)\tfrac{Y^2}{(w^1)^2}-V_l^1\tfrac{Y^1}{(w^1)^2}\right]\gamma^1=0
\end{aligned} \tag{4.9}$$

where μ is a non-negative scalar.[6] By substituting (4.7) and (4.8) into the (4.9) and rearranging, one obtains

$$\mu\left[V_l\left(\frac{Y^2}{w^1},m^2\right)\frac{Y^2}{(w^1)^2}\right]\frac{\varepsilon_{wr}^1-\varepsilon_{wr}^2}{r}w^1+t_S\left(\gamma_r^1 nl^1+\gamma_r^2(1-n)l^2\right)=0. \tag{4.10}$$

As for the linear case, (4.10) shows that the source-based tax is equal to zero only in two instances. The first is when $\varepsilon_{wr}^1=\varepsilon_{wr}^2$, that is the two types of labour bear the same tax burden. The second is when $\mu=0$, that is when the social optimum lies close to the competitive allocation so that the redistribution desired by the government is not bound by either of the two self-selection constraints. Consequently, the optimal allocation, represented by $\alpha^1=\alpha^2=1$, can be implemented through lump-sum taxes.

Proposition 4.1. *Under an optimal non-linear tax on labour income, the optimal source-based tax is different from zero if and only both $\varepsilon_{wr}^2\neq\varepsilon_{wr}^1$ and $\alpha^1\neq\alpha^2$ in competitive equilibrium with incentive- compatible lump-sum taxation.*

[6] The scalar μ is the ratio between the multiplier associated with the self-selection constraint and the multiplier associated with the budget constraint. Both multipliers are non negative (Stiglitz 1986).

Despite their similarity, the results stated in propositions (3.2) and (4.1) have rather different interpretations. In contrast with (3.20), the effects of a marginal increase in the source-based tax on consumers' net income does not enter into (4.10) since the government can achieve the optimal distribution of labour income that is consistent with the self-selection constraint through the nonlinear labour tax. What is left is the effect of the source-based tax on the self-selection constraint itself. If $\left|\varepsilon_{wr}^1\right| > \left|\varepsilon_{wr}^2\right|$, an increase in the source-based tax reduces the equilibrium gross wage of the skilled, so that they find it more costly to mimic the behaviour of the unskilled (as shown by the negative term inside the square brackets). Consequently, the self-selection constraint is relaxed and social welfare can be improved (recollect that μ is non negative). By the same token, when $\left|\varepsilon_{wr}^1\right| < \left|\varepsilon_{wr}^2\right|$, a reduction in the source-based tax relaxes the self-selection constraint and increases welfare.

As for the linear case, equation (4.10) shows that in equilibrium any positive effect on welfare, due to the relaxation of the self-selection constraint, must be counterbalanced by the revenue loss due to the variation in capital invested in the country. This condition makes it possible to determine the sign of the optimal tax rate. When $\left|\varepsilon_{wr}^1\right| > \left|\varepsilon_{wr}^2\right|$, the government levies a source-based tax. The tax rate is raised up to the point where the welfare gain due to relaxation of the self-selection constraint is exactly offset by the revenue loss due to capital outflow. When $\left|\varepsilon_{wr}^2\right| < \left|\varepsilon_{wr}^1\right|$, the government grants a subsidy. The subsidy rate is raised up to the point where the welfare gain due to relaxation of the self-selection constraint is exactly offset by the revenue loss due to capital inflow.

The case analysed in this section, in which the self-selection constraint is binding for the skilled, is usually regarded in the literature as the "normal" case. Yet, the possibility of the self-selection constraint being binding for the unskilled cannot be ruled out. Obviously, the foregoing analysis applies to this second case as well: by swapping the indices 1 and 2, one can conclude that a source-based tax is levied when $\left|\varepsilon_{wr}^2\right| < \left|\varepsilon_{wr}^1\right|$ and a subsidy granted when $\left|\varepsilon_{wr}^2\right| > \left|\varepsilon_{wr}^1\right|$. These results can be summarised using the fact that the self-selection is always binding for the group with the lower social marginal utility of income.

Proposition 4.2. *Under an optimal non-linear tax on labour income*[7],

$$\operatorname{sgn}(t_S) = \operatorname{sgn}\left(\left(\varepsilon_{wr}^2 - \varepsilon_{wr}^1\right) \times \left(\alpha^2 - \alpha^1\right)\right). \tag{4.11}$$

As explained in the preceding section, all the arguments developed for source-based taxation on capital income apply to uniform origin-based commodity taxation. However, it is worth investigating how the non-linear taxation of labour affects the structure of optimal differential commodity taxes. Differential origin-based commodity taxation can be introduced in problem 4.4 in the way described in section 3.3. The first-order conditions for Y^1, Y^2, t_O^1 and t_O^2 are respectively

$$\frac{\alpha_1}{V_m^1} n V_l^1 \frac{1}{w^1} - \left[-n - n t_O^1 \theta^1 \frac{1}{w^1}\right] + \mu V_l^1 \frac{1}{w^1} = 0 \tag{4.12}$$

$$\frac{\alpha_2}{V_m^2}(1-n) V_l^2 \frac{1}{w^2} - \left[-(1-n) - (1-n) t_O^2 \theta^2 \frac{1}{w^2}\right] - \mu\left[V_l\left(\frac{Y^2}{w^1}, m^2\right)\frac{1}{w^1}\right] = 0 \tag{4.13}$$

$$\begin{aligned} &\left\{\tfrac{\alpha_1}{V_m^1} n V_l^1 \tfrac{1}{w^1} - \left[-n - t_O^1 \theta^1 n \tfrac{1}{w^1}\right]\right\} \theta^1 l^1 - t_O^1 \theta_p^1 n l^1 \\ &-\mu\left[V_l\left(\tfrac{Y^2}{w^1}, m^2\right)\tfrac{Y^2}{(w^1)^2}\theta^1 - V_l^1 \tfrac{Y^1}{(w^1)^2}\theta^1\right] = 0 \end{aligned} \tag{4.14}$$

[7] *This result is equivalent to proposition 1 in Huber (1999).*

$$\left\{\frac{\alpha_2}{V_m^2}(1-n)V_l^2\frac{1}{w^2}-\left[-(1-n)-t_O^2\theta^2(1-n)\frac{1}{w^2}\right]\right\}\theta^2 l^2-t_O^2\theta_p^2(1-n)l^2-=0 \qquad (4.15)$$

By substituting (4.12) into (4.14) and (4.13) into (4.15) and rearranging one gets

$$\mu\left[V_l\left(\frac{Y^2}{w^1},m^2\right)\frac{1}{w^1}\right]+t_O^1\frac{\theta_p^1}{\theta^1}n\frac{w^1}{w^2}\frac{l^1}{l^2}=0 \qquad (4.16)$$

$$\mu\left[V_l\left(\frac{Y^2}{w^1},m^2\right)\frac{1}{w^1}\right]-t_O^2\frac{\theta_p^2}{\theta^2}(1-n)=0. \qquad (4.17)$$

These two conditions can be easily interpreted in the light of the arguments presented earlier. As for source-based capital taxation, the redistribution of income brought about by the two origin-based commodity taxes does not affect welfare if an optimal non-linear income tax is levied. Differential commodity taxation affects welfare through the changes in the self-selection constraint and government revenue. The two taxes have opposite effects on the self-selection constraint as an increase in the tax on commodity 1 always reduces the gross wage of skilled workers while the opposite is true for the tax on commodity 2. This has two implications. First, differential commodity taxation is optimal whenever the self-selection constraint is binding. Second, since the same reduction in the skilled wage can be achieved either through an increase in the tax rate on commodity 1 or through a reduction in the tax rate on commodity 2, in order to determine the optimal tax rate ratio, the effects on government revenue alone must be considered. This can be easily seen by substituting condition (4.16) into (4.16). Surprisingly, this substitution yields directly condition (3.34). Hence, we can state the following result:

Proposition 4.3. *Condition (3.36) represents the structure of optimal origin-based commodity taxation under both a linear and a non-linear income tax.*