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QUARK IN THE EXTERNAL GLUON FIELD OF MAGNETIC TYPE: COHERENT STATES

Abstract. The possibility of introducing coherent states of particles moving in constant homogeneous non-Abelian gauge fields is shown. An important feature of the model considered in the article is that each color degree of freedom corresponds to its characteristic size.

Keywords: coherent states, non-Abelian gauge fields, color charge, Dirac equation, Hermitian matrix, *SU*(3)-transformations, solutions

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КВАРК ВО ВНЕШНЕМ НЕАБЕЛЕВОМ КАЛИБРОВОЧНОМ ПОЛЕ МАГНИТНОГО ТИПА: КОГЕРЕНТНЫЕ СОСТОЯНИЯ

Аннотация. Показана возможность введения когерентных состояний частиц, движущихся в постоянном однородном неабелевом поле. Одной из особенностей рассмотренной модели является то обстоятельство, что в общем случае каждой из «цветовых» степеней свободы соответствует свой характерный размер.

Ключевые слова: когерентные состояния, неабелево калибровочное поле, «цветовой» заряд, уравнение Дирака, эрмитова матрица, *SU*(3)-преобразования, решения

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Introduction. In general, the problem of constructing coherent states for systems described by relativistic wave equations has not been solved. However, in the case of motion of a charged particle in a magnetic field, the relativistic problem is equivalent to a nonrelativistic problem because of the conservation of the momentum modulus in the classical case and the existence of the corresponding integrals of motion in the quantum-mechanical case. Thus, for the quantum-mechanical problem of the motion of a charged particle in the constant homogeneous magnetic field, coherent states can be introduced regardless of whether the relativistic or the non-relativistic case is considered. The approach to the description of coherent states in the magnetic field provides a clue for the possible connection of the hadron model proposed in [1, 2] with the field (quantum-chromodynamics) approach.

Indeed, the initial step in constructing of coherent states is the creation and annihilation operators

$$a_i^+ = \frac{1}{\sqrt{2}}\left(\xi_i - \frac{\partial}{\partial \xi_i}\right), \quad a_i^- = \frac{1}{\sqrt{2}}\left(\xi_i + \frac{\partial}{\partial \xi_i}\right), \tag{1}$$

where

$$\xi_i = \frac{x}{x_0}, \quad x_0 = \sqrt{\frac{\hbar}{m\omega}}. \tag{2}$$

In the expressions (2), m is the mass of a particle in the oscillator potential, and ω is the frequency of oscillations.

As known, transverse motion to the direction of the magnetic field of motion of a particle in classical and quantum-mechanical case are described by an oscillator with a Larmor frequency

$$\omega = \frac{eB}{mc}, \tag{3}$$

where e is the particle charge, c is the speed of light, m is the mass of the particle, and B is the induction of the magnetic field. Substituting (3) in to (2) for x_0, we obtain

$$x_0 = \sqrt{\frac{\hbar c}{2eB}}. \tag{4}$$

In the model of the hadrons developed in [1], the quantity x_0 plays the role of the hadron size R, and equating R to x_0 in (4), we obtain

$$R = \sqrt{\frac{\hbar c}{2eB}}. \tag{5}$$

Since nature R must be determined by strong interactions, it can be assumed that e in formula (5) it might be regarded as a charge of a quark, and B as a strength of a chromodynamics field.

Thus, the task of the present article is to investigate the existence of configurations of the chromodynamics field which allow the introduction of coherent states.

Below, we formulate and solve the problem of determining the coherent states in a non-Abelian gauge field.

Coherent states of quarks in the constant magnetic chromodynamics field. The Dirac equation describing three quarks in the gluon field has the form

$$i\gamma_\mu\left(p_\mu + gT^a A_\mu^a\right)\psi = m\psi. \tag{6}$$

Here γ_μ are the Dirac matrices μ = 0,1,2,3; A_μ^a is the vector potential of the external gluon field; a = 1,2…8; T^a are the generators of the group $SU(3)$ of the color symmetry group, $\psi = \psi^\beta$, $\beta = 1,2,3$ are bispinors with respect to the space-time symmetry transformations and the spinor (quark representation) with respect to the $SU(3)$ group, and A_μ^a is the vector with respect to the transformations of both groups.

Matrix representations for generators T^a have the form

$$T^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ T^2 = \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ T^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ T^4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},\ T^5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$T^6 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\ T^7 = \frac{1}{2}\begin{pmatrix} 0 & & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix},\ T^8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{7}$$

Now we introduce an constant homogeneous gluon field of the magnetic type. By analogy with the problem of the motion of an electrically charged Dirac particle in the constant uniform magnetic field [3], we choose the non-Abelian vector potential in the form

$$A_\mu^a = \left(0, 0, H^a x_1, 0\right). \tag{8}$$

The vector potential satisfies formally the homogeneous equations of a non-Abelian gauge field

$$\nabla_\mu F_{\mu\nu}^a = \partial_\mu F_{\mu\nu}^a + gf^{abc} F_{\mu\nu}^b A_\mu^c = 0, \tag{9}$$

where

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c. \tag{10}$$

By direct substitution, we verify that the 4-vector potential (8) satisfies equations (9), (10) and especially note that the nonlinear terms in equations (10) and (11) vanish.

The tension tensor contains a single component of the magnetic type:

$$\vec{F}^a = \left(0, 0, H^a, 0, 0, 0\right),$$

when the tensor is considered as a complex three-dimensional vector.

Thus, with the vector potential (8), the theory is linearized, but retains the symmetry with respect to the color group. The theory remains invariant under $SU(3)$-gauge transformations.

Substituting the expression for the vector potential (8) in (6), we obtain

$$i\gamma_\mu\left(p_\mu + gT^a H^a x_1\right)\psi = m\psi. \tag{11}$$

By a similarity transformation with the help of a constant unitary matrix, the Hermitian constant matrix can be reduced to a diagonal form, so equation (11) is written as

$$i\gamma_\mu\left(p_\mu + g\Lambda_k I x_1\right)\psi = m\psi, \tag{12}$$

where I is the unit matrix in the space of $SU(3)$-spinors, Λ_k (k = 1,2,3) are the eigenvalues of the matrix

$$T^a H^a = \frac{1}{2}\begin{pmatrix} H^3 + \frac{1}{\sqrt{3}}H^8 & H^1 - iH^2 & H^4 - iH^5 \\ H^1 + iH^2 & -H^3 + \frac{1}{\sqrt{3}}H^8 & H^6 - iH^7 \\ H^4 + iH^5 & H^6 + iH^7 & -\frac{2}{\sqrt{3}}H^8 \end{pmatrix}, \tag{13}$$

which are the roots of the cubic equation

$$\left|T^a H^a - \Lambda\right| = 0. \tag{14}$$

We will not use expressions of eigenvalues because they are cumbersome. Their specific expressions are not important. It is important that these roots are constant and real.

In the general case, all three roots can be different, so the equation splits into three separated equations with different eigenvalues Λ_k.

Equation (12) for stationary states $\psi \to e^{i\varepsilon t}\psi$ (here and everywhere $c = \hbar = 1$) for each Λ_k can be written in the form [3]

$$\begin{aligned} \vec{\sigma}(\vec{p} + g\Lambda_k I x_1)\varphi &= (\varepsilon + m)\chi \\ \vec{\sigma}(\vec{p} + g\Lambda_k I x_1)\chi &= (\varepsilon - m)\varphi \end{aligned}. \tag{15}$$

In the equations (15), $\vec{\sigma}$ is a vector of Pauli matrices, $\vec{p}$ is a vector from the projection operators of the quark momentum. Further we follow the monograph [3]. Eliminating χ in (15), we obtain the quadratic equation

$$\left\{\vec{p}^2 + g^2\Lambda_k{}^2 x_1{}^2 + g\Lambda_k(\sigma_3 + 2x_1 p_2)\right\}\varphi = (\varepsilon^2 - m^2)\varphi. \tag{16}$$

Choosing the spinor as an eigenfunction of the projection operator of the spin σ_3

$$\sigma_3\varphi = \mu\varphi,$$

where $\mu = \pm 1$,, we find the solution in the form

$$\Phi_{\Lambda_k \mu}(x_1) = e^{i(p_2 x_2 + p_3 x_3)}\Upsilon_{\Lambda_k \mu}(x_1). \tag{17}$$

The function $\Upsilon_{\omega_k \mu}(\xi)$ satisfies the Schrödinger-like equation for a harmonic oscillator

$$\left(-\frac{d^2}{d\xi^2}+\xi^2\right)\Upsilon_{\Lambda_k \mu}(\xi)=\frac{\varepsilon^2-m^2-p_3^2+g\Lambda_k\mu}{g\Lambda_k}\Upsilon_{\Lambda_k \mu}(\xi), \tag{18}$$

where

$$\xi=\sqrt{g\Lambda_k}\left(x-\frac{p_2}{g\Lambda_k}\right).$$

The energy spectrum is determined by the formula

$$\varepsilon^2=m^2+p_3^2+g\Lambda_k(2n+1-\mu). \tag{19}$$

We will not reproduce the wave functions of the problem here. They can be obtained from the wave functions in [3] by a change $eH \to g\Lambda_k$.

As known, coherent states arise naturally in the quantum-mechanical problem of a harmonic oscillator to which the problem has been reduced. Taking into account the definition of the creation and annihilation operators (1) and (2), the left side of the equation expressed in terms of the product of such operators and the wave function can be represented [4] as an integral over coherent states $|z\rangle$:

$$(2a^+a+1)\frac{1}{\pi}\int d^2z e^{-\frac{|z|^2}{2}}\Upsilon_{\Lambda_k \mu}(z^*)|z\rangle=\frac{\varepsilon^2-m^2-p_3^2+g\Lambda_k\mu}{g\Lambda_k}\frac{1}{\pi}\int d^2z e^{-\frac{|z|^2}{2}}\Upsilon_{\Lambda_k \mu}(z^*)|z\rangle. \tag{20}$$

Multiplying equation (18) by the conjugate coherent state $\langle z'|$ and using the definition of coherent states, we obtain the formula for the energy spectrum for (15).

Conclusion. Thus, we showed the principal possibility of introducing coherent states of particles moving in constant homogeneous non-Abelian gauge fields and interacting with these fields due to the presence of particles of the color type. An important feature of the considered model is that here each "color" degree of freedom corresponds to its characteristic size

$$R_k=\sqrt{\frac{\hbar c}{2g\Lambda_k}},$$

in contrast to the case of motion in a homogeneous constant magnetic field. We have, for clarity, returned to the notation with $\hbar$ and c. We also note that Λ_k, expressed in terms of the intensity of the gauge field (in this case of the magnetic type), which in principle are immeasurably considered as a single quantity in a product $g\Lambda_k$ with a charge and determine the dynamics of a particle in such a field.

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