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Alejandro Lavopa

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Alejandro Lavopa[†]

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Abstract

The success of nations in the path towards economic development hinges heavily on the emergence and dynamism of a modern sector capable of simultaneously absorbing an increasing share of the labour force while reducing the technological gap with the world's frontier. Failure to do so would eventually lead the economy to low- or middle- income traps, in which only a small fraction of the population would benefit from the gains of economic growth and technological progress.

Building on previous contributions from Post-Keynesian, Neo-Schumpeterian and Latin-American Structuralism literature, this paper sets up a theoretical model of catching-up among nations aimed at formalizing this idea by exploring the dynamic interactions between structural change and technological upgrading in the process of economic development. The focus of the model is on a "representative" nation of the South that is characterized by having: i) a dual structure (i.e., a large share of labour force working on low-productive-traditional activities that coexists with a small fraction of workers employed in modern activities); ii) a high degree of technological backwardness in the modern activities; and iii) a binding restriction on the external accounts. Under these circumstances, the dynamic behaviour of two key variables will determine the success or failure of this economy over time: the share of labour in the modern sector and the relative stock of technological knowledge of the modern sector compared to that of the world technological leader. Depending on initial conditions and underlying parameters, the southern economy would be attracted towards four different equilibrium points, each of them entailing extremely different implications in terms of long-run development.

After analysing the dynamic properties of the model, simple simulations are implemented in order to illustrate a number of structural trajectories that might shed new light on the complex forces acting behind the success or failure of economic development.

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1. INTRODUCTION

Since the early work of the development pioneers, back in the 1950s, one of the major concerns of the economic development discipline has been related to the ability of the economic system to absorb the whole working population in productive activities. According to these theorists, the dual character of less advanced nations imposed important restrictions on their development potential. In their view, only overcoming that duality through a process of structural change would lead them out of poverty.

More than half a century later, this issue continues to be at the core of policy and academic debates. In many cases, the long expected transformation of poor-rural societies into modern urbanized and industrial economies has left a bitter flavour. Huge urban conglomerations have absorbed increasing number of rural migrants who –instead of finding good-quality jobs in modern industries– ended up enlarging the pool of self-employed and informal workers in service activities. The sectoral structure of many countries has radically changed during these decades, but their dual nature has remained the same: labour markets are still sharply divided between a small fraction of good-quality, highly-paid jobs in modern industry and modern services and a large body of the working population employed in bad-quality, low-income activities, typically informal and in many cases oriented towards subsistence.

From a dual-economy perspective, this concern would actually be reflecting the need for higher rates of labour absorption in the modern sector of the economy. In a globalized world, this goal could hardly be achieved unless the modern sector is international competitive. In fact, it could be argued that the long-run survival of any working opportunity created in that sector would heavily rely upon its capacity to face global competition.

It follows that the ability of the modern sector to absorb labour needs to be evaluated together with its capacity to compete in world markets. This capacity, in turn, would ultimately be shaped by the innovation and technological capabilities of the country. Though price competitiveness might be a suitable mode of entry into international markets, there is today a widespread consensus that this is not a sustainable avenue towards development. Ultimately the factors that really matter for international competitiveness are quality upgrading, quality differentiation and technological change.

The present paper tries to develop this line of reasoning. The old question about creation of productive employment in dual economies is re-examined from a theoretical perspective that acknowledges the fundamental role played by innovation and technological capabilities in the process of development. In order to do so, it presents and evaluates a formal model that, combining elements from *Post-Keynesian*, *Neo-Schumpeterian* and *Latin-American Structuralist* literature, explores the factors that ultimately determine the possibilities of a developing economy to surpass its dual character and enter a path of successful catching up with the advanced economies. In the model, the developing economy is characterized by two distinguishing features. The first feature is the presence of an extended

traditional/informal sector (i.e., a large portion of its labour force is working in extremely low productive activities that use obsolete technologies and mainly produces for its own subsistence). The second feature is the existence of an important degree of technological backwardness in the non-traditional sectors (i.e., a significant technological gap in the modern activities as compared to the advanced economies).

The focus of the model is on the dynamical interaction between both features in the process of economic development. From this perspective, successful development entails a joint improvement in both dimensions up to the point at which modern activities not only become dominant but also manage to catch up with the world frontier. That is, they manage to close the technological gap while absorbing most of the workers from the traditional sector.

The paper is organized as follows. In the next section we briefly summarize the different contributions of the literature that have inspired the model. Next, in Section 3 we present the building blocks of the model and in Section 4 we analyse the dynamic behaviour of the system. In addition, we study the viability conditions and stability properties of the multiple equilibria obtained. Section 5 brings the mathematical formalization to a more tangible ground. By means of simple simulation it presents a set of structural trajectories that might shed light on the various obstacles that developing economies need to overcome before entering on a successful development path. Finally, in Section 6 we present the main conclusions that might be derived from the analysis and several lines along which our model could be extended. A mathematical appendix with some technicalities of the model is included at the end of the paper.

2. LITERATURE REVIEW

The idea that technology plays a central role in the process of economic growth would hardly be disputed by any economist nowadays. The specific ways in which new technologies are translated into the productive process and, more broadly, how their benefits can be reaped at national level, however, are still subject to debate.

A strand of literature that has significantly contributed to our understanding of these phenomena in the context of economic development is the so called *catching-up* literature. This literature originates in a series of contributions[1] that –from an economic historical perspective– investigated the process of catching-up in latecomer countries during the last two centuries, with special focus on the creation and international diffusion of new technologies (Castellacci (2007)). All these scholars share the idea that catching-up, far from being an automatic process, requires important efforts by the follower economies in order to build the necessary capabilities that will ultimately enable them to benefit from the technology created by the leading economy. In terms of Abramovitz (1994), these conditions fall in two broad categories: *technological congruence* and *social capability*. Technological congruence refers to the degree to which the follower and the leader share similar characteristics in terms of, for example, market size and factor supply, and therefore the technologies originating in the advanced economy are relevant or can be applied in the host economy. Social capability, in turn, refers to the capabilities that developing countries need to develop in order to catch-up, such as education and infrastructure. The latter concept has also been associated with the idea of *absorptive capacity*, which in the context of development economics would refer to the ability of developing countries to assimilate new knowledge (Fagerberg et al. (2010)).

Early attempts to formalize this body of ideas within a coherent mathematical framework can be found in Fagerberg (1988a), (1988b) and Verspagen (1991), (1993). The models proposed in these contributions try to explain macroeconomic growth from a catching-up perspective. A technologically backward economy (the follower) has a technology-gap that separates it from a technologically advanced economy (the leader), and the changes over time in that gap ultimately determines the growth potential of the less developed economy. Among other things, the follower can diminish this gap by exploiting its backward position, imitating or using advanced technologies developed by the leader. This process, however, is not costless and depends on the specific degree of technological congruence and social capability of the follower country. Catching up is, therefore, not guaranteed and the follower might even fall further behind.

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[1] Among others, we can mention: Abramovitz (1986); Freeman (1987); Gerschenkron (1962); Veblen (1915). For a recent review of this literature, see Fagerberg and Godinho (2005) and Fagerberg et al. (2010).

More recently, some authors have tried to build a more comprehensive framework, integrating the basic ideas of the catching-up tradition into the macroeconomic setting of *Post-Keynesian* cumulative-causation-type models. Following previous developments in this line, Castellacci (2002) and León-Ledesma (2002) [2] proposed models of catching-up in which demand-side factors play a more fundamental role than in the original contributions of the catching-up tradition. These models assume that growth is demand-led (in particular, export-led) and that there is a dynamic interactive process connecting the growth of aggregate demand and the growth of productivity. This process would emerge from two interrelated mechanisms: on one hand, the so called *Kaldor-Verdoorn* effects of increasing returns to scale that links productivity growth with demand growth [3]. On the other hand, the external causation mechanism that links demand growth with productivity growth through the effect of the latter on price-competitiveness and thus exports [4]. Therefore, an increase of output growth (due to export growth) would induce a higher increase in the growth of productivity that would feed through into lower inflation. This in turn, would improve price competitiveness, allow for higher exports and thus start the process again in a cumulative-causation fashion. Within this framework, catching up, as analysed in the traditional approach, can be retarded or stimulated by demand-side factors.

This sort of models is thus extremely useful to incorporate the complex interactions between supply and demand factors in the process of economic development, in contexts in which technological opportunities might lead to productivity gains that can feed back into higher demand and thus higher gains in productivity. Such a setting bring traditional models of catching-up much closer to the reality of developing economies, where demand factors play a fundamental role in the overall behaviour of the economy. Nevertheless, they still have at least two important limitations when depicting a follower economy that belongs to the developing world.

On one hand, they are based on an export-led framework that does not take into consideration major bottle-necks to aggregate growth, induced by the increase in imported goods that takes place as countries develop. That is, the external balance of payment restriction is not explicitly taken into consideration. In a recent review of *Post-Keynesian* models of aggregate growth, Robert Blecker demonstrates that the equilibrium solution of export-led cumulative-causation (ELCC) models is not sustainable in the long-run precisely because they lack a plausible external restriction (Blecker (2013)). In his view, models of growth based on a balance of payment constraint (BOPC) in the tradition of Thirlwall and Dixon (1979) are better suit to deliver sustainable long-run equilibrium outcomes. It follows that expanding the models proposed in Castellacci (2002) and León-Ledesma (2002) to a context of a BOPC economy would not only provide a picture that is closer to the reality of

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[2] Both authors builds on previous contribution from Amable (1993), De Benedictis (1998) and Targetti and Foti (1997).

[3] Following Boyer (1988), this has been typically labeled "productivity regime".

[4] The "demand regime", in Boyer's terminology.

developing countries but would also enable an equilibrium solution that is actually stable in the long run.

On the other hand –and perhaps more fundamentally– the aggregate nature of these models disregards a salient feature of the developing world, namely, the dual character of their productive structures. When it comes to developing nations, an aggregate view of the country might result in misleading conclusions. A well-documented fact about poor (and to a certain extent, middle-income) nations is the coexistence of modern, highly productive and technological advanced activities alongside low-productive, typically subsistence-oriented, traditional activities. A long strand of literature –ranging from the development pioneers[5] to the *Latin American Structuralists*[6]– has emphasized that each of these sets of activities have their own functioning mechanism and characteristics and they cannot be analysed as a single entity. Moreover, the very concept of structural change has been typically associated with the idea of shifting resources (primarily labour) from the low-productive-traditional sector to the high-productive-modern sector. An explicit consideration of this division in the follower country would therefore lead to a more clear understanding of the role of structural change in the process of catching-up.

In light of the above, recent contributions rooted in the *Neo-Structuralist* tradition have tried to combine the *Post-Keynesian* model of demand-led cumulative-causation growth with a Lewis-type model of dual economy (Cimoli et al. (2005); Ocampo et al. (2009); Rada (2007)). The goal of these models is to formalize the forces behind dynamic structural change, employment and growth in a dual economy with an abundant labour surplus. The economy is divided in two sectors. An established *modern capitalist* or *formal* sector (that typically comprises industry along with parts of agriculture and services) coexists with a *subsistence* or *informal* sector in which production relies only on low-wage labour. The modern sector functions in a *Kaldorian* demand-led cumulative-causation manner. The subsistence sector, instead, has decreasing (or at best, constant) returns to scale, and has an institutionally-based gap with the real incomes of the modern sector[7]. The underlying idea is that labour that is not employed in the modern sector survives by finding some sort of economic activity in the informal sector. This part of the labour force is thus under-employed and constitutes a sort of *reserve army*. Interestingly, these models manage to capture different mechanisms by which demand growth, productivity growth and dynamic structural change are related to each other. Moreover, they

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[5] Obliged references in this regard are Lewis (1954); Ranis and Fei (1961); Sen (1966). For recent reviews on dual models rooted in this tradition see Temple (2005) and Ranis (2012).

[6] The Latin American Structuralism has worked with the related concept of *Structural Heterogeneity*, according to which (in Latin America) there is not a sharp divide between subsistence and modern activities, but rather a continuous of activities with very different levels of technological sophistication. For a recent reviews on this literature see Cimoli (2005) and ECLAC (2012).

[7] In the model proposed by Cimoli et al. (2005) the wage differential between the two sectors arises from an efficiency wage formulation.

address a fundamental issue of development already highlighted in the introduction of this paper: the inability of the economic system to create enough *productive* employment. An important limitation, however, is that they only partially explore the role of innovation and technological catching-up in this process. In particular, neither the dynamics of the technological gap over time nor the possible productivity gains arising from international spillovers are modelled within these formulations. It follows that there is interesting ground to integrate this line of research with the catching-up models previously detailed.

In this paper we try to reconcile these traditions in a single, simplified framework. Our point of departure is a model along the lines of León-Ledesma (2002) and Castellacci (2002), in which the *Post-Keynesian* side is based on a BOPC model rather than a ELCC model. The main extension, however, lies in the characterization of the follower country as a dual economy. Accordingly, we split the developing economy into two sectors and we model the basic interactions among them along the lines proposed in Cimoli et al. (2005), Ocampo et al. (2009) and Rada (2007).

A series of recent papers has built models of similar inspiration. Botta (2009) presents a *structuralist North-South* model of economic convergence, that also incorporates elements from the *Post-Keynesian* and *Post-Schumpeterian* literature. Interestingly, the model distinguishes between manufacturing and non-manufacturing industries and studies the role of industrialization in the process of catching-up. The share of manufacturing in total GDP is used as the key variable representing the productive structure and the movements in time of this endogenous variable determines the growth potential of the *South*. In a similar vein, Cimoli and Porcile (2010) and (2013) present models also rooted in the *Neo-Structuralist* tradition and try to integrate elements from the *Post-Keynesian* and *Post-Schumpeterian* schools within that framework. A distinguishing feature of these models is the use of a multi-sectoral framework –based in Ricardian-type trade models– according to which the productive structure of each economy (*North* and *South*) is represented by a continuum of goods with different technological characteristics. By means of this setting, the models are able to analyse the particular outcomes of certain trade specialization patterns on the growth, productivity and technological potential of the follower economy.

In all these models, however, the dynamic interactions between the degree of duality and the possibilities for catching up are not fully explored. As we will see, the main contribution of our model relies precisely on the examination of this sort of interactions and its evolution along the process of economic development. Hence, our model contributes to the literature that has tried to bring together these related traditions in a coherent framework to analyse the basic problems of development. In the following section we detail the building blocks of this model.

3. THE MODEL

Following the literature previously reviewed, we propose a model in which a developing, technologically-backward economy tries to catch-up with an advanced, technologically-leading economy. The model is written from the perspective of the developing country that is also generically denoted as *the South*, *the follower* or *the domestic economy*. The advanced economy, in turn, is generically denoted as *the North*, *the leader* or *the foreign economy*, and is identified by the superscript f.

Besides being technologically backward, the follower has a dual productive structure. A low-productive traditional sector (identified with the subscript S), that is mainly oriented towards subsistence, coexists with a modern, high-productive, capitalist sector (identified with the subscript M). The model works with the simplifying assumption that each of these sectors produces a unique, homogeneous good. In reality, of course, these sectors are representing a wide set of heterogeneous activities producing very different goods. The traditional sector would typically encompass two broad sets of activities: subsistence agriculture and urban informal services. The remaining sectors (non-subsistence agriculture, industry and formal services) would all be contained within the modern sector aggregate. Although this heterogeneity within each broad sector is not explicitly modelled, it is implicitly captured by the various parameters that define the behaviour of the economy[8].

Since the focus of the model is placed on the dynamic behaviour of the southern economy along time, the setting is built in terms of the growth rates of a number of key variables. In particular, two variables stand out: the share of workers in the modern sector (denoted by the Greek letter λ) and the relative stock of technological knowledge in the modern sector as compared to the leading economy (denoted by the Greek letter ρ). The success or failure of the follower economy will ultimately be determined by the movement in time of these variables. Successful economic development will entail a joint increase in both variables up to the point where the modern sector not only becomes dominant (λ gets closer to one) but also manages to catch up with the technological frontier (ρ gets closer to one). This, however, would not be the only possible outcome of the model. Intermediate situations in which one or both of these variables remain trapped in a low level equilibrium are also possible.

Before entering into the detailed specification of the model, it is worth stressing some of the prime assumptions regarding the functioning of each sector.

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[8] This point will be developed further in the next sections.

<u>Modern sector:</u>

The modern sector is composed by firms that produce for the domestic and external demand and set prices applying a mark-up rule over unit labour cost. Wages in the sector, in turn, are in part determined by the size of the traditional sector that acts as a "reserve army". For this reason, increases in the share of labour in the modern sector will lead to higher pressures on wage inflation that might eventually erode price-competitiveness at the international level.

The technology in use in the modern sector has increasing returns to scale on labour. Productivity gains in this sector also depend on the growth of the technological knowledge applied to production. This knowledge, in turn, is assumed to depend in three major factors (that will be explained in detail later): a) domestic technological efforts; b) international spillovers; and c) induced innovations due to the modernization of the economy.

Finally, the sector is assumed to have a binding restriction on external accounts. That is, the rhythm of production is determined by the availability of foreign exchange. Since it is further assumed that there is no accumulation of financial capital, this implies that exports in this sector should grow at exactly the same rate as imports. Export potential, in turn, is assumed to depend on price and non-price factors. While the first are determined by the dynamics of wages and productivity, the second are directly determined by the level of technological sophistication of the sector, as captured by the technology gap. In particular, it is assumed that the income-elasticity of export demand has an endogenous component that depends negatively on the gap. The greater the distance to the technological frontier, the lower the income elasticity of the products that a country can produce.

<u>Traditional sector:</u>

The traditional sector, in contrast, is assumed to produce for its own demand, using only labour. Therefore, it does not provide any demand-push effect on the modern sector. The level of productivity (and thus, average income) is very low (by definition, lower than in the modern sector), and for this reason the workers of this sector are always willing to move to the modern sector if there are working opportunities there. The technology in use has non-increasing returns to scale. Income per worker will thus tend to increase –or at least, remain constant– when labour is absorbed by the modern sector. Last but not least, it is assumed that there is no interaction between this sector and the foreign economy. That is, there are neither imports nor exports from this sector.

This short summary of the main assumptions sets the ground to specify the detailed setting of the model. We start by describing the functioning of the modern sector, then we describe the functioning of the traditional sector and in the next section we study the dynamic properties of the model.

3.1. Economic Growth in the Modern Sector

Following conventional BOPC growth models, our point of departure to characterize the dynamic functioning of this economy is the external restriction. Ruling out the possibility of financial capital movements, net income payments and unilateral transfers, this restriction states that, in the long run, the value of exports should equal the value of imports[9]. Since the traditional sector is not involved in trade, total imports and total exports in the *South* will be equivalent to the value of exports (X_M) and imports (M_M) of the modern sector. Therefore, the restriction (expressed in domestic prices) can be stated as follow:

$$P_M * X_M = e * P_M^f * M_M \tag{1}$$

where, P stands for prices, e is the nominal exchange rate, the subscript M represents the modern sector and the superscript f represents the foreign economy.

Since our interest relies on the dynamic behaviour of the economy, our focus will be placed in the dynamic version of this restriction, according to which the value of exports should growth at the same rate than the value of imports. Log-differentiation of equation (1) yields:

$$\hat{P}_M + \hat{X}_M = \hat{e} + \hat{P}_M^f + \hat{M}_M \tag{2}$$

where, a *hat* over the variable represents the rate of growth.

Next, we introduce the specific equations for the growth rates in the volume of exports and imports. Following the literature, we assume that both rates are determined by changes in relative prices (real exchange rate) and income. That is,

$$\hat{X}_M = \eta\left(\hat{e} + \hat{P}_M^f - \hat{P}_M\right) + \varepsilon \hat{Z} \tag{3}$$

$$\hat{M}_M = -\nu\left(\hat{e} + \hat{P}_M^f - \hat{P}_M\right) + \chi \hat{Y}_M \tag{4}$$

where, η and ν are the price elasticities of demand for exports and demand for imports respectively; ε and χ are the income elasticities of demand for exports and demand for imports; $\hat{Z}$ stands for the

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[9] Financial flows could be easily incorporated into the analysis. Following Thirlwall and Hussain (1982) an extra term capturing the net capital inflows could be included in equation (1). Since our focus is on the dynamic version of the restriction, in such a case, our implicit assumption would be that these capital flows remain constant in the long run (i.e., there is no explosive accumulation of external debt or international reserves).

growth rate of foreign income, $\hat{Y}_M$ represents the growth rate of output in the modern sector[10] and the term within brackets $(\hat{e} + \hat{P}_M^f - \hat{P}_M)$ provides the rate of real currency depreciation.

Equations (3) and (4) are conventional equations for the growth rate of exports and import demand. Exports will depend positively on real currency depreciation and world income while imports will depend negatively on real currency depreciation and positively on the domestic income of the modern sector.

Making use of equations (3) and (4) in our dynamical external restriction (equation (2)) and solving for the growth rate of modern sector output yields:

$$\hat{Y}_M^{bp} = \frac{\overbrace{(\eta + \nu - 1)}^{a}\left(\hat{e} + \hat{P}_M^f - \hat{P}_M\right) + \varepsilon \hat{Z}}{\chi} = \frac{a\left(\hat{e} + \hat{P}_M^f - \hat{P}_M\right) + \varepsilon \hat{Z}}{\chi} \tag{5}$$

Equation (5) represents the conventional BOP constrained growth rate. It basically states that the output growth of the modern sector will depend positively on price competitiveness (as captured by the term: $a\left(\hat{e} + \hat{P}_M^f - \hat{P}_M\right)$) and foreign income growth (as captured by the term $\varepsilon\hat{Z}$), and negatively to its *appetite* for imports (as captured by the income-elasticity of imports, χ).

If the parameter a is positive, then the *Marshall-Lerner* condition holds and therefore an increase in the rate of real depreciation will put the modern sector on a higher growth path. If, however, $a = 0$ or relative *PPP* is assumed (that is: $\hat{e} + \hat{P}_M^f - \hat{P}_M = 0$), then the first part of the numerator becomes zero and equation (5) boils down to the so called *Thirlwall's law*:

$$\hat{Y}_M^{bp} = \frac{\varepsilon}{\chi}\hat{Z}$$

In this case, only changes in income elasticities or world income growth would have an impact in modern sector growth. Throughout this paper, however, we will assume that the Marshall-Lerner condition holds $(a > 0)$[11] and we will not impose any PPP condition[12]. Therefore output growth in the modern sector will also be affected by changes in price competitiveness.

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[10] It is important to remember that, by construction, we are assuming that the acquisition of foreign goods (import demand) is only done by domestic agents involved in the modern sector. For this reason, the growth of imports is only related to the output growth of the modern sector.

[11] The empirical evidence regarding the validity of this condition is mixed. A detailed recent review can be found in Bahmani et al. (2013). According to their analysis, in 56 of the 91 cases reviewed the condition holds. However, the authors also emphasize that many of these studies would not satisfy further significant tests.

[12] The validity of the PPP condition has also been long debated in the literature. In a recent review, Blecker (2013) concludes that PPP seems to hold only in the very long-run (half century or more) and mainly between countries that are structurally similar. The dynamics that we are interested to analyze in this paper are more likely to take place in periods that are shorter

An important feature of our model is that both elements (income elasticity and price competitiveness) will ultimately depend on the two key variables defined in the introduction of this section: ρ and λ. In what follows we introduce the dynamic equations for the various components that affect the growth rate stated in equation (5) and build a linear system that ultimately depends on these state variables.

To begin with, we endogenize the income elasticity of export demand (ε). Inspired in *Neo-Schumpeterian* and *Neo-Structuralist* literature, we assume that this elasticity depends negatively on the technological gap of the modern sector (G_M). That is:

$$\varepsilon = \xi - \psi G_M \tag{6}$$

where, $\xi > 0$ represents the income-elasticity of the *state-of-art* version of the good exported by the modern sector and $\psi > 0$ represents the penalty on that income-elasticity brought by the technological backwardness of the *South*.

The rationale behind equation (6) is that a certain good can be produced using a wide array of technologies, ranging from *state-of-art* technologies ($G_M = 0$) to technologies that are completely obsolete by international standards ($G_M = 1$). These technologies, in turn, will affect the characteristics of the good. In particular, we assume that the quality and nature of goods that an economy can produce changes as it come closer to the frontier. Since the demand elasticity of high quality and sophisticated goods tends to be higher than that of low quality goods, then a negative relationship can be traced between the income elasticity of exports and the technological gap. In a way, the term ψG_M would be capturing the ability of the country to achieve better quality and higher product differentiation in that particular exporting good.

The technological gap (G_M), in turn, is defined as one minus the relative stock of technological knowledge of the *South* as compared to that of the leading economy (ρ). That is,

$$G_M = 1 - \rho \tag{7}$$

$$\rho = \frac{T_M}{T_M^f} \tag{8}$$

where T stands for the stock of technological knowledge.

Next, we introduce an equation for the dynamic of prices in the modern sector. Following *Post-Keynesian* literature, we assume that prices in this sector are set by adding a mark-up over unit labour cost. The growth rate of prices will thus be represented by:

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than half a century and between very dissimilar countries. Therefore, not assuming PPP seems to be more in line with the available empirical evidence on the issue.

$$\hat{P}_M = \hat{\tau} + \widehat{W}_M - \widehat{R}_M \tag{9}$$

where $\hat{\tau}$ stands for the rate of change in (one plus) the mark-up over unit labour cost, $\widehat{W}_M$ represents the rate of wage inflation in the modern sector and $\widehat{R}_M$ stands for the rate of growth of productivity in the modern sector.

Following most literature on this topic, we assume that the mark-up is exogenously determined (by institutional factors) and constant in the medium/long run. Therefore the rate of change in the mark-up ($\hat{\tau}$) will be equal to zero. Wage inflation in the modern sector ($\widehat{W}_M$), instead, will be endogenous to the model. In particular, it is assumed to depend negatively on the relative size of the traditional sector. As long as the traditional sector is large, wage inflation will be small. However, as the modern sector expands and absorb workers from the traditional sector, pressures for wage increases will start to grow, partly eroding the price competitiveness advantage that the *South* might have at initial levels of development. The following equation captures this dynamical behaviour:

$$\widehat{W}_M = \omega + \theta\lambda \tag{10}$$

where ω represents wage inflation *exogenous* to the model (due to, for example, institutional factors not explicitly modelled) and θ is a positive parameter that captures the sensitivity of wage inflation to modern's sector share on total employment.

Plugging equations (6) to (10) in (5) we get a new expression for the BOPC growth rate of output that depends on our state variables (ρ and λ), the productivity growth of the modern sector and a set of exogenous variables:

$$
\begin{aligned}
\widehat{Y}_M^{bp} &= \frac{a\overbrace{\left(\hat{e} + \hat{P}_M^f - \omega\right)}^{b} - a\theta\lambda + \left[\overbrace{(\xi - \psi)}^{f} + \psi\rho\right]\hat{Z} + a\widehat{R}_M}{\chi} \\
&= \frac{\left(ab + f\hat{Z}\right) + \left(\psi\hat{Z}\right)\rho - (a\theta)\lambda + a\widehat{R}_M}{\chi}
\end{aligned} \tag{11}
$$

where $f = (\xi - \psi)$ stands for the income elasticity of the less technological sophisticated version of the good exported by the modern sector (good produced with $\rho = 0$) and $b = \left(\hat{e} + \hat{P}_M^f - \omega\right)$ encompasses the set of pressures for real exchange depreciation that are *exogenous* to the model[13].

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[13] Since the term b includes the rate of nominal depreciation of the domestic currency ($\hat{e}$), it has a very important role from a policy perspective. The management of the nominal exchange rate by the monetary authority will have an impact on the whole dynamic of the system through this term. Comparative-static exercises at the final section of the paper will illustrate further this point.

Looking at the numerator of Equation (11) we can already notice that the BOPC growth rate of the modern sector depends positively on the relative stock of technology, the growth rate of labour productivity and the set of exogenous variables affecting price-competitiveness and export demand. The share of the modern sector in total employment, instead, has a negative impact and therefore brings a counterbalancing effect that reduces the growth rate of output as the economy develops.

Using equation (11) and bearing in mind that, by definition, the growth rate of employment should be identical to the growth rate of output minus the growth rate of labour productivity ($\hat{L}_M \equiv \hat{Y}_M - \hat{R}_M$), we can derive an expression for the rate of growth of employment in the modern sector that is compatible with the external restriction:

$$\hat{L}_M^{bp} = \frac{(ab + f\hat{Z}) + (\psi\hat{Z})\rho - (a\theta)\lambda - (\chi - a)\hat{R}_M}{\chi} \qquad (12)$$

In order to fully characterize the modern sector, we still need to define the factors that determine productivity growth. Following Castellacci (2002) and León-Ledesma (2002) we assume that productivity gains in this sector are ruled by three main elements: embodied technological progress (proxied by the Investment-Output ratio, k)[14], increasing dynamic returns on labour (the so called *Kaldor-Verdoorn* effect) and improvements in the domestic technological knowledge ($\hat{T}_M$). That is:

$$\hat{R}_M = \mu k + \gamma_M \hat{L}_M + \alpha \hat{T}_M \qquad (13)$$

where μ, γ_M and α are positive parameters that capture the sensitivity of productivity gains to capital intensification, labour growth[15] and domestic technological knowledge growth respectively.

Following the catching-up literature, the growth rate of the stock of technological knowledge is assumed to depend on R&D domestic innovation efforts and on international spillovers stemming from the diffusion of technological knowledge generated by the leader. While the former is taken as exogenous to the model (represented by the parameter ζ), the latter is assumed to depend positively on the size of the technological gap (as captured by the term σG_M)[16]. In addition to these elements, our

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[14] Embodied technological progress might be better captured using more specific variables (accounting, for example, for the quality of the capital stock). Here, however, we preferred to keep the original formulation of the referred models.

[15] The parameter γ_M is equivalent to the traditional *K-V* coefficient on output growth but in terms of employment growth. Equation (13) could also be expressed with increasing dynamic returns on output. In that case, we would have: $\hat{R}_M = \phi_k k + \phi_L \hat{L}_M + \phi_T \hat{T}_M$, which would look very similar to the equations used in Castellacci (2002) and León-Ledesma (2002). Both equations are in fact equivalent, and the relationship between their parameters can be described as follows: $\phi_k = \mu/(1 + \gamma_M)$; $\phi_L = \gamma_M/(1 + \gamma_M)$; $\phi_T = \alpha/(1 + \gamma_M)$. Since our focus in the following sections will be placed on employment absorption, we prefer to express this equation in terms of employment growth rather than output growth. A discussion on the various forms of modelling the *K-V* effect can be found in Pieper (2003).

[16] Given this formulation, the larger is the gap, the larger is the growth rate of technological knowledge due to international spillovers. It is important to highlight that this relationship is assumed to be linear. Previous literature on this issue has also

model also incorporates a *Hicksean* induced-innovation channel in line with recent *Post-Keynesian* models on economic growth (See for example, Naastepad (2005); Palley (2012), (2013)). According to this literature, the rate of growth in the technological knowledge also depends on the availability of resources in the economy, and hence, a positive relationship can be established between the rate of employment and the rate of technological progress. In our model, this channel is represented by a positive relationship between the share of labour in the modern sector and the growth rate of technological knowledge (as captured by the term $\beta\lambda$): the absorption of labour in the modern sector reduces the size of the *reserve army* and therefore induces firms to increase their rates of innovation. The following equation summarizes these channels[17]:

$$\hat{T}_M = \zeta + \beta\lambda + \sigma G_M \tag{14}$$

Combining equations (13) and (14) and replacing G_M for its definition, we get a new expression for the growth rate of productivity in the modern sector:

$$\hat{R}_M = \mu k + \gamma_M \hat{L}_M + \alpha \left[\overbrace{(\zeta + \sigma)}^{d} - \sigma\rho + \beta\lambda \right] = \mu k + \gamma_M \hat{L}_M + \alpha(d - \sigma\rho + \beta\lambda) \tag{15}$$

where $d = (\zeta + \sigma)$ is a positive parameter representing the *autonomous* technological knowledge accumulation in the *South*.

The introduction of a Kaldor-Verdoorn coefficient in equation (13) give rise to the so called "cumulative causation" mechanism according to which an increase in the equilibrium growth rate of employment in equation (12) will lead to an increase in productivity growth that will reinforce the original increase in employment (through the last term of equation (12)) restarting the cycle. This cumulative cycle has been approached in the literature as an interactive process between two regimes: the demand regime (DR) and the productivity regime (PR). Taken together, these regimes constitute a system of two linear equations that in our case would be represented as follows:

$$\hat{L}_M^{DR} = \frac{(ab + f\hat{Z}) + (\psi\hat{Z})\rho - (a\theta)\lambda}{\chi} - \frac{(\chi - a)}{\chi}\hat{R}_M \tag{DR}$$

$$\hat{L}_M^{PR} = -\frac{(\mu k + \alpha d) - \alpha\sigma\rho + \alpha\beta\lambda}{\gamma_M} + \frac{1}{\gamma_M}\hat{R}_M \tag{PR}$$

To be stable, this system requires the (absolute) value of the slope of PR to be larger than the (absolute) value of the slope of DR[18]. That is:

$$\left|\frac{1}{\gamma_M}\right| > \left|-\frac{(\chi - a)}{\chi}\right| \Rightarrow \chi > |(\chi - a)\gamma_M| \tag{Cond. 1}$$

If Condition 1 holds, this setting for the dynamical behaviour of the modern sector will deliver stable equilibrium values for its output, employment and productivity growth that will ultimately depend on the state variables of our model (ρ and λ).

It is interesting to notice that the condition will always hold if γ_M is less than one and χ is larger than a. That is, if the increasing returns of labour are lower than one and the Marshall-Lerner condition is positive but lower than the income-elasticity of imports. Both assumptions seem to be in line with the literature, and therefore, in what follows we will assume that they hold[19]. This will already ensure that, if the dynamic behaviour of the state variables is stable, the final solution of the system will also be stable.

Solving the linear system formed by the DR and PR, and replacing the results in equation (11), we get the equilibrium values for output, productivity and employment growth in the modern sector:

$$\hat{Y}_M^* = \frac{(1 + \gamma_M)(ab + f\hat{Z}) + a(\mu k + \alpha d)}{g} + \frac{(1 + \gamma_M)\psi\hat{Z} - a\alpha\sigma}{g}\rho + \frac{a\alpha\beta - a\theta(1 + \gamma_M)}{g}\lambda \tag{16}$$

$$\hat{R}_M^* = \frac{\gamma_M(ab + f\hat{Z}) + \chi(\mu k + \alpha d)}{g} + \frac{\gamma_M\psi\hat{Z} - \chi\alpha\sigma}{g}\rho + \frac{\chi\alpha\beta - a\theta\gamma_M}{g}\lambda \tag{17}$$

$$\hat{L}_M^* = \frac{(ab + f\hat{Z}) - (\chi - a)(\mu k + \alpha d)}{g} + \frac{\psi\hat{Z} + (\chi - a)\alpha\sigma}{g}\rho - \frac{(\chi - a)\alpha\beta + a\theta}{g}\lambda \tag{18}$$

where $g = \chi + (\chi - a)\gamma_M$

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[18] To see this, we can introduce a lag structure in the equation DR, solve the system and analyze the stability of the equilibrium as if it was a single difference equation. In that case, the *eigenvalue* of the corresponding equilibrium would be: $[-(\chi - a)\gamma_M/\chi]$. It follows that the solution will be stable if and only if the absolute value of this expression is smaller than one. That is, if: $|-(\chi - a)\gamma_M/\chi| < 1$, that is equivalent to the condition stated in the text.

[19] Regarding the first assumption, most studies find a K-V coefficient (in output terms) of about 0.3 to 0.5, that would correspond to a coefficient in terms of employment growth lower than 1. As for the second condition, Wu (2008), for example, provides estimates for a and χ in a sample of 35 countries, and his results show that in 17 of the 23 countries where the Marshall-Lerner condition holds, χ is larger than a.

Since g is equivalent to the numerator of the slope differential between the DR and PR, it will always be positive. Moreover, since we have assumed that $\chi > a$ (in order to ensure that condition 1 holds), we can already notice that our state variables will have counterbalancing effects in the capacity of the modern sector to absorb labour: while increases in ρ will always lead to higher labour demand, increases in λ will always contract the growth rate of labour in the modern sector. This is so because the assumption that χ is larger than a implies that the negative impact of productivity gains on labour absorption cannot be compensated by the positive impact of the increasing demand due to improved price-competitiveness[20].

3.2. Economic Growth in the Traditional Sector

As mentioned before, a distinguishing feature of the southern economy in our model is the presence of a large traditional sector employing an important portion of the labour force. In fact, we work under the simplifying assumption that the labour force not employed in the modern sector finds some way of survival in the traditional sector. Therefore, our model resembles a full-employment model, but with the important remark that a major share of the labour population is under-employed in low-productive activities[21].

The dynamical behaviour of this sector is modelled in an extremely simplified fashion (in line with, for example, Ocampo et al. (2009)). We assume non-increasing returns to scale, with labour as the only input. In addition, we assume an exogenous growth rate in total labour force, and thus, labour growth in this sector is obtained as a residual. Under this setting, the whole dynamics of this sector is driven by the dynamics of labour growth in the modern sector (as stated in equation (18)).

The following equation represents the growth rate of productivity in the traditional sector:

$$\hat{R}_S = \iota + \gamma_S \hat{L}_S \tag{19}$$

where the subscript S identifies the traditional sector, ι is a positive parameter reflecting autonomous productivity gains in this sector and γ_S is the equivalent of γ_M for the traditional sector (that is, the sensitivity of productivity to changes in labour growth). In our setting, we assume that γ_S is non-positive, reflecting the fact that this sector –as opposed to the modern sector– does not benefit from increasing returns to labour.

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[20] To see this we should bear in mind that while productivity gains (by definition) have a one-to-one negative impact on labour growth, the positive impact on output (and thus labour) growth is partially restrained by the relative increase of imports associated with output (and thus income) growth. The positive impact of productivity gains on output growth is, according to equation (11), equal to (a/χ). If $\chi > a$, then this effect will always be lower than one, and therefore the net effect on employment absorption will be negative.

[21] From this perspective, unemployment could also be seen as an extreme case of under-employment with productivity equal to zero. Using this broad definition, unemployed population would also be included within the traditional sector.

By definition, the growth rate of total labour force can be derived as a weighted average of the sectoral growth rates of employment, with weights given by the respective shares on total employment. Bearing this fact in mind and assuming that total labour force grows at an exogenous rate n, then the growth rate of labour in the traditional sector can be obtained as a residual:

$$\hat{L}_S = \frac{n}{(1-\lambda)} - \frac{\lambda}{(1-\lambda)}\hat{L}_M \tag{20}$$

Finally, since by definition output growth should be identical to the sum of productivity and employment growth, combining equations (19) and (20) we can get an expression for the growth rate of output in the traditional sector. This equation, will depend on the growth rate of employment in the modern sector, and thus, on the state variables of our model:

$$\hat{Y}_S = \iota + (1 + \gamma_S)\left[\frac{n}{(1-\lambda)} - \frac{\lambda}{(1-\lambda)}\hat{L}_M\right] \tag{21}$$

As we can see, increases in the labour absorption of the modern sector might have a positive or negative impact on the output growth of the traditional sector depending on the magnitude of the decreasing returns on labour (γ_S). If $|\gamma_S|$ is lower than one, then an acceleration of labour absorption in the modern sector will always have a negative impact on traditional output growth. If, however, these returns are larger than the unity (in absolute terms) then the impact will be positive: the increase of productivity due to the migration of workers to the modern sector more than compensate the reduction of labour force. The last possibility seems quite implausible, and therefore in what follows we will work under the assumption that $|\gamma_S| < 1$[22].

Regardless of the assumptions imposed to this parameter, what really matters is that the dynamics of the traditional sector will ultimately depend on the growth rate of labour in the modern sector, and therefore, on the state variables of our system. To understand the dynamics of the southern economy we need therefore to analyse the dynamical behaviour of ρ and λ.

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[22] It is worth noting, however, that the assumptions imposed to this parameter will not have any relevant impact on the dynamic behaviour of the whole system.

4. DYNAMICAL BEHAVIOUR

According to the setting proposed in the previous section, the dynamic behaviour of the *South* will ultimately depend on the level of the relative stock of technological knowledge (ρ) and the share of the modern sector in total employment (λ). Hence, to examine this behaviour, we need to analyse the movement in time of these state variables. Recalling their definition,

$$\rho = \frac{T_M}{T_M^f} \tag{8}$$

$$\lambda = \frac{L_M}{L} \tag{22}$$

It follows that the growth rate of these variables will be given by:

$$\hat{\rho} = \hat{T}_M - \hat{T}_M^f \tag{23}$$

$$\hat{\lambda} = \hat{L}_M - \hat{L} \tag{24}$$

To close the system we need to specify a dynamic equation for the growth of technological knowledge in the leading economy ($\hat{T}_M^f$). For this purpose we make use of the same specification than in the *South*, but with two important remarks: since, by definition, the technological gap of the leader equals zero, the "catching-up" term disappears. In addition, since in the leading economy the modern sector is already dominant (in an extreme case, would could assume that λ^f equals one), the leader will always get a larger bonus for the *induced innovation* channel than the *South* (unless, of course, the *South* manages to completely catch-up with the *North*). Technological accumulation in the *North* will thus be given by the following expression:

$$\hat{T}_M^f = \zeta^f + \beta^f \lambda^f \tag{25}$$

where ζ^f and β^f are positive parameters capturing the domestic innovation efforts in the leading economy and the sensitivity of technological growth to modern sector labour share.

Plugging equations (14), (18) and (25) into equations (23) and (24), and rearranging terms we get a dynamic system that describes the movements in time of our state variables:

$$\begin{cases} \dot{\rho} = \rho\left(d - \hat{T}_M^f - \sigma\rho + \beta\lambda\right) \\ \dot{\lambda} = \lambda\left[\dfrac{(ab + f\hat{Z}) - (\chi - a)(\mu k + \alpha d) - ng}{g} + \dfrac{\psi\hat{Z} + (\chi - a)\alpha\sigma}{g}\rho - \dfrac{(\chi - a)\alpha\beta + a\theta}{g}\lambda\right] \end{cases}$$

or,

$$\begin{cases} \dot{\rho} = \rho(A + B\rho + C\lambda) & (26) \\ \dot{\lambda} = \lambda(D + E\rho + F\lambda) & (27) \end{cases}$$

In the context of our model, each of these terms has an important economic meaning (see Table 1).

Table 1. *Definition and interpretation of each term in the Dynamic System*

Term	Definition	Economic interpretation
A	$\left(d - \hat{T}_M^f\right)$	*Growth of the relative technological knowledge due to exogenous components (domestic investment in R&D minus technological growth in the frontier), assuming that the South is benefiting from all the potential spill-over from the North.*
B	$(-\sigma)$	*Deceleration in the growth rate of relative technological knowledge due to diminishing advantages of backwardness (decreasing technological gap).*
C	(β)	*Acceleration in the growth rate of relative technological knowledge due to the modernization of the economy (increasing modern sector labour share).*
D	*Growth of the modern sector share in total labour force due to exogenous components. These can be further divided into four terms:*	
	$\dfrac{ab}{g}$	*Labour absorption due to exogenous real exchange depreciation (brought by changes in nominal exchange rate, foreign inflation and/or domestic "institutional" inflation).*
	$\dfrac{f\hat{Z}}{g}$	*Labour absorption due to increases in world income, assuming the minimum potential export elasticity (maximum gap-penalty on income elasticity of exports)*
	$-\dfrac{(\chi - a)(\mu k + \alpha d)}{g}$	*Labour release due to productivity gains resulting from capital intensification and autonomous technological knowledge accumulation (assuming all potential spillovers).*
	$(-n)$	*Deceleration of sector's share growth due to total labour force expansion.*
E	*Acceleration in modern sector's absorption of labour due to increasing technological sophistication (decreasing technological gap). This can be further divided into two terms:*	
	$\dfrac{(\chi - a)\alpha\sigma}{g}$	*Labour absorption due to productivity growth deceleration brought by diminishing advantages of backwardness.*
	$\dfrac{\psi\hat{Z}}{g}$	*Labour absorption due to income elasticity improvements brought by increasing technological sophistication of exports.*
F	*Deceleration of labour absorption due to the modernization of the economy (increasing modern sector labour share). This can be further divided into:*	
	$-\dfrac{a\theta}{g}$	*Labour release due to real appreciation brought by wage-inflationary pressures.*
	$-\dfrac{(\chi - a)\alpha\beta}{g}$	*Labour release due to productivity growth acceleration brought by the modernization of the economy (increasing induced-innovation).*

4.1. Equilibrium points

If we keep the previous assumptions that the Marshall-Lerner condition holds ($a > 0$) but weakly (so that $a < \chi$), and we further assume that the Kaldor-Verdoorn effect is less than one ($\gamma_M < 1$), then we can already determine the signs of all terms except A and D:

$$A \gtreqless 0 \ ; \ B < 0 \ ; \ C > 0 \ ; \ D \gtreqless 0 \ ; \ E > 0 \ ; \ F < 0$$

Bearing this information in mind, we turn now to solve the system, find the potential equilibria and analyse their dynamic properties under different parameter conditions.

To find the equilibrium points (steady states) of the system, we set $\dot{\rho} = 0$ and $\dot{\lambda} = 0$ and solve for ρ and λ. The resulting equilibrium points are detailed in Table 2.

Table 2. *Equilibrium points*

Equilibrium	ρ^*	λ^*
E_1	$\rho_1^* = 0$	$\lambda_1^* = 0$
E_2	$\rho_2^* = -\dfrac{A}{B}$	$\lambda_2^* = 0$
E_3	$\rho_3^* = 0$	$\lambda_3^* = -\dfrac{D}{F}$
E_4	$\rho_4^* = \dfrac{CD - AF}{BF - CE}$	$\lambda_4^* = \dfrac{AE - BD}{BF - CE}$

The last equilibrium is the most interesting. Graphically, this equilibrium is reached in the intersection of the *isoclines* for which $\dot{\rho} = 0$ and $\dot{\lambda} = 0$. That is, the intersection of:

$$\lambda = -\frac{A}{C} - \frac{B}{C}\rho \qquad\qquad (\dot{\rho} = 0)$$

$$\lambda = -\frac{D}{F} - \frac{E}{F}\rho \qquad\qquad (\dot{\lambda} = 0)$$

The remaining equilibria would typically be characterized as being non-meaningful in economic terms and therefore, ignored. In the context of our model, however, these equilibria are also interesting because they can be associated with different stages in the development process. The first equilibrium, for example, would represent a low-income trap, in which the modern sector does not exist and the stock of domestic technological knowledge is negligible as compared with the advanced world. The other two equilibria would represent intermediate stages in which either there is some accumulation of

technological knowledge that has not yet given rise to a modern exporting sector (E_2) or there is a modern exporting sector that uses technologies that are extremely far from the leading economy (E_3). In graphical terms, E_1 is the origin while E_2 is the x-intercept of $\dot{\rho} = 0$ and E_3 is the y-intercept of $\dot{\lambda} = 0$ (See Figure 1).

Figure 1. *Graphical representation of the Equilibria*
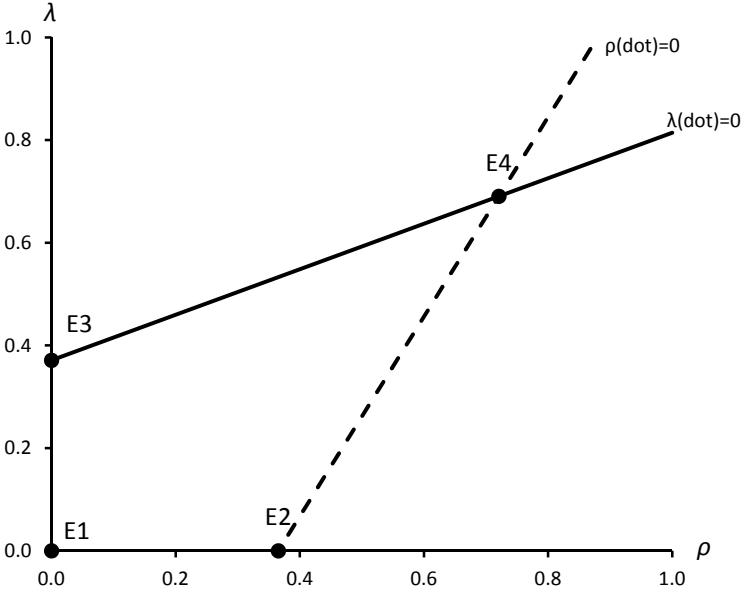
In what follows, our analysis will focus on the *Equilibrium 4*, but looking at the dynamic properties of all the equilibria taken together.

Before entering in the dynamic analysis, we need to establish a set of conditions that ensures the economic viability of *Equilibrium 4*. By definition, ρ and λ should be positive and less or equal than one. Therefore, we need to impose four "viability conditions":

Table 3. *Viability Conditions*

Condition	
VC 1	$\rho_4^* > 0$
VC 2	$\lambda_4^* > 0$
VC 3	$\rho_4^* \leq 1$
VC 4	$\lambda_4^* \leq 1$

The first two viability conditions will be met if and only if the numerator and denominator of ρ_4^* and λ_4^* have the same sign. *VC3* and *VC4*, instead, require that the absolute value of the denominator is larger than the absolute value of the numerator.

In the mathematical appendix we demonstrate that, given the signs of the terms that are already know, these conditions can be met under six different set of restrictions. Each set is defined by a different combination in the sings of the terms *A* and *D* and the sign of the slope differential between the equilibrium curves. The following table summarizes these sets and the particular conditions needed:

Table 4. *Viability Conditions under different restrictions on the signs of A, D and the slope differential*

				(slopes)				
	Sub-Cases					**Viability Conditions**		
	A	*D*	$\dot\rho=0$	$\dot\lambda=0$	*VC1*	*VC2*	*VC3*	*VC4*
Case 1.1:	+	+	$-\dfrac{B}{C}$	$> \quad -\dfrac{E}{F}$	*n.a.*	*n.a.*	$(BF-CE) \geq (CD-AF)$	$(BF-CE) \geq (AE-BD)$
Case 1.2:	+	-	$-\dfrac{B}{C}$	$> \quad -\dfrac{E}{F}$	$(CD-AF) > 0$	$(AE-BD) > 0$	$(BF-CE) \geq (CD-AF)$	$(BF-CE) \geq (AE-BD)$
Case 1.3:	-	+	$-\dfrac{B}{C}$	$> \quad -\dfrac{E}{F}$	$(CD-AF) > 0$	$(AE-BD) > 0$	$(BF-CE) \geq (CD-AF)$	$(BF-CE) \geq (AE-BD)$
Case 2.1:	-	-	$-\dfrac{B}{C}$	$< \quad -\dfrac{E}{F}$	*n.a.*	*n.a.*	$(BF-CE) \leq CD-AF$	$(BF-CE) \leq (AE-BD)$
Case 2.2:	+	-	$-\dfrac{B}{C}$	$< \quad -\dfrac{E}{F}$	$(CD-AF) < 0$	$(AE-BD) < 0$	$(BF-CE) \leq CD-AF$	$(BF-CE) \leq (AE-BD)$
Case 2.3:	-	+	$-\dfrac{B}{C}$	$< \quad -\dfrac{E}{F}$	$(CD-AF) < 0$	$(AE-BD) < 0$	$(BF-CE) \leq CD-AF$	$(BF-CE) \leq (AE-BD)$

In the table we distinguish two broad groups. In the first group (*Case 1*), the slope of $\dot\rho = 0$ is larger than the slope of $\dot\lambda = 0$ (as in the example presented in Figure 1). Within this group three further sub-cases are detailed according to different combinations in the sign of the terms *A* and *D* for which the viability conditions can be met[23]. In the second group, instead, the slope of $\dot\lambda = 0$ is larger. The sub-cases again distinguish different combinations for the signs of *A* and *D*.

The following figure illustrates each of these cases:

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[23] Notice that in this case a scenario in which the terms *A* and *D* are simultaneously negative could never satisfy the viability conditions and thus it is not included in the table.

Figure 2. *Graphical representation of the different cases*
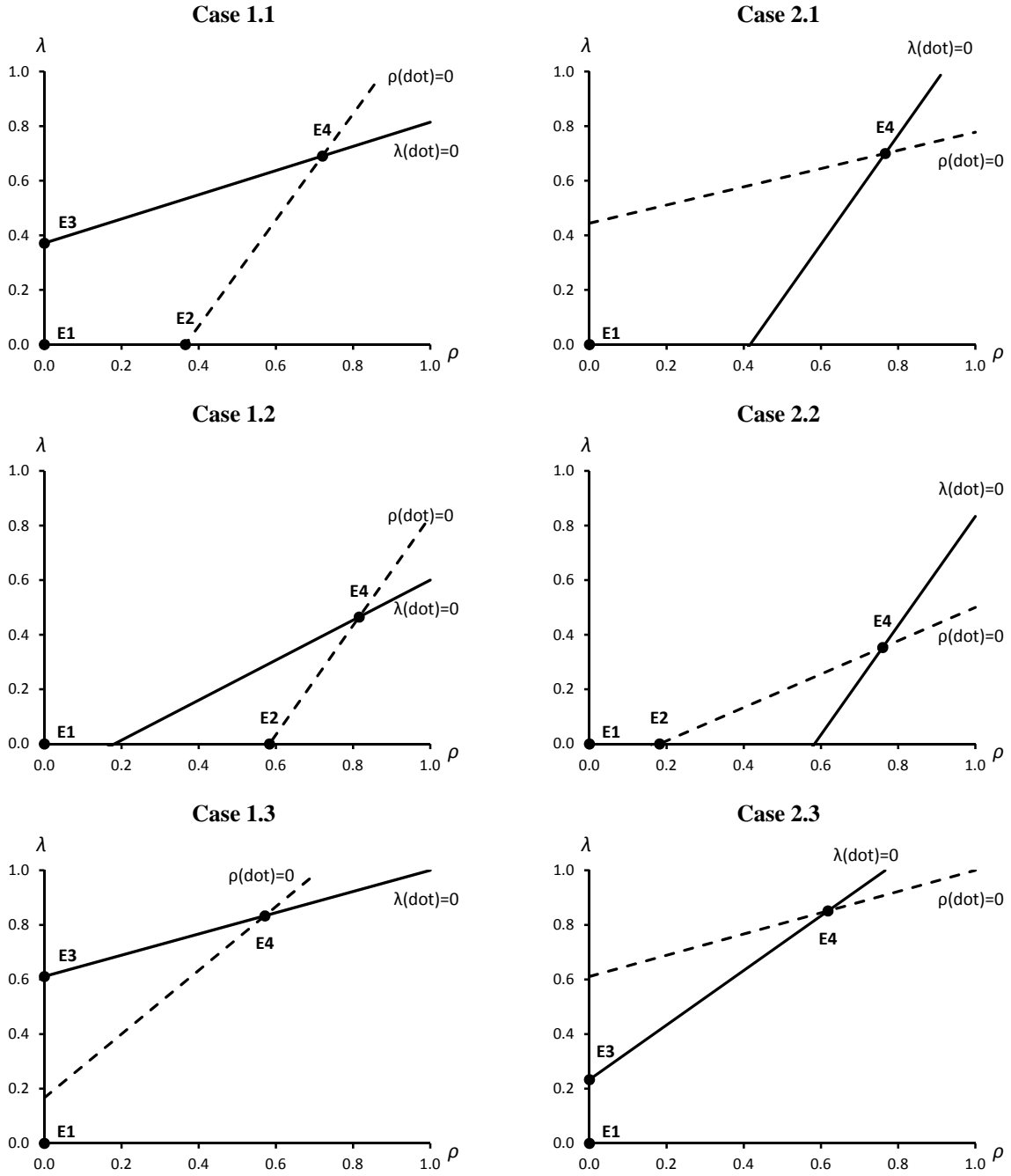
4.2. Dynamic Properties

We turn now to analyse the stability properties of each equilibrium under the different sets of restrictions defined in Table 4. Interestingly, if the viability conditions are met, it can be shown that the stability of the fourth equilibrium will only depend on the sign of the slope differential. If the slope

of $\dot{\rho} = 0$ is larger than the slope of $\dot{\lambda} = 0$ (*Case 1*), then the equilibrium will be stable. Otherwise, it will be a saddle point[24].

The remaining equilibria will never be stable when *Equilibrium 4* is stable. If this equilibrium is not stable, however, at least one of the remaining equilibria will be stable. In particular, in *Case 2.1*, the first equilibrium will be stable, in *Case 2.2* the second equilibrium will be stable and in *Case 2.3*, the third equilibrium will be stable[25]. The following table summarizes these results:

**Table 5.** *Stability properties of the Equilibria under different cases*

Sub-Cases				Equilibria			
	A	D	(slopes) $\dot{\rho}=0$ $\quad$ $\dot{\lambda}=0$	Eq. 1	Eq. 2	Eq. 3	Eq. 4
Case 1.1:	+	+	$-\dfrac{B}{C} > -\dfrac{E}{F}$	Unstable	Saddle	Saddle	**Stable**
Case 1.2:	+	-	$-\dfrac{B}{C} > -\dfrac{E}{F}$	Saddle	Saddle	Unstable	**Stable**
Case 1.3:	-	+	$-\dfrac{B}{C} > -\dfrac{E}{F}$	Saddle	Unstable	Saddle	**Stable**
Case 2.1:	-	-	$-\dfrac{B}{C} < -\dfrac{E}{F}$	**Stable**	Saddle	Saddle	Saddle
Case 2.2:	+	-	$-\dfrac{B}{C} < -\dfrac{E}{F}$	Saddle	**Stable**	Saddle	Saddle
Case 2.3:	-	+	$-\dfrac{B}{C} < -\dfrac{E}{F}$	Saddle	Saddle	**Stable**	Saddle

Another feature worth noting about Table 5 is that in the majority of the cases the non-stable equilibria are actually saddle points. That is, points that exhibit both stability and instability at the same time. In the vicinity of these points there are trajectories moving towards and away from them. However, there is only one trajectory that will lead the economy to that steady state. All other trajectories, even if they move towards that attractor for some time, will eventually turn and move away. The fact that *Equilibrium 4* constitutes a saddle point in *Case 2* (see last column of Table 5) is therefore quite interesting from policy perspective. It means that there might be a feasible trajectory to the *good equilibrium* but it is very difficult to *hit* it. This contrast with the *Case 1*, in which reaching the *good equilibrium* is much easier: it just requires that the economy moves out from the basis of attraction of the other low-income traps. The following figure illustrates these contrasting situations.

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[24] See Mathematical Appendix 8.1 for the formal demonstration.

[25] See Mathematical Appendix 8.2 for the formal demonstration.

Figure 3. *Illustrative trajectories under different stability properties in Equilibrium 4*

A) Saddle Point   *B) Stable Equilibrium*
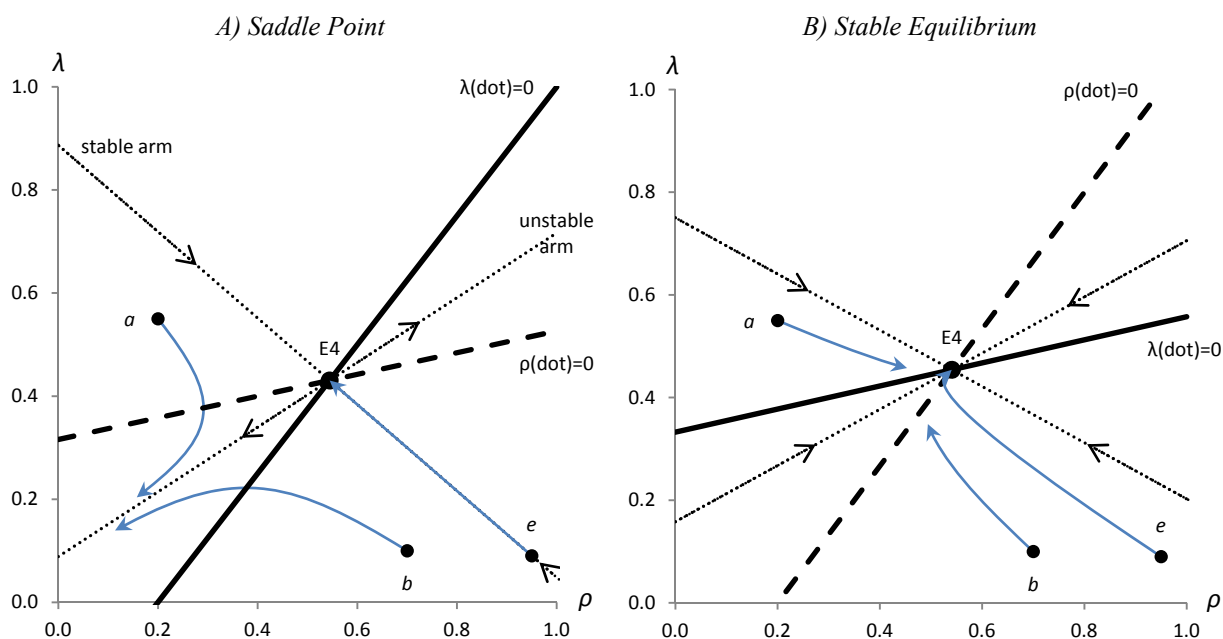
Panel A on Figure 3 illustrates a situation in which the *Equilibrium 4* is a saddle point (Case 2.1). Besides the regular *isoclines* (equations for which the movement of the state variables is zero), the figure also shows the so-called *separatrices* (dotted lines). These lines, which also pass through the steady-state, determine the direction of any trajectory moving in the vicinity of the equilibrium. One of them (the *stable arm*) has arrows pointing towards the equilibrium, while the other (the *unstable arms*) is pointing away from it. Due to these counterbalancing forces, the only way to reach the steady state is to be posited exactly on the stable arm. Any initial condition not situated in this arm, will lead the economy away from the steady state. This is illustrated in the figure by the trajectories starting in points *a* and *b*. As we can see, after initially heading towards the steady state, these trajectories bend and move towards the origin. The trajectory starting at the initial conditions depicted by point *e*, instead, manages to move smoothly towards the steady state. From a policy point of view, this would mean that if the economy starts with a very small modern sector (as represented, for example, in points *b* and *e*) and the *good* equilibrium is a saddle point, this equilibrium will only be reachable if the technological gap is extremely low. Specifically, for a modern sector that employs only 10 per cent of the labour force, the relative technological knowledge should be exactly 95 per cent of that of the leading economy in order to *hit* the stable path towards the equilibrium. Anything smaller will lead the economy towards a low-income poverty trap (as shown by the path starting in *b*).

In clear contrast, Panel B presents a situation in which the *Equilibrium 4* is stable (Case 1.1). In such a case, regardless of the initial conditions the economy will always head towards the steady state. We

can see, therefore, that in this case the trajectories starting with the initial conditions *a* and *b* will also move towards the steady state, significantly reducing the technological gap (trajectory starting in *a*) or expanding the share of the modern sector (trajectory starting in *b*).

4.3. Economic Interpretation

We focus now on the economic meaning of the various conditions that determine the different cases detailed in the previous section. These cases result from different combinations in the signs of the slope differential and the terms *A* and *D*. Recalling the definition of *B*, *C*, *E* and *F* (see Table 1), it is possible to see that the sign of the slope differential will be determined by the following expression:

$$\left(-\frac{B}{C}\right) - \left(-\frac{E}{F}\right) = \frac{(a\theta\sigma - \beta\psi\hat{Z})}{\beta[\alpha\beta(\chi - a) + a\theta]}$$

Under the assumption that $\chi > a$ the denominator will always be positive and the sign of the expression will ultimately be determined by the numerator. This means that *Equilibrium 4* will be stable (*Case 1*) if and only if:

$$a\theta\sigma > \beta\psi\hat{Z}$$

If this condition does not hold, then the equilibrium will be a saddle point (*Case 2*).

In order to get a better understanding of the economic meaning of this condition, we can re-write it as follows:

$$\frac{\sigma}{\beta} > \frac{\psi\hat{Z}}{a\theta} \Rightarrow \frac{(-\sigma)}{\beta} < \frac{\psi\hat{Z}}{(-a\theta)}$$

We can notice that the terms on the left-hand-side are related to the growth rate of technological knowledge, while the terms in the right-hand-side are related to the growth rate of output in the modern sector. In particular, $(-\sigma)$ captures the negative impact of the relative stock of technological knowledge (ρ) on the growth rate of technological knowledge (due to decreasing advantages of backwardness as the gap is reduced) and β captures the positive impact of the share of the modern sector (λ) on the growth rate of technological knowledge (the *induced-innovation* channel). On the other side, $\psi\hat{Z}$ reflects the "direct" positive impact of the relative stock of technological knowledge (ρ) on the growth rate of output in the modern sector[26] and $(-a\theta)$ captures the "direct" negative impact of

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[26] That is, without considering the "indirect" impact of ρ on output growth through relative-price changes brought by its effects on productivity growth.

the share of the modern sector (λ) on the growth rate of output in the modern sector due to wage-inflation[27].

In the light of these definitions, we can see that the condition is actually stating that the (direct) effect of ρ as compared to the (direct) effect of λ should be larger in the growth rate of technological knowledge than in the growth rate of output. This will always be the case if in each equation ($\hat{Y}_M$ and $\hat{T}_M$) the negative effect prevails. That is, if:

$$\sigma > \beta \ \land \ a\theta > \psi\hat{Z}$$

For this to happen, we need that the coefficients σ and a (combined with θ) are sufficiently large. Both coefficients play a key role in our model. The first one is capturing the scope by which the *South* can benefit from international technological spillovers (either in the form of technological transfer or directly via imitation). Since the absorptive capabilities of the *South* are not explicitly modelled in our technological knowledge equation (as it would be the case, for example, if we use a non-linear specification for the spillovers in line with Verspagen (1991), (1993)), these capabilities would be implicitly captured by σ. Increasing absorptive capacity would then increase σ and thus contribute to the stability of *Equilibrium 4*.

The coefficient a, on the other hand, is representing the so-called Marshal-Lerner condition, and therefore is capturing the cumulative effects of changes in the real exchange rate on competitiveness and long-run growth. We can see that the larger the size of this coefficient, the more likely that the equilibrium will be stable. In an extreme case (that we might call "strong Marshal-Lerner"), it is possible to show that if a is larger than χ and the difference ($a - \chi$) is larger than ($\psi\hat{Z}/a\sigma$), then the slope of $\dot{\lambda} = 0$ will be negative and the fourth equilibrium will always be stable. However, a strong Marshal-Lerner condition seems to be against most findings in the literature[28] and therefore we will keep the assumption that a is positive but smaller than χ.

We turn now to analyse the distinction between the sub-cases (1, 2 and 3). As we have already shown, each sub-case will depend on the particular sign of *A* and *D*.

Recalling the definition of *A* (see Table 1), we can see that this term will be positive if and only if:

$$d > h \Rightarrow (\zeta + \sigma) > (\zeta^f + \beta^f \lambda^f)$$

Bearing in mind that ζ^f (R&D investment in the leading economy) will be typically larger than ζ, and that λ^f will be typically close to one, it is possible to assume that A will always be negative at early stages of development. Only an extremely high value for σ could revert this situation (which would require, as we mentioned before, very high levels of domestic absorptive capacities).

The term D, on the other hand, will be positive if:

$$\left(ab + f\hat{Z}\right) > (\chi - a)(\mu k + \alpha d) + ng \Rightarrow$$

$$ab + (\xi - \psi)\hat{Z} > (\chi - a)[\mu k + \alpha(\zeta + \sigma)] + n[\chi + (\chi - a)\gamma_M]$$

That is, it will be positive if the autonomous forces leading to labour absorption in the modern sector (*exogenous* real depreciation and world income growth) more than compensate the autonomous forces leading to labour release (productivity gains resulting from capital intensification and *exogenous* technological accumulation) and the increase in total labour force.

At initial stages of development, D will typically be negative due to the combined effect of high population growth (n) and low income elasticity of demand for exports with poor technological sophistication ($\xi - \psi$). As a country develops, D will typically turn sign and become positive because the negative effect of n diminishes while the effects of capital intensification and exogenous technological accumulation remain low. At a certain stage of development, however, D might turn sign again: if domestic investments in capital and R&D rise significantly and/or absorptive capabilities are significantly improved, while income elasticity remain low (or increase less rapidly), D will become negative again.

As we will see in the next section, the correspondences that can be done between the signs of A and D and the different stages of development will be very useful in understanding, from an economic point of view, the dynamical behaviour of the model proposed.

To finish the characterization of the equilibria in the model, we state now the four viability conditions in terms of A, D and the deep parameters of the model:

$$\beta g D + [a\theta + (\chi - a)\alpha\beta]A > 0 \qquad\qquad \text{(VC 1)}$$

$$\beta[g D - (\chi - a)\alpha\sigma - \psi\hat{Z}] + [a\theta + (\chi - a)\alpha\beta](A - \sigma) \leq 0 \qquad\qquad \text{(VC 3)}$$

$$\sigma g D + [\psi\hat{Z} + (\chi - a)\alpha\sigma]A > 0 \qquad\qquad \text{(VC 2)}$$

$$\sigma[g D - (\chi - a)\alpha\sigma - a\theta] + [\psi\hat{Z} + (\chi - a)\alpha\sigma](A - \beta) \leq 0 \qquad\qquad \text{(VC 4)}$$

As we can see, the viability conditions are actually imposing some lower and upper boundaries to the relative sizes of the terms A and D.
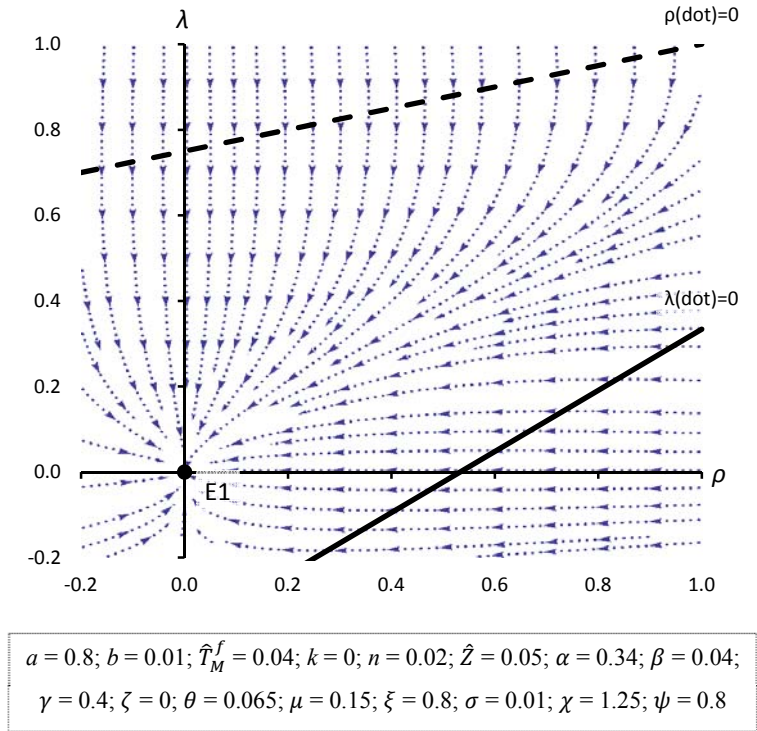
5. TRAJECTORIES AND UNDERDEVELOPMENT TRAPS

5.1. Structural Trajectories

An interesting feature of the model proposed in this paper is that it is able to reproduce, in a very simplified fashion, different structural trajectories which are in line with a long strand of appreciative theorizing on economic development. In order to exploit this feature, we present now a series of simulations that, based on the system form by equations (26) and (27), can illustrate this sort of trajectories and provide some intuition on the general conditions that need to be met in order to achieve success in the process of economic development.

In this regard, it is important to emphasize that the various scenarios defined before by looking at the signs of *A, D* and the slope differential (*Cases 1.1* to *2.3*) can actually be associated with different stages of development, and the multiple equilibria of the model can be associated with different sorts of low-income traps that need to be overcome before entering in a successful path towards economic development. That is, a path in which the economy manages to achieve structural modernization and, at the same time, reduce significantly the technological gap.

At very early stages of development, the economy would be dominated by the traditional sector and the stock of technological knowledge would be negligible when compared with the advanced world. In terms of our model, the economy would be situated very close to the basin of attraction of *Equilibrium 1* and, eventually, it will end up in a poverty trap where ρ and λ will be equal to zero. This equilibrium would be stable and therefore the economy will tend to stay in this trap unless an exogenous shock changes some of the underlying parameters that describe the functioning of the economy. As we have previously seen, *Equilibrium 1* will be stable if and only if the terms *A* and *D* are simultaneously negative. Such a situation can be represented as follows:

Figure 4. *Initial Stage*

$$a = 0.8; \ b = 0.01; \ \hat{T}_M^f = 0.04; \ k = 0; \ n = 0.02; \ \hat{Z} = 0.05; \ \alpha = 0.34; \ \beta = 0.04;$$
$$\gamma = 0.4; \ \zeta = 0; \ \theta = 0.065; \ \mu = 0.15; \ \xi = 0.8; \ \sigma = 0.01; \ \chi = 1.25; \ \psi = 0.8$$
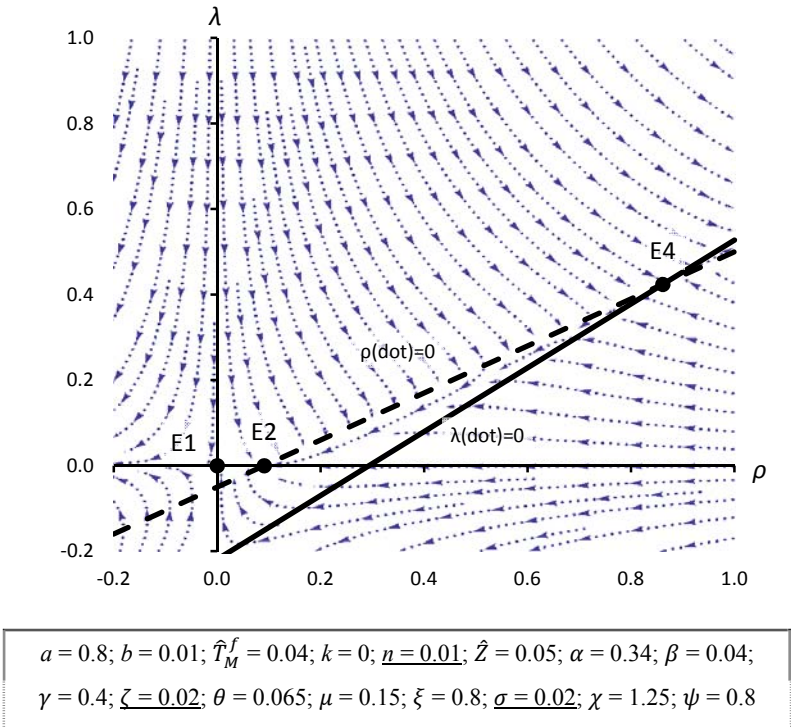
This figure present the usual *isoclines* in the space (ρ ; λ) and the corresponding relevant equilibrium. In addition, it shows the slope field of the system. That is, the direction of all possible trajectories starting from any point in the bi-dimensional plane of the state variables. This field provides a straightforward rule to determine the stability of the various equilibria: if an equilibrium is stable, then in the vicinity of that equilibrium all arrows should point towards it (as in *Equilibrium 1* in Figure 4). Below the figure we also detail the specific parameter-values used to build it.[29]

Under this particular set of parameters, the model illustrates an economy that –at initial levels of technological development– has no foreign demand for its product ($\xi = \psi$ and thus the income elasticity for exports is zero). In such a situation, regardless of how fast the world income is increasing (in the simulation we assume an increase of 5 per cent) the term D will probably be negative. Therefore, the absence of enough external demand will make the existence of a modern sector nonviable. Furthermore, by being at very initial stages of technological development the difference between the autonomous increase in domestic and foreign technology will be so large that the term A will be negative as well. In such a situation (terms A and D being simultaneously negative and the slope differential also being negative) the first equilibrium will be stable and regardless of the initial conditions the economy will end up in this poverty trap.

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[29] All the figures presented in this section have been calculated using *Wolfrand Mathematica 8.0*. The software scripts are available upon request.
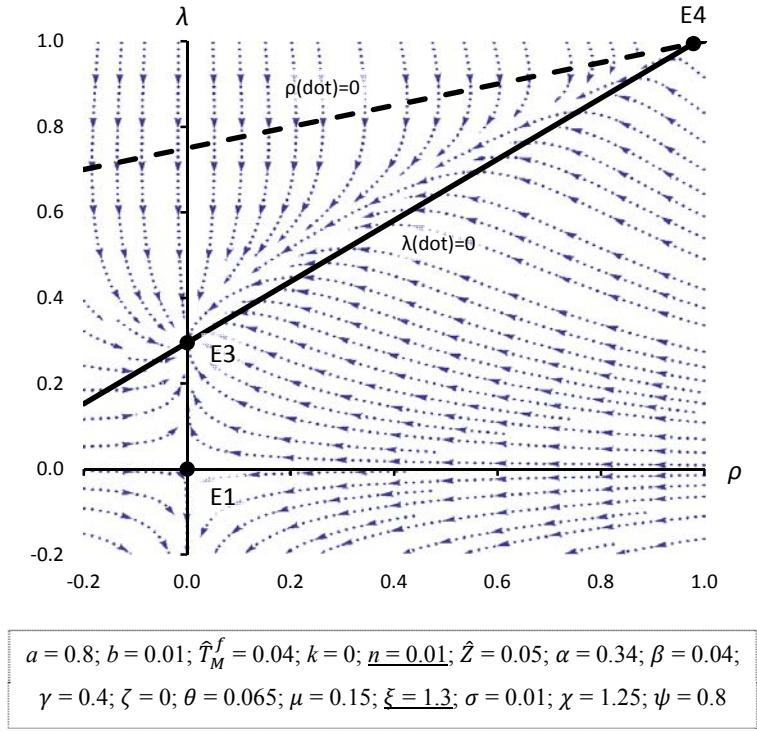
To *escape* from this equilibrium either *A* and/or *D* should become positive. That is, either the domestic investments on technological accumulation and the absorptive capacity of the economy should raise enough to surpass the technological growth of the *North* (thus turning positive the term *A*) and/or the autonomous forces leading to labour absorption in the modern sector should raise up to the point that they more than counterbalance the exogenous growth of total labour force (which would turn positive the term *D*). If the intensity of these changes is not as strong as to revert the instability of the *good* equilibrium, the economy will end up in an intermediate equilibrium that might also be associated with a low (or medium)-income trap. In the first case (increase in domestic technological capabilities) the economy will move towards the *Equilibrium 2* ($\rho_2^* > 0; \lambda_2^* = 0$): the traditional sector will continue to be dominant, but the technological gap will be reduced. In the second case, the economy will move towards the *Equilibrium 3* ($\rho_3^* = 0; \lambda_3^* > 0$): a modern exporting sector will emerge, but using obsolete technologies in international terms. The following figures illustrate each of these scenarios[30].

Figure 5. *Big-Push on technological investments*

$a = 0.8; b = 0.01; \hat{T}_M^f = 0.04; k = 0; \underline{n = 0.01}; \hat{Z} = 0.05; \alpha = 0.34; \beta = 0.04;$
$\gamma = 0.4; \underline{\zeta = 0.02}; \theta = 0.065; \mu = 0.15; \xi = 0.8; \underline{\sigma = 0.02}; \chi = 1.25; \psi = 0.8$

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[30] In each figure, the parameters that have changed with respect to the previous situation are underlined.

Figure 6. *Emergence of a modern exporting sector*

$a = 0.8$; $b = 0.01$; $\hat{T}_M^f = 0.04$; $k = 0$; $\underline{n = 0.01}$; $\hat{Z} = 0.05$; $\alpha = 0.34$; $\beta = 0.04$;

$\gamma = 0.4$; $\zeta = 0$; $\theta = 0.065$; $\mu = 0.15$; $\underline{\xi = 1.3}$; $\sigma = 0.01$; $\chi = 1.25$; $\psi = 0.8$

Starting from the set of parameters that defined Figure 4, in Figure 5 we have simulated a simultaneous increase of ζ and σ (a "big-push" on technological investments) and a decrease in n (the well-known decrease in population growth that goes hand to hand with economic development). In Figure 6, instead, we have simulated an increase of ξ (the autonomous income demand elasticity for exports) together with the decrease of n.

From these potential trajectories, the most reasonable would be the one represented by Figure 6. This trajectory is actually quite in line with the historical path followed by many developing countries, in which the emergence of a modern sector was typically associated with an exogenous shock that provided an export opportunity for the domestic production (in terms of our model, an increase in ξ so that, $\xi > \psi$)[31]. Other things equal, such an event will move the curve $\dot{\lambda} = 0$ upwards eventually turning positive the sign of the term D (and thus the y-intercept of this curve, as shown in Figure 6). This force might also be reinforced by a decrease in the growth rate of total labour force. Both factors will lead to an increasing participation of the modern sector in the absorption of total labour. At early

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[31] Think, for example, in the experience of the East and Southeast Asian economies during the post-war period. *Geostrategic events* (as Richard Stubbs put its) significantly helped these economies to find markets for their emerging manufacturing industries. The Korean war gave a huge boost to Japan's and Hong Kong's exports and, later, the Vietnam war boosted the sales of emerging manufacturing exports from Japan, South Korea, Taiwan and Singapore (Stubbs (1999), (2005)). In terms of our model, these exogenous shocks would have given new export opportunities that, in turn, boosted the emergence and consolidation of a modern exporting sector.
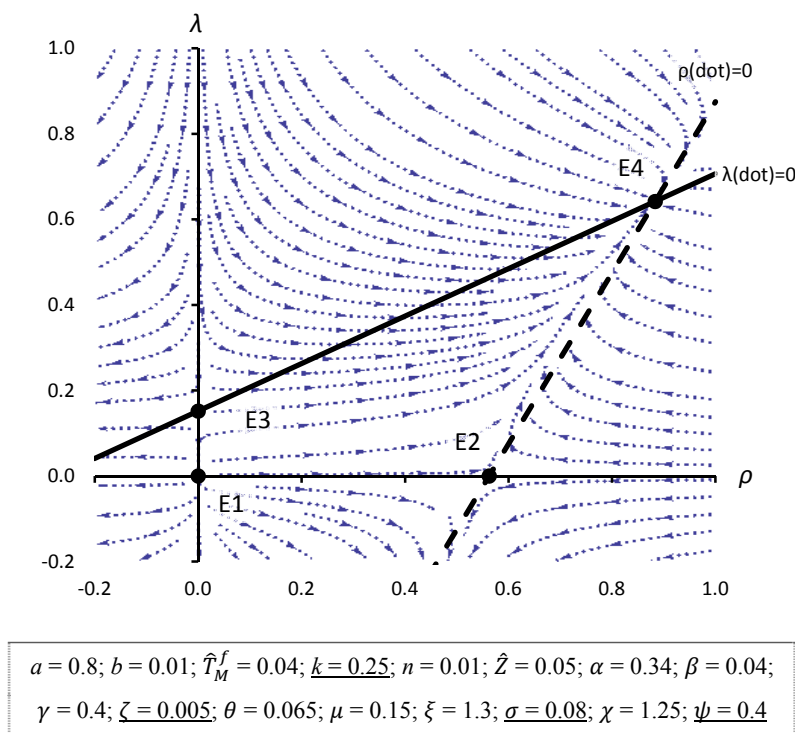
phases of this process, however, it is not surprising that the technology at use is obsolete in international terms, as compared with the world frontier.

The other case, instead, could be associated with a "big-push" in technological investments that succeeds in reducing the technological gap. This *push*, however, is unable to translate the increasing technological capabilities into the creation of a dynamic modern sector. In a way, demand factors (mainly related to a low income elasticity of exports) play against the final outcome of this trajectory. In terms of the figure, this is represented by a movement downwards of the curve $\dot{\rho} = 0$, up to a point in which A becomes positive (and thus the x-intercept of this curve). We can notice now the emergence of a stable equilibrium (the *Equilibrium 2* in our previous discussion) on the horizontal axis.

Once the economy reaches any of these intermediate equilibria, the challenge becomes different. To escape this sort of poverty (or middle)-income traps, the role of absorptive capacities and export performance become fundamental. If, for example, the economy manages to significantly increase the domestic absorptive capacity (or move towards sectors where imitation is easier) and at the same time increases the share of exportable goods with lower gap-punishment on demand (that is, goods with high income-elasticity of demand regardless of the degree of technological sophistication), then it might enter into a dynamic path towards the good equilibrium. This process will typically take place together with an intensification of the capital-output ratio (k) and an increase in R&D expenditures (ζ) that will lead to higher productivity gains and –*ceteris paribus*– would decrease the capacity of the modern sector to absorb labour. Therefore, the increase in exports should be large enough to more than compensate this negative impact in labour absorption.

In terms of our model, starting from Figure 6, this story could be reflected as a simultaneous increase in σ (which would be associated with an increase in the domestic absorptive capacity), k and ζ together with a decrease in ψ (the gap-punishment on income elasticity of exports). Figure 7 shows this new scenario.

Figure 7. *Path towards successful development*

$a = 0.8$; $b = 0.01$; $\hat{T}_M^f = 0.04$; $\underline{k = 0.25}$; $n = 0.01$; $\hat{Z} = 0.05$; $\alpha = 0.34$; $\beta = 0.04$; $\gamma = 0.4$; $\underline{\zeta = 0.005}$; $\theta = 0.065$; $\mu = 0.15$; $\xi = 1.3$; $\underline{\sigma = 0.08}$; $\chi = 1.25$; $\underline{\psi = 0.4}$

As we can see from Figure 7, if there are no further changes in the parameters, the economy will move towards a *good* equilibrium, in which the modern sector becomes dominant ($\lambda = 0.64$) and the technological gap is significantly reduced ($\rho = 0.88$). It follows that the structural trajectory described by the transition from Figure 4 to Figure 6 and then to Figure 7 can be characterized as a story of successful development. Such a trajectory would schematically illustrate the experience of the *East* and *Southeast Asian* economies. As it has been extensively documented by the literature, during the post-war period these economies managed to improve significantly their absorptive capacities and increase their stock of technological knowledge[32]. In parallel –and perhaps more importantly– they also managed to transform they economic structures towards the production of goods with higher degrees of technological sophistication and higher income-elasticity of demand in world markets[33]. That is, they managed to simultaneously achieve a significant increase of σ and a reduction of ψ. In the light of our model, these transformations would have been at the core of their successful developmental path.

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[32] See for example Cimoli et al. (2009); Hobday (1995); Kim and Nelson (2000)

[33] Gouvea and Lima (2010) provide interesting evidence in this regard. Using a multi-sectoral BOPC model they show that the acceleration of growth in the *Asian tigers* (as compared to *Latin America*) was primarily due to their increasing specialization in those goods for which the income-elasticity of exports was higher.

Once the economy manages to undertake the specific transformations needed to ensure the stability of *Equilibrium 4*, the particular attractor towards which it moves will depend on the specific values of the remaining parameters. Hence, it is important to examine how marginal changes in these parameters will impact on the equilibrium values of the state variables.

5.2. Comparative statics

The following table summarizes the impact that changes in the deep parameters of the model would have on the equilibrium values of the state variables in the vicinity of *Equilibrium 4* (assuming that it is stable and economically viable). It presents the sign of the partial derivative of ρ_4^* and λ_4^* with respect to each parameter of the model. Marginal increases of parameters that present a positive (negative) sign will improve (harm) the steady state of the *Southern* economy.

Table 6. *Comparative Statics in the vicinity of Equilibrium 4 (sign of the partial derivatives of $\boldsymbol{\rho_4^*}$ and $\boldsymbol{\lambda_4^*}$ with respect to the deep parameters of the model).*

Parameter	ρ_4^*	λ_4^*
a	? (+)	? (+)
b	+	+
$\hat{T}_M^f$	-	-
k	-	-
n	-	-
$\hat{Z}$	+	+
α	-	-
β	? (+)	+
γ	-	-
ζ	+	+
θ	-	-
μ	-	-
ξ	+	+
σ	+	? (+)
χ	-	-
ψ	-	? (-)

Note: The interrogation mark identifies those cases in which the sign of the derivative is undetermined under the assumptions already defined throughout the model. Between brackets it is specified the sign that these derivatives will take under some additional conditions. See Appendix 8.3 for further details.

Looking at the table, the first feature that stands out is the homogeneity of signs between the state variable. Under the setting proposed, the impact of marginal changes in the parameters will typically

go in the same direction both for ρ_4^* and λ_4^*. Only in very few cases, changes in the values of one parameter might have an opposite impact on the equilibrium values of the state variables[34]. In general terms, all signs are in line with the intuition and the literature on this topic. Increases in the size of the *Marshall-Lerner* condition (a), the exogenous rate of currency depreciation (b), the growth rate of foreign income $(\hat{Z})$ and the autonomous part of the income elasticity (ξ) will all have a positive effect on the growth rate of output and, through this demand channel, on the equilibrium values of the state variables. Improvements on domestic absorptive capacities (σ), local investments in R&D (ζ) and the size of the induced innovation coefficient (β) would also have a positive impact on the equilibrium values, in this case, through their direct effect on the rate of accumulation of technological knowledge and their indirect effect on competitiveness and export demand.

Increases in the remaining parameters would have negative impact on the equilibrium values. Higher income elasticity for imports (χ) would erode the net effects of any force that increases output, thus diminishing labour absorption and technological accumulation in the modern sector. Higher values in the invest-output ratio (k), or the sensitivity of productivity to captialization (μ), in turn, would accelerate productivity gains and lower the rates of labour absorption in the modern sector, decreasing the equilibrium values of ρ_4^* and λ_4^*. The same is true for the dynamic returns to scale parameter (γ_M) and the sensitivity of productivity gains to the technological stock growth (α).

Higher levels of technological accumulation in the leading economy $(\hat{T}_M^f)$ and rapid growth of the total labour force (n) would also tend to diminish the equilibrium values, due to their respectively negative effects on the relative technological stock and the share of the modern sector in total employment. In a similar vein, higher sensitivity of wages in the modern sector to the size of the *reserve army* (θ) would erode price-competitiveness of exports more rapidly and thus impact negatively on the equilibrium. Finally, a higher penalty to technological backwardness in the export elasticity (ψ) will tend to diminish the demand for exports and therefore would also have a negative impact on the equilibrium values.

In the cases of $a, \beta, \sigma,$ and ψ it is important to notice that the direction of the effects might not always be as described above. In the appendix we detail the specific additional conditions needed to get this particular impact. In most cases, these conditions are not very restrictive and thus likely to hold.

The effect of marginal changes in the deep parameters of the model can be illustrated by looking at two phenomena that have attracted special attention in the economic development literature in recent years: the acceleration of global technological change and the movements in the real exchange rates.

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[34] These would be the cases in which the additional conditions for a and β (in the case of ρ_4^*) and σ and ψ (in the case of λ_4^*) are not met. See Appendix 8.3 for the details.

Figure 8 present the case of a developing economy that, starting from an equilibrium position, face simultaneously an acceleration of technological change in the global economy (modelled as an increase in the technological knowledge growth rate of the leading economy, $\hat{T}_M^f$) and an acceleration of the rate of real appreciation of its domestic currency (modelled as a reduction of b, that could either respond to an increase in the rate of autonomous inflation, ω, or a reduction in the rate of nominal depreciation, $\hat{e}$). Other things equal, the first phenomena will make the term A fall and this will be reflected by a movement to the left of the ρ-isocline. In addition, the real appreciation would decrese the term D and therefore the λ-isocline will move downwards, as shown in the figure. The result of this two combined effect will be extremely harmful for the *Southern* economy. As we can see, the new equilibrium *E4'* will entail a significantly lower share of labour in the modern sector and a much wider technological gap.

Figure 8. *Acceleration of global technological change*
and real appreciation of the domestic currency
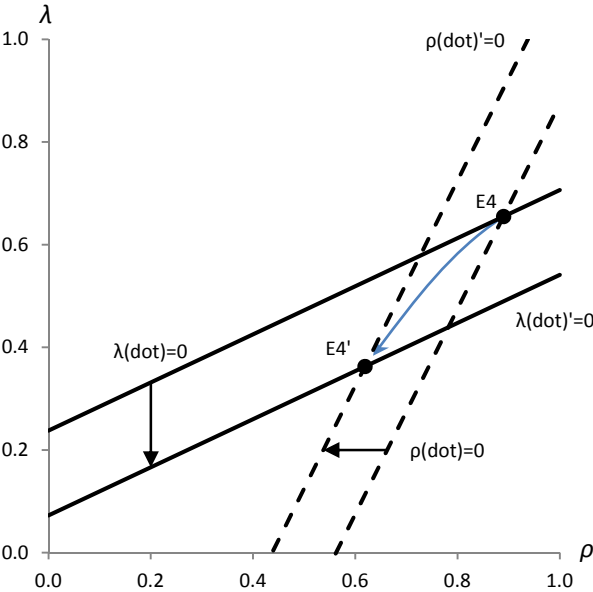
A trajectory as the one presented in Figure 8 could be associated, for example, to the experience of Latin America during the 1980s and 1990s where various episodes of strong real appreciation of domestic currencies (due to high inflation or stabilization policies anchored on appreciated nominal exchange rates) together with a failure to tap into the global acceleration of technological growth brought by the ICT-revolution, dramatically hampered the possibility of these countries to sustain a process of structural modernization and technological catching up.

6. CONCLUSIONS AND FUTURE STEPS

The *productive* absorption of labour has been long identified as one of the major challenges of developing economies. From the development pioneers back in the post-war period to the flagship publications of major international organization nowadays, this has been and continues being in the centre of the development agenda.

Despite its centrality, this issue has not been fully acknowledged in theoretical research about the sources and dynamics of economic growth. This paper tried to make a modest contribution in this line by setting up a model of catching up among nations in which the dual character of developing economies and the important challenges to absorb labour in the modern part of the economy stands out as one of the major features. By examining the dynamic interaction between technological catching-up and structural modernization, the model provides interesting insights on the different structural trajectories that an economy might follow in the process of economic development. Furthermore, it is able to deliver economically meaningful multiple equilibria in a simple linear setting. Hence, it can be easily solved and yields clear traceability of the main forces involved.

Interestingly, the multiple equilibria of the model can be associated with different types of low-income traps that need to be overcome in order to enter in a path towards successful development. Simple simulations of the model illustrated this feature. After surpassing an initial poverty trap –whether due to the emergence of a modern exporting activity or due to some boost in the domestic technological capabilities– the *Southern* economy would typically be attracted towards another *bad* steady state. In order to rectify this tendency, two fundamental transformations would be needed: a radical improvement in the export performance and a radical enhancement in the domestic absorptive capabilities. Failure to achieve any of these transformations will ultimately lead the developing economy to one of the remaining underdevelopment traps.

It remains open, however, the question about how exactly these transformation take place. The successful experience of *East* and *Southeast Asian* economies seems to suggest that a key element in this process is the upgrading of the modern sector towards the production of goods with higher degrees of technological sophistication and higher income-elasticity of demand in world markets. In the simplified framework proposed here this sort of dynamics could not be explicitly modelled. For this reason, an interesting extension of the model would lie in the introduction of a multi-sectoral structure within the modern sector of the *Southern* economy. Such an extension could shed new light in this issue and improve the analytical interpretation of the major results of the model. One way to do so would be to model the modern sector as producing a *continuum* of goods with different technological characteristics, as it is done, for example, in Cimoli and Porcile (2013). Alternatively, the modern sector could be modelled using a dynamic Input-Output framework, in line, for example, with Los and Verspagen (2006).

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A second line in which the model could be extended would be the inclusion of non-linearities in some of its building functional relations. This could significantly improve its capability to depict more closely the reality of certain economic phenomena. One step in this line would consist in introducing a non-linear specification for the international spillovers in the equation of technological knowledge accumulation. Following Verspagen (1991) and (1993), such a setting would capture better the ideas of the original catching-up theorists. Under this setting, the specific role of absorptive capacities and technological congruence at different stages of development could be fully explored. Preliminary simulations in this line give a richer set of possible outcomes when it comes to analyse the structural trajectories followed by the domestic economies. In particular, a non-zero low-level equilibrium also emerge, which might very well depict the reality of emerging economies that have been trapped at middle-income levels.

Another step in this line would be the introduction of a retardation mechanism in the Kaldor-Verdoorn coefficient according to which the size of this parameter would decrease as the economy develops. This is the approach used in Rada (2007), where the *K-V* coefficient depends (in a non-linear fashion) on the share of the modern sector in total employment. Such a setting would yield a more realistic pattern in the growth rate of output as countries become richer. If this coefficient diminishes with the size of the modern sector share, then more advanced economies would have a lower premium in terms of increasing returns to scale and –other things equal– a lower rate of output growth.

It follows that the model proposed can provide a well suited starting point for future research. As we have briefly detailed, some interesting extensions can be built upon the ground set by this model. Nevertheless, it should be emphasized that extending the model in any of these lines will significantly increase its complexity and reduce the intuitive traceability of the main forces involved, which has been one of the major concerns of this paper.

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8. MATHEMATICAL APPENDIX

8.1. Viability Conditions

In this appendix we derive the particular restrictions that need to be imposed to the parameters of the model in order to get a viable equilibrium with non-zero values for the state variables. That is, an equilibrium in which the four viability conditions defined in Section 4.1 are simultaneously satisfied.

As we have previously stated, *VC1* and *VC2* will be satisfied if and only if the numerator and denominator of ρ_4^* and λ_4^* are simultaneously positive or negative. That is:

Table A 1. *Cases in which VC1 and VC2 are satisfied*

Condition	CASE 1	CASE 2
VC 1: $\rho_4^* > 0$	$(BF - CE) > 0 \land (CD - AF) > 0$	$(BF - CE) < 0 \land (CD - AF) < 0$
VC 2: $\lambda_4^* > 0$	$(BF - CE) > 0 \land (AE - BD) > 0$	$(BF - CE) < 0 \land (AE - BD) < 0$

Regardless of the signs, *VC3* and *VC4* will be satisfied if and only if the absolute value of the denominator is larger or equal than the absolute value of the numerator. That is:

Table A 2. *Cases in which VC3 and VC4 are satisfied*

Condition	CASE 1 and 2
VC 3: $\rho_4^* \leq 1$	$|BF - CE| \geq |CD - AF|$
VC 4: $\lambda_4^* \leq 1$	$|BF - CE| \geq |AE - BD|$

Given that *VC1* and *VC2* can be satisfied under two sets of different conditions, we need to analyse each of them separately.

8.1.1. Case 1: positive denominator in Equilibrium 4

To satisfy *VC1* and *VC2* we need that $(CD - AF)$ and $(AE - BD)$ are simultaneously positive.

Bearing in mind that $B < 0, C > 0, E > 0$ and $F < 0$, this will never happen if $A < 0$ and $D < 0$. By the same token, both conditions will always hold if $A > 0$ and $D > 0$. If only one of this terms is negative (either A or D) then we need to explicitly specify the conditions stated before.

The following table summarizes the conditions needed in each sub-case:

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Table A 3. *VC1 and VC2 in Case 1*

Sub-Cases	VC 1	VC 2
Case 1.1: $(BF - CE) > 0 \wedge A > 0 \wedge D > 0$	*n.a.*	*n.a.*
Case 1.2: $(BF - CE) > 0 \wedge A > 0 \wedge D < 0$	$(CD - AF) > 0$	$(AE - BD) > 0$
Case 1.3: $(BF - CE) > 0 \wedge A < 0 \wedge D > 0$	$(CD - AF) > 0$	$(AE - BD) > 0$

To grasp the intuition behind these conditions, it is interesting to re-express them in terms of the slopes and intercepts of the system form by the equilibrium curves, and represented in Figure 1. That is:

Table A 4. *VC 1 and VC 2 in Case 1 (in terms of slopes and intercepts)*

Sub-Cases					VC 1 (y-intercept)		VC 2 (x-intercept)	
	A	D	(slopes) $\dot{\rho}=0$	$\dot{\lambda}=0$	$\dot{\rho}=0$	$\dot{\lambda}=0$	$\dot{\rho}=0$	$\dot{\lambda}=0$
Case 1.1:	+	+	$-\dfrac{B}{C} \quad > \quad -\dfrac{E}{F}$		*n.a.*		*n.a.*	
Case 1.2:	+	-	$-\dfrac{B}{C} \quad > \quad -\dfrac{E}{F}$		$-\dfrac{A}{C} \quad < \quad -\dfrac{D}{F}$		$-\dfrac{A}{B} \quad > \quad -\dfrac{D}{E}$	
Case 1.3:	-	+	$-\dfrac{B}{C} \quad > \quad -\dfrac{E}{F}$		$-\dfrac{A}{C} \quad < \quad -\dfrac{D}{F}$		$-\dfrac{A}{B} \quad > \quad -\dfrac{D}{E}$	

If the slope of $\dot{\rho} = 0$ is larger than the slope of $\dot{\lambda} = 0$, and *A* and *D* are both positive (*Case 1.1*) then conditions *VC1* and *VC2* will always hold. If, on the other hand, either *A* (*Case 1.2*) or *D* (*Case 1.3*) are negative, then we need to impose two additional conditions: that the y-intercept of $\dot{\rho} = 0$ is smaller than the y-intercept of $\dot{\lambda} = 0$ and that the x-intercept of $\dot{\rho} = 0$ is larger than the x-intercept of $\dot{\lambda} = 0$.

In each case, the conditions detailed will ensure that the equilibrium values for the non-zero steady state are positive. To ensure that they are also less or equal than one, we need to impose in addition *VC3* and *VC4*. In terms of the slopes and intercepts of the system these conditions become:

Table A 5. *VC 3 and VC 4 in Case 1, in terms of slopes and intercepts*

Sub-Cases				VC 3		VC 4	
	A	*D*	*(slopes)* $\dot\rho=0$ $\quad$ $\dot\lambda=0$	*(y-int. diff.)*	*(slope diff.)*	*(x-int. diff.)*	*(slope diff.)*
Case 1.1:	+	+	$-\frac{B}{C} > -\frac{E}{F}$	$\left(-\frac{D}{F}\right)-\left(-\frac{A}{C}\right)$ $\leq$	$\left(-\frac{B}{C}\right)-\left(-\frac{E}{F}\right)$	$\left(-\frac{A}{B}\right)-\left(-\frac{D}{E}\right)$ $\leq$	$\left(-\frac{F}{E}\right)-\left(-\frac{C}{B}\right)$
Case 1.2:	+	-	$-\frac{B}{C} > -\frac{E}{F}$	$\left(-\frac{D}{F}\right)-\left(-\frac{A}{C}\right)$ $\leq$	$\left(-\frac{B}{C}\right)-\left(-\frac{E}{F}\right)$	$\left(-\frac{A}{B}\right)-\left(-\frac{D}{E}\right)$ $\leq$	$\left(-\frac{F}{E}\right)-\left(-\frac{C}{B}\right)$
Case 1.3:	-	+	$-\frac{B}{C} > -\frac{E}{F}$	$\left(-\frac{D}{F}\right)-\left(-\frac{A}{C}\right)$ $\leq$	$\left(-\frac{B}{C}\right)-\left(-\frac{E}{F}\right)$	$\left(-\frac{A}{B}\right)-\left(-\frac{D}{E}\right)$ $\leq$	$\left(-\frac{F}{E}\right)-\left(-\frac{C}{B}\right)$

These conditions basically state that the slope differential should be greater than the intercept differentials. Regardless of the sub-case, if the slope differential is sufficiently large then ρ_4^* and λ_4^* will always be less than one.

We already know all the conditions needed to get a viable equilibrium under different assumptions on *A* and *D* for the case in which the slope of $\dot\rho = 0$ is larger than the slope of $\dot\lambda = 0$. We turn now to analyze the second case.

8.1.2. Case 2: negative denominator in Equilibrium 4

Following the same procedure than in the previous case, we arrive to these viability conditions:

Table A 6. *VC 1 and VC 2 in Case 2, in terms of slopes and intercepts*

Sub-Cases				VC 1		VC 2	
	A	*D*	*(slopes)* $\dot\rho=0$ $\quad$ $\dot\lambda=0$	*(y-intercept)* $\dot\rho=0$ $\quad$ $\dot\lambda=0$		*(x-intercept)* $\dot\rho=0$ $\quad$ $\dot\lambda=0$	
Case 2.1:	-	-	$-\frac{B}{C} < -\frac{E}{F}$	*n.a.*		*n.a.*	
Case 2.2:	+	-	$-\frac{B}{C} < -\frac{E}{F}$	$-\frac{A}{C} > -\frac{D}{F}$		$-\frac{A}{B} < -\frac{D}{E}$	
Case 2.3:	-	+	$-\frac{B}{C} < -\frac{E}{F}$	$-\frac{A}{C} > -\frac{D}{F}$		$-\frac{A}{B} < -\frac{D}{E}$	

Note that once the sign of the slope differential is changed, the signs of the remaining conditions change as well. In addition, it is interesting to mention that, in this case, if the terms *A* and *D* are simultaneously negative the equilibrium will always deliver positive values for ρ_4^* and λ_4^*, no matter the magnitude of *A* and *D*.

Turning now to analyse *VC3* and *VC4*, we find the same pattern. Now that the sign of the slope condition has change, the signs of the remaining conditions change as well:

Table A 7. *VC 3 and VC 4 in Case 2, in terms of slopes and intercepts*

Sub-Cases			(slopes)		VC 3		VC 4	
	A	D	$\dot{\rho}=0$	$\dot{\lambda}=0$	*(y-int. diff.)*	*(slope diff.)*	*(x-int. diff.)*	*(slope diff.)*
Case 2.1:	-	-	$-\frac{B}{C}$ <	$-\frac{E}{F}$	$\left(-\frac{D}{F}\right)-\left(-\frac{A}{C}\right) \geq$	$\left(-\frac{B}{C}\right)-\left(-\frac{E}{F}\right)$	$\left(-\frac{A}{B}\right)-\left(-\frac{D}{E}\right) \geq$	$\left(-\frac{F}{E}\right)-\left(-\frac{C}{B}\right)$
Case 2.2:	+	-	$-\frac{B}{C}$ <	$\frac{E}{F}$	$\left(-\frac{D}{F}\right)-\left(-\frac{A}{C}\right) \geq$	$\left(-\frac{B}{C}\right)-\left(-\frac{E}{F}\right)$	$\left(-\frac{A}{B}\right)-\left(-\frac{D}{E}\right) \geq$	$\left(-\frac{F}{E}\right)-\left(-\frac{C}{B}\right)$
Case 2.3:	-	+	$-\frac{B}{C}$ <	$\frac{E}{F}$	$\left(-\frac{D}{F}\right)-\left(-\frac{A}{C}\right) \geq$	$\left(-\frac{B}{C}\right)-\left(-\frac{E}{F}\right)$	$\left(-\frac{A}{B}\right)-\left(-\frac{D}{E}\right) \geq$	$\left(-\frac{F}{E}\right)-\left(-\frac{C}{B}\right)$

8.2. Stability properties

In this appendix we show that under *Case 1 Equilibrium 4* will always be stable and that the first, second and third equilibrium will be stable in *Case 2.1*, *2.2* and *2.3* respectively.

Before analysing each steady state it is important to recall that in a linear system of two dimensions, according to the *Routh-Hurwitz* criterion, an equilibrium point will be stable if the trace of the corresponding *Jacobian* matrix evaluated in that point is negative and the determinant is positive. That is, if:

$$tr(Jac^E) < 0$$

$$det(Jac^E) > 0$$

If, instead, both determinant and trace are positive then the equilibrium will be unstable. Finally, if the determinant is negative, the equilibrium will be a saddle point regardless of the sign of the trace.

In our case, the *Jacobian* of the system formed by equations (26) and (27) is:

$$Jac = \begin{bmatrix} A + 2B\rho + C\lambda & C\rho \\ E\lambda & D + E\rho + 2F\lambda \end{bmatrix}$$

Hence, the trace and determinant are given by the following expressions:

$$tr(Jac) = A + D + 2B\rho + E\rho + C\lambda + 2F\lambda$$

$$det(Jac) = C\lambda(D + 2F\lambda) + A(D + E\rho + 2F\lambda) + 2B\rho(D + E\rho + 2F\lambda)$$

Replacing the equilibrium values in the corresponding expressions of the trace and the determinant of the *Jacobian* of the system yields the following results:

Table A 8. *Trace and Determinant of the Jacobian for each equilibrium*

Equilibrium	Trace	Determinant
E_1	$A + D$	AD
E_2	$-\dfrac{(AE - BD)}{B} - A$	$\dfrac{(AE - BD)}{B}A$
E_3	$-\dfrac{(CD - AF)}{F} - D$	$\dfrac{(CD - AF)}{F}D$
E_4	$\dfrac{B(CD - AF) + F(AE - BD)}{(BF - CE)}$	$\dfrac{-(AE - BD)(CD - AF)}{-(BF - CE)}$

Now, by looking at the expressions of this table and recalling the conditions that defined each of the six sub-cases detailed in the previous appendix, it is possible to determine the sign of the trace and determinant and, therefore, the stability properties of each equilibrium in each sub-case. The following table summarizes these results.

Table A 9. *Trace and Determinant signs and stability properties of the Equilibria in each sub-case*

	Sub-Cases					Equilibria												
			Slopes	*VC1*	*VC2*	*Eq. 1*			*Eq. 2*			*Eq. 3*			*Eq. 4*		
	A	*D*	*(BF-CE)*	*(CD-AF)*	*(AE-BD)*	*Tr*	*Det*	*Stab*	*Tr*	*Det*	*Stab*	*Tr*	*Det*	*Stab*	*Tr*	*Det*	*Stab*
Case 1.1:	+	+	+	+	+	+	+	Unstable	?	-	Saddle	?	-	Saddle	-	+	**Stable**
Case 1.2:	+	-	+	+	+	?	-	Saddle	?	-	Saddle	+	+	Unstable	-	+	**Stable**
Case 1.3:	-	+	+	+	+	?	-	Saddle	+	+	Unstable	?	-	Saddle	-	+	**Stable**
Case 2.1:	-	-	-	-	-	-	+	**Stable**	?	-	Saddle	?	-	Saddle	-	-	Saddle
Case 2.2:	+	-	-	-	-	?	-	Saddle	-	+	**Stable**	?	-	Saddle	-	-	Saddle
Case 2.3:	-	+	-	-	-	?	-	Saddle	?	-	Saddle	-	+	**Stable**	-	-	Saddle

8.3. Comparative statics

In this appendix we present the partial derivatives of the equilibrium values of ρ_4^* and λ_4^* with respect to the deep parameters of the model. In most cases, the sign of these derivatives can be unambiguously determined using the conditions established along the paper. For some parameters, however, the direction of the effect will depend on some additional conditions.

In order to simplify the analysis, we express all derivatives in terms of the capital letters *A, B, C, D, E* and *F* defined in Section 4. This procedure makes much easier the derivation of the corresponding signs. As it is shown in Table A 10 the signs of the partial derivatives of each of these terms with respect to the deep parameters of the model is almost always unambiguously determined and therefore the analysis is significantly reduced.

Working under the assumption that $\chi > a$ and $\gamma < 1$, we already know the signs of four of the six terms ($B < 0; C > 0; E > 0; F < 0$). Moreover, since we are analysing changes in the vicinity of the fourth equilibrium, the slope condition should also hold ($S = a\theta\sigma - z\beta\psi > 0$). Recalling this information and making use of the partial derivatives detailed in Table A 10 we turn now to analyse the partial effects of changes in each of the deep parameters of the model.

8.3.1. Marshall-Lerner condition (a)

$$\frac{\partial \rho_4^*}{\partial a} = \frac{\overset{+}{\rho_4^*}\overbrace{\left(C\frac{\partial E}{\partial a} - B\frac{\partial F}{\partial a}\right)}^{-\chi\theta a + \gamma(z\beta\psi - \chi\theta\sigma) < 0} + \overbrace{\left(C\frac{\partial D}{\partial a} - A\frac{\partial F}{\partial a}\right)}^{?}}{\underset{+}{\underbrace{S}}} \gtreqless 0 \qquad \frac{\partial \lambda_4^*}{\partial a} = \frac{\overset{+}{\lambda_4^*}\overbrace{\left(C\frac{\partial E}{\partial a} - B\frac{\partial F}{\partial a}\right)}^{-\chi\theta a + \gamma(z\beta\psi - \chi\theta\sigma) < 0} + \overbrace{\left(A\frac{\partial E}{\partial a} - D\frac{\partial D}{\partial a}\right)}^{?}}{\underset{+}{\underbrace{S}}} \gtreqless 0$$

The first derivative ($\partial\rho_4^*/\partial a$) will be positive if and only if:

$$\rho_4^* < -\frac{C(\partial D/\partial a) - A(\partial F/\partial a)}{C(\partial E/\partial a) - B(\partial F/\partial a)}$$

Otherwise, it will be negative.

The second derivative ($\partial\lambda_4^*/\partial a$) will be positive if and only if:

$$\lambda_4^* < -\frac{A(\partial E/\partial a) - B(\partial D/\partial a)}{C(\partial E/\partial a) - B(\partial F/\partial a)}$$

Otherwise, it will be negative.

8.3.2. Exogenous rate of currency depreciation (b)

$$\frac{\partial \rho_4^*}{\partial b} = \frac{\overset{+}{C}\ \overset{+}{\frac{\partial D}{\partial b}}}{\underset{+}{\underbrace{S}}} > 0 \qquad\qquad\qquad \frac{\partial \lambda_4^*}{\partial b} = -\frac{\overset{-}{B}\ \overset{+}{\frac{\partial D}{\partial b}}}{\underset{+}{\underbrace{S}}} > 0$$

8.3.3. Technological accumulation in the leading economy $\left(\hat{T}_M^f\right)$

$$\frac{\partial \rho_4^*}{\partial \hat{T}_M^f} = -\underbrace{\frac{\overset{-}{\hat{F}}}{S}}_{+} \overset{-}{\frac{\partial \hat{A}}{\partial \hat{T}_M^f}} < 0 \qquad\qquad \frac{\partial \lambda_4^*}{\partial \hat{T}_M^f} = \underbrace{\frac{\overset{+}{\hat{E}}}{S}}_{+} \overset{-}{\frac{\partial \hat{A}}{\partial \hat{T}_M^f}} < 0$$

8.3.4. Investment-Output ratio (k)

$$\frac{\partial \rho_4^*}{\partial k} = \underbrace{\frac{\overset{+}{\hat{C}}}{S}}_{+} \overset{-}{\frac{\partial \hat{D}}{\partial k}} < 0 \qquad\qquad \frac{\partial \lambda_4^*}{\partial k} = -\underbrace{\frac{\overset{-}{\hat{B}}}{S}}_{+} \overset{-}{\frac{\partial \hat{D}}{\partial k}} < 0$$

8.3.5. Population growth (n)

$$\frac{\partial \rho_4^*}{\partial n} = -\underbrace{\frac{\overset{-}{\hat{F}}}{S}}_{+} \overset{-}{\frac{\partial \hat{A}}{\partial n}} < 0 \qquad\qquad \frac{\partial \lambda_4^*}{\partial n} = \underbrace{\frac{\overset{+}{\hat{E}}}{S}}_{+} \overset{-}{\frac{\partial \hat{A}}{\partial n}} < 0$$

8.3.6. World income $(\hat{Z})$

$$\frac{\partial \rho_4^*}{\partial \hat{Z}} = \underbrace{\frac{\overset{+}{\hat{C}}}{S}}_{+} \left(\overset{+}{\frac{\partial \hat{D}}{\partial \hat{Z}}} + \overset{+}{\rho_4^*} \overset{+}{\frac{\partial \hat{E}}{\partial \hat{Z}}} \right) > 0 \qquad\qquad \frac{\partial \lambda_4^*}{\partial \hat{Z}} = \frac{\overset{+}{\frac{\partial \hat{E}}{\partial \hat{Z}}} \left(\overset{+}{\hat{C}} \overset{+}{\lambda_4^*} + \overset{?}{\hat{A}} \right) - \overset{-}{\hat{B}} \overset{+}{\frac{\partial \hat{D}}{\partial \hat{Z}}}}{\underbrace{S}_{+}} \gtreqless 0$$

The second derivative $\left(\partial \lambda_4^* / \partial \hat{Z}\right)$ will be negative if and only if:

$$\lambda_4^* < \left(-\frac{A}{C}\right) + \overset{-}{\overline{\frac{B}{C} \frac{\partial D/\partial \hat{Z}}{\partial E/\partial \hat{Z}}}}$$

This means that λ_4^* should be smaller than the y-intercept of the curve $\dot{\rho} = 0 \left(-\frac{A}{C}\right)$. However, since both $\hat{\rho} = 0$ and $\hat{\lambda} = 0$ have positive slope, this can never happen. Therefore, the sign of the derivative will always be positive.

8.3.7. Sensitivity of productivity growth to knowledge accumulation (α)

$$\frac{\partial \rho_4^*}{\partial \alpha} = \frac{\rho_4^* \left(C \frac{\partial E}{\partial \alpha} - B \frac{\partial F}{\partial \alpha} \right) + \left(C \frac{\partial D}{\partial \alpha} - A \frac{\partial F}{\partial \alpha} \right)}{S}$$

$$= \frac{\overset{+}{\widehat{h\beta}} \overset{-}{\widehat{(a - \chi)}}}{\underbrace{(a\theta\sigma - z\beta\psi)}_{+}} < 0$$

$$\frac{\partial \lambda_4^*}{\partial \alpha} = \frac{\lambda_4^* \left(C \frac{\partial E}{\partial \alpha} - B \frac{\partial F}{\partial \alpha} \right) + \left(A \frac{\partial E}{\partial \alpha} - D \frac{\partial D}{\partial \alpha} \right)}{S}$$

$$= \frac{\overset{+}{\widehat{h\sigma}} \overset{-}{\widehat{(a - \chi)}}}{\underbrace{(a\theta\sigma - z\beta\psi)}_{+}} < 0$$

8.3.8. Induced innovation coefficient (β)

$$\frac{\partial \rho_4^*}{\partial \beta} = \overset{+}{\rho_4^*}\underbrace{\left(E - B\frac{\partial F}{\partial \beta}\right)}_{(z\psi/g)>0} + \left(\overset{?}{D} - \overset{?}{A}\frac{\overline{\partial F}}{\partial \beta}\right) \gtreqless 0 \qquad\qquad \frac{\partial \lambda_4^*}{\partial \beta} = -\frac{\overset{+}{\lambda_4^*}}{\underset{+}{S}}\left(\overset{-}{\tilde{B}}\frac{\overline{\partial F}}{\partial \beta} - \overset{+}{\tilde{E}}\frac{\overset{+}{\partial C}}{\partial \beta}\right) > 0$$

The first derivative ($\partial \rho_4^*/\partial \beta$) will be positive if and only if:

$$\rho_4^* > \frac{D - A(\partial F/\partial \beta)}{B(\partial F/\partial \beta) - E}$$

Otherwise, it will be negative.

8.3.9. Kaldor-Verdoorn Coefficient (γ)

$$\frac{\partial \rho_4^*}{\partial \gamma} = \frac{\rho_4^*\left(C\frac{\partial E}{\partial \gamma} - B\frac{\partial F}{\partial \gamma}\right) + \left(C\frac{\partial D}{\partial \gamma} - A\frac{\partial F}{\partial \gamma}\right)}{S}$$
$$= \frac{\overset{+}{\overbrace{n\beta}}\,\overset{-}{\overbrace{(a - \chi)}}}{\underbrace{(a\theta\sigma - z\beta\psi)}_{+}} < 0$$

$$\frac{\partial \lambda_4^*}{\partial \gamma} = \frac{\lambda_4^*\left(C\frac{\partial E}{\partial \gamma} - B\frac{\partial F}{\partial \gamma}\right) + \left(A\frac{\partial E}{\partial \gamma} - D\frac{\partial D}{\partial \gamma}\right)}{S}$$
$$= \frac{\overset{+}{\overbrace{n\sigma}}\,\overset{-}{\overbrace{(a - \chi)}}}{\underbrace{(a\theta\sigma - z\beta\psi)}_{+}} < 0$$

8.3.10. Domestic investments in R&D (ζ)

$$\frac{\partial \rho_4^*}{\partial \zeta} = \frac{\overset{+}{C}}{\underset{+}{S}}\frac{\overset{+}{\partial D}}{\partial \zeta} - \frac{\overset{-}{F}}{\underset{+}{S}}\frac{\overset{+}{\partial A}}{\partial \zeta} > 0 \qquad\qquad \frac{\partial \lambda_4^*}{\partial \zeta} = \frac{\overset{+}{E}}{\underset{+}{S}}\frac{\overset{+}{\partial A}}{\partial \zeta} - \frac{\overset{-}{B}}{\underset{+}{S}}\frac{\overset{+}{\partial D}}{\partial \zeta} > 0$$

8.3.11. Sensitivity of wage inflation to structural modernization (θ)

$$\frac{\partial \rho_4^*}{\partial \theta} = -\frac{\overset{-}{\overline{\partial D/\partial \theta}}}{\underset{+}{S}}\left(\overset{?}{\tilde{A}} + \overset{+}{\rho_4^*}\overset{-}{\tilde{B}}\right) \gtreqless 0 \qquad\qquad \frac{\partial \lambda_4^*}{\partial \theta} = -\overset{+}{\lambda_4^*}\frac{\overset{-}{\tilde{B}}}{\underset{+}{S}}\frac{\overset{-}{\overline{\partial F}}}{\partial \theta} < 0$$

The first derivative ($\partial \rho_4^*/\partial \theta$) will be positive if and only if:

$$\rho_4^* < -\frac{A}{B}$$

That is, if ρ_4^* is lower than the x-intercept of the curve $\dot{\rho} = 0$. Once more, since both curves have positive slopes, this will never happen. Therefore, the derivative will always be negative.

8.3.12. Sensitivity of productivity growth to capital intensification (μ)

$$\frac{\partial \rho_4^*}{\partial \mu} = \frac{\overset{+}{C}}{\underset{+}{S}}\frac{\overset{-}{\partial D}}{\partial \mu} < 0 \qquad\qquad \frac{\partial \lambda_4^*}{\partial \mu} = -\frac{\overset{-}{\tilde{B}}}{\underset{+}{S}}\frac{\overset{-}{\partial D}}{\partial \mu} < 0$$

8.3.13. Autonomous income elasticity of exports(ξ)

$$\frac{\partial \rho_4^*}{\partial \xi} = \frac{\rho_4^*\left(C\frac{\partial E}{\partial \xi} - B\frac{\partial F}{\partial \xi}\right) + \left(C\frac{\partial D}{\partial \xi} - A\frac{\partial F}{\partial \xi}\right)}{S}$$

$$= \frac{\overbrace{z\beta}^{+}}{\underbrace{(a\theta\sigma - z\beta\psi)}_{+}} > 0$$

$$\frac{\partial \lambda_4^*}{\partial \xi} = \frac{\lambda_4^*\left(C\frac{\partial E}{\partial \xi} - B\frac{\partial F}{\partial \xi}\right) + \left(A\frac{\partial E}{\partial \xi} - D\frac{\partial D}{\partial \xi}\right)}{S}$$

$$= \frac{\overbrace{z\sigma}^{+}}{\underbrace{(a\theta\sigma - z\beta\psi)}_{+}} > 0$$

8.3.14. Absorptive capacity (σ)

$$\frac{\partial \rho_4^*}{\partial \sigma} = \overbrace{\rho_4^*}^{+}\underbrace{\left(C\frac{\partial E}{\partial \sigma} + F\right)}_{(-a\theta/g)<0} + \underbrace{\left(C\frac{\partial D}{\partial \sigma} - F\right)}_{(a\theta/g)>0} \gtrless 0$$

$$\frac{\partial \lambda_4^*}{\partial \sigma} = \frac{\lambda_4^*\overbrace{\left(C\frac{\partial E}{\partial \sigma} + F\right)}^{(-a\theta/g)<0} + \left(\overset{+}{\hat{E}} + \overset{?}{\hat{D}}\right) + \overbrace{\left(A\frac{\partial E}{\partial \sigma} - B\frac{\partial D}{\partial \sigma}\right)}^{-\frac{\alpha\left(\hat{T}_M^f - \zeta\right)(\chi - a)}{g}<0}}{\underbrace{S}_{+}} \gtrless 0$$

The first derivative ($\partial \rho_4^*/\partial \sigma$) will be positive if and only if:

$$\rho_4^* < -\overbrace{\frac{C(\partial D/\partial \sigma) - F}{C(\partial D/\partial \sigma) + F}}^{1}$$

It follows that the derivative will always be positive unless the domestic economy manages to leap-frog the leading economy ($\rho_4^* > 1$).

The second derivative ($\partial \lambda_4^*/\partial \sigma$) will be positive if and only if:

$$\lambda_4^* < -\frac{(E + D)g - \alpha\left(\hat{T}_M^f - \zeta\right)(\chi - a)}{a\theta}$$

8.3.15. Income-elasticity of imports (χ)

$$\frac{\partial \rho_4^*}{\partial \chi} = \frac{\rho_4^*\left(C\frac{\partial E}{\partial \chi} - B\frac{\partial F}{\partial \chi}\right) + \left(C\frac{\partial D}{\partial \chi} - A\frac{\partial F}{\partial \chi}\right)}{S}$$

$$= \frac{\overbrace{\beta}^{+}\overbrace{\left[n(1 + \gamma) + \alpha\hat{T}_M^f + \mu k\right]}^{+}}{-\underbrace{(a\theta\sigma - z\beta\psi)}_{+}} < 0$$

$$\frac{\partial \lambda_4^*}{\partial \chi} = \frac{\lambda_4^*\left(C\frac{\partial E}{\partial \chi} - B\frac{\partial F}{\partial \chi}\right) + \left(A\frac{\partial E}{\partial \chi} - D\frac{\partial D}{\partial \chi}\right)}{S}$$

$$= \frac{\overbrace{\sigma}^{+}\overbrace{\left[n(1 + \gamma) + \alpha\hat{T}_M^f + \mu k\right]}^{+}}{-\underbrace{(a\theta\sigma - z\beta\psi)}_{+}} < 0$$

8.3.16. Gap punishment on income-elasticity of exports (ψ)

$$\frac{\partial \rho_4^*}{\partial \psi} = \frac{\overset{+}{\hat{C}}}{\underset{+}{\underbrace{S}}}\left(\overset{-}{\frac{\partial \hat{D}}{\partial \psi}} + \overset{+}{\hat{\rho}_4^*}\overset{+}{\frac{\partial \hat{E}}{\partial \psi}}\right) \gtrless 0$$

$$\frac{\partial \lambda_4^*}{\partial \psi} = \frac{\overset{+}{\frac{\partial \hat{E}}{\partial \hat{Z}}}\left(\overset{+}{\hat{C}}\overset{+}{\hat{\lambda}_4^*} + \overset{?}{\hat{A}}\right) - \overset{-}{\hat{B}}\overset{-}{\frac{\partial \hat{D}}{\partial \hat{Z}}}}{\underset{+}{\underbrace{S}}} \gtrless 0$$

The first derivative ($\partial \rho_4^*/\partial \psi$) will be positive if and only if:

$$\rho_4^* > -\frac{\overbrace{\partial D/\partial \psi}^{1}}{\partial E/\partial \psi}$$

It follows that the derivative will always be negative unless the domestic economy manages to leapfrog the leading economy ($\rho_4^* > 1$).

The second derivative ($\partial \lambda_4^*/\partial \psi$) will be positive if and only if:

$$\lambda_4^* > \left(-\frac{A}{C}\right) + \frac{B}{C}\frac{\overbrace{\partial D/\partial \hat{Z}}^{-1}}{\partial E/\partial \hat{Z}} \Rightarrow \lambda_4^* > -\frac{A+B}{C} \Rightarrow \lambda_4^* > \frac{h-\zeta}{\beta}$$

This would happen only at very high levels of ρ_4^* and therefore the sign will most probably be negative.

Table A 10. *Partial derivatives of terms A, B, C, D, E and F with respect to the deep parameters of the model*

Parameter	$\dfrac{\partial A}{\partial i}$	$\dfrac{\partial B}{\partial i}$	$\dfrac{\partial C}{\partial i}$	$\dfrac{\partial D}{\partial i}$	$\dfrac{\partial E}{\partial i}$	$\dfrac{\partial F}{\partial i}$
a	0	0	0	$\dfrac{[b(1+\gamma)+\mu k+\alpha(\zeta+\sigma)]\chi+\gamma(\xi-\psi)\hat{Z}}{[(\chi-a)\gamma+\chi]^2}>0$	$\dfrac{\gamma\psi\hat{Z}-\chi\alpha\sigma}{[(\chi-a)\gamma+\chi]^2}\lessgtr 0$	$\dfrac{[\alpha\beta-(1+\gamma)\theta]\chi}{[(\chi-a)\gamma+\chi]^2}\lessgtr 0$
b	0	0	0	$\dfrac{a}{(\chi-a)\gamma+\chi}>0$	0	0
$\hat{T}_M^f$	$-1<0$	0	0	0	0	0
k	0	0	0	$-\dfrac{(\chi-a)\mu}{(\chi-a)\gamma+\chi}<0$	0	0
n	0	0	0	$-1<0$	0	0
$\hat{Z}$	0	0	0	$\dfrac{(\xi-\psi)}{(\chi-a)\gamma+\chi}>0$	$\dfrac{\psi}{(\chi-a)\gamma+\chi}>0$	0
α	0	0	0	$-\dfrac{(\zeta+\sigma)(\chi-a)}{(\chi-a)\gamma+\chi}<0$	$\dfrac{\sigma(\chi-a)}{(\chi-a)\gamma+\chi}>0$	$-\dfrac{\beta(\chi-a)}{(\chi-a)\gamma+\chi}<0$
β	0	0	$1>0$	0	0	$-\dfrac{\alpha(\chi-a)}{(\chi-a)\gamma+\chi}<0$
γ	0	0	0	$-\dfrac{(\chi-a)\{ab+[\mu k+\alpha(\zeta+\sigma)](a-\chi)+(\xi-\psi)\hat{Z}\}}{[(\chi-a)\gamma+\chi]^2}<0$	$\dfrac{(a-\chi)[\alpha\sigma(\chi-a)+\psi\hat{Z}]}{[(\chi-a)\gamma+\chi]^2}>0$	$\dfrac{(\chi-a)[\alpha\beta(\chi-a)+a\theta]}{[(\chi-a)\gamma+\chi]^2}>0$
ζ	$1>0$	0	0	$-\dfrac{\alpha(\chi-a)}{(\chi-a)\gamma+\chi}<0$	0	0
θ	0	0	0	0	0	$-\dfrac{a}{(\chi-a)\gamma+\chi}<0$
μ	0	0	0	$-\dfrac{(\chi-a)k}{(\chi-a)\gamma+\chi}<0$	0	0
ξ	0	0	0	$\dfrac{\hat{Z}}{(\chi-a)\gamma+\chi}>0$	0	0
σ	$1>0$	$1>0$	0	$-\dfrac{\alpha(\chi-a)}{(\chi-a)\gamma+\chi}<0$	$\dfrac{\alpha(\chi-a)}{(\chi-a)\gamma+\chi}>0$	0
χ	0	0	0	$-\dfrac{[b(1+\gamma)+\mu k+\alpha(\zeta+\sigma)]a+(1+\gamma)(\xi-\psi)\hat{Z}}{[(\chi-a)\gamma+\chi]^2}<0$	$\dfrac{a\alpha\sigma-(1-\gamma)\psi\hat{Z}}{[(\chi-a)\gamma+\chi]^2}\lessgtr 0$	$\dfrac{a[(1+\gamma)\theta-\alpha\beta]}{[(\chi-a)\gamma+\chi]^2}\lessgtr 0$
ψ	0	0	0	$-\dfrac{\hat{Z}}{(\chi-a)\gamma+\chi}<0$	$\dfrac{\hat{Z}}{(\chi-a)\gamma+\chi}>0$	0

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