weak conformity in equilibrium, with both firms serving the market.

In this paper the linear transportation cost hypothesis is adopted, but it is combined with a quadratic formulation of the externality function. The choice of exploring the behavior of the market within this set-up turns out to be particularly useful in a double perspective. First, though both negative and positive externalities are jointly allowed for, the *régime* prevailing at equilibrium can be identified through a simple (a priory) comparison between the transportation cost size and the sign of the externality evaluated at the total population size. Second, and more important, it turns out that in this framework the location stage of the Hotelling game can be meaningfully solved even in the presence of linear transportation costs. The principle of minimum differentiation is confirmed, in a situation in which agglomeration coexists with firms earning positive profits. In particular, we show that firms endogenously choose to locate in the center of the interval, sharing the market with positive prices. If both imitation and congestion effects influence consumers' behavior, market power can be consistent with full market coverage, price competition and homogeneous products. The intuition behind this result is that at equilibrium, the consumers of both firms enjoy a *positive* but *decreasing* externality. At the margin congestion is perceived, and this allows firms to push up their prices.

The discussion is organized as follows. In section 1, we deal with the main characteristics of the consumption externality and study the strategic choices of firms with respect to prices and locations. Some conclusions are gathered in the last section.

1 The model

Consider the Hotelling (1929) model, where two stores 1 and 2 are respectively located at $x_1 \epsilon R$ and $x_2 \epsilon R$ on the real line, with $x_1 \leq x_2$. There is a continuum of consumers of mass N uniformly distributed over the interval $[0,1]$. A consumer located at $x \epsilon\, [0,1]$ bears a transportation cost of $t\,|x - x_i|$ for buying from the store located at x_i $(i = 1,2)$, where $t \geq 0$ is the transportation rate per unit distance. Let N_i denote the number of consumers patronizing store i. The consumer patronizing the store i is affected by the following consumption externality (Grilo et al., 2001):

$$C_i^{ext}(N_i) = \alpha N_i - \beta N_i^2 \tag{1}$$

in which $\alpha > 0$ expresses the incidence of the positive imitation (or conformity) effect, while $\beta > 0$ - which affects the degree of concavity of the externality function - is a measure of the incidence of the negative congestion (or vanity) effect. The externality function is defined over the domain $N_i \epsilon \, [0; N]$. Notice that if α be negative, the network effects would be negative over the entire domain, and each consumer would always suffer a loss from an increase in the number of consumers buying from the same store. If $\alpha > 0$, the sign of the overall externality may be negative or positive. Therefore we have two opposite forces at work: a positive imitation and a negative congestion effect. We assume that when evaluated at the total population size, the sign of the above externality is negative, i.e. $C_i^{ext}(N) < 0$. This allows to identify three possible regions. For $N_i \in \left]0; \frac{\alpha}{2\beta}\right[$ no congestion arises. In this range the externality is positive and increasing in the clientele size: only the imitation effect is observed and its (positive) maximum value is achieved for $N_i = \frac{\alpha}{2\beta}$. As the number of consumers patronizing store i further increases, the (positive) imitation effect is partially offset by a crowding effect. Therefore, for $N_i \in \left]\frac{\alpha}{2\beta}; \frac{\alpha}{\beta}\right[$ the externality is still positive, but its value decreases as the number of consumers patronizing the same store increases. Finally, for $N_i \epsilon \left]\frac{\alpha}{\beta}; N\right[$ the externality becomes negative and decreasing in the number of consumers: this means that the negative congestion effect dominates the positive imitation effect [1]. Notice that by imposing $C_i^{ext}(N) < 0$ we allow for the possibility that the negative congestion effect dominate the positive network effect, at least for a large size of the firm's clientele (figure 1).

As in the standard spatial model we assume that each consumer purchases one unit of the product. When buying from store i, the indirect utility of consumer located at x is then defined by:

$$V_i(x) = K - p_i + C_i^{ext}(N_i^e) - t \mid x - x_i \mid \qquad i = 1, 2 \tag{2}$$

where K denotes the gross intrinsic utility the consumer derives from consuming one unit of the product and N_i^e is the consumers' expectation about the effective number of customers served by store i.

It is well known that the price and location choice of the two firms can be seen as a two-stage game, the solution of which is a sub-game perfect Nash equilibrium in pure strategies. In the first stage the firms set their locations, in the second stage they choose their prices. We solve for the market equilibrium through backward induction.

We are interested in finding a consumer located at $\hat{x} \epsilon \, [0, 1]$ so that all consumers indexed by $x \epsilon \, [0, \hat{x}[$ patronize firm 1 and all consumers indexed by $x \epsilon \,]\hat{x}, 1]$ patronize firm 2. For $\hat{x}$ to be the location of the consumer indifferent to buy from firm 1 or firm 2, it must satisfy:

$$V_1(\hat{x}) = V_2(\hat{x})$$

[1] Notice that for $N_i = \frac{\alpha}{\beta}$ the two effects exactly offset each other.

that is:

$$-p_1 + \alpha N_1^e - \beta \left(N_1^e\right)^2 - t \mid \hat{x} - x_1 \mid = -p_2 + \alpha N_2^e - \beta \left(N_2^e\right)^2 - t \mid \hat{x} - x_2 \mid \quad (3)$$

If market is fully covered, then $N_1 + N_2 = N$. Moreover, the following conditions must be satisfied:

$$N_1 = N_1^e = N\hat{x} \qquad \text{and} \qquad N_2 = N_2^e = N(1 - \hat{x}) \tag{4}$$

These conditions mean that, in equilibrium, consumers' expectations about the network size are fulfilled. Making use of (3) and (4) and solving for $\hat{x}$, we have:

$$\hat{x} = \frac{(p_2 - p_1) + t\,(x_1 + x_2) - C^{ext}(N)}{2\,[t - C^{ext}(N)]} \tag{5}$$

Notice that, given our hypotheses about the sign of the externality evaluated at N, the denominator of (5) is always positive: $[t - C^{ext}(N)] > 0\ \forall \alpha, \beta, t$.

Before solving for equilibrium, we characterize the price pairs such that both firms have a positive market share. To do that, we find the sufficient and necessary conditions for $\hat{x} \epsilon\, [0, 1]$:

$$\hat{x} \geq 0 \text{ if } (p_1 - p_2) \leq t\,(x_1 + x_2) - C^{ext}(N) \tag{6}$$

and

$$\hat{x} \leq 1 \text{ if } (p_1 - p_2) \geq -t\,(2 - x_1 - x_2) + C^{ext}(N) \tag{6'}$$

When inequalities (6) and (6') simultaneously hold, both firms 1 and 2 have a positive demand, corresponding respectively to N_1 and N_2 in (4).

Moreover, we define the characterization of the price pairs for which only a single firm serves the market. Consider first the case in which $N_1 = N$ and $N_2 = 0$, i.e. the price pairs in which $V_1(\hat{x}) > V_2(\hat{x})$ for all $\hat{x} \epsilon\, [0, 1]$. This situation arises if and only if:

$$(p_1 - p_2) < -t\,(2 - x_1 - x_2) + C^{ext}(N) \tag{7}$$

Consider next the opposite case, $N_1 = 0$ and $N_2 = N$ with $V_1(\hat{x}) < V_2(\hat{x})$ for all $\hat{x} \epsilon\, [0, 1]$. This market configuration arises when:

$$(p_1 - p_2) > t\,(x_1 + x_2) - C^{ext}(N) \tag{8}$$

Inequalities (6-6'), (7) and (8) are mutually exclusive. Consider first the domain defined by (6) and (6'), where the demand functions of the two firms, linear and decreasing in the own price, are given by:

$$N_1(p_1, p_2) = N\hat{x} = N \frac{(p_2 - p_1) + t\,(x_1 + x_2) - C^{ext}(N)}{2\,[t - C^{ext}(N)]} \tag{9}$$

for firm 1, and

$$N_2(p_1,p_2) = N(1-\hat{x}) = N\frac{(p_1-p_2)+t\,(2-x_1-x_2)-C^{ext}(N)}{2\,[t-C^{ext}(N)]} \tag{10}$$

for firm 2. Assuming that production takes place at zero costs, the profit functions of the two firms are defined respectively by:

$$\pi_1 = p_1N_1(p_1,p_2) = Np_1\frac{(p_2-p_1)+t\,(x_1+x_2)-C^{ext}(N)}{2\,[t-C^{ext}(N)]} \tag{11}$$

$$\pi_2 = p_2N_2(p_1,p_2) = Np_2\frac{(p_1-p_2)+t\,(2-x_1-x_2)-C^{ext}(N)}{2\,[t-C^{ext}(N)]} \tag{12}$$

Firm 1 takes p_2 as given and chooses p_1 to maximize π_1; while firm 2 takes p_1 as given and chooses p_2 to maximize π_2. Given the location of the two stores, by differentiating π_i with respect to p_i, we obtain the following best-response functions:

$$\frac{\partial\pi_1}{\partial p_1} = 0 \Longleftrightarrow p_1 = \frac{1}{2}\left[p_2+t\,(x_1+x_2)-C^{ext}(N)\right] \tag{13}$$

$$\frac{\partial\pi_2}{\partial p_2} = 0 \Longleftrightarrow p_2 = \frac{1}{2}\left[p_1+t\,(2-x_1-x_2)-C^{ext}(N)\right] \tag{14}$$

Since the profit functions (11) and (12) are concave, (13) and (14) are the necessary and sufficient conditions for a maximum. Solving for (13) and (14) we obtain:

$$p_1^*(x_1,x_2) = \frac{t}{3}\,(2+x_1+x_2)-C^{ext}(N) \tag{15}$$

$$p_2^*(x_1,x_2) = \frac{t}{3}\,(4-x_1-x_2)-C^{ext}(N) \tag{16}$$

For (15) and (16) to be the equilibrium prices, it remains to check whether they satisfy the inequalities in (6) and (6'), that is:

$$-t\,(2-x_1-x_2)+C^{ext}(N) \leq -\frac{2}{3}t\,(1-x_1-x_2) \leq t\,(x_1+x_2)-C^{ext}(N)$$

It easily checked that these conditions are satisfied for any location such that:

$$-2 \leq x_1+x_2 \leq 4 \tag{17}$$

Therefore, as far as the two stores locate inside the interval, (6) and (6') always hold. Notice that the above range of locations pairs for which the equilibrium prices exist is the same as that identified by Grilo et al. (2001) in a framework where transportation costs are quadratic. Moreover, we could extend to this set-up their interpretation of the model in terms of vertical differentiation: in case firms locate outside the interval $[0,1]$ and produce two vertically differentiated products (figure 2), they could both survive because of the emergence of some congestion effect in consumption, in absence of which only the firm with the best quality good could enjoy a positive demand. In synthesis, market sharing occurs provided that (17) holds; it entails either the horizontal product differentiation if $0 \leq x_1 + x_2 \leq 2$ or the vertical product differentiation if $x_1 + x_2 \leq 0$ (or $x_1 + x_2 \geq 2$).

It is well know that in the Hotelling model with linear transportation costs a price equilibrium in pure strategies does not exist when stores are symmetric located in the inner quartiles. Differently, our example shows that positive equilibrium prices exist for any symmetric and asymmetric location when consumer preferences exhibit both imitation and congestion effects. Moreover, even if the stores were located at the same point, the usual Bertrand argument should not lead to the competitive outcome.

As a proof, payoffs of the two firms, valued at the equilibrium prices (15) and (16), are globally optimal if they are at least as great as the payoffs that firms would earn by undercutting the rival's price and suppling the whole market. Store 1 may gain the whole market undercutting its rival by setting $p_1^M = [p_2^* - t(2 - x_1 - x_2) + C^{ext}(N) - \epsilon]$, in this case profit amount to $\pi_1^M = \frac{2}{3}Nt[x_1 + x_2 - 1]$ for a small ϵ. The similar argument is valid for the store 2, undercutting the rival it would earns $\pi_2^M = \frac{2}{3}Nt[1 - x_1 - x_2]$. The conditions for such undercutting not to be profitable are:

$$\pi_1^* \geq \pi_1^M \iff \left[t(2 + x_1 + x_2) - 3C^{ext}(N)\right]^2 \geq 12t[x_1 + x_2 - 1]\left[t - C^{ext}(N)\right]$$

$$\pi_2^* \geq \pi_2^M \iff \left[t(4 - x_1 - x_2) - 3C^{ext}(N)\right]^2 \geq 12t[1 - x_1 - x_2]\left[t - C^{ext}(N)\right]$$

that are:

$$\left[t(4 - x_1 - x_2) - 3C^{ext}(N)\right]^2 \geq 0$$

$$\left[t(2 + x_1 + x_2) - 3C^{ext}(N)\right]^2 \geq 0$$

Therefore provided that (17) holds, a unique price equilibrium exists.

Proposition 1: *When consumer preferences exhibit both imitation and congestion effects, for any symmetric and asymmetric locations for which (17) hold, there exists a unique price equilibrium and it is described by (15) and (16).*

Substituting the equilibrium prices into (9) and (10) yields the number of consumers served by the two firms as function of their locations:

$$N_1(p_1^*, p_2^*) = N\frac{t(2 + x_1 + x_2) - 3C^{ext}(N)}{6[t - C^{ext}(N)]} \tag{18}$$

$$N_2(p_1^*, p_2^*) = N\frac{t(4 - x_1 - x_2) - 3C^{ext}(N)}{6[t - C^{ext}(N)]} \tag{19}$$

Therefore at the equilibrium prices, profits are given by:

$$\pi_1^*(x_1, x_2) = N\frac{\left[t\,(2 + x_1 + x_2) - 3C^{ext}(N)\right]^2}{18\left[t - C^{ext}(N)\right]} \tag{20}$$

$$\pi_2^*(x_1, x_2) = N\frac{\left[t\,(4 - x_1 - x_2) - 3C^{ext}(N)\right]^2}{18\left[t - C^{ext}(N)\right]} \tag{21}$$

We now investigate how externality function affects equilibrium. For any given location of the two stores, both prices and profits result decreasing with respect to α and increasing with respect to β and N. When consumer preferences are appreciably affected from positive network effects (high α), as expected competition is fiercer and it results in lower equilibrium prices (and profits). By contrast, when consumer preferences suffer from strong congestion effects (high β), the relaxing in competition allows firms to set higher prices (and profits). Also notice that for given α and β, an increase in consumer population (N) gives back an higher predictability to find overcrowd stores, hence it pushes up the equilibrium prices (and profits).

Proposition 2: *When consumer preferences exhibit both imitation and congestion effect, the equilibrium prices increase as:*
1) the consumer population raises (higher N),
2) the imitation effect becomes less significant (lower α),
3) the congestion effect becomes more significant (higher β).
Also we investigate how the network effects affects the stores' market shares. Through (5), the derivative of $\hat{x}(p_1^*, p_2^*)$ with respect to α yields:

$$\frac{\partial \hat{x}}{\partial \alpha} = N\frac{t(x_1 + x_2 - 1)}{6\left[t - C^{ext}(N)\right]^2}$$

Thus under asymmetric locations, an increase in α increases the clientele size of the store with positional advantage and decreases the clientele size of its rival. Differently, differentiating $\hat{x}$ with respect to β, we obtain opposite results:

$$\frac{\partial \hat{x}}{\partial \beta} = N^2\frac{t(1 - x_1 - x_2)}{6\left[t - C^{ext}(N)\right]^2}$$

an increase in β reduces the clientele size of the store with positional advantage and increases that one of its rival. Lastly, we investigate how the size of the total population influences the market shares of the two stores. Differentiating $\hat{x}$ with respect to N yields:

$$\frac{\partial \hat{x}}{\partial N} = N\frac{t(1 - x_1 - x_2)(2\beta N - \alpha)}{6\left[t - C^{ext}(N)\right]^2}$$

therefore since our assumption about $N > \frac{\alpha}{\beta}$ holds, an increase in the consumer population reduces the market share of the large store and increases the market share of the small one. Notice that when the stores are symmetrically located, the indifferent consumer between the two firms is always in the center of the interval $[0,1]$ therefore the size of the total population, N, and the two parameters, α and β, do not affect the market shares even if they still affect the equilibrium prices. Moreover, the derivatives of (18) and (19) with respect to N yields:

$$\frac{\partial N_1}{\partial N} = \frac{t\,(2 + x_1 + x_2) - 3C^{ext}(N)}{6\,[t - C^{ext}(N)]} + N\frac{t(1 - x_1 - x_2)(2\beta N - \alpha)}{6\,[t - C^{ext}(N)]^2}$$
$$\frac{\partial N_2}{\partial N} = \frac{t\,(4 - x_1 - x_2) - 3C^{ext}(N)}{6\,[t - C^{ext}(N)]} + N\frac{t(x_1 + x_2 - 1)(2\beta N - \alpha)}{6\,[t - C^{ext}(N)]^2}$$

and since (17) holds, both the first terms in the above derivatives are positive. Thus, while an increase in the size of the total population increases both market share and clientele size of the small store, the effect on the clientele size of the large store depends on parameters.

Proposition 3: *When consumer preferences exhibit both imitation and congestion effect, the market share of the store with positional advantage (figure 3) increases as:*
1) the imitation effect becomes more significant (higher α),
2) the congestion effect becomes less significant (lower β),
3) the population falls (lower N) even if the clientele size should be reduced.

Proposition 4: *When consumer preferences exhibit both imitation and congestion effect, the market share of the store with positional disadvantage increases as:*
1) the imitation effect becomes less significant (lower α),
2) the congestion effect becomes more significant (higher β),
3) the population raises (higher N).

At the first stage of the game, given the equilibrium prices, both store 1 and store 2 set their locations in order to maximize respectively (20) and (21).

The derivatives of (20) and (21) with respect respectively to $x_1$and x_2, yield:

$$\frac{\partial \pi_1^*(x_1, x_2)}{\partial x_1} = N\frac{t\,[t\,(2 + x_1 + x_2) - 3C^{ext}(N)]}{9\,[t - C^{ext}(N)]} > 0 \qquad (22)$$
$$\frac{\partial \pi_2^*(x_1, x_2)}{\partial x_2} = -N\frac{t\,[t\,(4 - x_1 - x_2) - 3C^{ext}(N)]}{9\,[t - C^{ext}(N)]} < 0$$

Since (17) holds, the firms earn higher profits by moving towards the center. Then, endogenously the two stores locate at the center of the interval $[0,1]$. In this case firms share the market selling a homogeneous product at the same positive price. Therefore, the Principle of Minimum Product Differentiation holds even for linear transportation costs.

Proposition 5: *When consumer preferences exhibit both imitation and congestion effects, even if consumers face linear transportation costs, there exists a unique symmetric equilibrium in which firms locate in the center of the interval setting positive prices:*
1) $p_1 = p_2 = [t - C^{ext}(N)]$,
2) $x_1 = x_2 = \frac{1}{2}$.

2 Conclusion

We have investigated price competition in the Hotelling location model with linear transportation costs when consumer preferences are affected by the number of consumers shopping at the same store. The introduction of a quadratic and concave consumption externality permits to consider at the same time, and not alternatively, both the imitation and the congestion effects which are opposite forces at work.

Some significant even if predictable results are been reached. While competition is relaxed when consumers have an appreciable degree of sensitivity to congestion effects, a fiercer price competition results when preferences are appreciably affected from imitative behaviors. The sensibility of consumer preferences to the two opposite network effects also have significant impacts on the market share of the stores in cases of positional advantages. The market share of the store with positional advantage increases either when the imitation effect becomes more significant and when the congestion effect becomes less important. Even if our approach is partial -perhaps not too general- in many aspects, it resolves the non-existence problem of the Hotelling model. Our model shows that positive equilibrium prices exist for any symmetric and asymmetric location. Moreover, even if the stores were located at the same point, the usual Bertrand argument should not lead to the competitive outcome. Furthermore given the possibility for firms to set the best locations, we confer new validity to the principle of minimum product differentiation when consumer preferences exhibit both imitation and congestion effects.