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Conflict & Cooperation under Stackelberg Assumption

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Abstract: : *In a Game Theory context, we consider partial cooperation between a portion of the players and the rest of the players who do not cooperate and play a Nash game. The players may decide their strategy simultaneously or in a two-stage model. In both cases, some properties of the partial cooperative equilibrium are studied and applied to a different practical situations.*

Keywords: non-cooperative games, cooperation, coalitions, Stackelberg model.

1. Partial Cooperation

We deal with an n-person normal form game $\Gamma = <n; X_1,..., X_n; f_1,..., f_n>$, being X_i the strategy space and f_i the payoff (or profit) function of player P_i defined on $X_1\times\ldots\times X_n$ for any $i=1,...,n$. If each player P_i chooses x_i in X_i, then he obtains a profit $f_i(x_1,..., x_n)$. All the players are profit maximizers.

If non-cooperative behavior is assumed between the n players, the equilibrium solution considered is the well known concept of Nash equilibrium, i.e. a vector $(x_1^*,..., x_n^*)$ such that for any i

$$f_i (x_1^*,..., x_n^*) \leq f_i (x_1^*,.., x_{i-1}^*, x_i, x_{i+1}^*,.., x_n^*) \qquad (1)$$

for any x_i in X_i (see, for example, Basar and Olsder, 1995). For any i, for any $x_{-i}=(x_1,.., x_{i-1}, x_{i+1},.., x_n)$ we define $B_i(x_{-i})$ as the best reply correspondence or reaction of player i mapping to any x_{-i} the set of the x_i in X_i satisfying (1). If there is only one element x_i satisfying (1) for all x_{-i}, $B_i(x_{-i})$ is the best reply function of player i. No player has an incentive to deviate from the profile $x^*=(x_1^*,..., x_n^*)$.

From the Nash equilibrium definition, we have that $x^*=(x_1^*,..., x_n^*)$ is a Nash equilibrium if and only if x_i^* is in $B_i(x_{-i}^*)$ for any i.

On the other hand, if all the players behave cooperatively they pursue the common interest and would maximize the aggregate profit

$$f_1 +\ldots+ f_n. \qquad (2)$$

In this case the players form a grand coalition and they jointly maximize the aggregate profit of the coalition.

As in several concrete situations, for example in international environmental problems, only a subset of the n players forms a coalition. In this case we speak about partial cooperation. This concept finds a natural framework in environmental problems (Section 2), but interesting applications can be found in Cournot oligopolistic games and public goods games (Ray and Vohra 1997, Yi 1997, Mallozzi and Tijs 2006, 2008).

Let us suppose that $P_{k+1},..., P_n$, decide to cooperate. The level of non-cooperation k is given. Cooperating players (or signatories) choose strategies by maximizing the aggregate welfare of the coalition members, i.e.

$$F=f_{k+1}+\ldots+f_n. \qquad (3)$$

The rest of the players (non-cooperating players or non-signatories) play as singletons and choose their strategies as a Nash equilibrium with payoffs $f_1,\ldots,f_k$.
In this case we define the partial cooperative equilibrium (Mallozzi and Tijs 2006, 2008). There are mainly two assumptions regarding the sequence of moves in the above scheme.

Nash-Cournot assumption

All the players choose simultaneously their strategies (Carraro and Siniscalco, 1992); the **partial cooperative equilibrium** is a strategy profile $x^*=(x_1^*,.., x_k^*, x_{k+1}^*,.., x_n^*)$ that is a Nash equilibrium of the k+1 person game with strategy spaces $X_1,..., X_k, X_{k+1}\times\ldots\times X_n$ and payoffs $f_1,\ldots,f_n$, F where F is given in (3). In this case the player k+1 is the group of signatory persons, whose strategy is a vector $y=(x_{k+1},.., x_n)$. A strategy profile $x^*=(x_1^*,..., x_k^*,y^*)$ is a Nash equilibrium if and only if x_i^* is in $B_i(x_{-i}{}^*)$ for any $i=1,\ldots,k$ and y^* is in player k+1's best reply $B_{k+1}(x_1^*,..., x_k^*)$, that is the set of $y^*=(x_{k+1}^*,.., x_n^*)$ satisfying $F(x_1^*,..., x_n^*) \le F(x_1^*,.., x_k^*, x_{k+1},.., x_n)$ for any $(x_{k+1},.., x_n)$. Sometimes $B_{k+1}(x_1^*,..., x_k^*)$ is called group best reply correspondence.

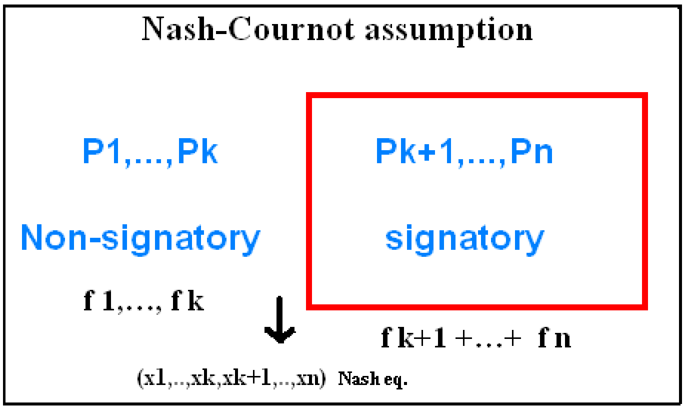

Figure 1: Signatories and non-signatories simultaneous decision

Stackelberg assumption

Signatories and non-signatories choose their strategies sequentially (Barret, 1994); the **partial cooperative equilibrium** is a strategy profile $x^*=(x_1^*,.., x_k^*, x_{k+1}^*,.., x_n^*)$ where $(x_{k+1}^*,.., x_n^*)$ is a Stackelberg leader strategy of the leading coalition, given $(x_1^*,.., x_k^*)$ the Nash equilibrium of the k person game –followers- with payoffs $f_1,\ldots,f_k$. More precisely, to any possible signatories decision $y=(x_{k+1},.., x_n)$, non-signatories react with a Nash equilibrium profile $(x_1,.., x_k)$. Suppose that this reaction is unique for any y, say $(x_1(y),.., x_k(y))$, the signatories maximize their joint profit F given in (3) given the non-signatories reaction, i.e. they solve the problem

$$\max_{y} F(x_1(y),.., x_k(y),y) \qquad (4)$$

In this model the Stackelberg leadership of the signatories has been assumed (Mallozzi and Tijs, 2008): signatories announce their joint strategy, non-signatories - the followers - act as singletons

and react by playing a non-cooperative game. The equilibrium x^* is determined by using a backward induction procedure.
It is possible that for a certain y that the non-signatories may react with several admissible profiles $(x_1,.., x_k)$ that are Nash equilibria. In this case, it is possible to define the partial cooperative equilibrium by using a selection in the set of the non-signatories reactions (Mallozzi and Tijs, 2008) and defining a suitable optimization problem for signatories, that corresponds to the problem (4) in the uniqueness case.

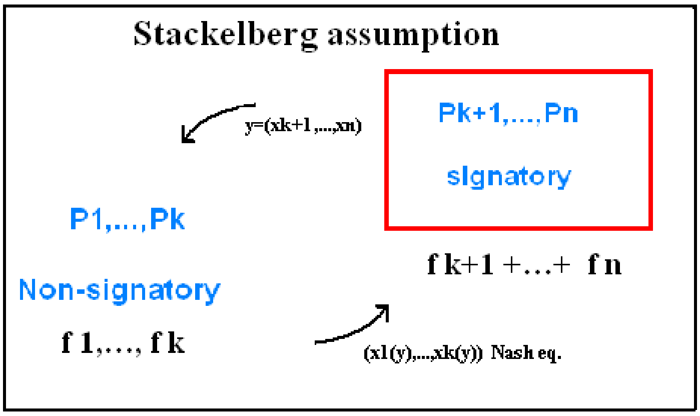

Figure 2: Signatories and non-signatories two-stage decision

In both assumptions 1.1 and 1.2, let us remark that for k=n all the players act as singletons and solve a Nash equilibrium problem and for k=0 all the players jointly maximize the sum of the profits given in (2). In the first case the definition of partial cooperative equilibrium coincides with the definition of Nash equilibrium, in the second case with the definition of social optimum (Finus 2001).

Suppose now that a partial cooperative equilibrium x^* is given. An interesting question is to compare the outputs in the two mentioned assumptions.
Let us call by $V_{NC} = f_{k+1}(x^*) +\ldots+ f_n(x^*)$ the signatories' aggregate profit under the Nash-Cournot assumption and by V_S the analogous under the Stackelberg one, assuming the uniqueness of the non-signatories reaction.

Proposition 1. We have that

$$V_{NC} \leq V_S \qquad (5)$$

This means that from the signatories point of view, it is better to be in the leading coalition group.
Analogous results can be given also without the uniqueness assumption of the non-signatories reaction. In Finus 2001 the inequality (5) is proved by direct calculation in the special case of identical (symmetric players) quadratic profit functions.

2. International Environmental Agreements (IEA)

In the last decades several papers on the economics of international environmental problems have been devoted to coalition analysis in a Game Theory context, particularly to coalition formation processes. See Yi (1997), Ray and Vohra (1997), Finus (2001) and the references therein. Recall for instance the Helsinki and Oslo Protocols on the reduction of sulphur signed in 1985 and 1994, and the Kyoto Protocol on the reduction of greenhouse gases causing global warming signed in 1997.

Since environmental problem are not of local nature, global welfare can be raised through cooperation. The usual situation in these problem is that only a portion of the countries involved sign an agreement, so the partial cooperative equilibrium concept represents a useful tool (Finus, 2001).

In the context of the formation of IEA the models consider three-stage games. In the first stage (coalition formation game) players decide whether participate in an agreement through binary variables: different coalitions may appear and a strategy of a player specifies to join or not join a specific coalition. In the second stage (partial cooperative game) players choose their emission levels: signatories choose within their coalition the group emissions and non-signatories act non-cooperatively. In the third stage (cooperative game), assuming asymmetric countries, the allocation of the welfare gains among the coalition members is decided according a sharing rule, for example the Nash bargaining solution or the Shapley value. In Section 1 a unique coalition between players has been considered and only the second stage or the partial cooperative game has been discussed in a general framework.

Let us consider n countries, i=1,…,n, and the welfare or profit π_i of the i-th country

$$\pi_i = \beta_i (e_i) - \varphi_i (\Sigma_i e_i) \qquad (6)$$

where the i-th country benefits from his own emission e_i receiving $\beta_i (e_i)$ and suffers damage $\varphi_i (\Sigma_i e_i)$ from its own e_i and foreign e_j, $j \neq i$ emissions. By choosing in formula (6) the following functions $\beta(t)$ and $\varphi(t)$ for any i (symmetric countries)

$$\beta_i (t) = \beta(t) = at - bt^2$$
$$\varphi_i (t) = \varphi (t) = ct^2 - dt$$

the partial cooperative equilibrium is given explicitly together with the aggregate emissions expressions, so comparison in the two above assumptions is straightforward (Finus 2001).

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