2 The model

Let us consider two regions, or two countries, north, n, and south, s. Both regions are inhabited by L unskilled workers. Moreover, H skilled workers are interregionally mobile. Following Baldwin et al. [1], we adopt the following normalizations for the number of workers: $H = 1$ and $L = (1-\mu)/(2\mu)$. As usual, μ represents expenditure share on manufacturing or industrial goods, with $0 < \mu < 1$. We notice that every time we use suffix r, $r = n, s$, and that if both r and v are used in the same expression, $r, v = n, s$ and $r \neq v$.

Each worker j, skilled or unskilled, consumes a traditional (or agricultural) homogenous good, and many varieties of a modern (or manufactured, industrial) good, which are partly locally produced and partly imported. Preferences, identical for all workers, are described by the following utility function

$$U(Q_{mjr}, Q_{ajr}) = Q_{mjr}^{\mu} {Q_{ajr}}^{1-\mu} \tag{1}$$

where Q_{ajr} is the traditional good consumption by individual j in r, and Q_{mjr} is the modern composite good consumption, which includes all locally produced and imported varieties. The composite manufacturing good, Q_m, is obtained by the aggregation of all industrial varieties i produced by n_r firms in region r, and n_v firms in region v, with

$$Q_m = \left(\int_{i=1}^{n_r+n_v} Q_{mi}^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

$\sigma > 1$ is the elasticity of substitution between any pair of industrial varieties. Moreover, we remind that $\rho = \frac{\sigma-1}{\sigma}$ represents an inverse measure of preference intensity for variety in the consumption of manufactured goods. Each worker in region r maximizes (1) given the budget constraint:

$$p_{mr} Q_{mjr} + p_{ar} Q_{ajr} = y_{jr} \tag{3}$$

where p_{mr} and p_{ar} are, respectively, the price of the composite industrial good and of the agricultural good in region r, while y_{jr} is j^{th} worker's income in r. As usual, all firms in a particular region are symmetric. With iceberg costs for the industrial goods, τ have to be shipped in order to sell one unit of them in the other region. Therefore, the industrial price index in region r is

$$p_{mr} = \left(n_r p_r^{1-\sigma} + n_v \tau^{1-\sigma} p_v^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{4}$$

From now on, following Baldwin et al. [1], we define $\phi = \tau^{1-\sigma}$, with $\phi \in [0, 1]$. ϕ is a measure of the "freeness" of trade, with ϕ equal to zero when trade costs are infinite, to one when they are null, and with ϕ that increases when trade costs decrease.

As usual, utility optimization yields demand for variety i produced in region r

$$Q_{mir} = p_r^{-\sigma} \left(\frac{1}{p_{mr}^{1-\sigma}} E_{mr} + \frac{1}{p_{mv}^{1-\sigma}} \phi E_{mv} \right) \tag{5}$$

where E_{mr} and E_{mv} are, respectively, expenditures on industrials goods in region r and in region v. Let us define w_h as skilled workers' wage, w_l as unskilled workers' wage and π_i as profits of firm i. Then, solving the utility maximization problem for each worker and aggregating expenditure on industrial goods in region r, we obtain regional expenditure on manufacturing goods

$$E_{mr} = \mu \left(w_{hr} H_r + w_{lr} L + n_r \pi_{ir} \right) \tag{6}$$

Expenditure levels in the agricultural good in region r and v, E_{ar} and E_{av}, are derived in a similar way

$$E_{ar} = (1 - \mu) \left(w_{hr} H_r + w_{lr} L + n_r \pi_{ir} \right) \tag{7}$$

Skilled workers are interregionally mobile and employed in the production of industrial varieties. Unskilled workers are not mobile and they are employed in the production of the agricultural good. To produce one unit of the traditional good, one unit of unskilled worker is employed. Therefore, with perfect competition, each region r produces $Q_{ar} = L$ units of the traditional good. This good is homogeneous and it is exchanged without trade costs. Therefore, its price must be equal in the two regions, and, given that it is chosen as the numéraire, we have that

$$w_{lr} = w_{lv} = p_{ar} = p_{av} = 1 \tag{8}$$

Each industrial variety is obtained with increasing returns to scale, which are internal to firms and derive from a fixed cost of production. Specifically, to produce Q_{mir} units of the i^{th} variety, firms have to employ β/a_r units of skilled workers for each unit produced, and α units of skilled workers independent of the production level. The variable cost may differ between the two regions, given that a_r may be different from a_v. Parameter a_r may be used as a measure of skilled workers productivity in a particular region. Obviously, if region r is more productive than v, then $a_r > a_v$. Hence, the H_{mir} workers required by the i^{th} firm to produce Q_{mir} units of the industrial goods are

$$H_{mir} = \alpha + \frac{\beta}{a_r} Q_{mir} \tag{9}$$

The cost function for each firm i in region r is

$$TC_{mir} = w_{hr}(\alpha + \frac{\beta}{a_r} Q_{mir}) \tag{10}$$

Notice that the average production cost is decreasing in regional productivity level a_r. Moreover, for α and β we adopt the following normalizations: $\alpha = 1/\sigma$ and $\beta = (\sigma - 1)/\sigma$.[4] Each firm maximizes its profits by taking the price indices p_m as given, and sets the mill price p_r with a mark up over the marginal production cost

$$p_r = \frac{\sigma\beta}{(\sigma - 1)\, a_r} w_{hr} = \frac{w_{hr}}{a_r} \tag{11}$$

with the price paid by consumers located in region v equal to

$$p_v = \tau p_r$$

Profits realized by each firm i in region r are

$$\pi_{ir} = \frac{w_{hr}}{\sigma} \left(\frac{Q_{mir}}{a_r} - 1 \right) \tag{12}$$

From the previous expression we know that each firm i in region r produces

$$Q_{mir} = \frac{\sigma a_r \pi_{ir}}{w_{hr}} + a_r \tag{13}$$

[4] We follow Puga [12].

The industrial good market is imperfectly competitive, and it is characterized by a free entry and exit assumption for firms. Therefore, profits must be null in equilibrium, and the equilibrium production level for each firm in r is

$$Q_{mir}^{*} = a_r \tag{14}$$

The equilibrium production level is higher, the higher the regional skilled workers' productivity level is. Increases in workers' productivity levels are translated not only in increases of produced quantities, but also in regional production competitiveness levels, given that for a given wage rate, manufactured goods' prices decrease. Finally, we notice that if region r has a higher skilled workers' productivity level, with $a_r > a_v$, then it has a comparative advantage in the production of the manufacturing sector.

Equating (13) and (5), and substituting manufacturing expenditure values in both regions when profits are null from (6) and (7), we obtain the following equilibrium condition for each industrial variety

$$a_r = p_r^{-\sigma} \mu \left[\frac{1}{p_{mr}^{1-\sigma}} \left(w_{hr} H_r + L \right) + \frac{1}{p_{mv}^{1-\sigma}} \phi \left(w_{hv} H_v + L \right) \right] \tag{15}$$

Since skilled workers are interregionally mobile and never unemployed, it must be verified that

$$H_r + H_v = H = 1 \tag{16}$$

Skilled workers' real wages in r, ϖ_r, are

$$\varpi_r = \frac{w_{hr}}{p_{mr}^{\mu}} \tag{17}$$

Finally, we observe that total incomes produced in both regions r and v are, respectively,

$$Y_r = H_r w_{hr} + \frac{(1-\mu)}{2\mu} \quad \text{and} \quad Y_v = (1 - H_r) w_{hv} + \frac{(1-\mu)}{2\mu} \tag{18}$$

As we can observe, the model so far described is the one proposed by Krugman [6], which has been modified in order to take into account potential interregional technological differences in skilled workers productivity levels when $a_r \neq a_v$.

The model is completed by the description of how regional productivity levels are determined. Equations that describe the values of a_r and a_v need to be continuous and differentiable around the symmetric equilibrium. Moreover, in the symmetric equilibrium both regions must be perfectly identical, because they are described by the same parameter values, and they have the same endogenous variable values. Particularly, in the symmetric equilibrium mobile workers and firms are uniformly distributed between the two regions, with $H_r = H_v = 1/2$, and regional productivity levels are equal, with $a_r = a_v = a$.

Following Krugman [8] we assume that labour productivity levels depend on the number of workers employed in a particular region. Particularly, we assume that regional productivity level, a_r, is a function of skilled worker density, H_r, with

$$a_r = f(H_r) \tag{19}$$

If skilled workers are uniformly distributed between the two regions, the productivity levels are equal with $a_r = a_v = a = 1$.[5]

Let us start from the symmetric equilibrium where $a_r = a_v = f(1/2) = 1$. Equation (19) tells us that if a certain number of skilled workers moves from the north to the south, the southern productivity level increases and, on the contrary, the northern productivity level decreases. Moreover, around the symmetric equilibrium it must be the case that for each region r

$$\frac{\partial a_r}{\partial H_r}\bigg|_{H_r = H_v = \frac{1}{2}} = \kappa \tag{20}$$

Geographically localized externalities may have different sources. They may have positive nature, with $\kappa > 0$, if they derive from *knowledge spillovers processes* or *learning by interacting processes* that foster higher productivity levels where workers density is higher. Vice versa, they may also have a negative nature, with $\kappa < 0$, if they derive from phenomena of *congestions* or of

[5] Note that we normalize to 1 regional productivity levels when skilled workers are uniformly distributed.

coordination problems. However, these interactions may be more complex with decreasing returns of regional productivity levels that may appear when workers density is sufficiently high, as shown, for instance, in figure 1, where we represent productivity levels, a_r and a_v, as a function of regional skilled workers density, H_r.[6]

Insert figure 1 about here

3 Centripetal and centrifugal forces in the core-periphery equilibrium

In this section we evaluate the sustainability of full agglomeration equilibria of the modern sector in one region and we discuss how different parameters concur in the determination of the intensities of centripetal and centrifugal forces at work.

As usual, the agglomeration of all firms in region v is a sustainable equilibrium only if the ratio between the sales that a firm could realize by relocating its production in region r, Q_{mir}, and those required to break even, Q^*_{mir}, is smaller than 1, that is if:

$$\frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \phi^{1+\frac{\sigma\mu}{\sigma-1}} \left[1 + \left(\frac{1}{\phi^2} - 1\right) \frac{(1-\mu)}{2}\right] < 1 \tag{21}$$

Expression (21) is derived considering the case in which real wages of skilled mobile workers are equal in the two regions in order to give them the incentive to work in both regions. It is well known that an expression similar to (21) can be derived if we assume that firms produce quantities that correspond to null profits, that is long run equilibrium quantities, and we examine if skilled workers have any incentive to move from the core v to the periphery r. Particularly, skilled workers do not move towards the periphery r when their real wage in the periphery r is smaller than in the core v. Therefore, the core periphery outcome with agglomeration in v is sustainable when

$$\varpi^\sigma_{hr} = a_v^{\sigma\mu} \left(\frac{a_v}{a_r}\right)^{1-\sigma} \phi^{1+\frac{\sigma\mu}{\sigma-1}} \left[1 + \left(\frac{1}{\phi^2} - 1\right) \frac{(1-\mu)}{2}\right] < a_v^{\sigma\mu} = \varpi^\sigma_{hv} \tag{22}$$

[6] The function for a_r is $a_r = 1 + 0.2H_r(1 - H_r)(H_r - 1/2)$, and for a_v is $a_v = 1 + 0.2H_v(1 - H_v)(H_v - 1/2)$ with $H_v = 1 - H_r$.