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Qualitative and quantitative models for ordinal data analysis

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Abstract: *In this paper, we explore and compare classical regression and ordinal data models when quantitative data are related to a qualitative assessment. Specifically, we test the approach on a data set of graduated students and we check the relative performance and the interpretative content of the models. Some further comments end the paper.*

Keywords: Ordinal data, CUB models, graduation data

1. Introduction

In current literature people look for adequate transformations of qualitative ordinal data into numerical values in order to apply standard statistical methods like regression models and multivariate analysis. However, when a genuine qualitative ordering is performed, a different approach may be more fruitful. Standard methods for modelling ordinal data stem from the logic of Generalized Linear Models (GLM) and, specifically, the class of ordinal logistic models has been applied, as discussed by Agresti (2002), Dobson (2002) and McCullagh (1980).
A different perspective has been recently introduced by D'Elia and Piccolo (2005) who proposed a direct formulation of the probability for a discrete ordinal choice. In this context, the paper analyses the graduated results as related to gender and duration of permanence at University and shows that a qualitative model can achieve useful results and more interesting interpretation than a plain regression analysis.

2. Data set and qualitative assessment of final grades

Our data set consists of the final assessment degrees received by n = 2324 graduates in Political Sciences at University of Naples Federico II after a 4-years course. It is a standard practice in Italy to express these evaluations on a 66-112 scale points (where 66 is the minimum to be graduated and 112 is arbitrarily set for first class honors). Experience shows that the final grade is determined both on average marks of exams and an overall judgment of the Commission with regard to the thesis defense; in fact, the final grade is expressed on a quantitative scale but it is to be considered as a qualitative assessment about the candidate.
Thus, it is important for a graduate to receive an evaluation V belonging to a class of merit, in correspondence with an ordinal variable R with $m=7$ values, as in Table 1.

Final grade	*Evaluation*	*Class (Rating)*
V=110 cum laude	First class honors	A *(R=7)*
V=110	Excellent	B *(R=6)*
$105 \le V < 109$	Very good	C *(R=5)*
$100 \le V < 104$	Good	D *(R=4)*
$90 \le V < 99$	Sufficient	E *(R=3)*
$80 \le V < 89$	Low	F *(R=2)*
$66 \le V < 79$	Very low	G *(R=1)*

Table 1: Correspondence among quantitative and qualitative evaluations of graduating marks.

The observed frequency distribution (dots in Fig.1) enhances a strong atypical value at *R=7* and confirms the importance of this modal value for explaining this variable.

3. Qualitative and quantitative models for final grades

In previous works (Iannario, 2007; Iannario and Piccolo, 2007), we found that for data set concerning ordinal values (ratings, evaluations, preference scores, etc.), a class of stochastic models (defined CUB) are adequate structures for explaining, fitting and testing such kind of data (Piccolo and D'Elia, 2008). Thus, given the considerations of section 2, an extended formulation of these models can be adapted to our data set. Specifically, the problem is to take into account the presence of an extreme frequency at class A (=First class honors) as compared with the remaining distribution.
Then, we introduce extended CUB model on the support of the first *m* integers by defining the following mixture probability distribution:

$$\Pr(R=r) = \pi_1 b_r(\xi) + \pi_2 \frac{1}{m} + (1-\pi_1-\pi_2) D_r^{(c)}, \qquad r = 1,2,\dots,m.$$

The first component is related to a *feeling* (with parameter ξ), the second one to *uncertainty* (with parameter π_2) and the third to a so-called *shelter effect*, modelled by a degenerate random variable that collapses at the value *(r=c)* of interest.
Here, we adopt the interpretation discussed in Iannario and Piccolo (2008) and we report significant evaluation and uncertainty components; moreover, we explain a *shelter effect* for the first class A (12.5% for men and 20.6% for women). Then, the performance of this model with *c=7* is shown in Fig.1 (circled points are estimated probabilities) and the remarkable fitting is confirmed by a normalised dissimilarity measure of *Diss*=0.058 (see: Piccolo, 2006; Piccolo e Iannario, 2008).
In our instance, both gender and duration turned out to be significant covariates for explaining the final results. In fact, CUB model is able to quantify the common experience that final grade is higher on average for women, while a sensible negative correlation comes out between grade and duration of the studies.
From a parametric point of view, in Table 2 we compare some estimated models by mean of *BIC* and *ICON* indicators defined for our models, respectively, by:

$$BIC = -2\,\mathrm{l}(\hat{\theta}) + 3\log(n)\,; \qquad ICON = 1 + \frac{\mathrm{l}(\hat{\theta})}{n * log(7)}\;;$$

where $\mathrm{l}(\hat{\theta})$ is the log-likelihood function computed at the maximum likelihood estimates, for the vector parameter $\theta = (\pi_1, \pi_2, \xi)'$. We notice that *ICON* is a sort of pseudo-R^2 (McFadden, 1974) and it measures the gain in terms of *I*nformation *CON*tent obtained by the estimated models with respect to the worst one, that is a discrete Uniform distribution over the set *{1,2,...,m}*.

Model	*Log-likelihood*	*BIC*	*ICON*
Uniform: discrete benchmark	-4522.3	9044.6	0.000
CUB(0,0): no covariates	-4294.5	8604.5	0.050
CUB(0,1): gender	-4251.6	8526.5	0.060
CUB(0,1): log(duration)	-4179.6	8382.5	0.076
CUB(0,2): gender+log(duration)	-4137.4	8305.8	0.085
CUB(0,3): gender+log(duration)+interaction	-4100.2	8239.2	0.093

Table 2: Fitting measures for estimated models for grades.

All listed models are significant if checked by asymptotic tests, and the last one improves the information content more than 9%, that is an important addition for overdispersed qualitative data.

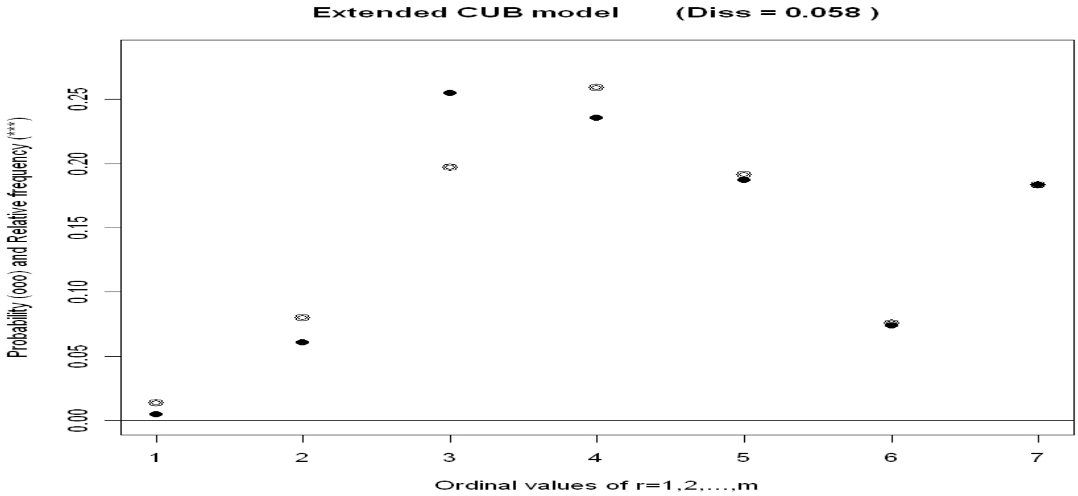

Figure 1: Distribution of grades (dots) and fitted probabilities (circles).

The final model shows that an intersection is significant between the covariates gender and duration. As a consequence, both covariates affect the final grade but for women a long duration of the studies acts as a major penalization.
Thanks to the estimated CUB(0,3) model we are able to assess the expected grade $E(R|\mathbf{w}_i)$ based on a qualitative ordering, given the subjects' covariates $\mathbf{w}_i$, as shown in the bottom panel of Fig.2. It seems evident that the expectation decreases with duration for both gender; however, for women the acceleration is more evident. Then, by a simple algebra, it is possible to estimate that the turning point for a different gender behavior happens at 10 years after the enrollment.

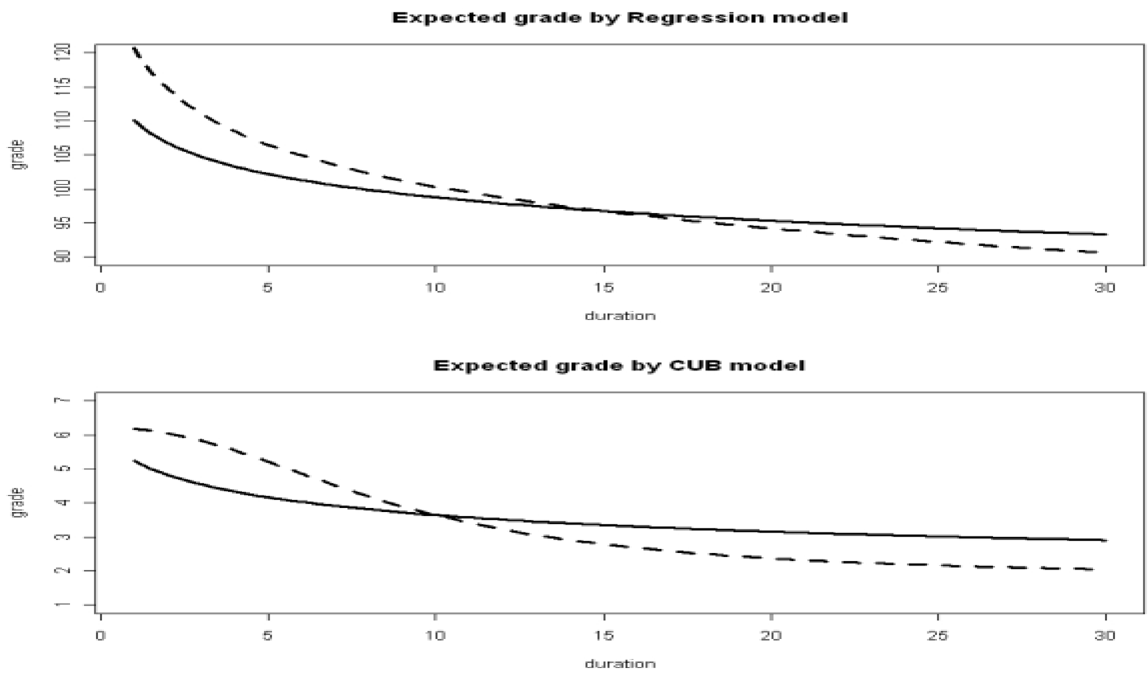

Figure 2: Expected rating versus duration, given the gender (Females: dashed lines).

Instead, if we model the quantitative grade V as a linear function of the same covariates we get the classical regression relationship with significant estimates. From these results, we obtain the corresponding expected grade versus duration, given the gender, as shown in the top panel of Fig.1. Although the patterns could suggest similar considerations for CUB and regression models, we notice that in the second model the turning point is deferred to 15 years after the enrollment and the best performance of the women in not clearly enhanced at the beginning of the University studies. Moreover, the regression model is unable to generate values less than 90 (classes F and G) while CUB model can also generate values belonging to class F $(V>80)$.

5. Concluding remarks

This case study confirms a better performance of qualitative models in the tails of the distribution given the robustness property of ordinal values.

From a practical point of view, we found that a qualitative data modelling should be preferred in our case study since the quantitative determination for graduating marks is derived from a qualitative assessment. In such cases, any choice stems from a composite procedure, where several latent variables are to be combined for explaining the final evaluation and the proposed mixture solution turned out to be adequate for interpreting and fitting the observed data.

The class of extended models that we have introduced in the educational context and evaluation analyses may be easily extended in any situation where some groups of raters concentrate their answers on specific choices; in these cases, it is useful to include their atypical behaviour in a single model and to simplify the interpretation of the results. Moreover, the extended CUB model allows to quantify and test the shelter effect within an inferential framework.

Finally, in further studies, we plan to deepen the comparative performance of qualitative and quantitative models by means of measures of predictive ability.

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