5 Stability and technological evolution

In this section, the stability analysis of the two types of equilibrium considered above is extended to the case in which the regional levels of technology (a_r) may change over time according to equation (23).

5.1 Agglomeration in region v

When the relative regional levels of technology may change, agglomeration in region v may be a sustainable equilibrium not only when (23) implies that the technological gap between the two regions gradually increases but also when it shrinks.

Let us first consider the case in which (23) entails a continuous increase in the technological gap $(a_v - a_r)$ between the two regions. In this case, starting from a situation in which the level of the technology of firms in region v is not smaller than that in region r $(a_v \geq a_r)$, agglomeration in region v is a sustainable equilibrium in the following two subcases: (i) trade costs are so high $(\tau > \bar{\tau})$ that they do not allow firms in the lagging region r to observe and assimilate potential knowledge spill-overs from the more productive manufacturing firms in region v and (ii) the lagging region's capabilities to assimilate potential technology spill-overs are too small because trade costs are too high for the given initial technological gap between the two regions $(a_v - a_r > \sqrt{\psi})$. In both cases, *firms in the lagging region are unable to recover from their lagged position during the technological development process, and agglomeration of the manufacturing sector in the leading region is the only sustainable equilibrium.* In fact, equation (23) implies that the technological advantage of region v continuously increases over time, and that a_v/a_r tends to infinity.

Let us rewrite the sustainability conditions for agglomeration in region v given by (24) and (26) as:

$$\frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} k$$

where k depends on the parameters. Since $\sigma > 1$, we observe that:

$$\lim_{a_v \to \infty} \frac{Q_{mir}}{Q^*_{mir}} = \lim_{a_r \to 0} \frac{Q_{mir}}{Q^*_{mir}} = 0 \tag{32}$$

Hence, when the relative technological advantage of region v increases continually over time, as in the two above-mentioned cases, agglomeration in region v is the only sustainable equilibrium because $Q_{mir}/Q^*_{mir} = 0 < 1$ for all admissible τ. In fact, in these two cases the agglomeration forces are strengthened because firms in region v become more and more productive compared to those in region r.

Figure 7 shows, for instance, what happens when the assimilation of interregional technology spill-overs is impeded because (23) implies that the relative technological level a_v/a_r continues to increase over time (with $\psi = 0$).[19] Particularly, Figure 7 shows that if a_v/a_r increases from the initial value 1 to 1.2, and then to 1.4, the curve Q_{mir}/Q^*_{mir} is shifted downward and to the right strengthening centripetal forces in region v.

Insert Figure 7 about here

Therefore, (23) and (32) imply that, when trade costs are higher than $\bar{\tau}$, the symmetric equilibrium is unstable. An exogenous increase in the level of technology a_v would result in a continuous increase of the technological advantage of region v, leading to the agglomeration of the modern sector in this region for any value of the trade costs.

Finally, we must consider the case in which the technological gap between the two regions can be closed, because learning capabilities are high enough to allow technology spill-overs to take place for the given initial technological gap ($\sqrt{\psi} > a_v - a_r$). In this case, agglomeration in region v may be a sustainable equilibrium only if $Q_{mir}/Q^*_{mir} < 1$ when the process of recovery of the technological lag is completed. Hence, in this case even though knowledge spill-overs take place, pecuniary externalities are strong enough to foster the concentration of firms in region v. On the

[19] The graphics are obtained for the following parameter values: $\gamma = 0.1$, $\mu = 0.4$, $\mu_c = 0.6$, $\sigma = 5$. Note that since $\mu_c > \mu^*_c$, $w_{lv} > 1$ and in region v is produced only the manufacturing good.

contrary, if $Q_{mir}/Q^*_{mir} \geq 1$ when the technological gap is closed, agglomeration of the modern sector is unsustainable, because centripetal forces are weaker than centrifugal forces.

5.2 Symmetric equilibrium

To study the stability of the symmetric equilibrium when technological change is possible, we must consider three differential equations. Two of them are given by expression (31), and the third is given by equation (23), which describes the change in the technology.

In this case, the Jacobian matrix is given by:

$$J_2 = \begin{bmatrix} \frac{\partial \dot{n}_n}{\partial n_n} & \frac{\partial \dot{n}_n}{\partial n_s} & \frac{\partial \dot{n}_n}{\partial a_s} \\ \frac{\partial \dot{n}_s}{\partial n_n} & \frac{\partial \dot{n}_s}{\partial n_s} & \frac{\partial \dot{n}_s}{\partial a_s} \\ \frac{\partial \dot{a}_s}{\partial n_n} & \frac{\partial \dot{a}_s}{\partial n_s} & \frac{\partial \dot{a}_s}{\partial a_s} \end{bmatrix}$$

Since $\frac{\partial \dot{a}_s}{\partial n_n} = \frac{\partial \dot{a}_s}{\partial n_s} = 0$, the Jacobian matrix (evaluated at the symmetric equilibrium denoted by $*$) is given by:

$$J_2^* = \begin{bmatrix} J_1^* & \left(\frac{\partial \dot{n}}{\partial a_s}\right)^* \\ 0 & -\psi \end{bmatrix}$$

Matrix J_2^* is a decomposable matrix and, therefore, two of its eigenvalues are given by the eigenvalues of matrix J_1^*, that is, $\delta\lambda_1$ and $\delta\lambda_2$, and the third eigenvalue is given by:

$$-\psi = -c(\bar{\tau} - \tau)$$

The symmetric equilibrium is stable when the three eigenvalues of matrix J_2^* are negative. As pointed out in section 4.2, the eigenvalues of matrix J_1^* are both negative - for a given level of integration between the two regions - when centrifugal forces for the given technology levels are stronger than centripetal forces.[20] The third eigenvalue ($-\psi$), instead, is negative when trade costs are sufficiently low to allow for successful technology spill-over processes, that is, when $\tau < \bar{\tau}$.

Table 1 reveals that the symmetric equilibrium is always unstable for low levels of economic integration, that is, for the highest range of trade costs values. More precisely, when $\tau > \bar{\tau}$, namely

[20] The symmetric equilibrium technology levels are $a_s = a_n = 1$

when the level of trade costs is higher than the value below which knowledge spill-overs may be assimilated by firms in the lagging region, the symmetric equilibrium is unstable.

Insert Table 1 about here

The fact that the symmetric equilibrium is unstable for the highest range of trade costs values is not in line with the general results of economic geography. While existing models predict that dispersion is always a stable equilibrium for high trade costs, the present work shows that this is not always the case. If trade costs are high enough, the lagging region cannot benefit from any potential knowledge spill-overs process, and the initial technological gap increases over time leading to the agglomeration of the manufacturing sector in the leading region, which is a sustainable equilibrium.

By contrast, even when the two regions are sufficiently integrated ($\tau < \bar{\tau}$), and therefore the process of technological catching up may be implemented through learning by interacting processes because $\psi > 0$, the other centripetal forces may be strong enough to make the dispersion of the economic activity unstable. This is the case when at least one of the eigenvalues $\delta\lambda_1$ and $\delta\lambda_2$ of J_2^* is positive.

Finally, if the economy is at the symmetric equilibrium, and this equilibrium is stable, technology may still evolve since this equilibrium is compatible with equal exogenous growth rates of a_r for the two regions (*steady state equilibrium*).

To summarize, we observe that *trade costs play a different role in the process of knowledge spill-overs and in the process of interaction between the standard centripetal and centripetal forces* (for given technology development levels). More precisely, from the point of view of technology spill-overs processes, a reduction in trade costs, when they are high, may enhance the recovery of the less developed region, but from the other point of view (that is, the interplay of centripetal and centrifugal forces evaluated at fixed technology levels), the reduction in trade costs may strengthen centripetal forces. As a consequence, in this case, *there is a trade-off in the role played* *by trade costs in knowledge spill-overs processes and in determining the result of the conflict between standard fixed-technology centripetal and centripetal forces.*

6 Conclusion

The model we develop in the first part of this paper can be considered as a generalization of the Core-Periphery models by Krugman (1991b) and Krugman and Venables (1995). More precisely, to derive both of them we have to assume that the regional technological levels are the same for the two regions ($a_s = a_n = 1$). Hence, Krugman's model corresponds to the case in which manufacturing firms employ only skilled mobile workers ($I_{mir} = H_r$). Krugman and Venables' model, instead, assume that firms employ only intermediate manufacturing varieties and immobile workers, who correspond to unskilled workers in the present model ($I_{mir} = L_{mir}^{1-\mu} D_{mir}^{\mu} / \left[(1-\mu)^{1-\mu} \mu^{\mu} \right]$).

For given technological levels, we find that full agglomeration of the manufacturing sector in a region is unsustainable for high trade costs because centrifugal forces are stronger than centripetal ones. By contrast, full agglomeration may be an equilibrium for low trade costs. Moreover, the introduction of two types of workers, characterized by different mobility assumptions, allows us to show that the existence of an immobile factor may give rise to a non-monotonic relationship between the sustainability of agglomeration and the levels of trade costs. In fact, we may encounter the $\cap$-shaped relationship found by Venables (1996) when parameter values are such that the wages of unskilled workers are higher in the region in which the agglomeration of the modern sector takes place.[21] When this is so, the existence of an immobile factor may lead to the dispersion of the economic activity for high level of integration because firms find it profitable to produce in the periphery where the wages of unskilled workers are lower than in the core region. However, we show that this happens only if the technological advantage of the core region is not too large, and if the wages of unskilled workers in the core are too high in relation to the technological gap.

For given equal technological levels, we find that the traditional result of a stable symmetric

[21] In particular, this may happen when the share of consumers' expenditure on manufactures μ_c is higher than the threshold value μ_c^*.