## 5. Further extensions

As noted in section 2, the framework of the present paper has two main shortcomings. First, residence-based taxes cannot be properly analysed as the domestic supply of capital is fixed. Second, since labour is sector specific, ad-hoc assumptions must be introduced in order to rule out the possibility of implementing the first-best solution by levying differential taxes at the firm level.

The first drawback can be overcome by extending the model to a two-period framework as in Bucovetstky and Wilson (1991) and Razin and Sadka (1991). In the first period no production or trade takes place. Consumers only choose how much to consume out of the same inherited lump-sum income. In the second period the economy is represented by the model analysed in this paper, except for the elastic saving supply, which stems from the choice made in the first period.

The arguments used to establish the optimality of source- and origin-based taxes in the static model<sup>8</sup> are still valid in the two-period framework with residence-based capital taxation. Source- and origin based taxes affect welfare exclusively through their impact on wages. They are not a substitute for residence-based taxation, as in Haufler (1997) and Lopez et al. (1996), since they do not influence the consumer interest rate. Consequently, the government resorts to them only if they can improve on the distribution of wages that can be implemented with the tax on labour income. For the source-based capital tax this is possible only if  $\varepsilon_{wr}^1 \neq \varepsilon_{wr}^2$ . Further, origin-based commodity taxes have opposite effects on the wage distribution. This implies that the optimal tax rate ratio is still defined by (3.36). As to the sign, the source-based capital tax

<sup>&</sup>lt;sup>8</sup>The formal analysis of the two period model is contained in a previous version of this paper, Arachi (1999).

can be positive or negative depending on the elasticities of wages with respect to the interest rate and on the social evaluation of labour income redistribution. However, since the latter depends on the residence-based capital tax, it is difficult to identify the precise conditions that yield a positive source-based capital tax. These remarks apply to destination-based commodity taxes as well. Destination-based taxes leave the wage distribution unaffected since they do not impinge on producer commodity prices. Hence, they only affect the choice of source- and origin-based taxes indirectly by altering the social evaluation of labour income redistribution.

The second limitation of the paper can be overcome by showing that the analysis developed in the preceding sections extends to the more general case where each sector employs both types of labour.

Condition (3.9) can be rewritten as follows

$$-\left(1-t_{L}^{1}\right)\frac{\partial w^{1}}{\partial r}\left\{\left(1-\alpha^{1}\right)nl^{1}-\left[t_{L}^{1}w^{1}nl_{w}^{1}+t_{S}\frac{\partial K^{1}}{\partial w^{1}}\right]\right\}-\left(1-t_{L}^{2}\right)\frac{\partial w^{2}}{\partial r}\left\{\left(1-\alpha^{2}\right)\left(1-n\right)l^{2}-\left[t_{L}^{2}w^{2}\left(1-n\right)l_{w}^{2}+t_{S}\frac{\partial K^{2}}{\partial w^{2}}\right]\right\}+$$

$$t_{S}\left(\frac{\partial K^{1}}{\partial r}+\frac{\partial K^{2}}{\partial r}\right)=0.$$
(5.1)

This condition must hold in the optimum even in the case where both types of labour enter into the production of each commodity. The differences between the general and the specific factor model lie in the derivatives of wages and domestic capital stock. In the general case, equation (3.3) must be replaced by<sup>9</sup>

$$\frac{\partial w^i}{\partial r} = \frac{K^j L_i^j - K^i L_j^j}{L_i^i L_j^i - L_i^i L_i^j}.$$
 (5.2)

<sup>&</sup>lt;sup>9</sup>When both sector employ the same proportions of skilled and unskilled, i.e.  $L_i^i L_j^j - L_j^i L_i^j$ , it is not possible to obtain the derivative of wages with respect to the interest rate from the no-profit conditions.

where  $L_j^i$  denotes the amount of labour of type i employed in sector j, while the function  $\gamma^i L^i$ , that yields the equilibrium capital stock in sector i, must be replaced by

$$K^{i} = c_{r}^{i} \frac{c_{w^{j}}^{j} L^{i} - c_{w^{i}}^{j} L^{j}}{c_{w^{j}}^{j} c_{w^{i}}^{i} - c_{w^{i}}^{j} c_{w^{j}}^{i}}.$$
(5.3)

The same argument applies to the remaining first-order conditions of problems (3.4), (3.15) and (4.4). Consequently, the optimal source-based capital income tax must still satisfy, in the more general framework, the crucial conditions (3.11), (3.20), and (4.10). A corollary of this conclusion is that the results stated in propositions (3.1), (3.2) and (4.1) do not depend on whether production requires just one or both types of labour. Further, the sign of the optimal source-based tax rate on capital can be determined on the basis of (3.24) and (4.11), provided the domestic capital stock is a decreasing function of the interest rate. Expression (5.2) implies that a positive source-based tax may actually increase one of the two wages. For example, the wage of the unskilled rises when the industry that employs the higher share of unskilled labour uses more skilled labour than capital (i.e.  $L_i^2/L_j^2 > L_i^1/L_j^1$  and  $K^j/K^i > L_j^1/L_i^1$ ). In this case the optimality of a source-based tax can be established without knowing wage elasticities, as the difference  $\varepsilon_{wr}^1 - \varepsilon_{wr}^2$  cannot be equal to zero.

The results obtained for origin-based commodity taxation hold in the general framework as well. However, the analysis becomes much more intricate as each tax affects both wages. The

<sup>&</sup>lt;sup>10</sup>With a linear labour income tax this guarantees a positive  $\Delta$  in (3.23).

first order conditions (3.31) and (3.32) must be replaced by two equations similar to (3.31), i.e.

$$(1 - t_L) \frac{\partial w^1}{\partial p^i} \left\{ (1 - \alpha^1) n l^1 - \left[ t_L^1 w^1 n l_w^1 + t_O^1 \frac{\partial X^1}{\partial w^1} + t_O^2 \frac{\partial X^2}{\partial w^1} \right] \right\} +$$

$$(1 - t_L) \frac{\partial w^2}{\partial p^i} \left\{ (1 - \alpha^2) (1 - n) l^2 - \left[ t_L^2 w^2 (1 - n) l_w^2 + t_O^1 \frac{\partial X^1}{\partial w^2} + t_O^2 \frac{\partial X^2}{\partial w^2} \right] \right\} +$$

$$t_O^1 \frac{\partial X^1}{\partial p^i} + t_O^2 \frac{\partial X^2}{\partial p^i} = 0$$

$$(5.4)$$

where

$$\frac{dw^{i}}{dp^{i}} = \frac{c_{wj}^{j}}{c_{wi}^{i}c_{xij}^{j} - c_{wi}^{j}c_{wj}^{j}}$$
(5.5)

$$\frac{dw^{i}}{dp^{j}} = -\frac{c_{w^{j}}^{i}}{c_{w^{i}}^{i}c_{w^{j}}^{j} - c_{w^{i}}^{j}c_{w^{j}}^{i}}$$

$$(5.6)$$

and

$$X^{i} = \frac{c_{w^{i}}^{j} L^{j} - c_{w^{j}}^{j} L^{i}}{c_{w^{j}}^{i} c_{w^{i}}^{j} - c_{w^{j}}^{j} c_{w^{i}}^{i}}.$$
(5.7)

It can be easily verified that the substitution of the first-order condition for the tax on labour income into (5.4) yields two equations similar to (3.20). These equations will contain two terms. The first is given by the product of one of the two curly bracket in (5.4) and the difference between the elasticities of the two wages with respect to the price of the taxed commodity. The second term is the effect of a change in the commodity price on government revenue. The difference between wage elasticities is never equal to zero since the right hand sides of (5.5) and (5.6) show that a price change has opposite effects on the two wages. Hence Proposition 3.4 can be restated in the general framework: the government resorts to differential origin-based commodity taxation whenever the expressions in square brackets in (5.4) are different from zero under an optimal linear labour tax.

By further substituting one of the two necessary conditions for optimal commodity taxes

into the other, one obtains an equation that defines the optimal ratio between the two tax rates. This equation shares the two main characteristics of (3.36). First, the optimal tax ratio does not depend on the shape of the social welfare functional as the expression in square brackets in (5.4) has been eliminated through the substitution. Second, the two tax rates have opposite signs, as they bring about opposite changes in wage distribution.

Following the same steps, one can also generalise the conclusions reached under an optimal non-linear income tax.

## 6. Concluding remarks

The analysis developed in the paper shows that source- and origin-based taxes are constrainedefficient instruments in a small open economy when consumers differ in skills and the government
cannot implement the optimal differential linear taxation of labour since it cannot directly
observe individuals' characteristics. The rationale for such taxes is quite different from the
one provided by the existing literature, which studies large open economies where source-based
taxation is seen as a substitute for residence-based taxation.

In a small open economy source- and origin-based taxes are shifted completely onto immobile labour. Where a linear tax is levied on labour income, source- and origin-based taxes can act as substitutes for differential taxation of labour if the various types of labour bear a different tax burden. By contrast, if a non-linear tax is levied on labour income, source- and origin-based taxes cannot directly improve income distribution as the two types of labour are taxed at two different marginal rates. However, they may still improve social welfare as they relax the self-selection constraint that binds the non-linear tax by changing the distribution of gross