



XXXVII IBERIAN LATIN AMERICAN CONGRESS
ON COMPUTATIONAL METHODS IN ENGINEERING
BRASÍLIA - DF - BRAZIL

AN OPTIMIZATION OF A SOLAR SAILCRAFT TRAJECTORY IN AN EARTH-MARS TRANSFER CONSIDERING THE ATTITUDE DYNAMICS

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Abstract. *The goal of this work was to optimize the trajectory of a solar sailcraft in an Earth-Mars transfer. Solar sailcraft is propulsion system with great interest in space engineering, since it uses solar radiation to propulsion. So there is no need for propellant to be used, thus it can remain active throughout the entire transfer maneuver. This type of propulsion system opens the possibility to reduce the cost of exploration missions in the solar system. In its simplest configuration, a Flat Solar Sail (FSS) consists of a large and thin structure generally composed by a film fixed to flexible rods. The performance of these vehicles depends largely on the sails attitude relative to the Sun. Using a FSS as propulsion, an Earth-Mars transfer optimization problem was tackled by the GEO_{real} algorithm (Generalized Extremal Optimization with real codification). This algorithm is an Evolutionary Algorithm (AE) based on the theory of Self-Organized Criticality. The FSS was able to perform up to 10 maneuvers until reach Mars. Two angles values are necessary to characterize the FSS film normal vector. Therefore, the GEO_{real} algorithm had to optimize up to 20 design variables in order to minimize the transfer time from Earth to Mars. Once the optimized control law and the FSS trajectory were obtained, the attitude equations of motion were considered. Finally, the impact of this consideration in the FSS control law performance was evaluated.*

Keywords: *Solar sailcraft, Trajectory optimization, Evolutionary algorithm, Attitude dynamics, Generalized Extremal Optimization.*

1 INTRODUCTION

The design and optimization of interplanetary transfer trajectory is one of the most important tasks during the design phase of an exploration mission due to the large increases of speed needed for the transfer. Low-thrust propulsion systems can improve significantly or even becomes feasible missions (Dachwald 2004). One of these kinds of propulsion system is solar sailcrafts.

The solar sailcraft, in its simplest configuration, is a large, thin and flexible structure. Generally, the films clamped at beams are thinner enough to grant its mass as lowest as possible. In other hand, the film needs to be resistant enough to handle with attitude maneuvers. Its function is to use the pressure of solar radiation to generate thrust on the vehicle. The performance of Flat Solar Sailcrafts (FSS) depends mainly on its attitude. FSS attitude control presents great difficulties because they use to be large and flexible. Moreover, obtaining optimal trajectories using low propulsion can become a job that requires excessive time work involving a lot of experience and expert knowledge in control theory and orbital mechanics (Dachwald 2004). In this sense, it is interesting to seek efficient ways to approach this type of optimization problem.

A method that has shown good performance by tackling complex optimization problems in engineering and science are Evolutionary Algorithms (EA) (Clark et al 2000, Dasgupta & Michalewicz 2001, Eiben & Smith 2003). De Sousa et al. (2003) developed an AE called Generalized Extremal Optimization (GEO - Generalized Extremal Optimization). It is easy to implement, it does not use derivatives and it can be used in problems with or without constraints. The design space can presents non-convex or even disjoint regions. This algorithm is able to handle problems with any combination of continuous variables, discrete or whole. Several versions of the GEO have been developed, including hybrid versions and multi-objective (Galski 2006), and this has proved to be very competitive when tackling test functions and design optimization problems.

Considering the characteristics presented by the GEO algorithm, this is a promising method to achieve good results for the problem proposed. Some works applying a new version of GEO algorithm to a two-dimensional trajectory optimization of FSS had already been done (Mainenti-Lopes et al., 2014 and Mainenti-Lopes et al., 2015).

In this work, a version of GEO able to work directly with real variables, called GEO_{real} (Mainenti et al. 2008), was used to tackle the problem of trajectory optimization of a FSS considering its attitude dynamics and three-dimensional orbits.

First, the algorithms were used to optimize the transfer time between Earth and Mars of a FSS. It was assumed that the forces acting on the sail are only the forces from Sun gravitational field and from solar radiation pressure generated by the FSS itself. Besides, all the mass of the Sun is concentrated at one point and it is a point light source. The maximum number of 10 attitude maneuvers to be performed by the FSS during transfer was imposed. Considering that the FSS solar radiation pressure force can be characterized using 2 angles and the launch date is a design variable, the GEO_{real} needed to optimize up to 21 design variables.

Once the GEO_{real} algorithm obtained an optimized trajectory, the attitude dynamics was incorporated to the FSS equation of motion. Besides, a Proportional Derivative (PD) control law was used to drive the FSS attitude to the reference attitude obtained by GEO_{real} during the

trajectory optimization. Each FSS attitude axis (roll, pitch and yaw) were guided by PD control law, so the GEO_{real} was used to optimize 6 design variables.

As result, the GEO_{real} was able to return an optimized FSS trajectory from Earth to Mars and an optimized control law that guided the FSS attitude. A comparative study between the solution with and without the attitude equations of motion was made. The study showed that the FSS transfer time using the attitude dynamics was the better. However, the best approximation was obtained without the attitude dynamics consideration.

2 METHODOLOGY

2.1 Mathematical model of the Flat Solar Sailcraft

In this work, the mathematical formulation of the FSS orbital motion was based on Dachwald. (2010) work. While, the attitude equations of motion was based on Wie and Murph (2007) work. Neither considered disturbance forces or torques during the transfer trajectory.

2.1.1 Orbital equations of motion

The orbital equations of motion made use of ecliptic reference frame formed by the radius the azimuth angle φ and elevation angle θ . Considering However, it is assumed that the sail film have a non-specular reflection. Furthermore, this study was considered a transfer between circular orbits in the same plane.

The orbital equations of motion made use ecliptic reference frame, so the orbital state vector is given by

$$x_e = \begin{bmatrix} r \\ \varphi \\ \theta \\ u \\ v \\ \omega \end{bmatrix} \quad (1)$$

where r is the distance between the FSS and the Sun, φ is the azimuth angle, θ is the elevation angle, u is radial velocity, v and ω are the angular velocities along the azimuth and elevation, respectively.

Considering the coordinates presented in Eq. (1), the orbital equations of motion can be written as

$$\dot{x}_e = \begin{bmatrix} u \\ v \\ \omega \\ r(\omega^2 + v^2 \cos^2 \theta) - \frac{\mu}{r^2} (1 - \lambda C_1) \\ -2 \left(\frac{uv}{r} \right) + 2v\omega \tan \theta + \frac{\mu \lambda}{r^3 \cos \theta} C_2 \\ -2 \left(\frac{u\omega}{r} \right) + 2v^2 \sin \theta \cos \theta + \frac{\mu \lambda}{r^3} C_3 \end{bmatrix} \quad (2)$$

where μ is the solar gravitational parameter, λ is the light number given by a_c/a_o (a_c is the FSS characteristic acceleration and a_o is sun's gravitational acceleration at Earth distance) and C_1 , C_2 and C_3 represent the solar radiation pressure force components in each unit vector $(e_u e_\varphi e_\theta)^T$, respectively, and can be write as follows

$$C_1 = \cos^3\beta \quad (3)$$

$$C_2 = \cos(\alpha + \gamma)\sin(\beta)\cos^2(\beta) \quad (4)$$

$$C_3 = \sin(\alpha + \gamma)\sin(\beta)\cos^2(\beta) \quad (5)$$

where α is the FSS clock angle, β is the cone angle and $\gamma = \arctan(\omega, v\cos\theta)$. Figure 1 presents a graphical representation of α and β angles.

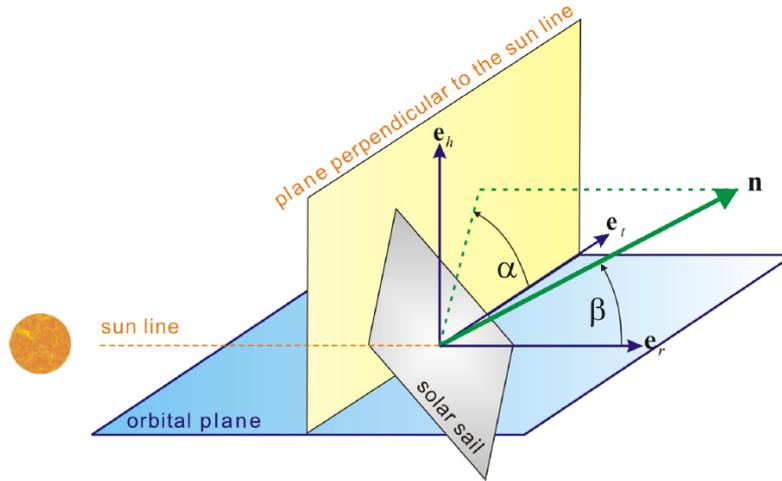


Figure 1. A graphical representation of the FSS clock angle α and the cone angle β (Dachwald, 2004).

2.1.2 Attitude equations of motion

The attitude equations of motion were obtained using Euler's equations, then the angular velocity along roll, pitch and yaw axis can be written as follows

$$\begin{bmatrix} \dot{\vartheta}_x \\ \dot{\vartheta}_y \\ \dot{\vartheta}_z \end{bmatrix} = \begin{bmatrix} \frac{1}{I_x} [(I_y - I_z)\vartheta_y\vartheta_z - \tau_x] \\ \frac{1}{I_y} [(I_z - I_x)\vartheta_z\vartheta_x - \tau_y] \\ \frac{1}{I_z} [(I_x - I_y)\vartheta_x\vartheta_y - \tau_z] \end{bmatrix} \quad (6)$$

For the description of angular orientations of sailcraft in a Sun centered orbit, a Local-Vertical reference frame with its origin at the center of mass of an orbiting sailcraft has a set of unit vectors (a_1, a_2, a_3) with a_3 locally vertical toward the Earth, a_1 along the locally horizontal (transverse) direction, and a_2 perpendicular to the orbit plane. The relative orientation of the sailcraft with respect to the Local-Vertical frame is described by three Euler

angles (Φ_x, Φ_y, Φ_z) of the rotational sequence of $R_1(\Phi_x) \leftarrow R_2(\Phi_y) \leftarrow R_3(\Phi_z)$ from the Local_Vertical frame to the body-fixed frame.

The angular velocity components of the sailcraft are then given by (Wie and Murph, 2007)

$$\begin{bmatrix} \vartheta_x \\ \vartheta_y \\ \vartheta_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\Phi_y \\ 0 & \cos\Phi_x & \sin\Phi_x \cos\Phi_y \\ 0 & -\sin\Phi_x & \cos\Phi_x \cos\Phi_y \end{bmatrix} \begin{bmatrix} \dot{\Phi}_x \\ \dot{\Phi}_y \\ \dot{\Phi}_z \end{bmatrix} - n \begin{bmatrix} \cos\Phi_y \sin\Phi_z \\ \sin\Phi_x \sin\Phi_y \sin\Phi_z + \cos\Phi_x \cos\Phi_z \\ \cos\Phi_x \sin\Phi_y \sin\Phi_z - \sin\Phi_x \cos\Phi_z \end{bmatrix} \quad (7)$$

where $n = \sqrt{\mu/r^3}$ is the orbital rate. With the Equations (6) and (7) the FSS attitude dynamics can be described by

$$\begin{bmatrix} \dot{\Phi}_x \\ \dot{\Phi}_y \\ \dot{\Phi}_z \\ \dot{\vartheta}_x \\ \dot{\vartheta}_y \\ \dot{\vartheta}_z \end{bmatrix} = \begin{bmatrix} \vartheta_x + \sin\Phi_x \tan\Phi_y \vartheta_y + \cos\Phi_x \tan\Phi_y \vartheta_z + n \frac{\sin\Phi_z}{\cos\Phi_y} \\ \cos\Phi_x \vartheta_y - \sin\Phi_x \vartheta_z + n \frac{\cos\Phi_x \cos\Phi_z}{\cos\Phi_y} \\ \frac{\sin\Phi_x}{\cos\Phi_y} \vartheta_y + \frac{\cos\Phi_x}{\cos\Phi_y} \vartheta_z + n \tan\Phi_y \sin\Phi_z \\ \frac{1}{I_x} [(I_y - I_z) \vartheta_y \vartheta_z - \tau_x] \\ \frac{1}{I_y} [(I_z - I_x) \vartheta_z \vartheta_x - \tau_y] \\ \frac{1}{I_z} [(I_x - I_y) \vartheta_x \vartheta_y - \tau_z] \end{bmatrix} \quad (8)$$

By using spherical trigonometry, the relations for α and β angles can be described as $\cos\beta = \cos\Phi_x \cos\Phi_y$ and $\alpha = \Phi_z - \arcsin[\sin\Phi_x / \sin\beta]$. Design a control law that drives $\Phi_x \rightarrow 0$ then $\Phi_y \rightarrow \beta$ and $\Phi_z \rightarrow \alpha$. Besides, the control law should drive the FSS attitude angular velocity ϑ_x , ϑ_y and ϑ_z to offset the orbital rate. The torques are given by

$$\begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} K_1 \Phi_x + K_2 (\vartheta_x - \vartheta_{xr}) \\ K_3 (\Phi_y - \beta) + K_4 (\vartheta_y - \vartheta_{yr}) \\ K_5 (\Phi_z - \alpha) + K_6 (\vartheta_z - \vartheta_{zr}) \end{bmatrix} \quad (9)$$

where K_i with $i = 1, 2 \dots 6$ are the proportional and derivatives gains and ϑ_{xr} , ϑ_{yr} and ϑ_{zr} are the reference attitude angular velocity with respect of roll, pitch, and yaw.

2.2 Units Convention for the Proposed Problem

In studies of interplanetary trajectories and transfers problems a specific set of units is widely used. Those units are: astronomical unit (AU) to distance and radians (rad) for time. The transformation of meters (m) to AU is trivial: $1 \text{ AU} = 1.4959787 \times 10^{11} \text{ m}$. The unit transformed of time variable is according with Earth orbital motion. It is known that the Earth

takes one year to turn around the sun and it represents a rotation of 2π rad. Therefore, it can be written as $1 \text{ year} = 2\pi \text{ rad}$. This relation in seconds is $1 \text{ s} = 1,99096677 \times 10^{-7} \text{ rad}$.

The main advantage of this unit transformation is to ensure that the values of distance and time will never be too big or too small over the entire numerical integration process. Thus, this strategy allows to avoid computational problems with numerical representation. Thus, the constant evolved in Equation 8 can be written using the following units:

- $\text{unit}[\mu] = \text{AU}^3/\text{rad}^2$;
- $\text{unit}[a_c] = \text{AU}/\text{rad}^2$.

Using the proposed units, it can be verified that $\mu = 1.0 \text{ UA}^3/\text{rad}^2$ and considering the characteristic acceleration value equal to 1.56 mm/s^2 , then $a_c = 0.2630696796 \text{ UA}/\text{rad}^2$.

2.3 Optimization Algorithms

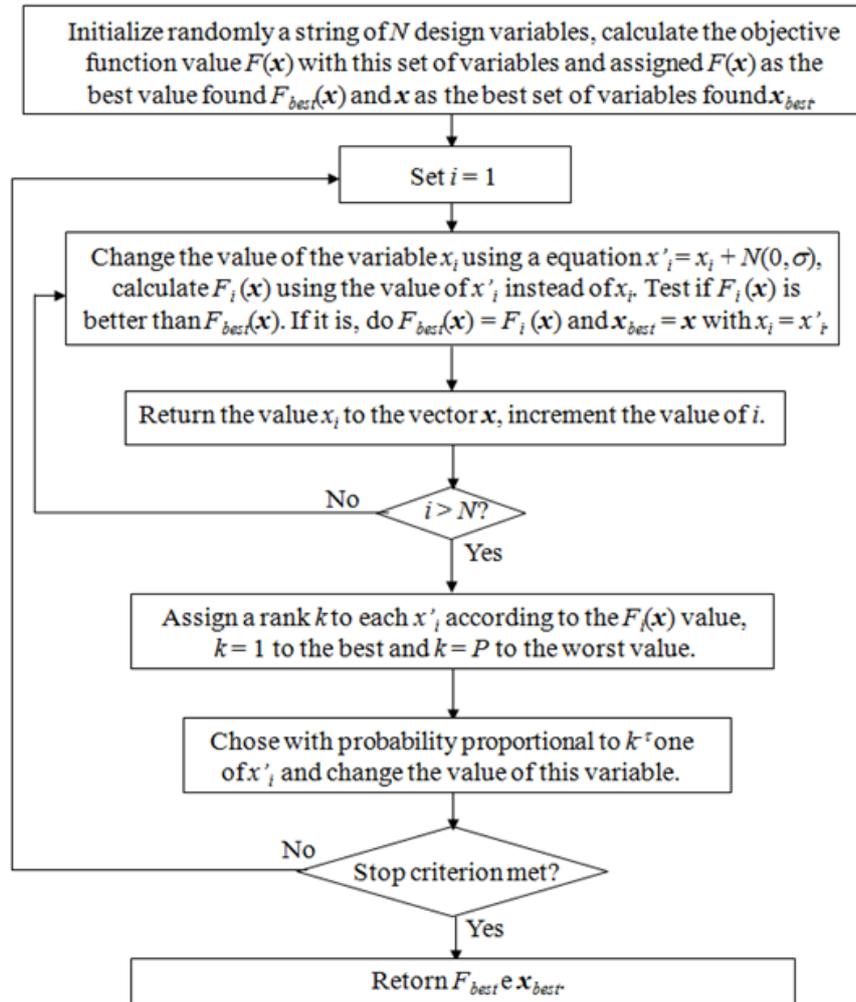
2.3.1 GEO_{real} Algorithm

In this work it was considered that the FSS can perform up to 10 attitude maneuvers, 2 angles were necessary to characterize the FSS film normal vector (α and β), the launch date was optimized and the time interval T_m between each attitude maneuver was considered constant. Therefore, optimization algorithms had to deal with up to 21 design variables. Besides, the control law gains were optimized to drive the FSS attitude angles to the reference α and β obtained during the trajectory optimization procedure.

All this work has been done by GEO_{real} . In this session the optimization algorithm used will be presented.

Although the GEO algorithm shows to be a competitive optimization tool, it has a limitation with regard to the design variables resolution. Since it uses binary encoding, it will always be necessary to define the search resolution previously. The resolution will be defined by the number of bits assigned to each variable. In other words, the binary encoding provides a set of quantized solutions. By flipping the less significant bit, the variable value changes without the possibility to test the intermediate values. This behavior may enable the algorithm to find the global minimum if it does not belong to the set of testable solutions. In order to avoid this limitation, a new version of the GEO algorithm that works directly with real variables was developed (Mainenti-Lopes *et al.*, 2014).

Called GEO_{real} , it was inspired by the basic operation of the GEO algorithm. The main change is the way to change the value of variables. GEO algorithm changes the variables values by flipping the bits, while in GEO_{real} , this change is made through a random perturbation with a Gaussian distribution in the design variable value. The flowchart shown in Figure 2 describes how GEO_{real} does the search for the optimal solution.

Figure 2. GEO_{real} algorithm flowchart.

2.4 Application of the algorithms to the optimization problem

The trajectory optimization of a FSS is obtained by defining its attitude with respect to time. That's mean, $\alpha^*(t)$ and $\beta^*(t)$ should be found, where $\alpha(t)$ and $\beta(t)$ is the function that describes the FSS attitude history. In general, this function can take any form as long as its image remain confined between $(-\pi)/2$ and $\pi/2$, which are the possible values for the FSS attitude angle in normal operation.

The approach adopted in this work considers that the functions $\alpha(t)$ and $\beta(t)$ were transformed in a sum of several step functions. Thus, to determine the function, it is necessary to define a set of values of each step. This strategy can be interpreted that $\alpha(t)$ and $\beta(t)$ functions were discretized and could be characterized by the vectors $\vec{\alpha}$ and $\vec{\beta}$, which are the vectors of the FSS attitude angles in each steps. Therefore, the number of elements N_m of these two vectors must be equal and represents the maximum number of maneuvers performed by the FSS during the transfer.

The trajectory optimization problem tackled was characterized as follows:

$$\text{Min } F_1 = T_{rend} + \varpi \cdot \text{Dist}_{best} \quad (10)$$

Subject to the restrictions

$$\left\{ \begin{array}{l} -\pi/4 \leq \alpha_j \leq \pi/4 \\ -\pi/2 \leq \beta_j \leq \pi/2 \\ 2457600 \leq Date_{Launch} \leq 2458380 \\ \text{with } j = 1, 2, 3, \dots, 10 \end{array} \right. \quad (11)$$

and the dynamics of solar sail given by Eq. (2). F_1 is the objective function, T_{rend} is the FSS transfer time to rendezvous with Mars, $\varpi = 0.003$ is a weight, $Dist_{best}$ is the best distance between Earth and Mars and $Date_{Launch}$ the FSS launch instant in Julian Date. The maximum $Date_{Launch}$ represents July 30, 2016 and the minimum represents September 18, 2018. The α maximum and minimum were defined as $-\pi/4$ and $\pi/4$, respectively, because the inclination of the Earth and Mars orbits are almost the same, then the FSS trajectory does not need to gain much inclination.

During the attitude control optimization, the GEO_{real} had to optimize the control law gains, then optimization problem tackled was characterized as follows:

$$Min F_2 = \sum_t \left[(\Phi_y - \beta)^2 + (\Phi_z - \alpha)^2 \right] \quad (12)$$

Subject to the restrictions

$$\begin{array}{l} 0 \leq K_j \leq 10000 \\ \text{with } j = 1, 2, 3, \dots, 6 \end{array} \quad (13)$$

The objective function F_2 represents the sum of the quadratic error for each integration instant t .

3 RESULTS

In this Section, the results for the trajectory and attitude control optimization problem described in section 2 will be present.

The FSS characteristic acceleration assumed was 1.56 mm/s^2 and its moments of inertia along each main axis of rotation are $I_x = 90 \text{ kg/m}^2$, $I_y = 45 \text{ kg/m}^2$ and $I_z = 45 \text{ kg/m}^2$.

For the FSS trajectory optimization several tests in order to adjust GEO_{real} free parameters. The best τ value found was 5.5 and the best σ found was 4. Then a run of 10^5 objective-function evaluation was started. As result the best transfer time found was 146.7 days (2.523 rad) with a distance precision greater than 0.001 AU. The history of $\alpha(t)$ and $\beta(t)$ obtained are presented in Figure 3.

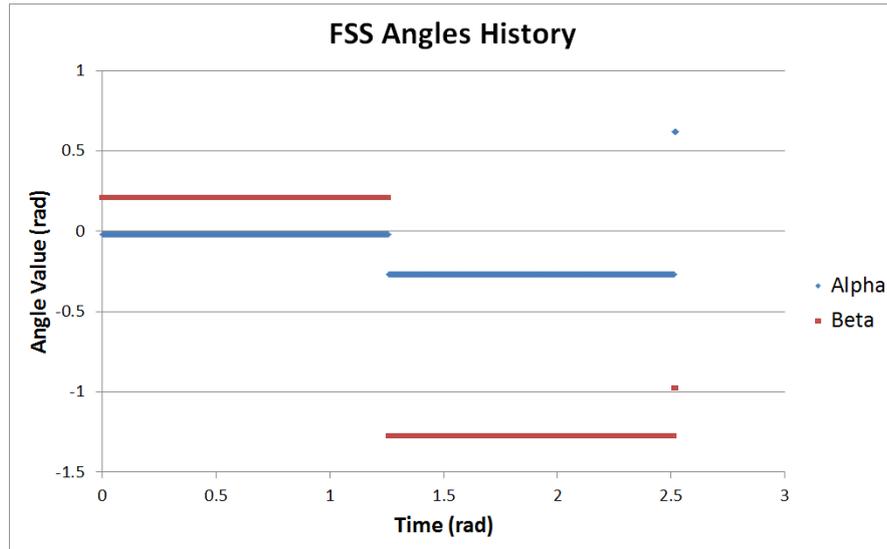


Figure 3. $\alpha(t)$ (blue line) and $\beta(t)$ (red line) angles history.

Once the optimized $\alpha(t)$ and $\beta(t)$ history were found, the gains of attitude control law need to be optimized so that it can drive the attitude angles to references $\alpha(t)$ and $\beta(t)$. The GEO_{real} algorithm was adapted to tackle this new problem. After several tests to adjust GEO_{real} free parameters, the best τ value found was 5.0 and the best σ found was 4. A run of 10^5 objective-function evaluation was made and a set of optimized attitude control law gains was obtained. These gains are presented in Table 1.

Table 1. Optimized attitude control law gains

K_j	Value
1	7634.96681675
2	141.19439497
3	9947.80402486
4	1195.84529878
5	8651.67921628
6	3367.73827787

After including the FSS attitude dynamics, the FSS reach the nearest point from Mars earlier than without the attitude dynamics consideration. It took 142.9 days (2.458 rad). However, the precision become much worse. The best distance was about 0.015 AU. This behavior might have happened because the difficult of the attitude control law to drive the α angle second reference angle, as it is shown in Figure 4.

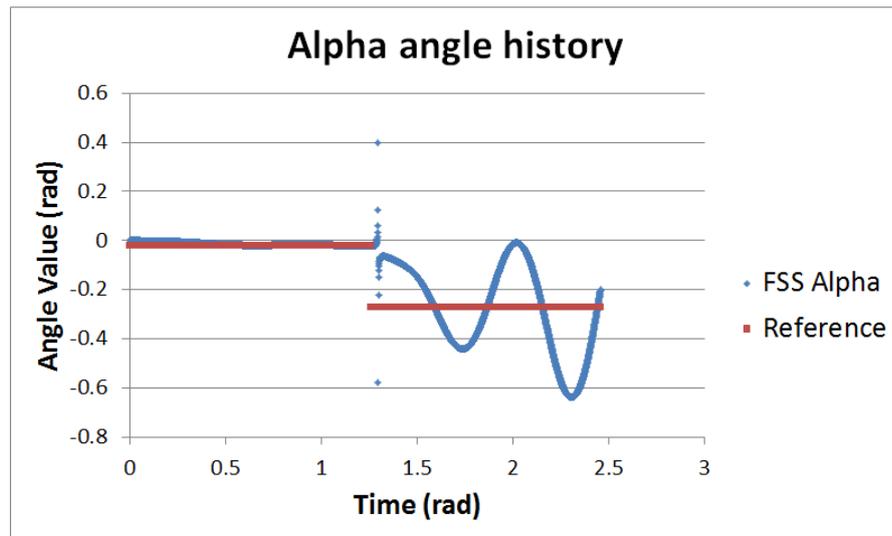


Figure 4. FSS $\alpha(t)$ angle (blue line) history and the reference angle (red line).

Although the difficult to drive the α angle, the attitude control law was capable to control the β angle, as it is presented in Figure 5. A possible explanation for the difficult to control α is the fact that α is inversely proportional to $\sin(\beta)$. Therefore, α presented a discontinuity at the same moment that $\beta = 0$, after that α starts to diverge. A possible strategy to correct this problem is to avoid situation where $\beta = 0$.

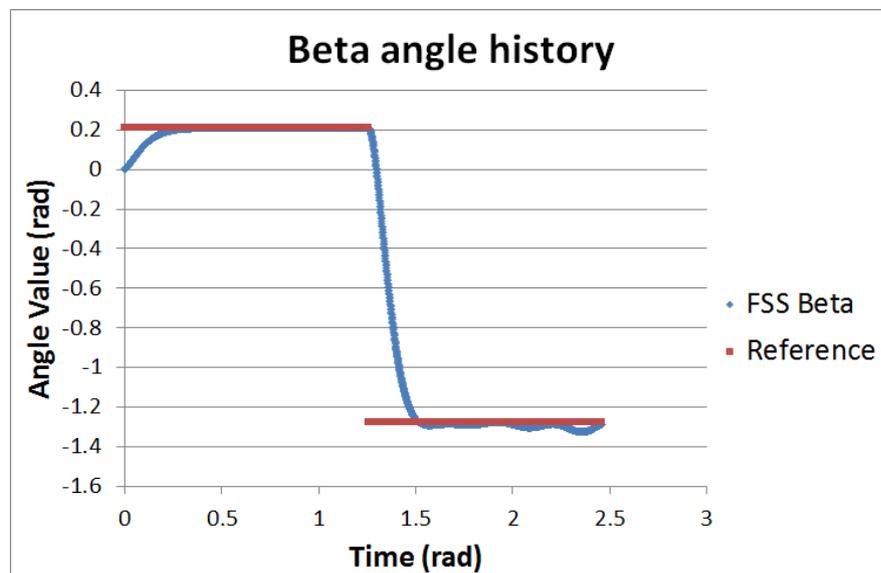


Figure 5. FSS $\beta(t)$ angle (blue line) history and the reference angle (red line).

In Figure 6 is presented the roll, pitch and yaw angles, $\Phi_x(t)$, $\Phi_y(t)$ and $\Phi_z(t)$. It seems that after the α angle discontinuity the roll angle $\Phi_x(t)$, that should stay around zero, tries to compensate $\Phi_z(t)$ lack of stability to ensure β stability.

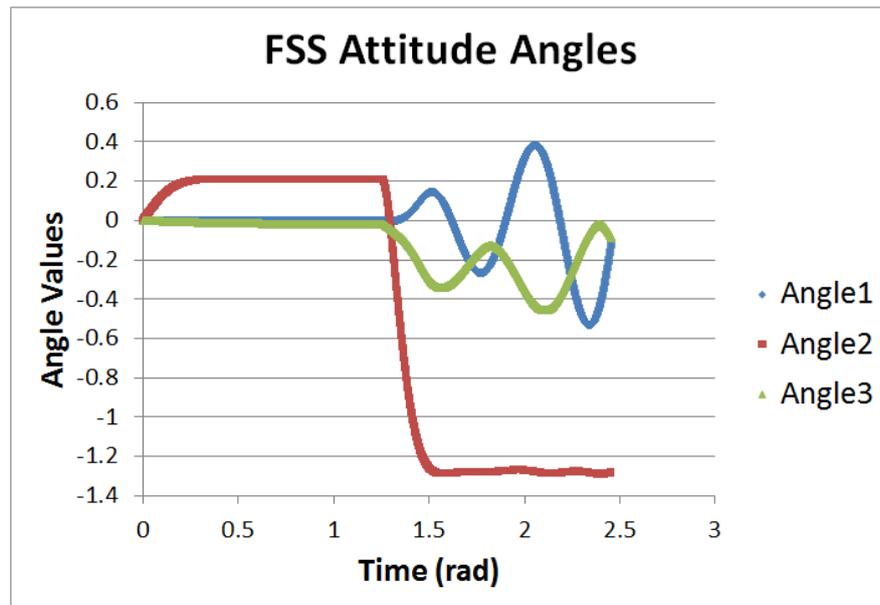


Figure 6. $\Phi_x(t)$ (blue line), $\Phi_y(t)$ (red line) and $\Phi_z(t)$ (green line) angles.

4 CONCLUSIONS

In this paper the problem of optimization of a FSS trajectory is proposed. The FSS should reach Mars from Earth in two conditions: considering only the orbital dynamics and considering the FSS attitude dynamics as well. In both approaches the GEO_{real} was used to optimize parameters. In the first case, it was used to optimize the FSS transfer time through the attitude angles α and β . In the second case, it should optimize the attitude control law gains such that could be able to drive the FSS attitude to the reference values of angles.

The study of the trajectory optimization presented good results. A solution was found that allow the FSS to reach Mars in 146.7 days. This solution was kept to use as reference for the second study.

Including to attitude dynamics, the FSS lose performance. Although it reach its nearest point from Mars earlier than without the attitude dynamics consideration, the precision become much worse. Besides, the attitude control law presented some difficult to stabilize the α angle. This behavior could be the reason for the FSS loss of performance. Further studies should be done to confirm these suppositions.

ACKNOWLEDGEMENTS

This research was developed with a CNPq-Brazil scholarship.

REFERENCES

Clarck, D.E, Mannhold, R., Kubinyi, H. & Timmerman, H. Evolutionary algorithms in molecular design. Weinheim: Wiley-VCH, 2000.

Dachwald, B. Low-Thrust Trajectory Optimization and Interplanetary Mission Analysis Using Evolutionary Neurocontrol. Doctor Thesis - Universität der Bundeswehr München Fakultät für Luft- und Raumfahrttechnik Institut für Raumfahrttechnik, 2004.

Dasgupta, D. & Michalewicz, Z. (Editores), Evolutionary Algorithms in: Engineering applications. Berlin, Heidelberg, New York: Springer, 2001.

De Sousa, F.L., Ramos, F.M., Paglione, P. & Girardi, R.M. New Stochastic Algorithm for Design Optimization, AIAA journal, Vol. 41, Number 9, pp. 1808-1818, 2003 (ISSN 0001-1452).

Eiben, A. E. & Smith, J.E. Introduction to evolutionary computing. Berlin: Springer, 2003.

Galski, R. L. Desenvolvimento de versões aprimoradas híbridas, paralela e multiobjetivo do método da otimização extrema generalizada e sua aplicação no projeto de sistemas espaciais. 2006-09-22. 279 p. (INPE-14795-TDI/1238). Doctor Thesis - Instituto Nacional de Pesquisas Espaciais, São José dos Campos. 2006.

Mainenti-Lopes, I., 2008, “Satellite Attitude Control of Rigid-Flexible Satellite Using the Generalized Extremal Optimization with a Multi-Objective Approach (in portuguese)”, Master Thesis, Nacional Institute for Space Research (INPE).

Mainenti-Lopes, I. 2013. An approach for multi-objective optimization solar sail trajectories. Doctor thesis (in portuguese), INPE, São José dos Campos.

Mainenti-Lopes, I., Souza, L . C . G ., De Sousa F . L . Application of Generalized Extremal Optimization (GEO) Technique to Design the Orbit Transfer Solar Sail Control System. Engineering Optimization 2014. CRC Press 2014. Pages 219–224. Print ISBN: 978-1-138-02725-1. eBook ISBN: 978-1-315-73210-7. DOI: 10.1201/b17488-41.

Mainenti-Lopes, I., Souza, L . C . G ., De Sousa F . L . 2015 A multi-objective optimization of a solar sailcraft trajectory in an Earth-Mars transfer using an evolutionary algorithm In: CILAMCE2015 - XXXVI Ibero-Latin American Congress on Computational Methods in Engineering, 2015, Rio de Janeiro.

Wie, B., Murph, D., 2007. Solar-Sail Attitude Control Design for a Sail Flight Validation Mission. JOURNAL OF SPACECRAFT AND ROCKETS Vol. 44, No. 4, July–August 2007 DOI: 10.2514/1.22996