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SOME MODIFIED EXPONENTIAL RATIO-TYPE ESTIMATORS IN THE PRESENCE OF NON-RESPONSE UNDER TWO-PHASE SAMPLING SCHEME

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***Abstract**: This paper addresses the problem of estimating the population mean using information on the auxiliary variable in the presence of non-response under two-phase sampling. On the lines of Bahl and Tuteja [1] and upadhyaya et al. [22], a class of modified exponential-ratio type estimators using single auxiliary variable have been proposed under two different situations of non-response of the study variable. The expressions for the bias and mean square error (MSE) of a proposed class of estimators are derived. Efficiency comparisons of a proposed class of estimators with the usual unbiased estimator by Hansen and Hurwitz [3] and other existing estimators are made. An empirical study has been carried out to judge the performances of the proposed estimators.*

***Keywords**: Auxiliary variable, bias, mean Square error, non-response, two-phase sampling, exponential-ratio type estimator.*

1. Introduction

Consider a finite population of size N. We draw a sample of size n from a population by using simple random sample without replacement (SRSWOR) sampling scheme. Let y_i and x_i be the observations on the study variable (y) and the auxiliary variable (x) respectively. Let $\bar{y} = \sum_{i=1}^{n} \frac{y_i}{n}$ and $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$ be the sample means corresponding to the population means $\bar{Y} = \sum_{i=1}^{N} \frac{y_i}{N}$ and $\bar{X} = \sum_{i=1}^{N} \frac{x_i}{N}$ respectively. When information on $\bar{X}$ is unknown then double

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sampling or two phase sampling is suitable to estimate the population mean. In first phase sample we select a sample of size n' by SRSWOR from a population to observe x. In second phase, we select a sample of size n from n' $(n < n')$ by SRSWOR also. Non-response occurs on second phase in which n_1 units respond and n_2 do not. From n_2 non-respondents, a sample of $r = n_2/k$; $k > 1$ units is selected, where k is the inverse sampling rate at the second phase sample of size n.

Sometimes it may not be possible to collect the complete information for all the units selected in the sample due to non-response. Estimation of the population mean in sample surveys when some observations are missing due to non-response not at random has been considered by Hansen and Hurwitz [3] is given by $\bar{y}^* = w_1\bar{y}_1 + w_2\bar{y}_{2r}$, where $\bar{y}_1 = \sum_{i=1}^{n_1} \frac{y_i}{n_1}$, $\bar{y}_{2r} = \sum_{i=1}^{r} \frac{y_i}{r}$, $w_1 = \frac{n_1}{n}$ and $w_2 = \frac{n_2}{n}$.

The variance of $\bar{y}^*$ is given by:

$$Var(\bar{y}^*) = \left(\frac{1-f}{n}\right) S_y^2 + W_2\left(\frac{k-1}{n}\right) S_{y(2)}^2, \tag{1}$$

where $f = \frac{n}{N}$ and $W_2 = \frac{N_2}{N}$, $S_y^2 = \sum_{i=1}^{N} \frac{\left(y_i - \bar{Y}\right)^2}{N-1}$ and $S_{y(2)}^2 = \sum_{i=1}^{N_2} \frac{\left(y_i - \bar{Y}_2\right)^2}{N_2 - 1}$.

It is well known that in estimating the population mean, sample survey experts use the auxiliary information to improve the precision of the estimates.

Similar to $\bar{y}^*$ one can write $\bar{x}^* = w_1\bar{x}_1 + w_2\bar{x}_{2r}$, where $\bar{x}_1 = \sum_{i=1}^{n_1} \frac{x_i}{n_1}$ and $\bar{x}_{2r} = \sum_{i=1}^{r} \frac{x_i}{r}$.

The variance of $\bar{x}^*$ is given by:

$$Var(\bar{x}^*) = \left(\frac{1-f}{n}\right) S_x^2 + W_2\left(\frac{k-1}{n}\right) S_{x(2)}^2, \tag{2}$$

where $S_x^2 = \sum_{i=1}^{N} \frac{\left(x_i - \bar{X}\right)^2}{N-1}$ and $S_{x(2)}^2 = \sum_{i=1}^{N_2} \frac{\left(x_i - \bar{X}_2\right)^2}{N_2 - 1}$.

The auxiliary information can be used both at designing and estimation stages to compensate for units selected for a sample that fails to provide adequate responses and for the population units missing from the sampling frame. Rao ([10], [11]), Khare and Srivastava ([4], [5], [6]), Okafar and Lee [9], Sarndal and Lundstrom [12], Tabasum and Khan ([20], [21]), Singh and Kumar ([13], [14], [15], [16], [17], [18]) and Singh et al. [19] have suggested some estimators for population mean $\bar{Y}$ of the study variable y using the auxiliary information in presence of non-response and studied their properties.

When there is non-response on the study variable y as well as on the auxiliary variable x, Cochran [2] suggested the conventional two-phase ratio and regression estimators for the population mean $\bar{Y}$ are defined as:

$$\hat{\bar{Y}}_{R(1)} = \bar{y}^* \frac{\bar{x}'}{\bar{x}^*}, \tag{3}$$

and

$$\hat{\bar{Y}}_{Reg(1)} = \bar{y}^* + b^*_{yx}\left(\bar{x}' \quad \bar{x}^*\right), \tag{4}$$

where $b^*_{yx} = s^*_{xy} / s^{*2}_x$ is the sample regression coefficient, whose population regression coefficient is $\beta_{yx} = S_{xy} / S^2_x$ at the first phase sampling. Here $s^*_{xy} = \frac{1}{(n-1)}\left(\sum_{i=1}^{n} x_i y_i + k\sum_{i=1}^{r} x_i y_i \quad n\bar{x}\,\bar{y}^*\right)$ and $s^{*2}_x = \frac{1}{(n-1)}\left(\sum_{i=1}^{n} x_i^2 + k\sum_{i=1}^{r} x_i^2 \quad n\bar{x}\,\bar{x}^*\right)$ are the sample covariance and sample variance respectively.

Recently Singh and Kumar [17] suggested the following estimator on the lines of Bahl and Tuteja [1] as:

$$\hat{\bar{Y}}_{Exp(1)} = \bar{y}^* \exp\left\{\frac{\bar{x}' - \bar{x}^*}{\bar{x}' + \bar{x}^*}\right\}. \tag{5}$$

To the first degree of approximation, the expressions for bias and mean square error of $\hat{\bar{Y}}_{R(1)}$, $\hat{\bar{Y}}_{Reg(1)}$ and $\hat{\bar{Y}}_{Exp(1)}$ are given by:

$$B(\hat{\bar{Y}}_{R(1)}) \cong \bar{Y}\left[\lambda''\left(1 - K_{yx}\right)C^2_x + \lambda^*\left(1 \quad K_{yx(2)}\right)C^2_{x(2)}\right], \tag{6}$$

$$B(\hat{\bar{Y}}_{\mathrm{Reg}(1)}) \cong \beta_{yx}\left[\lambda'' \frac{2N^2}{(N-1)(N-2)}\left(\frac{\mu_{30(2)}}{\mu_{11}} - \frac{\mu_{21}}{\mu_{12}}\right) + \lambda^*\left(\frac{\mu_{30(2)}}{\mu_{11}} \quad \frac{\mu_{21(2)}}{\mu_{12}}\right)\right], \tag{7}$$

$$B(\hat{\bar{Y}}_{EXP(1)}) \cong \frac{1}{2}\bar{Y}\left[\lambda''\left(\frac{3}{4} - K_{yx}\right)C^2_x + \lambda^*\left(\frac{3}{4} \quad K_{yx(2)}\right)C^2_{x(2)}\right], \tag{8}$$

$$MSE(\hat{\bar{Y}}_{R(1)}) \cong \bar{Y}^2\left[\lambda' C^2_y + \lambda''\left\{C^2_y + \left(1 - 2K_{yx}\right)C^2_x\right\} + \lambda^*\left\{C^2_{y(2)} + \left(1 \quad 2K_{yx(2)}\right)C^2_{x(2)}\right\}\right], \tag{9}$$

$$MSE(\hat{\bar{Y}}_{\mathrm{Reg}(1)}) \cong \bar{Y}^2\left[\lambda' C^2_y + \lambda''\left(1 - \rho^2\right)C^2_y + \lambda^*\left\{C^2_{y(2)} + K_{yx}\left(K_{yx} \quad 2K_{yx(2)}\right)C^2_{x(2)}\right\}\right], \tag{10}$$

$$MSE(\hat{\bar{Y}}_{EXP(1)}) \cong \bar{Y}^2 \left[\lambda' C_y^2 + \lambda'' \left\{ C_y^2 + \frac{1}{2}\left(\frac{1}{2} - 2K_{yx}\right) C_x^2 \right\} + \lambda^* \left\{ C_{y(2)}^2 + \frac{1}{2}\left(\frac{1}{2} - 2K_{yx(2)}\right) C_{x(2)}^2 \right\} \right], \quad (11)$$

where $K_{yx} = \frac{\beta_{yx}}{R} = \frac{\rho_{yx} C_y}{C_x}$, $K_{yx(2)} = \frac{\beta_{yx(2)}}{R} = \frac{\rho_{yx(2)} C_{y(2)}}{C_{x(2)}}$, $\beta_{yx} = \frac{S_{yx}}{S_x^2}$, $\beta_{yx(2)} = \frac{S_{yx(2)}}{S_{x(2)}^2}$,

$S_{xy} = \frac{\sum_{i=1}^{N}\left(x_i - \bar{X}\right)\left(y_i - \bar{Y}\right)}{N-1}$, $S_{xy(2)} = \frac{\sum_{i=1}^{N_2}\left(x_i - \bar{X}_2\right)\left(y_i - \bar{Y}_2\right)}{N_2 - 1}$, $C_y = \frac{S_y}{\bar{Y}}$, $C_{y(2)} = \frac{S_{y(2)}}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$,

$C_{x(2)} = \frac{S_{x(2)}}{\bar{X}}$, $\rho_{yx(2)} = \frac{S_{yx(2)}}{S_{x(2)} S_{y(2)}}$, $\lambda = \left(\frac{1-f}{n}\right)$, $\lambda' = \left(\frac{1-f'}{n'}\right)$, $\lambda'' = (\lambda - \lambda')$, $\lambda^* = \frac{W_2\left(k-1\right)}{n}$,

$R = \frac{\bar{Y}}{\bar{X}}$, $f = \frac{n}{N}$, $f' = \frac{n'}{N}$, $\mu_{vs} = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^v \left(y_i - \bar{Y}\right)^s$ and

$\mu_{vs(2)} = \frac{1}{N_2 - 1}\sum_{i=1}^{N_2}\left(x_i - \bar{X}_2\right)^v \left(y_i - \bar{Y}_2\right)^s$, (v, s) being non- negative integers.

When there is incomplete information on the study variable y and complete information on the auxiliary variable x, the conventional two-phase ratio, regression and exponential-ratio type estimators are respectively defined by:

$$\hat{\bar{Y}}_{R(2)} = \bar{y}^* \frac{\bar{x}'}{\bar{x}} \quad (12)$$

and

$$\hat{\bar{Y}}_{Reg(2)} = \bar{y}^* + b_{yx}^{**}\left(\bar{x}' - \bar{x}\right), \quad (13)$$

where $b_{yx}^{**} = s_{xy}^* / s_x^2$ is the sample regression coefficient, whose population regression coefficient is $\beta_{yx} = S_{xy} / S_x^2$ at second phase sampling and $s_x^2 = \frac{1}{\left(n-1\right)}\sum_{i=1}^{n}(x_i - \bar{x})^2$.

Singh and Kumar [14] defined the following exponential ratio type estimator:

$$\hat{\bar{Y}}_{Exp(2)} = \bar{y}^* \exp\left\{\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right\}. \quad (14)$$

To the first degree of approximation, the bias and mean square error of $\hat{\bar{Y}}_{R(2)}$, $\hat{\bar{Y}}_{Reg(2)}$ and $\hat{\bar{Y}}_{Exp(2)}$ are given by:

$$B(\hat{\bar{Y}}_{R(2)}) \cong \bar{Y}\lambda''\left(1-K_{yx}\right)C_x^2, \tag{15}$$

$$B(\hat{\bar{Y}}_{\mathrm{Reg}(2)}) \cong \lambda''\beta_{yx}\frac{2N^2}{(N-1)(N-2)}\left(\frac{\mu_{21}}{\mu_{11}}-\frac{\mu_{30}}{\mu_{20}}\right), \tag{16}$$

$$B(\hat{\bar{Y}}_{EXP(2)}) \cong \frac{1}{2}\lambda''\bar{Y}\left(\frac{3}{4}-K_{yx}\right)C_x^2, \tag{17}$$

$$MSE(\hat{\bar{Y}}_{R(2)}) \cong \bar{Y}^2\left[\lambda' C_y^2 + \lambda''\left\{C_y^2 + \left(1-2K_{yx}\right)C_x^2\right\} + \lambda^* C_{y(2)}^2\right], \tag{18}$$

$$MSE(\hat{\bar{Y}}_{\mathrm{Reg}(2)}) \cong \bar{Y}^2\left[\lambda' C_y^2 + \lambda'' C_y^2\left(1-\rho_{yx}^2\right) + \lambda^* C_{y(2)}^2\right], \tag{19}$$

$$MSE(\hat{\bar{Y}}_{EXP(2)}) \cong \bar{Y}^2\left[\lambda' C_y^2 + \lambda''\left\{C_y^2 + \frac{1}{2}\left(1-2K_{yx}\right)C_x^2\right\} + \lambda^* C_{y(2)}^2\right], \tag{20}$$

2. Proposed exponential-ratio type estimator

We propose the following modified exponential-ratio type estimator for estimating the populations mean $\bar{Y}$ under two-phase sampling scheme in two different situations.

2.1 Situation I

The population mean $\bar{X}$ is unknown, when non-response occurs on the study variable y and the auxiliary variable x. On the lines of Bahl and Tuteja [1] and Upadhyaya *et al.* [22], we propose the following estimator:

$$\hat{\bar{Y}}_{P(1)}^{(h)} = \bar{y}^* \exp\left(\frac{c\left(\bar{x}'-\bar{x}^*\right)}{\left(c\bar{x}'+d\right)+(h-1)\left(c\bar{x}^*+d\right)}\right), \tag{21}$$

where $(h>0)$; $c\,(\neq 0)$ and d are constants which can be coefficient of variation (C_x) or correlation coefficient (ρ_{yx}) or standard deviation (S_x).

Remarks:

(i) When $h=0$, the estimator $\hat{\bar{Y}}_{P(1)}^{(h)}$ reduces to

$$\hat{\bar{Y}}_{P(1)}^{(0)} = \bar{y}^* \exp(1), \tag{22}$$

which is a biased estimator with larger *MSE* than the usual estimator $\bar{y}^*$ due to the positive value of 'exp' and has multiplicative effect on the above estimator $\hat{\bar{Y}}_{P(1)}^{(0)}$.

(ii) When $h=1$, the estimator $\hat{\bar{Y}}_{P(1)}^{(h)}$ reduces to

$$\hat{\bar{Y}}_{P(1)}^{(1)} = \bar{y}^* \exp\left(\frac{c\left(\bar{x}' - \bar{x}^*\right)}{c\bar{x}' + d}\right). \tag{23}$$

(iii) When $h=2$, the estimator $\hat{\bar{Y}}_{P(1)}^{(h)}$ reduces to estimator:

$$\hat{\bar{Y}}_{P(1)}^{(2)} = \bar{y}^* \exp\left(\frac{c\left(\bar{x}' - \bar{x}^*\right)}{\left(c\bar{x}' + d\right) + \left(c\bar{x}^* + d\right)}\right). \tag{24}$$

To obtain bias and mean square error of the estimator $\hat{\bar{Y}}_{P(1)}^{(h)}$, we define:

$$\bar{y}^* = \bar{Y}\left(1+\varepsilon_0\right),\quad \bar{x}^* = \bar{X}\left(1+\varepsilon_1\right),\quad \bar{x}' = \bar{X}\left(1+\varepsilon_1'\right),\quad \bar{x} = \bar{X}\left(1+\varepsilon_2\right),$$

such that $E\left(\varepsilon_i\right)=0,\ (i=0,1,2)$ and $E\left(\varepsilon_i'\right)=0,$

$$E\left(\varepsilon_0^2\right)=\lambda C_y^2+\lambda^* C_{y(2)}^2,\ E\left(\varepsilon_1^2\right)=\lambda C_x^2+\lambda^* C_{x(2)}^2,\ E\left(\varepsilon_1'^2\right)=\lambda' C_x^2,\ E\left(\varepsilon_2^2\right)=\lambda C_x^2,$$
$$E\left(\varepsilon_0\varepsilon_1\right)=\lambda\rho_{yx}C_yC_x+\lambda^*\rho_{yx(2)}C_{y(2)}C_{x(2)},\ E\left(\varepsilon_0\varepsilon_1'\right)=\lambda'\rho_{yx}C_yC_x,\ E\left(\varepsilon_0\varepsilon_2\right)=\lambda\rho_{yx}C_yC_x,$$
$$E\left(\varepsilon_1\varepsilon_1'\right)=\lambda' C_x^2,\ E\left(\varepsilon_1\varepsilon_2\right)=\lambda C_x^2 \text{ and } E\left(\varepsilon_1'\varepsilon_2\right)=\lambda' C_x^2.$$

Expressing the estimator $\hat{\bar{Y}}_{P(1)}^{(h)}$ given in (21), in terms of ε's, we have:

$$\hat{\bar{Y}}_{P(1)}^{(h)} = \bar{Y}\left(1+\varepsilon_0\right)\exp\left(\frac{\left(\varepsilon_1' - \varepsilon_1\right)}{\left(\varepsilon_1' + \left(h-1\right)\varepsilon_1 + h\delta\right)}\right), \tag{25}$$

where $\delta = \left(\dfrac{c\bar{X}+d}{c\bar{X}}\right).$

Solving (25), neglecting terms of ε's having power greater than two, we have:

$$(\hat{\bar{Y}}_{P(1)}^{(h)} - \bar{Y}) \cong \bar{Y}\left[\varepsilon_0 + \frac{1}{h\delta}\left(\varepsilon_1' - \varepsilon_1\right) + \frac{1}{h\delta}\left(\varepsilon_0\varepsilon_1' - \varepsilon_0\varepsilon_1\right)\right.$$

$$+\frac{1}{h^2\delta^2}\left(\varepsilon_1'-\varepsilon_1\right)^2-\frac{1}{h^2\delta^2}\left(\varepsilon_1'^2+\left(h-2\right)\varepsilon_1'\varepsilon_1-\left(h-1\right)\varepsilon_1^2\right)\Big]. \tag{26}$$

Taking expectations on both sides of (26), we get the bias of $\hat{\bar{Y}}_{P(1)}^{(h)}$ which is given by:

$$B(\hat{\bar{Y}}_{P(1)}^{(h)}) \cong \bar{Y}\left[\lambda''\frac{1}{h\delta}\left\{\frac{1}{h\delta}\left(h-\frac{1}{2}\right)-K_{yx}\right\}C_{x2}^2+\lambda^*\frac{1}{h\delta}\left\{\frac{1}{h\delta}\left(h-\frac{1}{2}\right)-K_{yx(2)}\right\}C_{x(2)}^2\right]. \tag{27}$$

Squaring both sides of (26) and neglecting terms of $\varepsilon's$ involving power greater than two, we have:

$$(\hat{\bar{Y}}_{P(1)}^{(h)}-\bar{Y})^2 \cong \bar{Y}^2\left[\varepsilon_0^2+\frac{1}{h^2\delta^2}\left(\varepsilon_1'^2+\varepsilon_1^2-2\varepsilon_1'\varepsilon_1\right)+\frac{2}{h\delta}\left(\varepsilon_0\varepsilon_1' \quad \varepsilon_0\varepsilon_1\right)\right]. \tag{28}$$

Using (28), the *MSE* of $\hat{\bar{Y}}_{P(1)}^{(h)}$ to the first degree approximation is given by:

$$MSE(\hat{\bar{Y}}_{P(1)}^{(h)}) \cong \bar{Y}^2\left[\lambda' C_y^2+\lambda''\left\{C_y^2+A_1C_x^2\right\}+\lambda^*\left\{C_{y(2)}^2+A_2C_{x(2)}^2\right\}\right], \tag{29}$$

where $A_1=\frac{1}{h\delta}\left(\frac{1}{h\delta}-2K_{yx}\right)$ and $A_2=\frac{1}{h\delta}\left(\frac{1}{h\delta}-2K_{yx(2)}\right)$.

The $MSE(\hat{\bar{Y}}_{P(1)}^{(h)})$ is minimum when $h=\frac{\lambda''C_x^2+\lambda^*C_{x(2)}^2}{\left\{\lambda''K_{yx}C_x^2+\lambda^*K_{yx(2)}C_{x(2)}^2\right\}\delta}=h_0$ (say).

Thus the resulting minimum *MSE* of $\hat{\bar{Y}}_{P(1)}^{(h)}$ is given by:

$$MSE(\hat{\bar{Y}}_{P(1)}^{(h)})_{\min} \cong \bar{Y}^2\left[\lambda' C_y^2+\lambda'' C_y^2+\lambda^* C_{y(2)}^2 \quad \frac{\left(\lambda''C_x^2+\lambda^*C_{x(2)}^2\right)^2}{\lambda''K_{yx}C_x^2+\lambda^*K_{yx(2)}C_{x(2)}^2}\right]. \tag{30}$$

Table 1 shows some members of a proposed class of estimators $\hat{\bar{Y}}_{P(1)}^{(h)}$ of the population mean $\bar{Y}$ by taking $h=1$ and $h=2$, each at different values of c and d. Many more estimators can also be generated from the proposed estimator in (21) just by taking different values of h, c and d.

Table 1. Some members of a family of estimators $\hat{\hat{\bar{Y}}}_{P(1)}^{(h)}$ under Situation-I.

Estimator	h	c	d
$\hat{\hat{\bar{Y}}}_{P(1)}^{(1)(1)} = \bar{y}^* \exp\left(\dfrac{\bar{x}' - \bar{x}^*}{\bar{x}' + S_x}\right)$	1	1	S_x
$\hat{\hat{\bar{Y}}}_{P(1)}^{(1)(2)} = \bar{y}^* \exp\left(\dfrac{\bar{x}' - \bar{x}^*}{\bar{x}' + C_x}\right)$	1	1	C_x
$\hat{\hat{\bar{Y}}}_{P(1)}^{(1)(3)} = \bar{y}^* \exp\left(\dfrac{\bar{x}' - \bar{x}^*}{\bar{x}' + \rho_{yx}}\right)$	1	1	ρ_{yx}
$\hat{\hat{\bar{Y}}}_{P(1)}^{(1)(4)} = \bar{y}^* \exp\left(\dfrac{C_x\left(\bar{x}' - \bar{x}^*\right)}{C_x\bar{x}' + S_x}\right)$	1	C_x	S_x
$\hat{\hat{\bar{Y}}}_{P(1)}^{(2)(1)} = \bar{y}^* \exp\left(\dfrac{\bar{x}' - \bar{x}^*}{\left(\bar{x}' + S_x\right) + \left(\bar{x}^* + S_x\right)}\right)$	2	1	S_x
$\hat{\hat{\bar{Y}}}_{P(1)}^{(2)(2)} = \bar{y}^* \exp\left(\dfrac{\bar{x}' - \bar{x}^*}{\left(\bar{x}' + C_x\right) + \left(\bar{x}^* + C_x\right)}\right)$	2	1	C_x
$\hat{\hat{\bar{Y}}}_{P(1)}^{(2)(3)} = \bar{y}^* \exp\left(\dfrac{\bar{x}' - \bar{x}^*}{\left(\bar{x}' + \rho_{yx}\right) + \left(\bar{x}^* + \rho_{yx}\right)}\right)$	2	1	ρ_{yx}
$\hat{\hat{\bar{Y}}}_{P(1)}^{(2)(4)} = \bar{y}^* \exp\left(\dfrac{C_x\left(\bar{x}' - \bar{x}^*\right)}{\left(C_x\bar{x}' + S_x\right) + \left(C_x\bar{x}^* + S_x\right)}\right)$	2	C_x	S_x

The expressions of mean square error of the above estimators (Table 1) are given by:

$$MSE(\hat{\hat{\bar{Y}}}_{P(1)}^{(1)(i)}) \cong \bar{Y}^2\left[\lambda' C_y^2 + \lambda''\left\{C_y^2 + A_3 C_x^2\right\} + \lambda^*\left\{C_{y(2)}^2 + A_4 C_{x(2)}^2\right\}\right], \quad (31)$$

where $A_3 = \dfrac{1}{\delta_i}\left(\dfrac{1}{\delta_i} - 2K_{yx}\right)$ and $A_4 = \dfrac{1}{\delta_i}\left(\dfrac{1}{\delta_i} - 2K_{yx(2)}\right)$ $(i = 1, 2, 3, 4)$ and

$$MSE(\hat{\hat{\bar{Y}}}_{P(1)}^{(2)(i)}) \cong \bar{Y}^2\left[\lambda' C_y^2 + \lambda''\left\{C_y^2 + A_5 C_x^2\right\} + \lambda^*\left\{C_{y(2)}^2 + A_6 C_{x(2)}^2\right\}\right], \quad (32)$$

where $A_5 = \frac{1}{2\delta_i}\left(\frac{1}{2\delta_i} - 2K_{yx}\right)$, $A_6 = \frac{1}{2\delta_i}\left(\frac{1}{2\delta_i} - 2K_{yx(2)}\right)$ $(i = 1,2,3,4)$, $\delta_1 = \left(\frac{\bar{X} + S_x}{\bar{X}}\right)$,
$\delta_2 = \left(\frac{\bar{X} + C_x}{\bar{X}}\right)$, $\delta_3 = \left(\frac{\bar{X} + \rho_{yx}}{\bar{X}}\right)$ and $\delta_4 = \left(\frac{C_x\bar{X} + S_x}{C_x\bar{X}}\right)$.

2.2 Situation II

The population mean $\bar{X}$ is unknown, when non-response occurs on the study variable y and complete response on the auxiliary variable x. The estimator is given by:

$$\hat{\bar{Y}}_{P(2)}^{(g)} = \bar{y}^* \exp\left(\frac{c(\bar{x}' - \bar{x})}{(c\bar{x}' + d) + (g-1)(c\bar{x} + d)}\right), \tag{33}$$

where $(g > 0)$.

Remark:

(i) When $g = 0$, the estimator $\hat{\bar{Y}}_{P(2)}^{(g)}$ reduces to

$$\hat{\bar{Y}}_{P(2)}^{(0)} = \bar{y}^* \exp(1), \tag{34}$$

which is a biased estimator with larger *MSE* than the usual estimator $\bar{y}^*$.

(ii) When $g = 1$, the estimator $\hat{\bar{Y}}_{P(2)}^{(g)}$ reduces to

$$\hat{\bar{Y}}_{P(2)}^{(1)} = \bar{y}^* \exp\left(\frac{c(\bar{x}' - \bar{x})}{c\bar{x}' + d}\right). \tag{35}$$

(iii) When $g = 2$, the estimator $\hat{\bar{Y}}_{P(2)}^{(g)}$ reduces to the estimator

$$\hat{\bar{Y}}_{P(2)}^{(2)} = \bar{y}^* \exp\left(\frac{c(\bar{x}' - \bar{x})}{(c\bar{x}' + d) + (c\bar{x} + d)}\right). \tag{36}$$

To obtain bias and mean square error of $\hat{\bar{Y}}_{P(2)}^{(g)}$, in terms of $\varepsilon's$, we have:

$$\hat{\bar{Y}}_{P(2)}^{(g)} = \bar{Y}(1 + \varepsilon_0)\exp\left(\frac{(\varepsilon_1' - \varepsilon_2)}{(\varepsilon_1' + (g-1)\varepsilon_2 + g\delta)}\right). \tag{37}$$

Solving (37), neglecting terms of $\varepsilon's$ and having power greater than two, we have:

$$\hat{\bar{Y}}_{P(2)}^{(g)} \cong \bar{Y}\Big[\varepsilon_0 + \frac{1}{g\delta}(\varepsilon_1' - \varepsilon_2) + \frac{1}{g\delta}(\varepsilon_0\varepsilon_1' - \varepsilon_0\varepsilon_2) + \frac{1}{g^2\delta^2}(\varepsilon_1' - \varepsilon_2)^2 - \frac{1}{g^2\delta^2}\left(\varepsilon_1'^2 + (g-2)\varepsilon_1'\varepsilon_2 - (g-1)\varepsilon_2^2\right)\Big]. \tag{38}$$

The bias of $\hat{\bar{Y}}_{P(2)}^{(g)}$, to first order of approximation, is given by:

$$B(\hat{\bar{Y}}_{P(2)}^{(g)}) \cong \bar{Y}\left[\lambda''\frac{1}{g\delta}\left\{\frac{1}{g\delta}\left(g-\frac{1}{2}\right)-K_{yx}\right\}C_x^2\right]. \tag{39}$$

Squaring both sides of (38) and neglecting terms of $\varepsilon's$ involving power greater than two, we have:

$$(T_{R(2)}^{(g)}-\bar{Y})^2 = \bar{Y}^2\left[\varepsilon_0^2+\frac{1}{g^2\delta^2}\left(\varepsilon_1'^2+\varepsilon_2^2-2\varepsilon_1'\varepsilon_2\right)+\frac{2}{g\delta}\left(\varepsilon_0\varepsilon_1' \quad \varepsilon_0\varepsilon_2\right)\right]. \tag{40}$$

Using (40), the mean square error of $\hat{\bar{Y}}_{P(2)}^{(g)}$ to the first degree of approximation is given by:

$$MSE(\hat{\bar{Y}}_{P(2)}^{(g)}) \cong \bar{Y}^2\left[\lambda' C_y^2+\lambda''\left\{C_y^2+\frac{1}{g\delta}\left(\frac{1}{g\delta}-2K_{yx}\right)C_x^2\right\}+\lambda^* C_{y(2)}^2\right]. \tag{41}$$

The $MSE(\hat{\bar{Y}}_{P(2)}^{(g)})$ is minimum when $g=\dfrac{1}{\delta K_{yx}}=g_0$(say).

Thus the resulting minimum *MSE* of $\hat{\bar{Y}}_{P(2)}^{(g)}$ is given by:

$$MSE(\hat{\bar{Y}}_{P(2)}^{(g)})_{\min} \cong \bar{Y}^2\left[\lambda' C_y^2+\lambda^* C_{y(2)} 2\lambda''+C_y^2\left(1 \quad \rho_{yx}^2\right)\right]. \tag{42}$$

In Table 2, for $g=1$ and $g=2$, we propose a family of estimators $\hat{\bar{Y}}_{P(2)}^{(g)}$ of the population mean $\bar{Y}$ by taking at different choices of c and d respectively. Many more estimators can also be generated from the proposed estimator in (33) just by putting different values of g, c and d.

Using Table 2, the *MSE* of $\hat{\bar{Y}}_{P(2)}^{(1)(i)}$ and $\hat{\bar{Y}}_{P(2)}^{(2)(i)}$ $(i=1,2,3,4)$ to first degree of approximation are given by:

$$MSE(\hat{\bar{Y}}_{P(2)}^{(1)(i)}) \cong \bar{Y}^2\left[\lambda' C_y^2+\lambda''\left\{C_y^2+A_3 C_x^2\right\}+\lambda^* C_{y(2)}^2\right], \tag{43}$$

and

$$MSE(\hat{\bar{Y}}_{P(1)}^{(2)(i)}) \cong \bar{Y}^2\left[\lambda' C_y^2+\lambda''\left\{C_y^2+A_5 C_x^2\right\}+\lambda^* C_{y(2)}^2\right]. \tag{44}$$

Table 2. Some members of a family of estimators $\hat{\hat{Y}}_{P(2)}^{(g)}$ under Situation-II.

Estimator	g	c	d
$\hat{\hat{Y}}_{P(2)}^{(1)(1)} = \bar{y}^* \exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + S_x}\right)$	1	1	S_x
$\hat{\hat{Y}}_{P(2)}^{(1)(2)} = \bar{y}^* \exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + C_x}\right)$	1	1	C_x
$\hat{\hat{Y}}_{P(2)}^{(1)(3)} = \bar{y}^* \exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \rho_{yx}}\right)$	1	1	ρ_{yx}
$\hat{\hat{Y}}_{P(2)}^{(1)(4)} = \bar{y}^* \exp\left(\frac{C_x(\bar{x}' - \bar{x})}{C_x\bar{x}' + S_x}\right)$	1	C_x	S_x
$\hat{\hat{Y}}_{P(2)}^{(2)(1)} = \bar{y}^* \exp\left(\frac{\bar{x}' - \bar{x}}{(\bar{x}' + S_x) + (\bar{x} + S_x)}\right)$	2	1	S_x
$\hat{\hat{Y}}_{P(2)}^{(2)(2)} = \bar{y}^* \exp\left(\frac{\bar{x}' - \bar{x}}{(\bar{x}' + C_x) + (\bar{x} + C_x)}\right)$	2	1	C_x
$\hat{\hat{Y}}_{P(2)}^{(2)(3)} = \bar{y}^* \exp\left(\frac{\bar{x}' - \bar{x}}{(\bar{x}' + \rho_{yx}) + (\bar{x} + \rho_{yx})}\right)$	2	1	ρ_{yx}
$\hat{\hat{Y}}_{P(2)}^{(2)(4)} = \bar{y}^* \exp\left(\frac{C_x(\bar{x}' - \bar{x})}{(C_x\bar{x}' + S_x) + (C_x\bar{x} + S_x)}\right)$	2	C_x	S_x

3. Efficiency comparisons

3.1 Situation I

(a) When the constant $'h'$ is unknown:

To compare the estimator $\hat{\hat{Y}}_{P(1)}^{(h)}$ with the usual estimators $\bar{y}^*$, $\hat{\hat{Y}}_{R(1)}$ and $\hat{\hat{Y}}_{Exp(1)}$ when the value of constant $'h'$ does not coincide with its optimum value $'h_0'$, we have

(i) $Var(\bar{y}^*) - MSE(\hat{\hat{Y}}_{P(1)}^{(h)}) > 0$ if $h > \max\left\{\frac{1}{2\delta K_{yx}}, \frac{1}{2\delta K_{yx(2)}}\right\}$.

(ii) $MSE(\hat{\hat{Y}}_{R(1)}) - MSE(\hat{\hat{Y}}_{P(1)}^{(h)}) > 0$ if

$$\min\left\{\frac{1}{\delta},\frac{1}{\delta\left(2K_{yx}-1\right)},\frac{1}{\delta\left(2K_{yx(2)}-1\right)}\right\}<h<\max\left\{\frac{1}{\delta},\frac{1}{\delta\left(2K_{yx}-1\right)},\frac{1}{\delta\left(2K_{yx(2)}-1\right)}\right\}.$$

(iii) $MSE(\hat{\bar{Y}}_{Exp(1)})-MSE(\hat{\bar{Y}}^{(h)}_{P(1)})>0$ if

$$\min\left\{\frac{2}{\delta},\frac{2}{\delta\left(4K_{yx}-1\right)},\frac{2}{\delta\left(4K_{yx(2)}-1\right)}\right\}<h<\max\left\{\frac{2}{\delta},\frac{2}{\delta\left(4K_{yx}-1\right)},\frac{2}{\delta\left(4K_{yx(2)}-1\right)}\right\}.$$

(b) When the constant $'h'$ is known:

(i) $Var(\bar{y}^{*})-MSE(\hat{\bar{Y}}^{(h)}_{P(1)})_{\min}>0$ if $\dfrac{\left(\lambda'' K_{yx}C_x^2+\lambda^{*}K_{yx(2)}C_{x(2)}^2\right)^2}{\lambda'' C_x^2+\lambda^{*}C_{x(2)}^2}>0.$

(ii) $MSE(\hat{\bar{Y}}_{R(1)})-MSE(\hat{\bar{Y}}^{(h)}_{P(1)})_{\min}>0$ if

$$\left(\frac{\left(\lambda'' K_{yx}C_x^2+\lambda^{*}K_{yx(2)}C_{x(2)}^2\right)^2}{\lambda'' C_x^2+\lambda^{*}C_{x(2)}^2}+\lambda''\left(1-2K_{yx}\right)C_x^2\right)>0 \text{ and } K_{yx(2)}<\frac{1}{2}.$$

(iii) $MSE(\hat{\bar{Y}}_{Exp(1)})-MSE(\hat{\bar{Y}}^{(h)}_{P(1)})_{\min}>0$ if

$$\left(\frac{\left(\lambda'' K_{yx}C_x^2+\lambda^{*}K_{yx(2)}C_{x(2)}^2\right)^2}{\lambda'' C_x^2+\lambda^{*}C_{x(2)}^2}+\lambda''\left(\frac{1}{4}-K_{yx}\right)C_x^2\right)>0 \text{ and } K_{yx(2)}<\frac{1}{4}.$$

(iv) $MSE(\hat{\bar{Y}}_{Reg(1)})-MSE(\hat{\bar{Y}}^{(h)}_{P(1)})_{\min}>0$ if

$$\left(\frac{\left(\lambda'' K_{yx}C_x^2+\lambda^{*}K_{yx(2)}C_{x(2)}^2\right)^2}{\lambda'' C_x^2+\lambda^{*}C_{x(2)}^2}-\lambda''\rho_{yx}^2C_y^2\right)>0 \text{ and } K_{yx}>2K_{yx(2)}.$$

3.2 Situation II

(a) When the constant $'g'$ is unknown:

To compare the estimator $\hat{\bar{Y}}^{(g)}_{P(2)}$ with the usual estimators $\bar{y}^{*}$, $\hat{\bar{Y}}_{R(2)}$ and $\hat{\bar{Y}}_{Exp(2)}$ when the value of constant $'g'$ does not coincide with its optimum value $'g_0'$, we have

(i) $Var(\bar{y}^{*})-MSE(\hat{\bar{Y}}^{(g)}_{P(2)})>0$ if $g>\dfrac{1}{2\delta K_{yx}}.$

(ii) $MSE(\hat{\bar{Y}}_{R(2)})-MSE(\hat{\bar{Y}}^{(g)}_{P(2)})>0$ if

$$\min\left\{\frac{1}{\delta}, \frac{1}{\delta\left(2K_{yx}-1\right)}\right\} < g < \max\left\{\frac{1}{\delta}, \frac{1}{\delta\left(2K_{yx}-1\right)}\right\}.$$

(iii) $MSE(\hat{\bar{Y}}_{Exp(2)}) - MSE(\hat{\bar{Y}}^{(g)}_{P(2)}) > 0$ if

$$\min\left\{\frac{2}{\delta}, \frac{2}{\delta\left(4K_{yx}-1\right)}\right\} < g < \max\left\{\frac{2}{\delta}, \frac{2}{\delta\left(4K_{yx}-1\right)}\right\}.$$

(b) When the constant $'g'$ is known:

(i) $Var(\bar{y}^*) - MSE(\hat{\bar{Y}}^{(g)}_{P(2)})_{\min} > 0$ if $K_{yx} > 0$.

(ii) $MSE(\hat{\bar{Y}}_{R(2)}) - MSE(\hat{\bar{Y}}^{(g)}_{P(2)})_{\min} > 0$ if $K_{yx} < 1$ and $K_{yx} > 1$.

(iii) $MSE(\hat{\bar{Y}}_{Exp(2)}) - MSE(\hat{\bar{Y}}^{(g)}_{P(2)})_{\min} > 0$ if $K_{yx} < \frac{1}{2}$ and $K_{yx} > \frac{1}{2}$.

(iv) $MSE(\hat{\bar{Y}}_{Reg(2)}) - MSE(\hat{\bar{Y}}^{(g)}_{P(2)})_{\min} = 0$.

The proposed estimators in Situations I and II are more efficient than the other considered estimators if above conditions are satisfied.

4. Empirical study

We use two data sets for efficiency comparison.

Population 1: (source: Khare and Sinha [7])
The data on physical growth of upper socio-economic group of 95 school children of Varanasi under an ICMR study, department of Pediatrics, B. H.U., during 1983-84 has been taken under study. The first 25% (i.e. 24 children) units have been considered as non-responding units.
Let $y =$ Weights (kg) of children and $x =$ Skull circumference (cm) of the children.
For this population, we have:

$N = 95, n' = 70, n = 35, W_2 = 0.25, \bar{Y} = 19.4968, \bar{X} = 51.1726, C_y = 0.15613, C_{y(2)} = 0.12075,$
$C_x = 0.03006, C_{x(2)} = 0.02478, \rho_{yx} = 0.328, \rho_{yx(2)} = 0.477$.

Population-II: (Source: Murthy [8])
Consider the data on number of workers and output for 80 factories in a region. The middle 20% units in the population have been treated as non-responding units.
Let $y =$ output and $x =$ number of workers in the factory.
For this population, we have:

$N = 80, n' = 45, n = 20, W_2 = 0.20, \bar{Y} = 5182.64, \bar{X} = 285.125, C_y = 0.35419, C_{y(2)} = 0.07110,$
$C_x = 0.94846, C_{x(2)} = 0.08519, \rho_{yx} = 0.914, \rho_{yx(2)} = 0.691.$

We have computed the percent relative efficiency (*PRE*) of different estimators with respect to usual unbiased estimator $\bar{y}^*$ for different values of k .

Table 3. ***PRE*** **of different estimators with respect to** $\bar{y}^*$ **for different values of** k **under Situation-I.**

Estimator	Population-I				Population-II			
	(1/*k*)				(1/*k*)			
	(1/5)	(1/4)	(1/3)	(1/2)	(1/5)	(1/4)	(1/3)	(1/2)
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\hat{\bar{Y}}_{P(1)}^{(1)(1)}$	112.21	111.49	110.55	109.28	209.91	208.98	208.03	207.07
$\hat{\bar{Y}}_{P(1)}^{(1)(2)}$	112.48	111.74	110.78	109.47	40.56	40.19	39.81	39.44
$\hat{\bar{Y}}_{P(1)}^{(1)(3)}$	112.43	111.69	110.73	109.43	40.55	40.18	39.81	39.44
$\hat{\bar{Y}}_{P(1)}^{(1)(4)}$	106.88	106.52	106.05	105.40	219.38	218.51	217.63	216.73
$\hat{\bar{Y}}_{P(1)}^{(2)(1)}$	106.70	106.35	105.89	105.26	250.66	251.59	252.55	253.54
$\hat{\bar{Y}}_{P(1)}^{(2)(2)}$	106.84	106.52	106.04	105.40	220.57	219.71	218.83	217.94
$\hat{\bar{Y}}_{P(1)}^{(2)(3)}$	106.84	106.48	106.01	105.36	220.56	219.70	218.82	217.93
$\hat{\bar{Y}}_{P(1)}^{(2)(4)}$	103.57	103.39	103.16	102.84	246.36	247.27	248.22	249.19
$\hat{\bar{Y}}_{R(1)}$	112.49	111.75	110.78	109.47	38.36	38.08	37.79	37.52
$\hat{\bar{Y}}_{Reg(1)}$	117.17	115.95	114.38	112.27	256.22	257.68	259.18	260.73
$\hat{\bar{Y}}_{Exp(1)}$	106.88	106.52	106.05	105.40	193.96	194.02	194.08	194.14
$\hat{\bar{Y}}^{(h)}$	117.80	116.41	114.65	112.37	256.25	257.69	259.19	260.73

In Table 3 under Population-I, it is observed that the *PRE* of all estimators decreases as the value of $(1/k)$ increases. In this table under Population-II, the estimators $\hat{\bar{Y}}_{P(1)}^{(1)(2)}$, $\hat{\bar{Y}}_{P(1)}^{(1)(3)}$ and $\hat{\bar{Y}}_{R(1)}$ show the poor performances as compared to all other considered estimators. Also under Population-II, the *PRE* of estimators $\hat{\bar{Y}}_{P(1)}^{(2)(2)}$, $\hat{\bar{Y}}_{P(1)}^{(2)(4)}$, $\hat{\bar{Y}}_{\mathrm{Re}g(1)}$, $\hat{\bar{Y}}_{EXP(1)}$ and $\hat{\bar{Y}}_{P(1)}$ increases as the value of $(1/k)$ increases whilst *PRE* of estimators $\hat{\bar{Y}}_{P(1)}^{(1)(1)}$, $\hat{\bar{Y}}_{P(1)}^{(2)(2)}$ and $\hat{\bar{Y}}_{P(1)}^{(2)(3)}$ decreases as the value of $(1/k)$ increases.

In Table 4, *PRE* of all estimators increases as the value of $(1/k)$ increases under both Populations I and II except in Population-II where the estimators $\hat{\bar{Y}}_{P(2)}^{(1)(2)}$, $\hat{\bar{Y}}_{P(2)}^{(1)(3)}$, $\hat{\bar{Y}}_{R(2)}$ perform badly.

Table 4. ***PRE*** **of different estimators with respect to $\bar{y}^*$ for different values of k under Situation-II.**

Estimator	Population-I (1/k)				Population-II (1/k)			
	(1/5)	(1/4)	(1/3)	(1/2)	(1/5)	(1/4)	(1/3)	(1/2)
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\hat{\bar{Y}}_{P(2)}^{(1)(1)}$	103.70	104.23	104.94	105.95	197.46	199.49	201.60	203.80
$\hat{\bar{Y}}_{P(2)}^{(1)(2)}$	103.76	104.31	105.03	106.06	40.07	39.83	39.57	39.32
$\hat{\bar{Y}}_{P(2)}^{(1)(3)}$	103.75	104.30	105.02	106.05	40.07	39.83	39.57	39.32
$\hat{\bar{Y}}_{P(2)}^{(1)(4)}$	102.24	102.56	102.98	103.57	205.99	208.29	210.69	213.20
$\hat{\bar{Y}}_{P(2)}^{(2)(1)}$	102.18	102.49	102.91	103.48	239.32	242.84	246.54	250.44
$\hat{\bar{Y}}_{P(2)}^{(2)(2)}$	102.24	102.56	102.98	103.57	207.06	209.39	211.83	214.37
$\hat{\bar{Y}}_{P(2)}^{(2)(3)}$	102.23	102.54	102.97	103.55	207.05	209.38	211.82	214.36
$\hat{\bar{Y}}_{P(2)}^{(2)(4)}$	101.20	101.37	101.60	101.91	235.61	238.98	242.52	246.26
$\hat{\bar{Y}}_{R(2)}$	103.77	104.31	105.04	106.06	38.22	37.98	37.73	37.48
$\hat{\bar{Y}}_{Reg(2)}$	104.57	105.24	106.14	107.40	245.89	249.68	253.67	257.89
$\hat{\bar{Y}}_{Exp(2)}$	102.24	102.56	102.98	103.57	186.95	188.65	190.43	192.28
$\hat{\bar{Y}}^{(g)}$	104.57	105.24	106.14	107.40	245.89	249.68	253.67	257.89

From Tables 3 and 4, it is observed that the proposed estimators $\hat{\bar{Y}}_{P(2)}^{(h)}$ and $\hat{\bar{Y}}_{P(2)}^{(g)}$ are more efficient as compared to the usual Hansen and Hurwitz [3] estimator, classical ratio, exponential-ratio type estimators and all other considered estimators in their respective situations under optimum conditions. It is also observed that the difference between $\hat{\bar{Y}}_{P(1)}^{(h)}$ and $\hat{\bar{Y}}_{Reg(1)}$ is either small or equal in Situation-I and are equally efficient in Situation-II. Overall Situation-I is preferable as compared to Situation-II.

From the range of constants i.e. (h and g) in efficiency comparisons, it has been observed that the proposed estimators $\hat{\bar{Y}}_{P(1)}^{(h)}$ and $\hat{\bar{Y}}_{P(2)}^{(g)}$ are more desirable over all the considered estimators even if the guessed values of the scalars 'h' and 'g' depart substantially from the exact optimum values i.e. 'h_0'and 'g_0' respectively.

5. Conclusion

We have developed a general class of exponential ratio type estimators under two different situations of nonresponse. Theoretical and numerical comparisons show that the proposed class of estimators $\hat{\hat{\bar{Y}}}_{P(1)}^{(h)}$ and $\hat{\hat{\bar{Y}}}_{P(2)}^{(g)}$ are more efficient than the estimators $\bar{y}^*$, $\hat{\hat{\bar{Y}}}_{R(i)}$ and $\hat{\hat{\bar{Y}}}_{EXP(i)}$ $(i = 1, 2)$ for both data sets. In Table 4, $\hat{\hat{\bar{Y}}}_{P(2)}^{(g)}$ is exactly equal to the regression estimator $\hat{\hat{\bar{Y}}}_{Reg(2)}$.

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