Introduction

The Multicast problem is a Combinatorial Optimization problem whose aim is to connect by wired or wireless links a set of required vertices at the minimum cost. There are several contexts in which such a problem finds its application, one of these is the Multicast routing in communication [66]. The main objective, in this case, is to ensure that an information generated by a node of a network, called source, reaches a multicast group which is a set of selected elements of the network, minimizing the usage of resources, in particular the energy or the power employed in the communication.

The major part of the presented results is devoted to a particular type of network, the Ad-Hoc wireless network (see e.g. [60], [72], [84]). The vertices of these networks are electronic devices (sensors, computers, radio transmitters etc.) which transmit radio signals without using a fix infrastructure and without a centralized administration. This type of network is expected to be used in several fields going from natural disasters to battlefields, where the existing infrastructures are damaged or unusable.

The devices of an Ad-Hoc network are supposed to be stationary and they are equipped with an omnidirectional antenna in such a way that the signal is spread radially from the nodes. A device may communicate with a single–hop, i.e. directly, with any other terminal which is located within its transmission range. In order to communicate with the terminals placed out of this range a multi–hop communication has to be performed: it simply consists in making use of intermediate devices, called routers, that retransmit the received messages to the directly unreachable terminals ([72], [84]).

A crucial issue in this context consists in assigning a transmission power to each node in order to ensure the connectivity of the network while minimizing the total power expenditure over the network. Determining the optimal transmission power for each node is, indeed, desirable since a high power value will achieve a wide transmission range and, therefore, reach many nodes via a direct link, but at the same time will require higher consumption and will increase the interference level. On the other hand, low energy value may isolate one or more nodes causing the network to be disconnected. Both Cagalj *et al.* and Clementi *et al.* have shown that the Multicast problem in wireless Ad-Hoc networks is an NP-hard problem ([13], [20]).

In case of Multicast problem in wired networks, we take into account a Quality of Service in the routing of the communication. Indeed, in many application [66] there may be the further request of delivering the information generated by a source and directed to a set of destinations within a maximum delay. Naturally, the Quality of Service constraints and in the specific the maximum delay constraints impose a restriction on an acceptable Multicast tree. Only recently the Delay-constrained Steiner Tree problem has been object of study (see [49], [66]), indeed, with the developments of the multimedia technology, the real-time applications need to transmit information within a certain amount of time and so a message generated by one source of the network has to reach a set of target devices for delivering the same information in a fixed delay limit.

The first chapter is a preliminary chapter in which all the concepts: definitions, properties and problems that are used all along the dissertation are presented.

With the second chapter, we begin to consider the Minimum Power Multicast problem in wireless Ad-Hoc networks. We present a Set Covering formulation for the problem and we show that it is better than the formulation proposed in [53] and better than two adaptations to the Multicasting case of formulations proposed in [3] and in [60] for the Broadcasting problem (where the Broadcasting problem is a particular Multicast problem in which all the nodes of the network must be connected to the source). We propose also two exact procedures for solving the problem that use the properties of a Set Covering formulation and we present some computational results on randomly generated graphs with size ranking from 5 to 100 nodes and an increasing number of destinations.

In the third chapter, we study the properties of the Set Covering polytope of the Multicast problem in wireless Ad-Hoc networks. Specifically, we describe two heuristics for finding particular valid inequalities of the first Chvátal closure in order to strengthen the linear relaxation of the formulation. We compare the results on the improvement of the lower bounds obtained by solving the linear relaxation of the formulation with the addition of the constraints generated by these heuristics with the results produced minimizing over the first Chvátal closure polytope [19].

In the fourth chapter, we deal with the Broadcasting problem in which the minimization of the power cost and the achievement of a robust routing are considered. Indeed, we take into account the possibility that the devices may be subject to a temporary damage or a permanent failure and so they are assigned a probability of being active. We propose three mixed integer linear programming formulations whose optimal solution not only minimizes the total transmission power over the network, but also guarantees a certain reliability level. The optimal solution provides a broadcasting structure robust enough to guarantee, in case of failure of some terminals, a reliable connectivity for the remaining terminals.

The study of a generalization of the Steiner tree problem is, instead, the topic of the fifth chapter. In particular the Delay constrained Steiner Tree problem is analysed, there, in wired networks. We present several valid mixed integer programming formulations that provide a tree spanning the source and the required nodes with the minimum cost and that satisfies a maximum delay threshold. We compare the respective linear relaxations of the formulations and we describe some preprocessing procedures to reduce the size of the problems. We present exact and approximate solution procedures with some computational results.

At the end, there is an appendix in which we briefly define some of the symbols used in the dissertation.