



# A Measure of Variability for the Customer Satisfaction Index

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**Abstract:** In this paper we deal with the problem of identifying heterogeneity indices for the purpose of improving the analysis of customer satisfaction observing the phenomenon through a new perspective. Our work introduces some indices that may be used for measuring heterogeneity in Customer Satisfaction framework and an application on real data is illustrated.

**Keywords:** heterogeneity, response pattern, intrinsic heterogeneity profile

## 1. Introduction

Customer satisfaction (CS) is a central concept in marketing and is adopted as an important outcome measure of service quality by service industries. Most service companies have research programs designed to measure service quality and/or customer satisfaction. Such programs are designed to provide essential information to guide efforts to reduce variability in service quality. As highlighted by Giancrifaro *et al.* (2007), the concept of quality is strictly related to variability.

This paper deals with the problem of identifying heterogeneity indices for the purpose of improving the analysis of customer satisfaction observing the phenomenon through a new perspective. In particular, in Section 2, we introduce the basic ideas, several indices and methods that may be used for measuring heterogeneity in Customer Satisfaction framework, while Section 3 deals with a case study in which the diversity profiles are used to mark differences among two subpopulations.

## 2. Defining and measuring customer satisfaction heterogeneity

The semantic of a few terms used throughout this paper are addressed. The term response patterns is a convenient label for a set of distinct responses of a subject to a set of items (e.g. 5124 is a response pattern for four items coded from 1 to 5).

Suppose we have an  $n$  by  $p$  data matrix of values of  $p$  ordered categorical variables,  $x_1, x_2, \dots, x_p$  with  $m_i$  categories ( $i=1, 2, \dots, p$ ) for  $n$  individuals. Any row of the data matrix is referred to as a response pattern and in general there are  $m_1 \times m_2 \times \dots \times m_p$  possible response patterns. If the sample size is much larger than  $m_1 \times m_2 \times \dots \times m_p$ , many of the response patterns will be repeated and they may be summarized by a matrix as a list of the observed response patterns together with their associated frequencies.

Furthermore, the terms of heterogeneity and homogeneity need to be introduced even if they are concepts widely used in the field of statistics. The concept of homogeneity addresses the case in which every unit belonging to a population manifests the same category with respect to a statistical variable  $X$ . If this does not occur then heterogeneity is indicated by absence of homogeneity. Therefore the degree of heterogeneity obviously depends on the number of categories observed as well as on their associated frequencies. In particular the heterogeneity is at a minimum if the



distribution presents a single category with a relative frequency equal to 1. On the other hand heterogeneity is at a maximum if the variable is equally distributed on all categories.

Starting from these notions, various indicators about heterogeneity were proposed of which only the most commonly used will be reported:

$$\delta_{Sh} = -\sum_{j=1}^s p_j \log p_j \quad \text{Shannon} \quad (1)$$

$$\delta_{Si} = 1 - \sum_{j=1}^s p_j^2 \quad \text{Gini-Simpson} \quad (2)$$

where  $s$  denotes the cardinality of a collection of response patterns for a particular population and  $p_j$  ( $j=1,2,\dots,s$ ) is the relative frequency with respect to the  $j$ th response pattern.

In order to illustrate how these indices may be applied in the customer satisfaction framework, we present a hypothetical example of three groups of customers that give responses to three items based on a five point scale. In Table 1, the relative frequency of each response pattern is given, the mean and variance of Customer Satisfaction Index (CSI),<sup>1</sup> and the heterogeneity indices calculated from these relative frequencies also are shown for the three different groups.

The example shows that Group 1 has the highest CSI, while Group 2 has the highest variability (i.e. coefficient of variation). In making comparisons, it is necessary to view also diversity among response patterns in order to capture multidimensional complexity onto a one-dimensional ordinal scale with possibly right conclusions in terms of variability. Analysing the different indices, we may observe that  $s(\text{Group 1}) > s(\text{Group 2})$ , but  $\delta_{Sh}(\text{Group 1}) < \delta_{Sh}(\text{Group 2})$  and  $\delta_{Si}(\text{Group 1}) < \delta_{Si}(\text{Group 2})$ . This comparison illustrates how one may be led to the conclusion that a group with fewer response patterns (Group 2) may be more heterogeneous than one with one more response patterns (Group 1) using Shannon's or Simpson's index. Similar inconsistencies may be found by comparing Group 1 and Group 3. The only comparison that is consistently ordered with all indices is among Group 2 and Group 3.

Response Patterns	Relative frequency		
	Group 1	Group 2	Group 3
543	0.50	0.25	0.35
432	0.30	0.25	0.33
422	0.10	0.25	0.32
444	0.05	0.25	0.00
312	0.05	0.00	0.00
Total	1.00	1.00	1.00
Mean	3.37	3.42	3.24
Variance	0.38	0.36	0.33
	Heterogeneity index		
$s$	5	4	3
$\delta_{Sh}$	1.24	1.39	1.10
$\delta_{Si}$	0.65	0.75	0.67

Table 1: Three hypothetical groups composed of five or fewer response patterns

<sup>1</sup> For the sake of simplicity we have not transformed the ordinal into the metric scale types. For more details, see Zanella (2001)



In the ecological field the most useful way to compare heterogeneity between communities is by the concept of intrinsic diversity ordering (Patil and Tallie, 1982). In CS framework, the same approach may be adopted in order to evaluate the heterogeneity in the collection of the response patterns that is measured by

$$P_j = \sum_{i=j+1}^s p_i^{\#} \quad j=1, \dots, s-1 \quad (3)$$

where  $P_s=0$  and  $P_0=1$ ,  $\mathbf{p}^{\#}$  represents the ranked relative frequency from greatest to least so that  $\mathbf{p}^{\#} = (p_1^{\#} \dots p_s^{\#})$ , where  $p_1^{\#} \geq \dots \geq p_s^{\#}$ .

When we plot  $P_j$  against  $j$  the resulting curves are termed intrinsic heterogeneity profiles.

Figure 1 gives the comparative heterogeneity profile for the three groups in Table 1. The profile for Group 1 crosses both those for Group 2 and 3, but the profile for Group 2 is everywhere above that for Group 3. It follows that Group 2 is intrinsically more heterogeneous than Group 1.

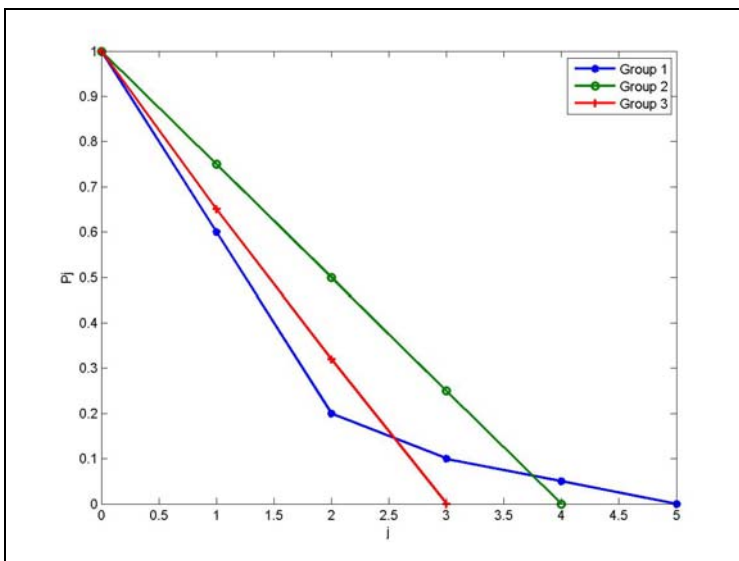


Figure 1: Right tail-sum ( $P_j$ ) profiles for the three hypothetical groups in Table 1

### 3. Application

We used data from Student Satisfaction Survey carried out in 2005 at the Business and Management of the University of Chieti-Pescara. The survey was designed to collect information about satisfaction on learning facilities, campus facilities, organization, library facilities and overall satisfaction rating. Items are measured using 5 point Likert scales that range from “very dissatisfied” to “very satisfied”. In order to show the comparative heterogeneity profiles for the collections of response patterns relative to the overall satisfaction measured using three items addressing the administrative staff, university facilities and educational services, two subpopulations (male and female) are compared with heterogeneity profiles.

Table 2 presents the six most frequent response patterns with their relative frequencies, the mean and variance of CSI and heterogeneity indices for the two groups. Since the two subpopulations present approximately the same CSI and coefficient of variation, we might be lead to the conclusion that the two groups are equal in terms of responses relatively to the three items.

Indeed, inspection of Figure 2 reveals that every data point on the male comparative heterogeneity profiles plots above the female profile. Thus, heterogeneity is intrinsically greater in the male group respect with the female subpopulation. The various heterogeneity indices reported in Table 2 agree on the heterogeneity ordering between the two subgroups.



Male				Female			
Response Patterns	$p_j$	Response Patterns	$p_j$	Response Patterns	$p_j$	Response Patterns	$p_j$
433	23.94	333	7.98	433	27.12	333	7.63
443	17.37	544	6.57	444	17.80	434	7.63
444	14.55	543	5.16	443	16.10	431	3.39
Mean		3.57		Mean		3.51	
Variance		0.31		Variance		0.29	
Number of observ.		213		Number of observ.		118	
Heterogeneity index				Heterogeneity index			
$s$		32		$s$		22	
$\delta_{Sh}$		2.54		$\delta_{Sh}$		2.33	
$\delta_{Si}$		0.87		$\delta_{Si}$		0.85	

Table 2: The six most frequent response patterns for the two collections of response patterns (male and female) and their relative heterogeneity indices

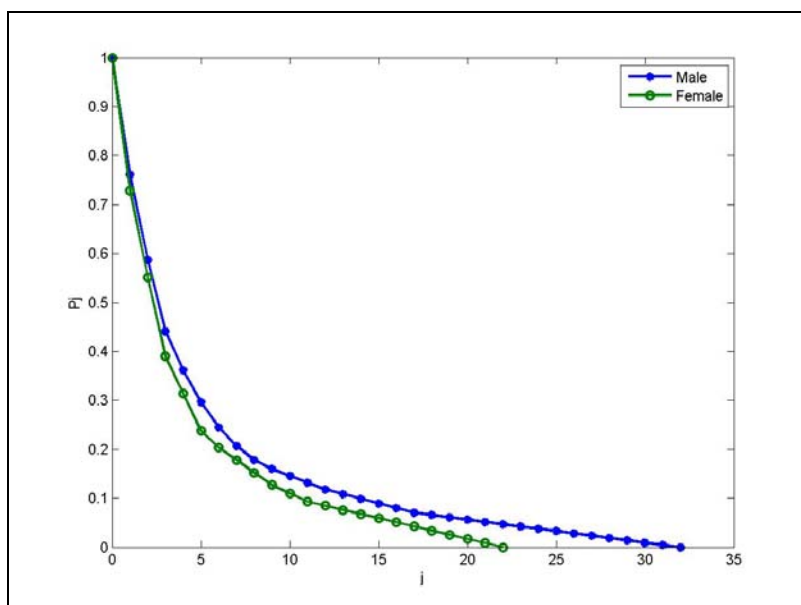


Figure 2: Right tail-sum ( $P_j$ ) diversity profiles for the two collections of response patterns (male and female)

## Bibliography

- Giancristofaro R.A., Bonnini S., Salmaso L. (2007), Qualità dei servizi e disomogeneità di prestazioni: procedure inferenziali per valutazioni comparative, Valutazione e Customer Satisfaction per la Qualità dei Servizi, Roma, 12-13 Aprile 2007.
- Gove J.H., Patil G.P., Swindel B.F., Taillie C. (1994), Ecological diversity and forest management. In: G.P. Patil, C.R. Rao, and N.P. Ross, (eds.), *Handbook of Statistics Volume 12: Environmental Statistics*. North Holland/Elsevier Science Publishers, New York. p. 409-462.
- Patil G. P., Taillie C. (1982) Diversity as a concept and its measurements, *Journal of the American Statistical Association*, 77: 548-561.
- Zanella A. (2001), Valutazione e modelli interpretativi di Customer Satisfaction: una presentazione di insieme, Università Cattolica del S. Cuore, Istituto di Statistica, Serie E. P. N. 105.