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RELIABILITY ANALYSIS OF A STEEL BEAM USING THE MONTE CARLO METHOD

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Abstract. *This paper aims to show the feasibility of structural analysis in steel beams, based on the precepts of reliability. We assessed the reliability and security of a steel I-beam profile (I 254 (10'') x 37,7), MR250, subject to an applied bending moment. The purpose was to evaluate the appropriateness of the component in handling specific project stresses. First we provide a dimensioning analysis based on Brazilian structural standards and then a verification of the beam's relative safety, in terms of the reliability index* β. *The adopted failure function is related to gross and net areas of the flange, submitted to traction stresses. The system's reliability index, constituted by the failure function is also determined. Considering the statistical results, the failure rate in the structure, demonstrates that the solicited project loadings are sustained, also determining the capacity of the structure to exceed the applied load, while maintaining structural safety of the steel beam. In the reliability analysis, the use of randomized integer generation, assisted by computational resources (Mathcad), makes it feasible to infinitely test the metallic structure. The Monte Carlo method was used, based on determined probability distributions (involved variables), to obtain the probability of structural failure. The random variables used in the reliability analysis, are delineated by the Joint Committee for Structural Safety (JCSS).*

Keywords: *probabilistic design, I-beam, Monte Carlo Method, structural reliability analysis.*

1 INTRODUCTION

The design of steel structures, according to the Brazilian technical standards (NBR 8800/08, *Design of steel structures and composite structures of steel and concrete*), are based on semi-probabilistic analytical methods. The semi-probabilistic method uses limit state design, defined by the random variables of the problem (Shooman, 1968). The overestimations made (both in terms of stresses as in structural design) are intended to increase structural safety, inputting values that would be probabilistically unlikely. This increase, weighting factor, seeks, through the study of improbable events, to reduce the failure rate associated with the current limit state. The procedure mentioned above does not permit quantification of the probability of structural failure. Thus, traditional sizing and designing (using deterministic as well as semi-probabilistic methods), do not consider the global security of the analyzed structure leading to continuously structural oversizing, increased costs and inflated budgets, as well as deficiencies, whilst not significantly decreasing the risk of systemic failure. The reliability analysis of structures enables the determination of the probability of deterioration or collapse associated with a limit state, as well as providing data for a more coherent design with such perspectives (Papadrakakis, 1996).

According to Pinto (2004), reliability functions return the probability of a performing system or component failing during a specific period of time (estimated design working life). The reliability function represents a downward curve, showing regular wear as well as structural fatigue. Also, reliability functions are generated from statistical distributions associated with collapse (exponential, Weibull, lognormal, Poisson, etc.).

As a proposed case study, a steel I-beam subjected to bending moment (foreseen in the project), is evaluated based on reliability standards. Initially, the beam’s performance while undergoing the proposed loading is analyzed in order to verify its adequacy in terms of project requirements. Hence, according to the semi-probabilistic method, the occurrence of deficiencies, failure rate, is considered using reliability analysis methods. The beam is tested, based on failure rate equations demonstrated in the works of M. and W. Pfeil (2009) (proposed by NBR 8800/08), using the Monte Carlo method.

As described by Eckhardt (1987), the Monte Carlo method (MMC) was developed in the period after World War II, during the Manhattan Project. Introduced by the works of Jon Neuman and S. M. Ulam, precursors of the atomic bomb, the method is based on the use of randomness and statistics for troubleshooting. As a parametric approach, the MMC estimated probability distributions to extract random samples from a population, using adhesion tests to suit a function already known.

Random variables considered in the reliability analysis, are derived from the permanent and variable stresses, breaking strength and yield strength of steel. In order to adjust the sample to a certain probability function, adherence tests were used (Chi-square and Kolmogorov-Smirnov). The flow and rupture tension of steel, behave similarly to the patterns show in the normal probability distribution. Therefore, the performed reliability analysis is based on the First Order Reliability Method, or FORM (Lee 2007). Additionally, an algorithm was developed to be implemented in Mathcad to enable the fulfillment of the proposed hypotheses. This program makes it feasible to carry out the Monte Carlo method, enabling its use in engineering studies and structural projects.

2 FAILURE FUNCTIONS AND STRENGTH ANALYSIS EXPRESSIONS.

2.1 Expressions shown in NBR 8800/08 for bending moment stresses.

The Brazilian Technical Standard n. 8800/2008 presents determinations of weighting factors, as well as a general conditional equation, known as the failure function. If the conditions set by the equation are not met, another expression is used (new failure function) in order to determine the bending moment that causes the breakdown of the structure (ultimate limit state). The proposed I-beam includes pre-defined scaling values, thus allowing its test before the equations presented in the problem (proposals 1, 2 and 3) in terms of both service limit states (SLS) and ultimate limit states (ULS).

Conditional equation, moment of resistance:

$$M_R < 1.5 \cdot W \cdot (\frac{f_y}{y_{al}}) \qquad (1)$$

Where:

M_R = moment of resistance;

W = minimum elastic section modulus;

f_y = steel yield strength;

γ_{al} = ponderation factor.

Having to comply with the following expressions to be valid:

$$f_u \cdot A_{fn} \geq Y_t \cdot f_y \cdot A_{fg} \qquad (2)$$

In which:

f_u = steel ultimate strength;

A_{fn} = net area of the flange;

Y_t = deterministic factor;

f_y = steel yield strength;

A_{fg} = b_f . t_f ; gross area of the flange, calculated based on the dimensions of the observed beam.

$$M_R \leq (\frac{1}{\gamma_{al}}) \cdot (f_u \cdot \frac{A_{fn}}{A_{fg}}) \cdot W_t \qquad (3)$$

Where:

M_R = moment of resistance;

γ_{al} = ponderation factor;

f_u = steel ultimate strength;

A_{fn} = net area of the flange;

A_{fg} = gross area of the flange;

W_t = traction elastic section modulus.

2.2 Failure functions and reliability analysis

The failure function, as shown below, is expressed by a vectorial addition of all the random variables considered, both resistance and applied moments, given by loading stresses (divided into variable and permanent actions)):

$$f(G) = M_R - M_S \quad (4)$$

Where:

f(G) = failure function;

M_R = moment of resistance;

M_S = applied moment.

According to Brazilian technical standards, the expressions of the ultimate limit state (ULS) is based on a semi-probabilistic method of analysis, replacing random variables with deterministic values, found on calculations, identified as weighting factors. Therefore, the featured failure function follows this standard and is thus associated with ULS expressions in the subsequent format:

$$f(G) \leq 0 \quad (5)$$

In which:

f(G) = failure function.

Where the failure rate, P_f, associated with the failure function, is obtained by integrating the probability density function *f(G)* within positive real numbers integers:

$$Dom: f(G) \rightarrow (0; \infty) \quad (6)$$

With:

f(G) = failure function.

$$P_f = P(f(G) \leq 0) \quad (7)$$

With:

P_f = failure rate.

Applying this methodology to practical problems in engineering, the presence of several interdependent variables with complex failure functions is notable. Therefore, the numerical determination of Eq. 5 is not trivial. Thus, alternative methods are customarily applied in their evaluation, ensuring satisfactory results. In the present study, the Monte Carlo method (analytical method based on reliability) was used, using randomized integer generation, aided by the Mathcad software.

According to the Joint Committee on Structural Safety (JCSS), the target reliability indexes of a structure (β_{target}) are as shown in Table 1, in terms of a reference period of one year, by calculating the design working life.

Table 1. Target reliability indexes (β_{target}) – one-year reference period

Relative cost of security measures	Modest failure consequences	Moderate failure consequences	Great failure consequences
Great (A)	β_{target} = 3.1	β_{target} = 3.3	β_{target} = 3.7
Moderate (B)	β_{target} = 3.7	β_{target} = 4.2	β_{target} = 4.4
Modest (C)	β_{target} = 4.2	β_{target} = 4.4	β_{target} = 4.7

In the case study of an I-beam, part of a structural element, the relative costs stipulated, classify the structure as average (in terms of global structural importance) with moderate failure consequences, therefore the reliability rate recommended (for a one-year reference period) is β_{target} = 4.2. Based on a design working life of fifty years (n) for the metal structure, the estimated target index is obtained (Equation 9):

$$P_{fn} = 1 - (1 - P_f)^n \tag{9}$$

Where:

P_f =failure rate;

n = years (design working life)

Using the Mathcad software, a value of β_{target} = 3.90 is achieved, for the target reliability index.

3 CASE STUDY: I 254 (10”) X 37.7 STEEL BEAM

The analyzed component, a steel I-beam with applied bending moment in the region of the rigid connection to the column (Figure 1) with lateral restraints, thickness t_f = 12.5 mm, width b_f = 118.4 mm and height h = 254 mm (Figure 2), is as presented:

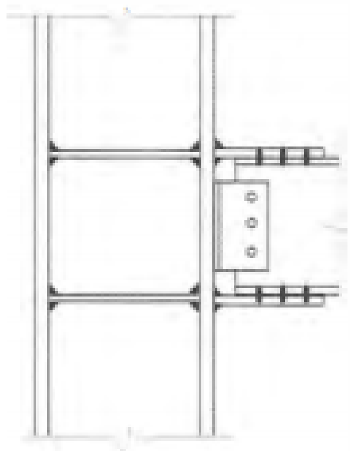


Figure 1. Beam restraints (M. and W. Pfeil (2009)).

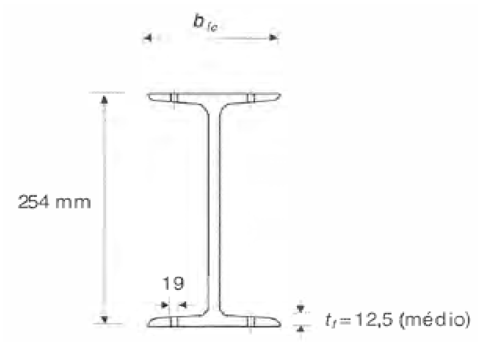


Figure 2. Proposed beam (M. and W. Pfeil (2009)).

3.1 Determination of variables used in the case study (Pfeil, 2009) using NBR 8800/08 and NBR 15980/11

It is considered that the analyzed I-beam has soldering spaces or use of structural fasteners, thus having gross and net area surfaces. The geometry data and values of deterministic variables of the problem (dimensions, factors and elastic moduli) are delineated in Table 2:

Table 2. Deterministic variables

Deterministic variable	Description	Value	Unit
b_f	Flange width	118.40	mm
h	Height of the structure	254.00	mm
t_f	Flange thickness	12.50	mm
A_{fg}	Gross area of the flange	0.001480	m^2
A_{fn}	Net area of the flange	0.000917	m^2
γ_{al}	Ponderation factor	1.10	-
Y_t	Deterministic factor	1.00	-
W	Minimum elastic section modulus	90.00	mm^3
W_t	Traction elastic section modulus	117.00	mm^3

3.2 Reliability analysis of the beam

The random variables employed in the reliability analysis are as provided below (probabilistic variations, JCSS):

Table 3: Random variables with their statistical parameters.

Random variable	Description	Unit	Average value	Probability distributions
M_g	Dead load (dead)	kN.m	70.00	Normal
δ_{Mg}	Variability factor dead load	-	0.05	Normal
σ_{Mg}	Standard deviation dead load	-	2.50	Normal
M_q	Live load (live)	kN.m	5.00	Normal
δ_{Mq}	Variability factor live load	-	0.30	Normal
σ_{Mq}	Standard deviation live load	-	60.00	Normal
f_u	Steel ultimate strength	MPa	400.00	Lognormal
δ_{fu}	Variability factor ultimate strength	-	0.25	Lognormal
σ_{fu}	Standard deviation ultimate strength	-	100.00	Lognormal
f_y	Steel yield strength	MPa	250.00	Lognormal
δ_{fy}	Variability factor yield strength	-	0.25	Lognormal
σ_{fy}	Standard deviation yield strength	-	100.00	Lognormal

Based on a structural design working life of fifty years:

Determination of the applied and resistance moments, respectively:

$$M_S = M_q + M_g \tag{10}$$

Where:

M_S = applied moment;

M_q = live load (live);

M_g = dead load (dead).

$$M_R = f_u \cdot (\frac{A_{fn}}{A_{fa}}) \cdot W_t \tag{11}$$

In which:

M_R = moment of resistance;

f_u = steel ultimate strength;

A_{fn} = net area of the flange;

A_{fg} = gross area of the flange;

W_t = Traction elastic section modulus.

The reliability evaluation, using the Monte Carlo method for the failure function (Eq. 4), is obtained by the number of random generations (*x*), in order to observe the structural behavior and, consequently, account for events where the profile fails. By using an algorithm (shown in (12)), it is possible to count the number of events where the structure failed (x_f) out of the (*x*) tests. With the below failure rate (13):

$$
\begin{aligned}
x_f = \;& \big|\, v \leftarrow 0 \\
& \quad \text{for } i \in 1...x \\
& \quad\quad v \leftarrow v+1 \text{ if } f(G)_i < 0 \\
& \big|\, v
\end{aligned}
\qquad (12)
$$

$$P_f = \frac{x_f}{x} \qquad (13)$$

Where:

x_f = number of events in which the profile fails;

x = number of tests;

P_f = rate of failure.

Based on the probability of failure, the reliability index (β) is obtained, using a standard probability distribution. It is observed that said index is obtained by generating the opposite of the inverse normal probability distribution function (-qnorm (P_f, μ, σ) - shown in (14) -.

$$\beta = -\text{qnorm}\,(P_f, 0, 1) \qquad (14)$$

Where:

β = reliability index;

P_f = rate of failure;

Analyzing the proposed case study (I-beam with an applied bending moment of 83.0 kN.m), with a million tests undergone ($x = 10^6$), a probability of failure of $P_f = 3.2.10^{-5}$ was obtained for the beam with a reliability index of $\beta_{beam} = 3.98$, proving well suited in comparison to the target reliability index, $\beta_{target} = 3.90$, being larger than the target minimum. The comparison between the indexes must always be in terms of the target rate (control rate), for the target reliability index represents an appropriate parameter for the design working life proposal of 50 years. Thus, in cases where the component's reliability index ($\beta_{beam} = 3.98$) is larger than the target value ($\beta_{target} = 3.90$), the viability of the I-beam profile is confirmed, otherwise, the component is considered to be undersized. In that sense, when the reliability index is larger than the target value, the structure can be submitted to greater stresses (loads), without compromising structural safety.

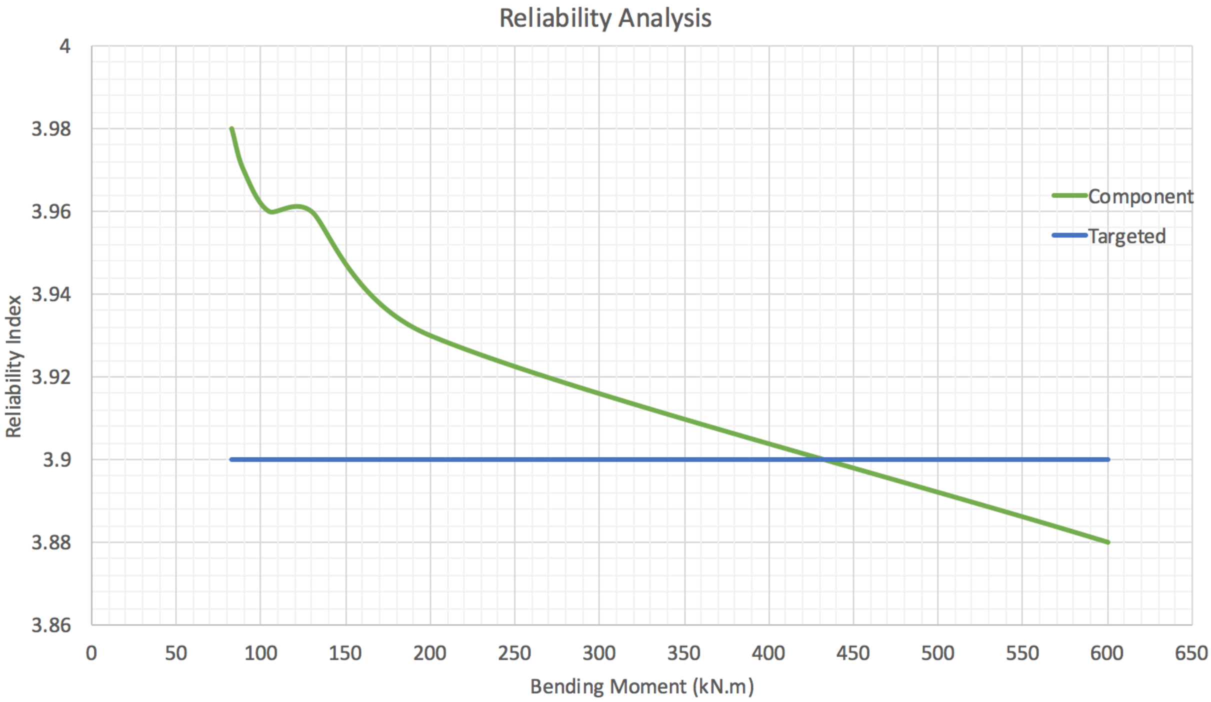
Based on structural reliability analysis, the addressed I-beam is notably oversized, thus, can be submitted to increased loading stresses without damaging the global structure. Hence, in terms of permanent and live loadings, the applied stresses are increased, without compromising the technical performance requirements, while still observing the likelihood of failure generated in the component and system. The dimensioning of the profile, based on a probabilistic reliability method, seeks to approximate the most unfavorable reliability index associated (β_{beam}) with the target reliability index (β_{target}). Therefore, for various stress values, the profile was analyzed in terms of its failure probabilities, being obliged to have its reliability index higher than the minimum target ratio (to ensure overall security). By progressively increasing load stresses, the component failed only once, when an overload four times higher than required by the proposed case was applied. Results in the profile's obtained analysis tests are shown below (Table 4):

Table 4: Calculations by iterative process in an applied moment, generation of β_{beam}

Input – live load	Input – dead load	Input - moment	MMC Results	JCSS
M_q (kN.m)	M_g (kN.m)	M_s (kN.m)	β_{beam}	β_{target}
77.3	5.7	83.0	3.98	3.90
80.0	10.0	90.0	3.97	3.90
90.0	15.0	105.0	3.96	3.90
100.0	30.0	130.0	3.96	3.90
150.0	50.0	200.0	3.93	3.90
450.0	150.0	600.0	3.88	3.90

With the results presented, collected by means of simulations using the Monte Carlo Method, performed in the Mathcad software, a graphical interpretation of the results (as shown in Figure 3) is expected. In accordance with the proposed case, dimensioning within reliability (probabilistic analysis) presents quite adequate and consistent results, as seen in the implementation (the profile could withstand large applied moments), rarely failing.

Figure 3. Graph representing the iterative process applied in order to determine the maximum feasible applied load



4 CONCLUSIONS

Under the proposed case study, the suitability of the chosen structural profile is verified, in terms of applied stresses, seeking to facilitate structural dimensioning based on reliability. An I-beam (I 254 (10 ") x 37.7) in MR250 steel, laterally restrained, was submitted to an applied bending moment in the range of the rigid connection to the column. By using randomized integer generation (computational method of testing), the oversizing of the component is widely amplified by the medium outputs of the program (exposing extremely low failure rates in various loadings). Based on the probabilistic method of structural reliability chosen, Monte Carlo method, it was concluded that the structure rarely failed, given a considerable number of randomized integers generated (about one million (10^6) tests, within every modification in loading stresses). Based on known probability distributions, as well as adhesion tests to examine the validity of the generated data, in terms of structural safety analysis were carried out iteratively, in order to observe the behavior of the component. Based on principles of the Brazilian technical standards, NBR 8800/2008 and NBR 15980/2011, failure functions were stipulated with regard to the appropriate analysis of the structure. Implementing an algorithm, elaborated in Mathcad, generating randomized integers, structural tests were concluded by modifying submitted stresses to assess the withstanding capacity of the beam, adapting the average iteration values (probability distributions, mean, standard deviation parameters). Based on a given probability index (β_{target}), it was possible to verify the validity of the suggested modifications, given the use of the structure (re-dimensioning). The case study proposed, confirms the feasibility of dimensioning structures using reliability-based analysis', due to large increases in loading stresses and undergone tests, whilst not affecting structural safety, considering the beam's valid performance during applied loadings of more than 200% the solicited stress.

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