2 Conjectural variations and market equilibrium

In this section market equilibrium is analyzed under the hypothesis that each variety is produced by a mono-product firm, which competes with both the other producers within its own group, and the producers belonging to other groups. In particular, we assume that there are n_i mono-product firms for each group i of products ($i = 1, ..., M$), and that each of them produces a brand (indexed with $j = 1, ..., n_i$) of the i-th group.

Since there are n_i varieties per group, each firm simultaneously faces two different competitive environments. Horizontally, each firm competes with others producing imperfect substitutes of degree σ at the inter-group level. At the intra-group level, however, it competes with other firms producing imperfect substitutes with degree of substitutability δ[6]. Therefore, there is an inter-group competition between firms of different groups, and an intra-group competition within the same group. We assume that prices are the firms' strategic variable.

The j-th mono-product firm ($j \in [1, n_i]$), of the i-th group ($i \in [1, M]$), produces the ij-th variety according to a linear technology. Hence for the ij-th firm, the cost function is $C(x_{ij}) = cx_{ij}$, where c is the constant marginal cost. Throughout the analysis, we normalize it to one (i.e. $c = 1$). Each firm sets its own price in order to maximize profits:

$$\pi_{ij} = x_{ij}(p_{ij})p_{ij} - x_{ij}(p_{ij}) \tag{17}$$

The first order condition for profit maximization can be written in terms of the Lerner index of monopoly power:

$$\frac{\partial \pi_{ij}}{\partial p_{ij}} = 0 \Leftrightarrow \frac{p_{ij} - 1}{p_{ij}} = \frac{1}{-\frac{\partial x_{ij}}{\partial p_{ij}} \frac{p_{ij}}{x_{ij}}} \tag{18}$$

where $-\frac{\partial x_{ij}}{\partial p_{ij}} \frac{p_{ij}}{x_{ij}} = \eta^f_{x_{ij},p_{ij}}$ is the demand price elasticity as perceived by the ij-th firm[7]:

$$\eta^f_{x_{ij},p_{ij}} = \delta + (\sigma - \delta)\Delta_{ij} + (1 - \sigma)\Phi_{ij} \tag{19}$$

In the elasticity formula, Δ_{ij} and Φ_{ij} measure, respectively, the effects of the ij-th price variation on the own group price-index (group price-index-effect) and on the industry price-index (industry price-index-effect). The first is given by $\Delta_{ij} = \frac{\partial q_i}{\partial p_{ij}} \frac{p_{ij}}{q_i}$; while the second is $\Phi_{ij} = \frac{\partial q}{\partial p_{ij}} \frac{p_{ij}}{q}$.

[6] In particular, intra-group competition could be involved differences in quality, so that intra-group competition may turn to vertical product differentiation.

[7] Notice the difference between demand elasticity in (14) and demand elasticity as perceived by firms.

Clearly, the firm's demand elasticity affects its market power, being related to the competitive environment perceived by firms. In particular, different market structures can be seen as the outcome of different assumptions about the impact of each firm's price decision on the rivals' behavior. Consider the effect of changing of p_{ij} both on q_i (i.e. Δ_{ij}) and on q (i.e. Φ_{ij}). If $\Delta_{ij} = 0$ and $\Phi_{ij} = 0$, then the firm's price decisions have no effect respectively at the group and at the industry level; on the contrary, $\Delta_{ij} = 1$ and $\Phi_{ij} = 1$ denote full effects[8]. When the individual price decision influences the group price-index, an oligopolistic intra-group competition arises[9]. Differently, the intra-group competition is monopolistic when the firm's price decisions are negligible and they do not influence the group price-index.

Moreover, the perceived effect on q synthesizes the nature of competition at the inter-group level. When the effect is not negligible, inter-group competition is oligopolistic; while inter-group monopolistic competition arises when such effect is neglected[10].

2.1 The perceived market structure

In the standard monopolistic competition literature, the attention has often been focused on markets where the existence of a large number of operating firms implies that each individual decision is negligible in the previous sense. In particular the Dixit-Stiglitz (1977) model has been used to examine a wide range of issues.

However, the assumption of *competitive* behavior is independent of the number of agents in the market[11]: as long as the agents behave competitively, the competitive equilibrium can be solved for any number of firms. The idea that the competitive behavior and the negligibility assumption are related to the existence of a large number of sellers, depends on two main reasons. First, price-index-taking behavior seems more reasonable when the number of firms is large; second, the equilibrium in non-competitive market structures converges to the competitive one when the number of firms increases. Nevertheless, the definition of market structure is indeed independent of the number of firms. Rather, the specific environment faced by firms is closely related to the beliefs, the conjectures, about the rivals' reactions (Bresnahan

[8] The admitted ranges are: $0 \le \Delta_{ij} \le 1$ and $0 \le \Phi_{ij} \le 1$.

[9] Di Cintio (2005) studies a similar industry stucture, where each group is composed by homogeneous products.

[10] Reasonably, if firms neglect the effect on q_i they cannot take into account the effect on q.

[11] In frameworks of product differentiation, the term *competitive* is obviously refered to "monopolistic *competitive* behavior".

(1981), Kamien-Schwartz (1983), Perry (1982)).

As a useful starting point, we first reinterpret the well-known market outcomes (commonly analyzed in the literature) in terms of conjectural variations. In this model two conjectural variations are considered. The first is the intra-group conjectural derivative: $\frac{\partial p_{ik}}{\partial p_{ij}} = \lambda_{ik}^{ij}$ ($\forall i$ and $\forall k \neq j$), which measures what the typical *ij-th* firm believes about the relationship between its own price variation and the price change of rivals of the same group. The second is the inter-group conjecture $\frac{\partial p_{hk}}{\partial p_{ij}} = \mu_{hk}^{ij}$ ($\forall h \neq i$ and $\forall k$), which measures what the typical *ij-th* firm believes about the relationship between its own price change and the reaction of rivals of any other group.

For $\lambda_{ik} = 0$ and $\mu_{hk} = 0$, we are in the standard Bertrand case: each firm expects that if it changes its price, the rivals will not change theirs. In this model we also allow for non-zero conjectures. In particular, negative conjectures mean that each firm believes that the rivals will react to a price increase through a reduction of their prices; while positive conjectures mean that rivals will react in the same direction of the *ij-th* price change.

Taking into account the above definitions, we may express the effect (of a change in the *ij-th* price) on the group and industry price indices, in terms of the conjectural variations λ and μ:

$$\Delta_{ij} = \frac{p_{ij}}{q_i^{1-\delta}} \left(p_{ij}^{-\delta} + \sum_{k \neq j}^{n_i} p_{ik}^{-\delta} \lambda_{ik} \right) \tag{20}$$

and

$$\Phi_{ij} = \frac{p_{ij}}{q^{1-\sigma}} \left[q_i^{\delta-\sigma} \left(p_{ij}^{-\delta} + \sum_{k \neq j}^{n_i} p_{ik}^{-\delta} \lambda_{ik} \right) + \left(\sum_{h \neq i}^{M} q_h^{\delta-\sigma} \left(\sum_{k=1}^{n_h} p_{hk}^{-\delta} \mu_{hk} \right) \right) \right] \tag{21}$$

By making use of conjectural variations, we allow for the dependence of market structure on the beliefs of agents, thus mitigating the *predictive* power of those theories which link the structure of the market to the number of active firms. Any market structure is therefore conceivable for any given number of firms, and different competitive environments may result.

From equations (18) and (19), we can derive the general solution of the model for the equilibrium price, quantity and profits, under generic conjectures:

p_{ij}^*	x_{ij}^*	π_{ij}^*
$\frac{\delta+(\sigma-\delta)\Delta_{ij}+(1-\sigma)\Phi_{ij}}{(\delta-1)+(\sigma-\delta)\Delta_{ij}+(1-\sigma)\Phi_{ij}}$	$Y \left(\frac{p_{ij}}{q_i}\right)^{1-\delta} \left(\frac{q_i}{q}\right)^{1-\sigma} \frac{(\delta-1)+(\sigma-\delta)\Delta_{ij}+(1-\sigma)\Phi_{ij}}{\delta+(\sigma-\delta)\Delta_{ij}+(1-\sigma)\Phi_{ij}}$	$Y \frac{\left(\frac{p_{ij}}{q_i}\right)^{1-\delta} \left(\frac{q_i}{q}\right)^{1-\sigma}}{\delta+(\sigma-\delta)\Delta_{ij}+(1-\sigma)\Phi_{ij}}$

If we confine our attention to the symmetric equilibrium with the same number of products in each group ($n_i = n$), we have that all $p_{ij} = p \; \forall i, j$, $\Delta = \frac{1+\lambda(n-1)}{n}$ and $\Phi = \frac{1+\lambda(n-1)+\mu n(M-1)}{Mn}$.

In the limit, if $\lambda = 1$ ($\Delta = 1$) and $\mu = 1$ ($\Phi = 1$)[12] each firm expects that its own price change will be followed exactly by rivals; this means that in the market a perfect coordination of decisions is achieved (full intra- and inter-coordination).

When $\lambda = 1$ ($\Delta = 1$) and $-\frac{1}{M-1} < \mu < 1$ ($0 < \Phi < 1$) we have some kind of inter-group oligopolistic competition, while firms of the same group behave cooperatively in order to maximize the profits of the group (intra-group coordination).

For $\lambda = -\frac{1}{n-1}$ ($\Delta = 0$)[13] each firm believes that its price variation will be offset by a price reduction of all its rivals in the same group, aimed at maintaining the group price index constant. But if a firm may not affect the group price-index, it is reasonable to assume that it cannot influence the industry price-index (i.e. if $\Delta = 0$ then $\Phi = 0$), thus we have both intra-group and inter-group monopolistic competition.

For all λ such that $-\frac{1}{n-1} < \lambda < 1$ ($0 < \Delta < 1$), each firm enjoys some kind of market power within its own group; therefore if at the inter-group level $\mu = -\frac{1+(n-1)\lambda}{n(M-1)}$[14] (so that $\Phi = 0$), we obtain intra-group oligopolistic competition and inter-group monopolistic competition.

Obviously, the standard Bertrand competition arises for $\lambda = 0$ $\left(0 < \left(\Delta = \frac{1}{n}\right) < 1\right)$ and $\mu = 0$ $\left(0 < \left(\Phi = \frac{1}{Mn}\right) < 1\right)$. Lastly, all other values of λ and μ, for which still $0 < \Delta < 1$ and $0 < \Phi < 1$[15], allow for 'unusual' intra- and inter-group oligopolistic competition.

Summing up, each firm may perceive the own price change as relevant or not with respect to the group price-index and it may preserve some degree of market power at the group perspective. However, when firms of different groups react in order to maintain the industry sales unchanged, an individual price change does not affect the industry price-index q (i.e. $\Phi = 0$) and, as a consequence, we have an *inter-group monopolistic competition*[16].

[12] $\lambda = 1$ and $\mu = 1$ are the upper limits of both conjectures.
[13] The lower limit for λ.
[14] The lower limit for μ.
[15] That is $-\frac{1}{n-1} < \lambda < 1$ and $-\frac{1+(n-1)\lambda}{n(M-1)} < \mu < \frac{M}{M-1} - \frac{1+(n-1)\lambda}{n(M-1)}$.
[16] In this case, elasticity in (19) yields: $\eta^{f}_{x_{ij},p_{ij}} = \delta + (\sigma - \delta)\frac{p_{ij}}{q_i^{1-\delta}}\left[p_{ij}^{-\delta} + \sum_{k \neq j} p_{ik}^{-\delta}\lambda_{ik}\right]$

In the following table we confine our attention to situations of inter-group monopolistic competition, allowing for different values of λ, i.e. for different intra-group competitive environments. Prices, quantities and profits are then evaluated at the symmetric equilibrium.

$\lambda = -\frac{1}{n-1} \Rightarrow \Delta = 0$	$-\frac{1}{n-1} < \lambda < 1 \Rightarrow \Delta = \frac{1}{n}$	$\lambda = 1 \Rightarrow \Delta = 1$
$\pi^{Mc} = \frac{Y}{Mn\delta}$	$\pi^{Oc} = \frac{Y}{M}\frac{1}{(1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma}$	$\pi^{C} = \frac{Y}{Mn\sigma}$
$p^{Mc} = \frac{\delta}{\delta-1}$	$p^{Oc} = \frac{(1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma}{(1-\lambda)[\sigma+\delta(n-1)]+n(\lambda\sigma-1)}$	$p^{C} = \frac{\sigma}{\sigma-1}$
$x^{Mc} = \frac{Y}{Mn}\frac{\delta-1}{\delta}$	$x^{Oc} = \frac{Y}{Mn}\frac{(1-\lambda)[\sigma+\delta(n-1)]+n(\lambda\sigma-1)}{(1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma}$	$x^{C} = \frac{Y}{Mn}\frac{\sigma-1}{\sigma}$

where superscript denotes respectively Monopolistic competition, Oligopolistic competition and Coordinated behavior at the intra-group level. As expected, under all possible conjectures, profits are decreasing in the number of varieties and in the elasticities of substitutions (δ, σ, or both).

However, if we allow for inter-group oligopolistic competition, i.e. if we allow for strategic interaction both at the intra-group and at the inter-group level, we have to consider the price-index effect on both price indices q_i and q. In this case, for any $-\frac{1+(n-1)\lambda}{n(M-1)} < \mu \leq 1$, firms' market power arises both within groups and in the entire market [17].

In this case of *inter-group oligopolistic competition* (i.e. for $0 < \Phi \leq 1$), if $0 < \Delta \leq 1$ we have the following expressions for the symmetric equilibrium prices, quantities and profits in terms of λ and μ:

p	$\frac{M[(1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma]+(1-\sigma)[1+\lambda(n-1)+\mu(M-1)]}{M[(1-\lambda)[\sigma+\delta(n-1)]+n(\lambda\sigma-1)]+(1-\sigma)[1+\lambda(n-1)+\mu(M-1)]}$
x	$\frac{Y}{Mn}\frac{M[(1-\lambda)[\sigma+\delta(n-1)]+n(\lambda\sigma-1)]+(1-\sigma)[1+\lambda(n-1)+\mu(M-1)]}{M[(1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma]+(1-\sigma)[1+\lambda(n-1)+\mu(M-1)]}$
π	$\frac{Y}{M[(1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma]+(1-\sigma)[1+\lambda(n-1)+\mu(M-1)]}$

Under Bertrand behavior ($\mu = 0$, $\lambda = 0$) we have $\Delta = \frac{1}{n}$ and $\Phi = \frac{1}{Mn}$, while full coordination arises for $\lambda = 1$ and $\mu = 1$. In this latter case firms choices have a full effect on the market ($\Delta = 1$ and $\Phi = 1$) and the perfect coordination between all firms allows them to extract the total consumers' surplus.

[17] In this case, elasticity in (19) yields: $\eta^{f}_{x_{ij},p_{ij}} = \delta +$

$$+(\sigma-\delta)\frac{p_{ij}}{q_i^{1-\delta}}\left[p_{ij}^{-\delta} + \sum_{k\neq j} p_{ik}^{-\delta}\lambda_{ik}\right] +$$

$$+(1-\sigma)\frac{p_{ij}}{q^{1-\sigma}}\left[q_i^{\delta-\sigma}\left(p_{ij}^{-\delta} + \sum_{k\neq j}^{n_i} p_{ik}^{-\delta}\lambda_{ik}\right) + \left(\sum_{h\neq i}^{M} q_h^{\delta-\sigma}\left(\sum_{k=1}^{n_h} p_{hk}^{-\delta}\mu_{hk}\right)\right)\right]$$

The following table summarizes the equilibrium profits for specific values of μ and λ:

$\mu = -\frac{1+\lambda(n-1)}{n(M-1)}$, $-\frac{1}{n-1} < \lambda < 1$	$\mu = 0$, $\lambda = 0$	$\mu = 1$, $\lambda = 1$
$\pi^{Mc} = \frac{Y}{M}\frac{1}{\delta+(\sigma-\delta)(1+\lambda(n-1))}$	$\pi^{Bc} = \frac{Y}{(1-\sigma)+M[\sigma+\delta(n-1)]}$	$\pi^{PC} = \frac{Y}{Mn}$

where again superscripts denote respectively Monopolistic competition, Bertrand Competition and Perfect Coordinated behavior.

3 The multiproduct firms

The literature on multiproduct firms has mainly focused on the incentive to create brands portfolios as opposed to mono-product strategies. Indeed, the production of an entire product line may be a powerful tool to deter entry and to escape from a too much intense competition (Schmalensee, 1978).

However, the literature has paid a relatively little attention to the optimal price policies of large companies selling an entire product line; moreover, it has not provided a full motivation of two alternative organizational structures: there are companies which directly control prices from the above and companies which delegate the price decisions to independent PMs. Many papers on mergers have shown that it is profitable to allow for independent divisions when the capacity constraints play a fundamental role, such as in the cigarette market and in the automobile industry, while it is better to control each decision centrally under price competition - examples being the fast-food and mineral water industries. In the fast-food industry, all customers of the Mac-Donald and Burger King groups know that prices are defined centrally and that no autonomy is left to the single division (store). On the contrary, Williamson (1975) and Milgrom-Roberts (1992) have stressed the importance of giving independence to product divisions of the same company. There is significant evidence that Philip Morris tobacco, General Motors, Fiat, and Ford encourage competition across their own divisions, and that the same applies to Procter-&-Gamble and Mitsubishi (Nikkei Weekly 1994), to the firms of the cosmetics sector (Low 1994) and to those offering high-tech services (Forbes 1992).

Whether and when a system of PMs decentralized decisions is better than a mechanism with a centralized GD is not a trivial question. The analytical framework developed in this paper may provide an adequate tool to deal with this problem on the basis of a key distinction: the profitability of one or the other organizational structure may depend on the characteristics of the multiproduct firm's product line: market segmentation or market interlacing.