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N. N. Guschinsky[1], O. Battaia[2], A. Dolgui[3]

[1]United Institute of Informatics Problems of the National Academy of Sciences of Belarus, Minsk, Belarus
[2]ISAE-Supaéro, Toulouse, France
[3]Ecole des Mines de Nantes, Nantes, France

ON OPTIMIZATION OF PROCESSES FOR SEQUENTIAL BATCH MACHINING

A problem of optimal design of processes of sequential machining of multiple parts on rotary table machines is considered. Batches are processed in a given sequence. Parts of the same batch are located at the working positions of rotary table and are machined simultaneously. Operations are divided into groups which are performed by spindle heads or by turrets. Constraints on the design of spindle heads, turrets, and working positions, as well as on the order of operations are given. The problem is to minimize the estimated cost of machine equipment while reaching a given output. The proposed method to solve the problem is based on its formulation in terms of mixed integer linear programming. Computational results are reported.

Keywords: rotary table machine, optimization, sequential batch machining.

Н. Н. Гущинский[1], О. Баттайа[2], А. Долгий[3]

[1]Объединенный институт проблем информатики Национальной академии наук Беларуси, Минск, Беларусь
[2]Высшая национальная школа аэронавтики и космоса, Тулуза, Франция
[3]Горная школа Нанта, Нант, Франция

К ОПТМИЗАЦИИ ПРОЦЕССОВ ПОСЛЕДОВАТЕЛЬНОЙ ОБРАБОТКИ ПАРТИЙ ДЕТАЛЕЙ

Рассматривается задача оптимального проектирования процессов последовательной обработки партий деталей на станках с поворотным столом. Последовательность обработки партий задана. Детали одной и той же партии устанавливаются на рабочих позициях станка и обрабатываются одновременно. Множество технологических переходов для обработки всех деталей разбивается на группы, которые выполняются с помощью шпиндельных или револьверных головок. Заданы ограничения, связанные с разбиением переходов по шпиндельным и револьверным головкам, рабочим позициям станка, а также порядком выполнения переходов. Задача заключается в минимизации оценки стоимости оборудования станка при обеспечении заданной производительности. Предлагаемый метод решения задачи основан на ее формулировке в терминах смешанного целочисленного линейного программирования. Приводятся результаты вычислительных экспериментов.

Ключевые слова: станок с поворотным столом, оптимизация, последовательная обработка партий.

Introduction. This paper deals with a problem of the optimal design of a rotary transfer machine with turrets for sequential machining of multiple parts. Such a machine is multi-positional, i. e. parts are sequentially machined on m (1, 2, …, m) working positions. One position of the machine (zero) is exclusively used for loading new billets and unloading finished parts. At each working position, several machining modules (spindle heads) can be installed to process the operations assigned to this position. They are activated sequentially or simultaneously. Simultaneous activation is possible if machining modules are related to the different sides of the part and work in parallel. Sequential activation is realized by the use of turrets. There are horizontal and vertical spindle heads and turrets to access to different sides of parts on a working position.

We consider the case when there is only one vertical turret mounted at one position or one spindle head common for all working positions. There are several horizontal spindle heads or turrets. However, there is only one horizontal spindle head or turret per position.

At the preliminary design stage, the following decisions must be made: the choice of orientations of parts, the partitioning of the given set of operations into positions and machining modules, and the choice of cutting modes for each spindle head and turret.

Few studies on rotary transfer machines were published. Configuration of semi-automated systems with multi-turn rotary table was discussed in [1]. Mathematical models of transfer machines with rotary table were proposed in [2, 3]. The first mathematical model for the design of rotary transfer machines with turrets for machining a single part was presented in [4]. MIP models for parallel and sequential machining of multiple parts were considered in [5, 6].

Batch scheduling problems have been treated by many researchers. For an extensive review, we refer to [7]. Two types of batch machines can be distinguished: sum-batch and max-batch machines. On a sum-batch machine jobs are completed sequentially and the processing time of a batch is equal to the sum of the processing times of all the jobs in the batch [8]. A max-batch machine treats the jobs simultaneously and the processing time of a batch is equal to the time of the longest job [9].

1. Problem statement. We consider the problem of design of a rotary transfer machine with m working positions for machining d_0 types of parts with required output O^d, $d = 1, 2,\dots, d_0$. After finish of processing of O^d parts of type d the rotary transfer machine is reconfugured, i.e. the fixtures of parts are changed and some spindles are mounted or dismounted if necessary.

Let $\mathbf{N}^d$ be the set of machining operations needed for machining of elements of the d^{th} part $d = 1, 2,\dots, d_0$, located on n_d sides and N_s^d, $s = 1, 2,\dots, n_d$, is a subset of opertations for machining of elements of the s^{th} side of the part. The part d can be located at zero position in different orientations $\mathbf{H}(d)$ but elements of no more than one side can be machined by vertical spindle head or turret. All elements of other sides of the part have to be assigned to horizontal spindle heads or turrets. $\mathbf{H}(d)$ can be represented by matrix of dimension $r_d \times n_d$ where $h_{rs}(d)$ is equal j, $j = 1,2$, if the elements from N_s^d can be machined by spindle head or turret of type j for such an orientation of the part d.

Let $\mathbf{N}=\bigcup_{d=1}^{d_0} \mathbf{N}^d$. All operations $p\in\mathbf{N}$ are characterized by the following parameters:

– the length $\lambda(p)$ of the working stroke for operation $p\in\mathbf{N}$;

– range $[\gamma_1(p), \gamma_2(p)]$ of feasible values of feed rate;

– set $H(p)$ of feasible orientations of the part (indexes $r\in\{1, 2,\dots, r_d\}$ of rows of matrix $\mathbf{H}(d)$) for execution of operation $p\in N_s^d$ by spindle head or turret of type j (vertical if $h_{rs}(d) = 1$ and horizontal if $h_{rs}(d) = 2$).

Obviously, there is no solution exists if $\bigcap_{p\in N_s^d} H(p) = \varnothing$ for some $d\in\{1, 2,\dots, d_0\}$ and $s\in\{1, 2,\dots, n_d\}$.

Let N_k, $k = 1,\dots,m$, be a subset of operations from $\mathbf{N}$ assigned to the k^{th} working position; N_{k1} and N_{k2} be the sets of operations assigned to working position k that are concerned by vertical and horizontal machining, respectively; b_{kj} be the number of machining modules of type j installed at the k^{th} working position; N_{kjl}, $l = 1,\dots,b_{kj}$, be subsets of operations from N_{kj} assigned to the l^{th} machining module of type j at the k^{th} working position.

This assignment has to take into account the following technological constraints:

– possible sequences of operations for machining parts (precedence constraints);

– the necessity to perform some pairs of operations from $\mathbf{N}$ at the same working position, by the same turret, by the same machining module (inclusion constraints);

– the impossibility to perform some pairs of operations from $\mathbf{N}$ at the same working position, by the same turret, by the same machining module (exclusion constraints);

– the maximal number m_0 of working positions and the maximal number b_0 of machining modules in a turret;

– feasible orientations of the part for execution of each operation;

– the impossibility to perform operations from N_s^d by machining modules of different types;

– the productivity providing the given output.

The precedence constraints can be specified by a directed graph $G^{OR} = (\mathbf{N},D^{OR})$ where an arc $(p,q)\in D^{OR}$ if and only if the operation p has to be executed before the operation q. It should be noted that if such operations p and q belong to different sides of the part then they cannot be executed at the same position without violating the precedence constraint.

The inclusion constraints can be given by undirected graphs $G^{SM} = (\mathbf{N},E^{SM})$, $G^{ST} = (\mathbf{N},E^{ST})$, and $G^{SP} = (\mathbf{N},E^{SP})$ where the edge $(p,q)\in E^{SM}$ ($(p,q)\in E^{ST}$, $(p,q)\in E^{SP}$) if and only if the operations p and q must be executed in the same machining module (turret, position).

The exclusion constraints can also be defined by undirected graphs $G^{DM} = (\mathbf{N},E^{DM})$, $G^{DT} = (\mathbf{N},E^{DT})$, and $G^{DP} = (\mathbf{N},E^{DP})$ where the edge $(p,q)\in E^{DM}$ ($(p,q)\in E^{DT}$), $(p,q)\in E^{DP}$)) if and only if the operations p and q cannot be executed in the same machinning module (turret, position).

It is assumed that infeasible combinations of part orientations are given by a set E^{DH}, each element of which $e = \{(d_1,r_1),(d_2,r_2),\ldots,(d_k,r_k)\}$ represents a collection of pairs (part number d and row number of $\mathbf{H}(d)$) that prohibit simultaneously orientation r_1 for part d_1, orientation r_2 for part d_2, and orientation r_k for part d_k. Obviously, the set E^{DH} includes $\{(r',d'), (r'',d'')\}$ if there exist $p\in N_{s'}^{d'}$, $s'\in\{1,\ldots,n_{d'}\}$, $q\in N_{s''}^{d''}$, $s''\in\{1,\ldots,n_{d''}\}$ such that $(p,q)\in E^{SM}\cup E^{ST}$ and $h_{r's'}(d') \neq h_{r''s''}(d'')$.

Let $P = \langle P_1,\ldots,P_k,\ldots,P_m\rangle$ is a design decision with $P_k = (P_{1k11},P_{2k11},\ldots,P_{d_0k11},\ldots,P_{1k1b_{k1}}, P_{2k1b_{k1}},\ldots,P_{d_0k1b_{k1}}, P_{1k21},P_{2k21},\ldots,P_{d_0k21},\ldots,P_{1k2b_{k2}},P_{2k2b_{k2}},\ldots,P_{d_0k2b_{k2}})$, $P_{dkjl} = (N_{dkjl},\Gamma_{dkjl})$, $P_{dkj} = (P_{dkjl}|l=1,\ldots,b_{kj})$, $P_{dk} = (P_{dkj}|j = 1,2)$, and $\mathbf{N}_j = \bigcup_{d=1}^{d_0}\bigcup_{k=1}^{m}\bigcup_{l=1}^{b_{kj}} N_{dkjl}$, $j = 1,2$.

The execution time $t^b(P_{dkjl})$ of operations from N_{dkjl} with the feed per minute $\Gamma_{dkjl}\in[\max\{\gamma_1(p)|p\in N_{dkjl}\}, \min\{\gamma_2(p)|p\in N_{dkjl}\}]$ is equal to $t^b(P_{dkjl}) = L(N_{dkjl})/\Gamma_{dkjl}+\tau^a$, where $L(N_{dkjl}) = \max\{\lambda(p)|p\in N_{dkjl}\}$, and τ^a is an additional time for advance and disengagement of tools. We assume that only time needed for rotation of the turret between nonempty sets N_{dkjl} is taken into account and the execution time is equal to $t^h(P_{dkj}) = \tau^g(l_{\max}^d(P_{dkj}) - l_{\min}^d(P_{dkj}))+\sum_{l=1}^{b_{kj}} t^b(P_{dkjl})$, $j = 1, 2$, where τ^g is an additional time for one rotation of turret, $l_{\max}^d(P_{dkj}) = \max\{l = 1,2,\ldots,b_{kj}|N_{dkjl}\neq\varnothing\}$ and $l_{\min}^d(P_{dkj}) = \min\{l=1,2,\ldots,b_{kj}|N_{dkjl}\neq\varnothing\}$, respectively. The execution time $t^p(P_{dk})$ is defined as $t^p(P_{dk}) = \tau^r+\max\{t^h(P_{dkj})|j = 1,2\}$, where τ^r is an additional time for table rotation. Then the time t_d for machining all the elements of d^{th} part is equal to $t^d(P) = \max\{t^p(P_{dk})| k = 1,\ldots,m\}$.

We assume that the given productivity is provided, if the total time $T(P)$ for machining O^d parts does not exceed the available time T_0, i. e. $T(P) = \sum_{d=1}^{d_0} t^d(P)(O^d + m - 1)\leq T_0$. We take into account that at the beginning and the end of machining of O^d parts not all the working positions are occupied.

It is easy to see that the constraint on the productivity is provided if and only if it satisfied for $\Gamma_{dkjl} = \min\{\gamma_2(p)|p\in N_{dkjl}\}$, $d = 1,\ldots,d_0$, $k = 1,\ldots,m$, $j = 1, 2$, $l = 1,\ldots,b_{kj}$.

Let C_1, C_2, C_3, and C_4 be the relative costs for one position, one turret, one machining module of a turret, and one spindle head respectively. Since the vertical spindle head (if it presents) is common for several positions its size (and therefore the cost) depends on the number of positions to be covered. Let $k_{\min}^h$ and $k_{\max}^h$ be the minimal and the maximal position of the common vertical spindle head. Then its cost can be estimated as $C_4 + (k_{\max}^h - k_{\min}^h)C_5$ where C_5 is the relative cost for covering one additional position by vertical spindle head. If the vertical spindle turret is installed its cost can be estimated by $C_2+C_3b_{k1}$. In the similar way the cost $C(b_{k2})$ for performing set of operations N_{k2} by associated b_{k2} machining modules can be assessed as follows:

$$C(b_{k2}) = \begin{cases} 0 \text{ if } b_{k2} = 0, \\ C_4 \text{ if } b_{k2} = 1, \\ C_2 + C_3 b_{k2} \text{ if } b_{k2} > 1. \end{cases}$$

The machine cost $Q(P)$ is calculated as the total cost of all equipment used i. e.

$$Q(P) = C_1 m + C_4 \operatorname{sign}(|N_1|)(1 - \sum_{k=1}^{m} \operatorname{sign}(|N_{k12}|)) + \sum_{k=1}^{m} \operatorname{sign}(|N_{k12}|)(C_2 + C_3 b_{k1}) +$$
$$+ C_5 (k_{\max}^h - k_{\min}^h) + \sum_{k=1}^{m} C(b_{k2})$$

where sign(a) = 1 if $a > 0$, and sign(a) = 0 if $a \le 0$.

The studied problem is to determine:

the number of positions m;

orientations of parts $H(d)$;

the number b_{kj} of machining modules of type j (j = 1 for vertical and j = 2 for horizontal) installed at the k^{th} position, $k = 1,...,m$;

subsets N_{dkjl} of operations from $\mathbf{N}^d$ assigned to the l^{th} machining module of type j at the k^{th} position, $d = 1,2,\ldots,d_0$, $k = 1,...,m$, $l = 1,...,b_{kj}$;

the feed per minute Γ_{dkjl} for each subset N_{dkjl}, $d = 1,2,\ldots,d_0$, $k = 1,...,m$, $j = 1,2$, $l = 1,\ldots, b_{kj}$ in such a way that the machine cost is small as possible and all the constraints are not violated.

It is easy to see that if P is an optimal solution of the considered problem then the design decision P' with $P'_{dkjl} = (N_{dkjl}, \min\{\gamma_2(p)|p\in N_{dkjl}\})$ is also optimal. This property is used in MIP formulation of the problem.

2. MIP formulation. Let us introduce the following notation:

X_{pkjl} – a decision variable which is equal to 1 if the operation p from $\mathbf{N}$ is assigned to the l^{th} machining module of spindle head or turret type j at the k^{th} position;

Y_j^{ds} – an auxiliary variable which is equal to 1 if at least one operation from N_s^d is executed by spindle head or turret of type j;

Y_{kjl}^d – an auxiliary variable which is equal to 1 if at least one operation for machining elements of the d^{th} part is executed in the l^{th} machining module of spindle head or turret type j at the k^{th} position;

Y_{kjl} – an auxiliary variable which is equal to 1 if the l^{th} machining module of spindle head or turret type j is installed at the k^{th} position;

$l_{kj\max}^d$ – an auxiliary variable for estimation of the last machining module of spindle head or turret type j at the k^{th} position for machining elements of the d^{th} part;

$l_{kj\min}^d$ – an auxiliary variable for estimation of the first machining module of spindle head or turret type j at the k^{th} position for machining elements of the d^{th} part;

$Y_{1\min}$ – an auxiliary variable which is equal to k if k is the minimal position covered by vertical spindle head or turret;

$Y_{1\max}$ – an auxiliary variable which is equal to k if k is the maximal position covered by vertical spindle head or turret;

Y_1 – an auxiliary variable which is equal to 1 if the vertical spindle head or turret is installed;

Z_k – an auxiliary variable which is equal to 1 if at least one operation is assigned to the k^{th} position;

h_r^d – an auxiliary variable which is equal to 1 if elements of the d^{th} part are machined with the r^{th} orientation;

F_{kjl}^d – an auxiliary variable for determining the time of execution of operations from $\mathbf{N}^d$ in the l^{th} machining module of spindle head or turret type j at the k^{th} position;

F^d – an auxiliary variable for determining the time of execution of all the operations from $\mathbf{N}^d$;

T_k^d – an auxiliary variable which is equal to F^d if the k^{th} position exists and 0 otherwise;

t_{pq} – minimal time necessary for execution of operations p and q in the same machining module, $t_{pq} = \max(\lambda(p), \lambda(q))/\min(\gamma_2(p),\gamma_2(q))+\tau^a$.

It is assumed that $(p,q)\in E^{DM}$ if $\min(\gamma_2(p),\gamma_2(q)) < \max(\gamma_1(p),\gamma_1(q))$.

The number of variables and constraints can be reduced by using set $\mathbf{N}'$ instead of $\mathbf{N}$. The set $\mathbf{N}'$ is built based on graph G^{SM}. Let $G_i^{SM} = (N_i^{SM}, E_i^{SM})$, $i=1,\ldots,n^{SM}$, be connectivity components of G^{SM}

including isolated vertices. Only one vertex (operation) $\wp_i$ is chosen from each N_i^{SM} and included into $\mathbf{N}'$. Later $\chi(p)=\wp_i$ for all $p\in N_i^{SM}$.

2.1. Cost calculation. The objective can be represented as follows:

$$\min C_1\sum_{k=1}^{m_0}Z_k+C_4\sum_{k=1}^{m_0}Y_{k21}+(C_2+2C_3-C_4)\sum_{k=1}^{m_0}\sum_{j=1}^{2}Y_{kj2}+C_3\sum_{k=1}^{m_0}\sum_{j=1}^{2}\sum_{l=3}^{b_0}Y_{kjl}+C_4Y_1+C_5(Y_{1\max}-Y_{1\min}). \quad (1)$$

If the horizontal turret is installed at position k then $Y_{k21} = Y_{k22} = 1$ and $C_4Y_{k21}+(C_2+2C_3-C_4)Y_{k22}=C_2+2C_3$. If the horizontal spindle head is installed at position k then $Y_{k2l} = 0$, $l=2,\dots,b_0$, and $C_4Y_{k21}+(C_2+2C_3-C_4)Y_{k22}=C_4$. If the vertical turret is installed at position k then $Y_{k11} = Y_{k12} = 1$, $Y_1 = 1$, $Y_{1\min} = Y_{1\max}$ and $(C_2+2C_3-C_4)Y_{k12}+C_4Y_1+C_5(Y_{1\max}-Y_{1\min})=C_2+2C_3$. If the vertical spindle head is installed common for positions $k_1 = Y_{1\min},\dots,k_v = Y_{1\max}$ then $Y_1 = 1$, $Y_{k1l} = 0$, $l = 2,\dots,b_0$, $k = 1,\dots,m_0$ and $C_4Y_1+(C_2+2C_3-C_4)\sum_{k=1}^{m_0}Y_{k12}=C_4$.

Variables Z_k, $k = 1,\dots,m_0$ and Y_1 should satisfy the following constraints:

$$Y_1\le\sum_{m=1}^{m_0}Y_{k11}, \quad (2)$$

$$\sum_{m=1}^{m_0}Y_{k11}\le m_0Y_1, \quad (3)$$

$$Z_k\le Y_{k11}+Y_{k21},\ k=1,\dots,m_0, \quad (4)$$

$$Y_{k11}+Y_{k21}\le 2Z_k,\ k=1,\dots,m_0. \quad (5)$$

Since the objective function (1) is minimized variables $Y_{1\min}$ and $Y_{1\max}$ can be defined by the following constraints:

$$(m_0-k+1)Y_{k11}+Y_{1\min}\le m_0+1,\ k=1,\dots,m_0, \quad (6)$$

$$Y_{1\max}\ge kY_{k11},\ k=1,\dots,m_0, \quad (7)$$

$$Y_{1\max}\le m_0Y_1, \quad (8)$$

$$Y_{1\min}\le m_0Y_1. \quad (9)$$

If there is no vertical machining in the design decision ($Y_1 = 0$) then both $Y_{1\max}$ and $Y_{1\min}$ are equal to 0 due to (8) and (9). If the vertical turret is installed at position k ($Y_{k11} = 1$, and $Y_{k'11}=0$, $k'\ne k$) then $Y_{1\min}\le k$ and $Y_{1\max}\ge k$ due to (6) and (7). In this case $C_5(Y_{1\max}-Y_{1\min})$ is minimal if $Y_{1\max} = Y_{1\min} = k$. If the vertical spindle head is installed common for positions $k_1 = k',\dots,k_v = k''$ then $Y_{1\min}\le k'$ and $Y_{1\max}\ge k''$ due to (7) and (8) and $C_5(Y_{1\max}-Y_{1\min})$ is minimal if and only if $Y_{1\min} = k'$ and $Y_{1\max} = k''$.

2.2. Time calculation. The time of execution of operations from $\mathbf{N}^d$ by the l^{th} machining module of spindle head or turret type j at the k^{th} position cannot be less than the time of execution of any operation from $\mathbf{N}^d$ assigned to this machining module:

$$F_{kjl}^d\ge t_{qq}X_{\chi(q)kl},\ q\in\mathbf{N}^d,\ d=1,\dots,d_0,\ k=1,\dots,m_0,\ j=1,2,\ l=1,\dots,b_0. \quad (10)$$

The time of execution of operations from $\mathbf{N}^d$ by the l^{th} machining module of spindle head or turret type j at the k^{th} position cannot be less than the time of execution of any pair of operations from $\mathbf{N}^d$ assigned to this machining module:

$$F_{kjl}^d\ge t_{pq}(X_{\chi(p)kl}+X_{\chi(q)kl}-1),\ p,q\in\mathbf{N}^d,\ d=1,\dots,d_0,\ (p,q)\notin E^{DP}\cup E^{DB},\ p<q,\ k=1,\dots,m_0,\ j=1,2,\ l=1,\dots,b_0. \quad (11)$$

The time of execution of operations from $\mathbf{N}^d$ cannot be less than the time of execution of vertical and horizontal spindle head or turret at each of m_0 positions:

$$F^d\ge\tau^r+\sum_{l=1}^{b_0}F_{kjl}^d+\tau^g(l_{kj\max}^d-l_{kj\min}^d),\ d=1,\dots,d_0,\ k=1,\dots,m_0,\ j=1,2, \quad (12)$$

where variables $l_{kj\max}^d$ and $l_{kj\min}^d$ can be defined by the following constraints:

$$(b_0-l+1)Y^d_{kjl}+l^d_{kj\min} \le b_0+1,\ d = 1,\ldots,d_0,\ k = 1,\ldots,m_0,\ j = 1,2,\ l = 1,\ldots,b_0, \quad (13)$$

$$l^d_{kj\max} \ge lY_{k11},\ d = 1,\ldots,d_0,\ k = 1,\ldots,m_0,\ j = 1,2,\ l = 1,\ldots,b_0, \quad (14)$$

$$l^d_{kj\max} \le b_0\sum_{l=1}^{b_0} Y^d_{kjl},\ d = 1,\ldots,d_0,\ k = 1,\ldots,m_0,\ j = 1,2, \quad (15)$$

$$l^d_{kj\min} \le b_0\sum_{l=1}^{b_0} Y^d_{kjl},\ d=1,\ldots,d_0,\ k = 1,\ldots,m_0,\ j = 1,2. \quad (16)$$

If there no operations from $\mathbf{N}^d$ assigned to spindle head or turret of type j at the k^{th} position ($Y^d_{kjl} = 0$, $l = 1,\ldots,b_0$) then $l^d_{kj\max} = l^d_{kj\min} = 0$ due to (15) and (16). If operations from $\mathbf{N}^d$ are executed by machining modules $l_1 = l',\ldots,l_v = l''$ then $l^d_{kj\min} \le l'$ and $l^d_{kj\max} \ge l''$ due to (13) and (14).

Variables T^d_k are defined by constraints:

$$T^d_k \ge F^d-T_0(1-Z_k),\ d = 1,\ldots,d_0,\ k = 1,\ldots,m_0. \quad (17)$$

Then the productivity constraint can be expressed as follows:

$$\sum_{d=1}^{d_0}(F^dO^d + \sum_{k=1}^{m_0} T^d_k - F^d) \le T_0. \quad (18)$$

2.3. Assignment constraints. Each operation is assigned to one block

$$\sum_{k=1}^{m_0}\sum_{j=1}^{2}\sum_{l=1}^{b_0} X_{pkjl} = 1,\ p\in\mathbf{N}'. \quad (19)$$

Each predecessor q of the operation p assigned to the l^{th} machining module of spindle head or turret of type j at the k^{th} position has to be executed at the previous positions or the previous machining module of the corresponding turret

$$\sum_{k'=1}^{k-1}\sum_{j=1}^{2}\sum_{l=1}^{b_0} X_{\chi(q)k'j'l'} + \sum_{l'=1}^{l-1} X_{\chi(q)kjl'} \ge X_{\chi(p)kjl},\ (p,q)\in D^{OR},\ p,q\in\mathbf{N},\ k = 1,\ldots,m_0,\ j = 1,2,\ l = 1,\ldots,b_0. \quad (20)$$

Inclusion and exclusion constraints for working positions, turrets, and machining modules are expressed by (21)–(26).

$$\sum_{j=1}^{2}\sum_{l=1}^{b_0} X_{\chi(p)kjl} = \sum_{j=1}^{2}\sum_{l=1}^{b_0} X_{\chi(q)kjl},\ (p,q)\in E^{SP},\ p,q\in\mathbf{N},\ k = 1,\ldots,m_0, \quad (21)$$

$$\sum_{l=1}^{b_0} X_{\chi(p)kjl} = \sum_{l=1}^{b_0} X_{\chi(q)kjl},\ (p,q)\in E^{ST},\ p,q\in\mathbf{N},\ k = 1,\ldots,m_0,\ j = 1,2, \quad (22)$$

$$\sum_{j=1}^{2}\sum_{l=1}^{b_0} X_{\chi(p)kjl} + \sum_{j=1}^{2}\sum_{l=1}^{b_0} X_{\chi(q)kjl} \le 1,\ (p,q)\in E^{DP},\ p,q\in\mathbf{N},\ k = 1,\ldots,m_0, \quad (23)$$

$$\sum_{l=1}^{b_0} X_{\chi(p)kjl} + \sum_{l=1}^{b_0} X_{\chi(q)kjl} + Y_{kj2} \le 2,\ (p,q)\in E^{DT},\ p,q\in\mathbf{N},\ k = 1,\ldots,m_0,\ j = 1,2, \quad (24)$$

$$X_{\chi(p)kjl}+X_{\chi(q)kjl} \le 1;\ (p,q)\in E^{DM},\ p,q\in\mathbf{N},\ k = 1,\ldots,m_0,\ j = 1,2,\ l = 1,\ldots,b_0. \quad (25)$$

For operations p that cannot be executed in spindle head or turret type j

$$X_{\chi(p)kjl} = 0;\ p\in N^d_s,\ d = 1,\ldots,d_0,\ s = 1,\ldots,n_d,\ k = 1,\ldots,m_0,\ \{h_{rs}(d) = j|\ r = 1,\ldots,r_d\} = \varnothing,\ l = 1,\ldots,b_0. \quad (26)$$

Operations p from N^d_s have to be assigned to the same type of spindle head or turret

$$\sum_{k=1}^{m_0}\sum_{l=1}^{b_0} X_{\chi(p)kjl} = \sum_{k=1}^{m_0}\sum_{l=1}^{b_0} X_{\chi(q)kjl},\ p,q\in N^d_s,\ j = 1,2,\ d = 1,\ldots,d_0,\ s = 1,\ldots,n_d. \quad (27)$$

The following constraints define variables Y_{kjl}^{d}, Y_{kjl} and Y_{j}^{ds}. They take 1 if and only if the corresponding sums are not equal 0.

$$Y_{kjl}^{d} \le \sum_{p\in \mathbf{N}^{d}} X_{\chi(p)kjl},\ d = 1,\dots,d_0,\ k = 1,\dots,m_0,\ j = 1,2,\ l = 1,\dots,b_0, \quad (28)$$

$$\sum_{p\in \mathbf{N}^{d}} X_{\chi(p)kjl} \le |\mathbf{N}^{d}| Y_{kjl}^{d},\ d = 1,\dots,d_0,\ k = 1,\dots,m_0,\ j = 1,2,\ l = 1,\dots,b_0, \quad (29)$$

$$Y_{kjl} \le \sum_{d=1}^{d_0} Y_{kjl}^{d},\ k = 1,\dots,m_0,\ j = 1,2,\ l = 1,\dots,b_0, \quad (30)$$

$$\sum_{d=1}^{d_0} Y_{kjl}^{d} \le d_0 Y_{kjl},\ k = 1,\dots,m_0,\ j = 1,2,\ l = 1,\dots,b_0, \quad (31)$$

$$Y_{j}^{ds} \le \sum_{p\in N_s^{d}} \sum_{k=1}^{m_0} \sum_{l=1}^{b_0} X_{\chi(p)kjl},\ d = 1,\dots,d_0,\ j = 1,2, \quad (32)$$

$$\sum_{p\in N_s^{d}} \sum_{k=1}^{m_0} \sum_{l=1}^{b_0} X_{\chi(p)kjl} \le |N_s^{d}| Y_{j}^{ds},\ d = 1,\dots,d_0,\ j = 1,2. \quad (33)$$

Operations from at most one side for d^{th} part can be assigned to the vertical spindle head or turret

$$\sum_{s=1}^{n_d} Y_{1}^{ds} \le 1,\ d = 1,\dots,d_0. \quad (34)$$

The constraints which prohibit empty machining modules:

$$Y_{kjl-1} \ge Y_{kjl},\ k = 1,\dots,m_0,\ j = 1,2,\ l = 2,\dots,b_0. \quad (35)$$

Vertical turret cannot be located with horizontal tables

$$Y_{k12} + Y_{k21} \le 1,\ k = 1,\dots,m_0. \quad (36)$$

Variables h_r^d can be defined by the following constraints:

$$h_r^{d} \ge 1 - \sum_{s=1}^{n_d} \sum_{j=1,\, j\ne h_{rs}(d)}^{2} Y_{j}^{ds},\ d = 1,\dots,d_0,\ r = 1,\dots,n_d, \quad (37)$$

$$\sum_{r=1}^{r_d} h_r^{d} = 1,\ d = 1,\dots,d_0, \quad (38)$$

$$\sum_{(r,d)\in e} h_r^{d} \le |e| - 1,\ e\in E^{DH}. \quad (39)$$

If the orientation $(h_{r1}(d), h_{r2}(d),\dots,h_{rn_d}(d))$ is chosen for the part d, then variables $Y_{j}^{ds'} = 0$ for $d = 1,\dots,d_0$, $s' = 1,\dots,n_d$, $s' \ne s$, $j = 1,2$, $j \ne h_{rs}(d)$. Therefore $\sum_{s=1}^{n_d} \sum_{j=1,\, j\ne h_{rs}(d)}^{2} Y_{j}^{ds} = 0$, $h_r^d = 1$ due to (37), and $h_{r'}^{d} = 0$ for $r' = 1,\dots,r_d$, $r' \ne r$, due to (38). Constraints (39) forbid to choose infeasible combinations of part orientations.

2.4. Bound constraints.

$$X_{pkjl} \in \{0,1\};\ p\in \mathbf{N}',\ k = 1,\dots,m_0,\ j = 1,2,\ l = 1,\dots,b_0, \quad (40)$$

$$Y_{j}^{ds} \in \{0,1\};\ k = 1,\dots,m_0,\ d = 1,\dots,d_0,\ s = 1,\dots,n_d, \quad (41)$$

$$Y_{kjl}^{d} \in \{0,1\};\ k = 1,\dots,m_0,\ j = 1,2,\ l = 1,\dots,b_0,\ d = 1,\dots,d_0, \quad (42)$$

$$Y_{kjl} \in \{0,1\};\ k = 1,\dots,m_0,\ j = 1,2,\ l = 1,\dots,b_0, \quad (43)$$

$$l_{kj\max}^{d},\ l_{kj\min}^{d} \in \{0,1,\dots,b_0\};\ k = 1,\dots,m_0,\ j = 1,2,\ d = 1,\dots,d_0, \quad (44)$$

$$Y_{1\min}, Y_{1\max} \in \{0,1,..., m_0\};\ k = 1,\dots,m_0,\ j = 1,2,\ l = 1,\dots,b_0, \tag{45}$$

$$Y_1 \in \{0,1\}, \tag{46}$$

$$Z_k \in \{0,1\};\ k = 1,\dots,m_0, \tag{47}$$

$$F_{kjl}^{d} \in [0,\bar{t}^{d} - \tau^{r}];\ k = 1,\dots,m_0,\ j = 1,2,\ l = 1,\dots,b_0,\ d = 1,\dots,d_0, \tag{48}$$

$$F^d \in [\underline{t}^{d},\bar{t}^{d}],\ d = 1,\dots,d_0, \tag{49}$$

where $\underline{t}^{d} = \max\{\lambda(p)/\gamma_2(p)+\tau^a+\tau^r | p\in\mathbf{N}^d\}$ and $\bar{t}^{d} = (T_0 - \sum_{d'=1,d'\neq d}^{d_0} \underline{t}^{d'} O^{d'}) / O^{d}$.

2.5. Estimation of m_0. It is obviously that the number of variables and the number of constraints in model (1)–(49) depends on m_0. This value can be refined as $\lfloor (UB\text{-}LB)/C_1 \rfloor$ where *UB* is an upper bound of the objective function (1) and *LB* is a lower bound of the cost of equipment i.e. the cost of turrets, spindle heads and machining modules needed to accomplish machining of all the parts. For finding *UB* the heuristics [10] can be used. *LB* can be determined in the following way.

Let $\mathbf{H} = \{H = (H(1), H(2),\dots, H(d_0)|H(d)\in\mathbf{H}(d), d=1,2,\dots,d_0\}$ be the set of all possible orientations of parts. Then $LB = \min\{LC_1(H)+LC_2(H)|H\in\mathbf{H}\}$ where $LC_1(H)$ and$LC_2(H)$ are lower bounds on the equipment cost for vertical and horizontal machining with the orientation *H* of parts. They can be estimated as follows:

$$LC_1(H) = \begin{cases} C_4 + C_5(LM_1(H)-1), \text{ if } LM_1(H) > b_0, \\ \min[C_4 + C_5(LM_1(H)-1), C_2 + C_3 LM_1(H)], \text{ otherwise,} \end{cases}$$

$$LC_2(H) = \min[C_4 LM_2(H), C_2\lceil LM_2(H)/b_0\rceil + C_3 LM_2(H)],$$

where $LM_1(H)$ and $LM_2(H)$ are lower bounds on the number of machining modules for vertical and horizontal machining with the orientation *H* of parts. They can be calculated using algorithms [3].

3. Experimental study. The purpose of this study is to evaluate the effectiveness of the proposed techniques. There were generated series of 100 test instances for batches of 4, 6, 8 and 10 parts. Their characteristics are presented in Table 1 where |**N**| is the number of operations, OSP is the order strength of precedence constraints; DM, DT, DP, and SM are the densities of graphs G^{DM}, G^{DT}, G^{DP}, and G^{SM} respectively. Constraints were generated using tools [11]. Experiments were carried out on ASUS notebook (1.86 Ghz, 4 Gb RAM) with academic version of CPLEX 12.2.

Table 1. **Parameters of problems**

Number of parts	Parameters of problems	|N|	OSP	DM	DT	DP	SM
4	Minimal value	44	0.034	0.064	0.026	0	0.027
	Maximal value	95	0.525	0.659	0.659	0.242	0.067
	Average value	69	0.106	0.373	0.348	0.024	0.04
6	Minimal value	89	0.029	0.003	0.002	0	0.024
	Maximal value	159	0.471	0.462	0.462	0.205	0.088
	Average value	124	0.29	0.228	0.197	0.027	0.043
8	Minimal value	118	0.023	0.003	0.002	0	0.024
	Maximal value	216	0.456	0.438	0.417	0.214	0.09
	Average value	165	0.288	0.197	0.168	0.025	0.045
10	Minimal value	251	0.023	0.025	0	0	0.014
	Maximal value	255	0.447	0.58	0.47	0.194	0.071
	Average value	254	0.164	0.326	0.183	0.032	0.024

First, we compare the effectiveness of refinement of m_0 in accordance with section 2.5 for tests with 4 parts. The results are presented in Table 2. With refinement of m_0 the total time for solving all test instances was reduced in 3 times and 49 instances were solved in 20 sec. For one instance, m_0 was not refined and its time solution was greater than without refinement.

Table 2. **Effectiveness of refinement of m_0**

Parameters	$m_0 = 8$	Refined m_0
Minimal time (sec)	10.890	5.321
Maximal time (sec)	2039.800	2101.840
Average time (sec)	173.903	58.399
Total time (sec)	17390.300	5839.910

Then we change the procedure of refinement of m_0. We set m_0 to be equal to the number of positions of heuristic solution. Finally, we present the results of solving 4 series of 100 test instances for 4, 6, 8, and 10 parts with such a refinement. The maximal available time was set to 2 hours (7200 sec). The calculation results are presented in Table 3. Two test instances for 8 parts were not solved optimally during this time (gap is 21 and 27 %). For 10 parts only 35 test instances were solved optimally. Moreover academic version of CPLEX was not capable to solve 54 test instances (out of memory).

Table 3. **Time solution of test instances**

Parameters	4 parts	6 parts	8 parts	10 parts
Minimal time (sec)	3.78	6.98	0.93	15.90
Maximal time (sec)	713.80	7193.40	7200	7200
Average time (sec)	21.23	630.23	1210.97	1185.60
Number of solved problems	100	100	100	46
Number of problems with proven optimality	100	100	98	35

Conclusion. A problem of design of rotary transfer machines has been studied. The problem is to choose the orientation of parts and to assign the manufacturing operations to positions in order to minimize the equipment cost. The improved version of MIP formulation is proposed. Experiments show that the MIP approach is applicable up to 8 parts and 200 operations. Further development will concern the design of machining lines consisting of several rotary transfer machines.

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Информация об авторах

Гущинский Николай Николаевич – кандидат физико-математических наук, заведующий лабораторией исследования операций, Объединенный институт проблем информатики Национальной академии наук Беларуси (ул. Сурганова, 6, г. Минск, 220012, Республика Беларусь). E-mail: gyshin@newman.bas-net.by

Баттайа Ольга – доктор наук, профессор, профессор кафедры проектирования сложных технических систем, Высший институт аэронавтики и космоса (пр. Э. Белин, 10, 31400, г. Тулуза, Франция). E-mail: olga.battaia@isae.fr

Долгий Александр – доктор наук, профессор, заведующий кафедрой автоматики, информатики и управления производством, Горная школа Нанта (ул. Альфреда Кастлера, 4, 20722, г. Нант, Франция). E-mail: alexandre.dolgui@mines_nantes.fr

Information about authors

Guschinsky Nikolai Nikolaevich – Ph. D. (Physics and Mathematics), Head of the Operational Research Laboratory, United Institute of Informatics Problems of the National Academy of Sciences (6, Surganov Str., 220012, Minsk, Republic of Belarus). E-mail: gyshin@newman.bas-net.by

Battaia Olga – Dr. habil., Professor, Professor of the Department of Complex Systems Engineering, Institut Supérieur de l’Aéronautique et de l’Espace (10, Ave. E. Belin, 31400, Toulouse, France). E-mail: olga.battaia@isae.fr

Dolgui Alexadre – Dr. habil., Professor, Head of the Automation, Production and Computer Sciences Department. Ecole des Mines de Nantes (4, rue Alfred Kastler – B.P. 20722, F-44307 NANTES Cedex 3, France). E-mail: alexandre.dolgui@mines_nantes.fr

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