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Creativity for Invention Insights: Corporate Strategies and Opportunities for Public Entrepreneurship*

by

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Abstract: This paper introduces and describes the invention-insight sample space and uses it to describe the creative process of discovering invention insights—the essential combinations of elements of knowledge to envision the basic working configurations of inventions, the working ideas for new technologies. Evidence about invention insights and about corporate strategies to promote them is viewed in the context of the paper's description of the invention-insight discovery process. Then that description is used (1) to identify a novel new opportunity—initiation of policies to stimulate invention insights that directly combine unusually large numbers of knowledge elements—for public sector entrepreneurship to speed the pace of technological progress and the opening up of altogether new areas of science and technology, and (2) to delimit the appropriate form of policy—promotion of competition and the free exchange of ideas—to exploit the opportunity. With sufficient uncertainty in the search for insights, pre-invention-insight ideas in themselves should ideally be freed from the restrictions of intellectual property.

Keywords: creativity, invention, innovation, public sector entrepreneurship, research and development (R&D), technology

JEL Classifications: O31, O32, O34, O38

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I. Introduction

This paper describes the discovery of invention insights in the context of a description of the invention-insight sample space. An invention insight is defined as the essential combination of elements of knowledge to envision the basic working configuration of an invention—the working idea for a new technology.[1] Section II describes invention insights and gives examples. Section III describes the invention-insight sample space and the creative process of discovering invention insights. Section IV uses the description of the creative process to explain how competition—defined as potential inventors who can freely enter the discovery process and freely share ideas as they strive to discover invention insights—increases the discovery of invention insights. Section V discusses evidence consistent with the description of the creative process that emerges from the discussion of the invention-insight sample space. Section VI concludes by using Section III's description of the invention-insight discovery process to identify a novel new opportunity for public entrepreneurship that would speed the pace of technological progress, and by using Section IV's observations about competition to delimit the role of government in supporting invention-insight discovery and the opening up of altogether new areas of science and technology.

II. Invention Insight

Invention is born with insight that brings together in the inventor's mind the essential elements of knowledge in a vision of the working configuration of the invention. As described by Usher (1929, p. 11),

> Invention finds its distinctive feature in the constructive assimilation of preexisting elements into new syntheses, new patterns, or new configurations of behavior.

The inventor is prepared for the insight that launches an invention; the necessary elements of knowledge gained from education and experience are taken from the shelves of the inventor's mind (Duggan, 2007, p. 60, p. 173) or, in Arthur's terms (2009, pp. 221-223), from the inventor's "store of functionalities" that have been accumulated with study and experience. Duggan (2007, p. 61) explains that in the flash of insight "selected elements

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[1] Following Arthur (2009, pp. 27-30, p. 38, pp. 50-51, pp. 53-54, p. 60, p. 203), a technology is a use of phenomena to achieve human purposes, and a given technology combines many technologies. This paper focuses on inventions—"novel technologies that are deliberately created" (Arthur, p. 106).

from various past examples come together in a new combination", and he (2007, p. 20) quotes Thomas Kuhn describing his own flash of insight and observing that "fragments in my head sorted themselves out in a new way" with "the pieces suddenly sorting themselves out and coming together in a new way." Or as observed by Usher (1929, p. 17, p. 19),

> Close attention to the detailed accounts of particular inventions affords a clue to the general character of the circumstances that promote the achievement of a new configuration. It is well-nigh indispensable that certain data of experience should be presented to the mind of the inventor in such a fashion as to suggest their connection with the problem. All the elements essential to the accomplishment must be brought together sufficiently to facilitate their organization into a new circuit or configuration. . . . [The] unity involved in the individual act of invention is brought to a close with the achievement of a single new concept, design, pattern, or configuration. The variety of words that may be used is indicative of the difficulty of adequately conveying the full connotation of the technical term "configuration."

Arthur (2009, p. 116) describes insight for invention:

> The insight comes as a removal of blockage, often stumbled upon, ... It comes in a moment of connection, always a connection, because it connects a problem with a principle that can handle it. Strangely, for people who report such breakthroughs, the insight arrives whole, as if the subconscious had already put the parts together. And it arrives with a "knowing" that the solution is right—a feeling of its appropriateness, its elegance, its extraordinary simplicity. The insight comes to an individual person, not to a team, for it wells always from an individual subconscious. And it arrives not in the midst of activities or in frenzied thought, but in moments of stillness.
>
> This arrival is not the end of the process, it is merely a marker along the way. The concept must still be translated into a working prototype of a technology before the process is finished.

Arthur's foregoing description of the invention insight is itself replete with many important insights that are used in this paper. The invention insight entails connection and it arrives whole—it puts parts together; in Section III the insight is a collection of t elements of knowledge. Further, an individual, not a team, has the insight. Section V's discussion of evidence plays on the idea that the insight comes to an individual, yet collaboration, in the form of shared knowledge and interaction among individuals who are pursuing the invention, is important for the achievement of the invention insight.

Duggan (2007) describes and analyzes the creative process of "strategic intuition" that brings the "flash of insight" and invention. As many examples—in Duggan (2007), Arthur (2009), Isaacson (2014), and Johnson (2014), among many other sources—from the history of invention show, the insight underlying invention may come to the inventor in a flash, but also the insight typically comes after a long period of study and thought that prepares the inventor's mind for having the insight. Johnson (2014, p. 72, p. 170) illustrates the point—that the initial insight may take hold in an inventor's mind slowly over decades—with the examples of Birdseye's invention of commercially viable frozen food and Galileo's invention of the first pendulum clock. Yet, after the period (sometimes quite long) of mulling over the problem, the examples from the history of technology provide evidence of the flash of insight when the inventor sees the essential combination of ideas in the basic working configuration and the invention insight occurs.

Arthur (2009) uses many examples to explain that while invention is often born in the well-prepared mind with a flash of insight revealing the invention's basic working configuration, many additional years of research and development (R&D) may be needed to create from the initial vision a new technology that successfully combines existing technologies in a commercialized innovation. Johnson (2014, pp. 206-215) describes the arduous R&D process over roughly eight decades as the numerous inventors who had the invention insight of the electric lightbulb's three essential elements struggled to create from the initial insight the successful innovation that Edison and his Menlo Park team ultimately achieved.

The present paper describes the possibilities for the invention insight as distinguished from the possibilities for the outcomes of the R&D that develops the invention insight.[2] Section III describes the *invention-insight sample space*—in the example of the Wright brothers' invention of their 1903 flying machine, the invention-insight sample space describes the set of possible combinations of essential elements of knowledge as the brothers searched for the right combination of knowledge elements. Using Arthur's (2009, p. 120) conceptualization of their invention along with McCollough's (2015, pp. 38-39) description, we can observe that the Wright brothers discovered the right combination of the four elements of knowledge— "means of control and stability of flight", "wing sections with good lift", "a lightweight propulsion system", and "a high-efficiency propeller" based

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[2] Scott and Scott (forthcoming) use the example of the Wright brothers' 1903 flying machine to illustrate the *R&D sample space* and apply it to understanding how the technological complexity of an entrepreneur's idea affects the probability of securing outside finance for the R&D to create a commercial product from the idea.

on the underlying principles of propulsion, lift, and wing-warping control—that formed their invention.

Importantly, an element of knowledge (that is used in combination with other elements for an invention insight) can be a complex combination of separately identifiable elements of knowledge. Analogous to the elements in the periodic table that are composed of atoms which themselves are composed of subatomic particles, the elements of knowledge combined in an invention insight may be composed of other, less aggregated elements of knowledge. Elements of knowledge include phenomena and facts about nature that are used by technologies, as described by Arthur (2009), to achieve human purposes, and also include existing technologies. As Arthur explains (2009, pp. 54-56), thinking of technologies as combinations of technologies can even be appropriate when the elements of knowledge to form an invention are not physical phenomena and the physical technologies (such as radar or the laser) that are "purposed systems" based on physical phenomena, but instead are in the realms of "non-physical purposed-system" technologies in the arts or in economic and legal systems where the phenomena underlying technologies are behavioral rather than physical. Thus, the description of creativity and invention in this paper is applicable to invention outside the realm of technologies based on physical phenomena, although the focus is on invention of new technologies grounded in physical phenomena.

Table 1 lists some important inventions in quite different technological areas and for each lists the elements of knowledge that were combined to reach the invention insight. An invention insight will typically be one instance of many invention insights supporting a sequence of inventions associated with a broad definition of the technology—necessarily so, because any new technology is a combination of other technologies and because subsequent technologies will build on that new technology. For one example from Table 1, in Johnson's (2014, pp. 45-85) detailed history of the evolution of technology to produce and use cold temperatures, the invention of refrigeration achieving temperatures far below freezing had to occur before commercially viable frozen foods could be invented. It is for that reason that associating a particular inventor or inventors with a particular invention is problematic. As Arthur (2009, p. 120) observes about another example in Table 1, the Wright brothers' great achievement: "Their 1903 powered flight was not so much a demonstration of "an invention"; it was a marker along a lengthy path trodden by others before them."

For invention in general, Arthur (2009, p. 125), echoing the observations of many others, observes, ". . . multiple efforts and filling in of key pieces in fact make it difficult to speak of "invention" in the sense of being first." Thus, although Table 1 identifies by name individual inventors associated with inventions, the sources cited in the table explain that the named individual inventors discovered the invention insights because of the availability of many other technologies and many other thinkers' ideas. As Duggan (2007, p. 99) observes in his description of another of Table 1's examples, the Google story, Page and Brin did not invent the idea of downloading the Internet to a set of powerful computers to allow full-text search, or the idea of academic citations analogous to the reverse links they observed in the downloaded Internet, or the idea of data mining algorithms, or the idea of presenting advertisements in simple lists in the same way that search results are presented, "but they are the ones who combined them, over four years in a series of flashes of insight."

*****TABLE 1 ABOUT HERE*****

Thus, introducing notation to be used in Section III, the initial act of creativity is defined as the recognition of the essential combination of the *t* elements from among the many, *s*, knowledge elements that the inventor has accumulated with study and experience from the universe of *n* knowledge elements. The *t* elements are a set of juxtaposed essential elements for an invention, and Table 1 illustrates with some examples showing such sets of essential elements. Among those examples is the invention of commercially successful frozen food; as Johnson (2014, p. 74, italics in original) explains, "Birdseye's breakthrough was not a single insight, but a *network* of other ideas, packaged together in a new configuration. What made Birdseye's idea so powerful was not simply his individual genius, but the diversity of places and forms of expertise that he brought together."

Because an existing set of knowledge elements can be combined to form new technologies that themselves become elements of knowledge, the number, *n*, of available elements of knowledge grows over time even if there are no altogether new elements because of newly discovered phenomena (and new technologies combining existing technologies and the new phenomena). Thus, even without altogether new elements of knowledge, the number *n* at a subsequent time includes all the knowledge elements available in the present time plus any discovered combinations of those to form new knowledge elements.[3] To use another of Table 1's examples, today the personal computer (PC) is an element of knowledge, but in 1974 it was not, and Bill Gates and Paul Allen, as described by Duggan (2007, pp. 84-92) and Isaacson (2014, pp. 313-343), combined in an invention insight the four knowledge elements of the computer language BASIC developed by Dartmouth College professors Kemeny and Kurtz, the PDP-8 minicomputer from Digital Equipment Corporation (DEC), the Intel 8080 chip (microprocessor), and Robert's desktop microcomputer Altair from Robert's company MITS to launch their new company Micro-Soft, a software company for microcomputers. As Microsoft, the company dominated the market for operating software for mass-market PCs.

Also, the number of knowledge elements grows over time because, as Arthur describes (2009, pp. 61-67), discoveries of new phenomena open up the possibilities for new technologies. For example, the scientific fields of physics and anatomy developed the knowledge that sound waves moved through air and then on entering the human ear vibrated the eardrum; that knowledge made possible Bell's telephone (Johnson, 2014, pp. 90-97).

Thus, knowledge elements grow through time, in part because older technologies are combined to create new ones and in part because that process is expanded as new useful phenomena are discovered. Weitzman (1998) places the growth of useful technologies in the context of the neoclassical economic growth model and observes the self-sustaining possibilities for economic growth based on combinations of technologies that then become the basis for subsequent technologies. More generally, as Arthur (2009, p. 174) explains, ". . . if new technologies lead to further technologies, then once the numbers of elements in the collective pass some rough threshold, the possibilities of combination begin to explode. With relatively few building blocks the possibilities become vast." Arthur (2009, pp.174-189) explains how growth would actually occur as opportunities for technologies based in both human needs and technological needs evolve with the growth of knowledge elements

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[3] As pointed out by a referee, countering the tendency for the number, *n*, of knowledge elements to grow over time is the possibility that some knowledge once acquired will be forgotten—thus, there can be a loss of knowledge elements through time. Such loss of knowledge elements is especially important and likely because so much knowledge has a large tacit component.

and technologies in what Arthur calls combinatorial evolution. Kohn's (2014) sweeping economic history of preindustrial Europe emphasizes that a theory of self-perpetuating technology that builds on earlier technologies required the entrepreneur as the actuator of economic progress, identifying and exploiting the new opportunities as markets expanded, reorganizing production, introducing technology and increasing productivity. Arthur (2009, p. 174, p. 179) uses the invention of the transistor as an example of how a new technology can itself be the fundamental element of knowledge that when combined with other knowledge generates a great number of other technologies. Isaacson (2014) provides the historical details and insightful observations about the creation of the transistor—with the invention insights at Bell Labs in late 1947 by Bardeen and Brattain and in early 1948 by Shockley—and about the digital revolution spawned by the transistor.

Before describing the sample space for the invention insight (as distinguished from the sample space for R&D), we can use the invention of the transistor to explain that the distinction between the process that culminates in an invention insight and the subsequent R&D that culminates in an innovation is often blurred in practice; and moreover, even the statement that a new technology is a combination of existing technologies may obscure the complicated nature of the research culminating in an invention insight. The process of discovering the invention insight can not only take decades before everything comes together (in "the flash of insight" or "moment of connection"). The process of using phenomena and creating a new technology from existing technologies can be complicated because both new phenomena and new technologies, to be combined with older ones, may be discovered in the research process. Such coincidence during the creation of a new technology—coincidence of the discovery of both new phenomena and new technologies to be used—is seen in Isaacson's (2014, pp. 131-149) careful account of the historical details of the invention of the transistor.

Bardeen and Brattain were working with Shockley's insight about the phenomenon of a field effect by which an electrical field in proximity with a semiconductor would pull electrons to the semiconductor's surface and thereby allow electrical current to move through the semiconductor, thus allowing the possibility that a semiconductor in the presence of a low-power source could be used to switch on or off the flow of a high-power current through the semiconductor (Isaccson, 2014, p. 141). To have their invention insight for their invention of the point-contact transistor, Bardeen and Brattain had to discover the phenomenon of the shield effect where electrons trapped on the surface of the semiconductor blocked the influence of the positive electric charge of the electric field near the semiconductor and prevented electrons from moving freely in the semiconductor, thus blocking the necessary field effect phenomenon. They then had to devise a technology to overcome the shield effect. They discovered the shield effect, and with considerable effort and creativity devised a technology to overcome it (Isaacson, 2014, pp. 141-142).

The story of their invention insight cautions that thinking of invention of a new technology as a combining of previous technologies, while true, may hide complexities because the inventor is sometimes discovering phenomena and inventing new technologies along the way to the ultimate new technology developed. The example of the invention of the transistor—Bardeen's and Brattain's point-contact version and then Shockley's p-n junction version as listed in Table 1—illustrates that the distinction between achieving the invention insight and the subsequent R&D is often blurred because the invention insight may require R&D on a new technology that is invented as a building block along the way to the ultimate invention. Thus, Bardeen and Brattain carried out a very substantial and creative R&D project to develop their initial solution for the shield effect. They came up with the invention insight for the intermediate technology of an electrode jabbed into the semiconductor as a way to solve the shield effect, but developing that invention insight into a workable intermediate technology required R&D resulting in several advances that culminated in the two well-insulated and extraordinarily closely-placed gold foil electrodes inserted into the semiconductor (Isaacson, 2014, pp. 141-144). The path to the invention insight for their point-contact transistor included developing knowledge about the phenomenon of the shield effect and developing new intermediate technology to overcome the shield effect.

III. The Invention-insight Sample Space and the Creative Process of Discovering Invention Insight

Weitzman (1998, pp. 334-335) has several noteworthy statements from writers who have explained that the creative process of invention is a process of combining ideas and, moreover, combining quite diverse ideas. Among others, he quotes the mathematician Poincaré who in 1908 observed, as quoted by Weitzman (1998, p. 335):

> To create consists precisely in not making useless combinations and in making those which are useful and which are only a small minority. Invention is discernment, choices. . . . Among chosen combinations the most fertile will often be those formed of elements drawn from domains which are far apart.

The invention-insight sample space introduced in this paper is based on the idea that invention and discovery result from combining elements of knowledge, an idea that Weitzman (1998) embeds within an economic model of growth based on the productivity-enhancing combinations of elements of knowledge that emerge through time. Because the new combinations themselves become elements of knowledge that can be further combined to create even more new technology, with a sufficiently high rate of productive new combinations emerging through time, economic growth can be self-sustaining as Weitzman emphasizes.

This paper describes the needle—of an invention insight—in the haystack—of knowledge elements—and discusses implications of the description of the invention-insight sample space for the creative process that finds that needle. Weitzman abstracts from that creative process to focus on the explanation of how invention, seen as the discovery over time of new useful combinations of elements of knowledge, fits within the neoclassical model of economic growth. As Weitzman (1998, pp. 346-347) explains, his focus is achieved by:

> . . . abstracting away from many details such as inventive inspiration, the process of idea selection, possible distinctions between macro-inventions and micro-inventions, the degree of the appropriability of knowledge, the role of the entrepreneur, the role of basic research, property rights, private and public incentives, market structure, competition, et cetera, et cetera.

In the context of the invention-insight sample space, the present paper focuses on the process of finding the essential combination of knowledge elements, on how competition affects that process of idea selection, and also addresses some of the other "details" such as appropriability, intellectual property, and incentives.

We turn now to a description of the invention-insight sample space and its implications for the creative process. The invention-insight sample space—the set of possibilities in the search for the right combinations of essential elements of knowledge in the configuration envisaged for an invention—is quite different from the sample space for research and development (R&D)—the set of possibilities in the search for combinations of consistent components to achieve a fully-functioning, working technology from the essential working configuration envisaged in the invention insight.[4] At the risk of losing the organic-whole character of Duggan's strategic intuition or flash of insight, this paper decomposes the process of discovering invention insights.

This paper's detailed descriptions of both the sample space for invention insights and the decomposition of the process of invention-insight discovery are consistent with very different creative processes. Sometimes the pursuit of invention insight is purposive as was the case with the Wright brothers' focused search for the elements comprising the invention insight for their 1903 flyer, or with the Bell Labs' pursuit of the invention insight for the transistor. Sometimes the discovery of invention insight is serendipitous as with Fleming's discovery of the invention insight for penicillin (Le Fanu, 2012, pp. 15-17); Fleming's accidental discovery stands out as both extraordinarily accidental and extraordinarily momentous for human welfare, but serendipitous discoveries abound (Kennedy, 2016). An invention insight may be stumbled upon when the inventor is looking for something else or even when not consciously looking at all. Sometimes adding just a single element of knowledge to a set of well understood elements will complete an invention insight as was the case for laser printers that awaited the key principle (an element of knowledge in our description of the invention-insight sample space) allowing them to supplant the early approach to computer printing (Arthur, 2009, p. 33, p. 108). Sometimes an invention is waiting in plain sight where it has a use in another industry and then is discovered for use in a new industry as was the case with the manufactured pins in Adam Smith's famous description of the pin factory with its division of labor—a process that sprang from the observation of the stiff wire bristles in the carding comb used to process wool fibers for their use in spinning.[5] All of these examples can be described in the context of the description of the sample space for invention insight and the decomposition of the process of invention insight discovery that are developed here in Section III. The parameters of the sample space description will differ across the different types of invention insight discovery, and the way

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[4] The R&D sample space is described and applied in Scott (1991) and Scott and Scott (2015a).

[5] Jacobs (1969, p. 81), as cited and interpreted in Kohn (2014, Chapter 5, "Entrepreneurs and Cities," p. 20).

that an inventor accomplishes the steps of the decomposed creative process that we shall describe will differ (the inventor may progress methodically and purposively through the steps, or may instead accomplish the steps in one fell serendipitous swoop), but the essential description of the sample space and of the decomposition of the process of finding the invention insight remain.

Let *n* denote the number of knowledge elements from which innovative ideas are formed; a knowledge element may be a combination of other knowledge elements in a technology, or it may be a phenomenon used by a technology—such as the phenomenon of sound waves moving through air and vibrating the eardrum.

For an inventor, *s* is the number of knowledge elements provided by the inventor's own learning and experience and selected from among the *n* knowledge elements about which an inventor might learn. At the archetypal starting point, or initial condition, in the discovery process, there is complete uncertainty. The potential inventor, beginning the process of reducing the uncertainty, browses—a cursory perusal or inspection or scanning—through the *n* elements of knowledge and chooses *s* of the elements to study carefully and master.

For the particular technology to be invented, the number of elements in the essential element set for the original, initial insight (that constitutes the invention insight that will launch the development of the essential insight into a new working technology) is $t \leq s$. Let $C_{x,y}$ denote the combination of *x* things taken *y* at a time where $C_{x,y} = x!/(y!(x-y)!)$ with $x!$ denoting *x* factorial, the product of the integer *x* with all the positive integers smaller than *x*.

For the inventor who attempts to create the invention by envisaging the set of *t* elements that together form the essential insight of the invention, let $p(f)$ denote the proportion of all possible combinations of elements that is taken by those cases where *f* of the *s* elements in the inventor's mind coincide with *f* elements in the set of *t* elements that together define the invention.[6] Then, $p(f)$ is given by the formula:

$$(1) \qquad p(f) = \frac{C_{t,f} C_{n-t,s-f}}{C_{n,s}}$$

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[6] The formulation here is a special case (and a very different application) of the more general formulation introduced in Scott (1982) and applied in Scott (2000, 2001a).

Given the particular set of t elements that constitute the essential idea of the invention, $C_{t,f}$ denotes the number of different combinations of f elements that are in the sought after invention's collection of elements that could be (and may or may not be) among the elements in the inventor's knowledge set. Conditional on the two sets coinciding for a particular f elements, $C_{n-t,\, s-f}$ is the number of possibilities for s minus f elements that do not coincide with the t elements that constitute the invention. $C_{n,s}$ is the total number of ways to have a knowledge set for the inventor whose knowledge and experience have resulted in a knowledge set with s elements. Thus, conceiving of a potential inventor as someone whose experience and training prepares the inventive mind by acquiring s elements of knowledge from the n elements available, $p(f)$ gives the proportion of all possible cases taken by those cases where the inventor's set of knowledge elements and the invention's set of t elements coincide in f elements. The summation of $p(f)$ from $f = 0$ to t equals 1, and the set of all the possible outcomes that comprise these proportions is the invention-insight sample space. The proportions are not probabilities; the set of possible outcomes—the invention-insight sample space—has been described, but a numerical value for probability has not been associated with each potential outcome. For example, if all potential outcomes were equally likely, then the proportions here would be probabilities, but that is not in general the case.

The successful inventor has the insight to "see" the essence of the new technology—namely, the successful inventor has the creative insight bringing together the t elements of the new technology. For the inventor to be able to discover the essential combination of elements—i.e., for it to be possible for those elements to be brought together in the inventor's mind, the t essential elements must be among the s elements in the inventor's mind. Thus, in equation (1), $f = t$, and thus $p(f = t)$ is given by the formula:

$$(2) \qquad P_t \equiv p(f = t) = \frac{C_{t,t}C_{n-t,s-t}}{C_{n,s}} = \frac{C_{n-t,s-t}}{C_{n,s}} = \frac{\prod_{i=0}^{t-1}(s-i)}{\prod_{i=0}^{t-1}(n-i)} = \prod_{i=0}^{t-1}\frac{s-i}{n-i}$$

Thinking about (1) and (2), observe the following points about the creative process of invention for a potential inventor who is conceived of as having chosen, from among the n knowledge elements available, a set of s elements with which to work. First, a successful inventor is rather special, because obviously the successful inventor must have the knowledge and experience that includes the necessary t elements for the invention, yet, a very large proportion of the invention-insight sample space does not include the essential *t* knowledge elements. Second, the successful inventor must not only have the knowledge that includes the necessary *t* elements for the invention, but also the inventor must recognize that those elements in the available knowledge set belong together and together constitute the essential idea of the sought-after invention. That recognition is what Duggin (2007) called “strategic intuition.” Having such intuition and “seeing” the desired invention is not a trivial task because *n* and *s* are large relative to *t* and so the essential-knowledge-element set constitutes a miniscule proportion of the knowledge sets potentially available to the inventor. For a simple example, if the universe of knowledge elements contains a million elements ($n = 1{,}000{,}000$), and the essential idea for an invention brings together three elements ($t = 3$) of knowledge, and a potential inventor’s chosen knowledge set contains a thousand elements ($s = 1{,}000$), the proportion of the invention-insight sample space for which the inventor’s mastered set of *s* elements contains the essential idea is 499/500498999000 or $9.97005\text{x}10^{-10}$—about one billionth of the sample space. In this context, the fact that invention insights are so often perceived as having been stumbled upon and seen as accidental discoveries is not surprising.[7]

The invention-insight sample space can be used to decompose the discovery process for the invention insight into five steps.

- First, there is an exclusion step; — an individual focuses on subsets of the complete universe of *n* knowledge elements.
- Second, a diversity step; — the choice of a larger number *s* of elements to master is the second step or the diversity step.
- Third, a focus step; — the focus step, when the number of elements *s* under consideration is reduced.
- Fourth, a composition step; — the composition step reduces the number of possible combinations by ruling out as a possibility a specific number of elements in the essential idea.
- Fifth, a recognition step; — seeing and recognizing the essential *t* elements of knowledge together as the working configuration of the invention is the “flash of

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[7] Accidental discovery is an overarching theme of Le Fanu’s (2012) description of the discoveries of the invention insights underlying what he terms the definitive moments of modern medicine.

insight" or "moment of connection" that culminates the discovery process described with the invention-insight sample space and the exclusion, diversity, focus, composition, and recognition steps.

We now discuss these steps—exclusion, diversity, focus, composition, and recognition—in turn.

For the exclusion step, an individual focuses on subsets of the complete universe of knowledge elements; in essence the successful inventor partitions the complete set of *n* knowledge elements and ignores large chunks of the set of knowledge elements, selecting subsets of the *n* elements within which the crucial set of *t* elements will be found. The proportion of the invention-insight sample space, that includes the unique combination of elements of knowledge within the inventor's mastered set of *s* elements of knowledge, will increase when a potential inventor chooses *s* elements of knowledge to master from just a subset—that contains the *t* elements of the essential idea—of the universe of *n* knowledge elements. For the universe of *n* knowledge elements, we have the proportion, P_t, and if *z* elements are eliminated successfully (i.e., elements of the knowledge set that are eliminated from consideration do not include any elements among the *t* elements that together constitute the essential idea) from consideration as possible knowledge elements to be studied and mastered and be among the inventor's *s* knowledge elements, when $z = 1$, the change in P_t is:

$$(3) \quad \Delta P_t \,|\, (z=1) = \left(\frac{n}{n-t}\right)P_t - P_t = \left(\frac{t}{n-t}\right)P_t > 0$$

Thus, starting with the initial proportion P_t of the sample space, if $z = 1$ of the *n* elements of knowledge is successfully eliminated from consideration, the proportion of the sample space for which the inventor's mastered set of *s* knowledge elements contains the solution is:

$$(4) \quad P_t \,|\, (z=1) = \left(\frac{n}{n-t}\right)P_t$$

where the first term is the ratio of the first term (which is dropped when $z = 1$) in the denominator of the final single-product expression for P_t given in equation (2) to the additional new term which is now included in the denominator of the final single-product expression in equation (2).

Using intuition to reduce successfully by x elements the number of elements in the set of possibilities from which the inventor acquires understanding of s elements, and still with complete uncertainty about the usefulness of the remaining elements, the proportion of the invention-insight sample space for which a set of s elements will overlap completely with the t elements of the essential idea is:

$$(5) \quad P_t \mid (z = x) = \left(\frac{n}{n-t}\right) P_t \left(\frac{\prod_{i=0}^{x-1} (n-1-i)}{\prod_{i=0}^{x-1} (n-1-t-i)} \right)$$

where the proportion is now the product of the proportion given $z = 1$ with a new ratio term. The product in the numerator of that new ratio term—the last term of (5)—is the product of the additional terms (beyond the first that was deleted to form equation (4)) deleted from the product in the denominator of P_t in equation (2), and the product in the denominator of the last term of (5) is the product of the new additional terms (beyond the first that was added to form equation (4)) in the denominator that were not in the denominator of P_t in equation (2). Thus, the solution for the proportion of the sample space for which the inventor's mastered set of s elements includes the invention insight, after reducing by x the number of elements in the universe on which the potential inventor focuses, is:

$$(6) \quad \hat{P}_t = P_t \mid (z = x) = \left(\frac{n}{n-t}\right) P_t \left(\prod_{i=0}^{x-1} \frac{n-1-i}{n-1-t-i} \right)$$

For our example, if intuition allows the potential inventor to eliminate 10% of the universe of knowledge elements, leaving 900,000 elements from which the essential idea must come, the proportion of the sample space taken by the complete overlap—of the one thousand (s) elements in the potential inventor's chosen knowledge set with the three elements ($t = 3$) necessary for the essential idea—rises from 9.97×10^{-10} or about 1 in a billion to 1.37×10^{-9} or about 1.37 in a billion. If intuition allows the focus on just half of the elements in the universe of knowledge, the proportion of the sample space with the answer rises to 7.98×10^{-9} or about 8 in a billion. If the potential inventor has the intuition to focus successfully on just 20 percent of the universe of knowledge, the proportion of the sample space for which the investor's set of s mastered elements includes the solution increases to 1.25×10^{-7} or about 125 in a billion.

The focus obtained with the exclusion step by reducing the amount of the universe of knowledge elements that is considered for search—by using intuition to eliminate successfully from consideration portions of the universe of knowledge that do not contain the elements needed for the invention—at first increases slowly the proportion of the invention-insight sample space for which the inventor's mastered element set contains the solution. Table 2 uses our example to illustrate how using the exclusion step to eliminate successfully portions of the universe of n knowledge elements increases the proportion $\hat{P}_t$ of the invention sample space for which $f = t$—that is, for which the essential combination of t elements necessary for the essential initial insight is included in the inventor's knowledge set of s elements. The proportion $\hat{P}_t$ increases at an increasing rate, with very large gains as the number of the elements from the universe of knowledge that are considered falls close to the number of elements s in the inventor's collection of knowledge elements that are acquired from the entire knowledge set. The proportion $\hat{P}_t$ equals 1.0 when the number of elements considered is reduced successfully (i.e., the elements eliminated from consideration do not contain any that are among the t elements necessary for the essential initial insight) to a number equal to s.

*****TABLE 2 ABOUT HERE*****

Figure 1 plots $\hat{P}_t$ as a function of z (smoothing the curve for the discrete z) to illustrate the benefits of the exclusion step. Over a large range for z, there are very low proportions of the sample space for which the inventor has the solution—the invention insight—among the s mastered elements, even given very large successful reductions in the number of the n (equal 1,000,000 in our example) elements of the universe of knowledge that are under consideration. Figure 1 shows the increase to 1.0 of the proportion as the reduction in the knowledge elements considered goes in the limit to 999,000 and the effective n has been reduced to s. The inventor's knowledge set of s elements is the mastered knowledge of the inventor. The mastered knowledge elements are in general a proper subset of all of the knowledge that could be learned, and although in principle the inventor could increase the s mastered elements so they are closer in number to the n

elements available, in practice, the inventor eliminates portions of the *n* elements of knowledge and lowers below *n* the number of elements from which the *s* elements, to be mastered through study and experience, are selected. So, for our example, it is a matter of selecting *s* elements from the 1,000,000, or instead selecting them from some subset of that; focusing on a subset of the *n* elements available is the first step—the exclusion step.

*****FIGURE 1 ABOUT HERE*****

The inventor can choose a set of *s* knowledge elements to master, and there are many combinations of *s* elements from which to choose, even given the elimination from consideration of many of the *n* elements available. The diversity of the set of *s* knowledge elements can be increased by having more knowledge elements in the set of *s* elements studied by the inventor, and the choice of a larger number of elements to master is what we have called the second step or the diversity step.

Increasing *s* can be thought of as bringing in diverse viewpoints. There is a limit on *s* for an individual because there is a limit on the sensible amount of learning to acquire—or to use a colloquialism, to “bite off,” as in, “don’t bite off more than you can chew and digest.” The ideal diversity solution here is to get good ideas into the individual inventor’s set of *s* mastered knowledge elements while keeping *s* as small as possible while including the essential *t* elements. To address the difficulty of finding the essential invention insight among the large set of knowledge elements, consider first the possibility of increasing the diversity of thought by simply increasing *s* for the potential inventor or—and this is the more practical solution given that an individual’s ability to learn new elements of knowledge is limited—a group of individuals working together by combining their knowledge and experience. Such diversity amounts to an increase in *s*, the number of elements in the inventor’s or inventing group’s knowledge set.

For the universe of *n* knowledge elements, we have the proportion, P_t, and if the number of elements in *s* is increased by one unit, the change in P_t is:

$$(7) \quad \Delta P_t \,|\, (\Delta s = 1) = \left(\frac{s+1}{s-t+1}\right) P_t - P_t = \left(\frac{t}{s-t+1}\right) P_t > 0$$

Starting with complete uncertainty in the initial condition, with the initial proportion P_t of the sample space, using an increase in diversity of thought to increase by *v* elements the number of elements in the inventor's or inventing group's knowledge set of *s* elements, the proportion $P_{t,v}$ of the sample space for which the set of mastered elements will overlap completely with the *t* elements of the essential idea is:

$$(8)\quad P_{t,v} = \left(\frac{s+1}{s-t+1}\right)P_t\left(\prod_{v=0}^{v-1}\frac{s+2+v}{s-t+2+v}\right)$$

Using our earlier example, with a million (*n*) elements in the universe of knowledge elements, and with a thousand (*s*) elements in the inventor's knowledge set, and with three (*t*) elements comprising the essential initial idea, the proportion P_t of the sample space for which the mastered set of *s* elements contains the essential idea is 499/500498999000 or $9.97005\text{x}10^{-10}$ or 1 in a billion. If the strategy of increasing the diversity of thought is used to add $v = 100$ elements to the original *s* elements in the inventor's knowledge set, then the proportion $P_{t,v}$ of the invention-insight sample space for which the inventor's mastered set of elements has the answer becomes 1.33×10^{-9} or 1.33 in a billion. If 1000 elements are added, the proportion becomes 7.99×10^{-9} or 8 in a billion. If 10,000 elements are added, the proportion is 1.33×10^{-6} or 1333 in a billion or 1.33 in a million. In the limit, with 999,000 elements added to the potential inventor's knowledge set, the proportion of the invention-insight sample space for which the mastered set includes the essential initial insight—the essential $t = 3$ knowledge elements in our example—is 1.0. Figure 2 illustrates the effect of increasing the number of elements in the inventor's knowledge set on the proportion of the inventor's sample space that contains the solution; in our example, the proportion reaches 1.0 when $v = 999{,}000$.

*****FIGURE 2 ABOUT HERE*****

Thus, if *s* is increased to the point that $s = n$, then $P_{t,v} = 1$. But since *s* is much larger than *t*, there is still a huge problem to solve. So, good intuition requires "effective diversity" in order to increase the chance that intuition recognizes the solution among the many possible combinations of knowledge elements. Thus, there is a third step, the focus step, to make a good choice of the *s* elements to master; simply increasing *s* is not a good strategy.

Whether because *s* has been increased to *n* and $P_{t,v} = 1$, or more plausibly *n* has been successfully (i.e. keeping the *t* essential elements) reduced to *s* and $\hat{P}_t = 1$, there are huge gains from reducing successfully (i.e., conditional on the *t* essential elements of knowledge being included in the *s* elements of knowledge chosen for study) the number of elements in *s*. Given the probability is 1.0 that the essential *t* elements are included in the *s* elements, for the inventor working with *s* elements of knowledge that contain the *t* essential elements, for a reduction of only one element, from $s = 1000$ in our example to $s = 999$, the number of possible combinations of the inventor's mastered elements of knowledge is cut in half. That effect of reducing *s* by 1 remains at essentially a one half reduction in the possible combinations over a large range of *s*. Only when *s* is so small that the researcher essentially has the answer in hand does the reduction in possible combinations by essentially half, for each element removed, change.

For the third step, the focus step, when *s* is reduced by 1, the number of possible combinations of knowledge elements removed as a proportion α of the number of possible combinations before the removal of an element of knowledge from the *s* elements is:

$$(9) \qquad \alpha = \frac{\sum_{i=1}^{s} C_{s,i} - \sum_{i=1}^{s-1} C_{s-1,i}}{\sum_{i=1}^{s} C_{s,i}} = \frac{2^{s-1}}{2^{s}-1}$$

The total number of possibilities for combining the *s* elements of knowledge into distinct groups with one or more elements is $\sum_{i=1}^{s} C_{s,i}$ which equals $2^{s}-1$.[8] In the numerator,

$\sum_{i=1}^{s-1} C_{s-1,i} = 2^{s-1} - 1$, so $\sum_{i=1}^{s} C_{s,i} - \sum_{i=1}^{s-1} C_{s-1,i} = 2^{s} - 2^{s-1} = 2^{s-1}(2-1) = 2^{s-1}$.

From equation (9), the impact (on the proportion of possible combinations that is removed) of reducing *s* by just one knowledge element is remarkably large. The limit for α

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[8] Adding 1 (for the null technology where no elements of knowledge are present) yields what Weitzman (1998, p. 340, p. 345) refers to as the power function, a level of explored combinations that would, in his model, perhaps be obtained when R&D effort has increased without bound. That $\sum_{i=0}^{s} C_{s,i} = 2^{s}$ follows directly from the binomial theorem for $(a + b)^{s}$ with $a = b = 1$, and the sum is also found as the sum of the elements in the ($s + 1$)th row of Pascal's triangle (Apostol, 1967, p. 44). Arthur (2009, p. 173), discussing multi-element technologies, explains that in a new combination, each of the elements of knowledge may be present or not, giving two ways the first could occur in the new combination times the two ways the second could occur and so on for a total of 2^{s} ways to get a new combination. Excluding the null technology and the *s* single-element technologies gives $2^{s} - s - 1$ combinations.

as s becomes large is obviously 0.5. Moreover, as s increases from low levels, the proportion α decreases toward 0.5 very quickly and approaches its asymptotic value over very low levels for s. For example, when $s = 8$, $\alpha = 0.501961$. Figure 3 illustrates the proportion α as a function of s.

*****FIGURE 3 ABOUT HERE*****

The huge gains from the third step of successfully focusing on a smaller number of elements of knowledge may have something to do with the often made observation that it is the individual who invents—has the flash of insight combining elements of knowledge that are essential for the invention. Recall Arthur's (2009, p. 116) observation: "The insight comes to an individual person, not to a team, for it wells always from an individual subconscious." Successfully reducing the number of elements of knowledge under consideration has a dramatic effect reducing the possible outcomes in the invention sample space, and it seems reasonable to assume that an individual is more likely to have the desired focus than the collective inventive mind of an organization (or other inventive group of people). The organization's diversity of thought and experience and knowledge can provide good candidates to be on what Duggan (2007, p. 60, p. 173) refers to as the shelves of the individual inventor's mind, but the individual's focus on relatively few of the elements of knowledge must play some role in successful strategic intuition as described by Duggan. Even in the immensely successful environment of great diversity at Bell Labs during the period that the transistor was invented, the research director at Bell Labs, Kelly, who championed collaboration and brought together the diverse talents and experience in the solid-state team of Shockley, Brattain, Bardeen and others to invent a replacement for the vacuum tube using semiconductors (Isaacson, 2014, pp. 132-140), observed (Isaacson, 2014, p. 134): "With all the needed emphasis on leadership, organization and teamwork, the individual has remained supreme—of paramount importance. . . . It is in the mind of a single person that creative ideas and concepts are born." Isaacson (2014, p. 134) interprets the history and observes: "The key to innovation—at Bell Labs and in the digital age in general—was realizing that there was no conflict between nurturing individual geniuses and promoting collaborative teamwork. It was not either-or."

Thus, one way to focus the search for the discovery of an invention insight—to reduce successfully the number of elements in the set of mastered elements of knowledge—is the third or focus step. Thinking further about the search for the "moment of connection" (Arthur, 2009, p. 116) or the "flash of insight" (Duggan, 2007, p. 61) in the case whether because s has been increased to n and $P_{t,v} = 1$, or more plausibly n has been successfully (i.e. keeping the t essential elements) reduced to s and $\hat{P}_t = 1$, another way to reduce the number of possible combinations is the fourth step—the composition step. The composition step reduces the number of possible combinations by ruling out as a possibility a specific number of elements in the essential idea. So, for example, the researcher might rule out the cases where t would equal 10. Thus, in terms of reducing the number of possible knowledge-element combinations, eliminating the cases for which the numbers of elements j are in the middle of the range from 1 to s will have the biggest effect because $C_{s,j}$ will be largest for the values of j in the middle of the range.

The reduction in the number of possible combinations, from the fourth or composition step of eliminating a particular number j of elements as the number in the essential combination constituting the invention, as a proportion β_j of the number of possible combinations before such elimination, is

$$(10)\quad \beta_j = \frac{C_{s,j}}{\sum_{i=1}^{s} C_{s,i}} = \frac{C_{s,j}}{2^s - 1}$$

Figure 4 (smoothing the curve over the discrete j-element cases eliminated) shows, for our example where $s = 1000$, the proportion β_j as a function of the value j (as a possibility for the number t of essential elements of knowledge in the invention insight) that is eliminated from consideration. Observe that almost all of the possible combinations are concentrated in the middle range of values for j. For example,

$$(11)\quad \sum_{j=450}^{550} \beta_j = \frac{\sum_{450}^{550} C_{1000,j}}{2^{1000} - 1} = 0.998608$$

*****FIGURE 4 ABOUT HERE*****

The fifth and final step in the decomposition of the process of discovering the invention insight is the recognition step. Seeing and recognizing the essential *t* elements of knowledge together as the working configuration of the invention is the "flash of insight" or "moment of connection" that culminates the discovery process that we have described with the invention-insight sample space and the exclusion, diversity, focus, composition, and recognition steps.

IV. Competition and Discovery of Invention Insights

With the five-step decomposition of the discovery process in hand, a simple story emerges for how competition increases the discovery of invention insights.[9] Competition—in the sense of many potential inventors striving to discover the invention insight in "free and open competition" by which we mean freely sharing ideas among potential inventors who can freely enter or leave the activity of pursuing invention insights—is important for the discovery of invention insights because with more competition a potential inventor can learn from the ideas of many other potential inventors.[10] Observing the efforts of others informs an individual inventor's choices about steps 1, 2, 3, and 4. Perusing *n* and eliminating portions of that complete set of knowledge elements—the exclusion step—is easier when ideas from many other inventors can be used. Adding diverse elements to *s*—the diversity step—is easier. Subtracting unneeded elements from *s*—the focus step—is easier. Eliminating *j*-element compositions for the invention insight—the composition

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[9] R&D investment occurs subsequent to the discovery of an invention insight, although as explained in Section II with the story about the R&D projects embedded in the pursuit at Bell Labs of the invention insight for the transistor, the invention-insight discovery process and the R&D investment process may overlap. Explanation of how rivalry affects industrial R&D investment is addressed in Scott (2009) and Scott and Scott (2014) and is distinct from the focus in the present paper on competition and the process of discovering invention insights. R&D rivalry also increases the diversity of the R&D investments and such diversity can improve R&D investment performance (see Boudreau (2012) emphasizing the tradeoff between crowding effects and network effects, and Scott (1991) emphasizing the diversity of research paths). Even further removed, from the competition to discover invention insights, is the competition and learning among firms jockeying to discover and ride the winning technology to commercially successful innovation subsequent to the major R&D investments that develop competing invention insights into alternative technologies that compete to become the dominant technology for a new type of product. Eggers (2014) describes and analyses such competition in the final stages of the development of flat panel displays as liquid crystal (LCD) technology emerged as the winning technology in competition with plasma technology.

[10] Note well that competition as defined here is an idealized situation; in actuality, competition for invention insights is not entirely open in the sense of sharing. A referee notes the stories of Silicon Valley firms that will fire a scientific employee on the basis of even a rumor that he or she is talking to another firm.

step—is easier. Freely observing the work of many other inventors allows steps 1, 2, 3, and 4 work better with competition; hence, the probability that the typical potential inventor discovers the invention insight—takes step 5, the recognition step—increases. The probability that at least one potential inventor recognizes the invention insight is 1.0 minus the probability that all potential inventors fail to take step 5, and the probability that they all fail decreases with the number of competing potential inventors as long as the outcomes for the inventors are not perfectly positively correlated, and it decreases all the more since the probability that each individual inventor fails will fall with more competition.

Competition affects the search for invention insight on both an extensive and an intensive margin. On the extensive margin, competition increases the inventors' collective set of *s* knowledge elements and the collective set of *t*-element combinations being considered. The competition on the extensive margin increases the probability that the answer is in the set being searched. On the intensive margin, competition winnows unsuccessful *s* and *t* combinations, narrowing the combinations considered and making more likely the recognition of the invention insight.

Certainly mistakes can be made in excluding elements of knowledge to consider.[11] The implicit assumption in the story about the benefits of sharing ideas among potential inventors is that at the level of the world-wide collection of inventors' ideas (much as with the case of world-wide sharing of ideas among aviation experts in the era leading up to the Wright brothers' 1903 flyer), the best way to winnow elements of knowledge will be the sharing of ideas among numerous, independent, competing inventors. They will have incentives to be different, pursuing different approaches (Scott, 1991), and when very numerous it is likely that the critical eyes of many would detect an erroneous report of failure. Yet, when something is tried and reported as a failure, knowing to what the failure should be attributed may be difficult. Conceivably, potential inventors may make mistakes (either individually or collectively) in attributing failure, and they may do so in a way that, if everyone were sharing information, might preclude the chance of ever finding a solution. That might be especially likely when the "failure" is driven by erroneous presumptions about the evolution of technology.

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[11] For the insights in this paragraph, I am greatly indebted to J. P. Eggers, whose thoughts have shaped the views expressed, although any shortcomings in my sanguine view of the benefits of sharing ideas among a very large group of independent competitors are mine alone.

For example, Eggers (2014) observes that in the development of flat panel displays early assumptions were made and published reporting that liquid crystal display (LCD) technology would never approach a size suitable for use in personal computers. IBM then chose to largely ignore LCD technology and focus on plasma technology. But the underlying assumptions about the evolution of the technological building blocks for LCD were wrong. If everyone had believed the original assumptions, then we never would have found LCD. Thus, within a single firm, the difficulty of assessing the validity of a report of failure is certainly a potential reason why not having one big search process, but at least a limited number of compartmentalized search processes, might produce better outcomes, and it is one reason that firms use parallel search processes in many cases, even without telling the groups running along different paths what the other is working on—they do not want group-think biases creeping into their search processes.[12]

However, the argument for the efficacy of shared information among large numbers of competing and independent inventors is based on the idea that with those large numbers and with independence, the errors are more likely to be recognized and effective research paths chosen. Indeed, LCD technology did ultimately become the dominant technology for flat panel displays in personal computers. And, ultimately, with the freely flowing ideas about aviation, the Wright brothers found the way despite the many unproductive approaches that had attracted attention. The truth will out, and be more likely to do so the more freely that potential inventors can enter the invention-insight discovery process in an environment of freely shared ideas.

After the discovery of invention insight, at the R&D end of the spectrum from the pursuit of invention insight to the R&D investment to bring the insight to commercial reality, there is the often analyzed tradeoff of the advantage of greater rivalry that comes from the increased probability that at least one of the rivals succeeds and the lessened expectations of appropriating a return from innovative investment as in the Loury (1979) and Lee and Wilde (1980) winner-take-all model (and also in the extension of the model by Stewart (1983) to have shared rewards rather than having the winner take all) simulated and analyzed in Scott (1993, pp. 93-112). More rivalry increases the R&D investment up to the

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[12] Note that in the context of science, such erroneous group-think is what Le Fanu (2012, p. 400) observes as the self-delusion of expert committees protecting the reputations of established scientists and setting an unproductive trajectory for research.

point where the appropriability problem overtakes the incentives to invest that rivalry provides. The rivalrous equilibrium R&D investment may overshoot or fall short of the socially optimal level depending on the relative strengths of the appropriability problem and the rivalrous incentive to invest and the structure of the R&D investment costs. Boudreau, Lacetera, and Lakhani (2011) and Boudreau (2012) examine the R&D problem of software developers creating new apps and find that in highly uncertain research environments the benefit of having more competitors and hence increasing the probability of at least one successful innovation dominates the negative impact of lessened investment because of lessened prospects for appropriating returns. When research results are less uncertain, the effort-reducing effects of competition dominate and innovative performance will improve with less rivalry than would be optimal when research results are less certain.[13]

Returning to the focus on the discovery of invention insight that precedes the R&D investment problem, at this idea stage, inventors are motivated by the joy of being part of the pursuit of the invention insight and also by the hope of being first or among the first to discover the insight. To some extent, just being first matters regardless of any financial reward. In the search for invention-insight, it may not be important that expected financial reward—the commercial value—falls as competition increases. Priority is key. Meyer (2015) observes that during the period of great uncertainty about the ultimate solution—the initial working configuration of the invention—the satisfaction received by the potential inventor in the search for invention-insight may support intense activity regardless of the expectation of financial reward. In such an environment, how does competition affect the pace of invention?

With no one else searching, there is some probability that the potential inventor will find the answer, but it is certainly not random with probability equal to the number of favorable outcomes over the total. We expect it to be much more than that given effective focus reducing part of the *n* elements considered and using knowledge and experience to

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[13] Bhaskarabhatla and Hegde (2014) observe that in the R&D-end of the spectrum from pursuit of the invention insight to the development to achieve an innovation, the choice between patent-oriented policy versus sharing of knowledge is determined by forces within organizations such as a change in leadership or simply inertia. Here we are focused on the pursuit of the invention insights and the possibility that knowledge will be willingly shared among potential inventors.

give greater weight to looking in various ways at the remaining elements.[14] That is, for reasons embedded in the knowledge and experience of the potential inventor, not all elements, from the original *n*, that remain under consideration get equal weight in the search even though they are not among those getting zero weight.

So, the probability distribution is certainly not randomness in the sense of the probability of a particular outcome being the number of favorable outcomes divided by the total possible. If we knew *n* and had a sample for the typical cases of invention with more or less simultaneous independent discoveries of the essential idea, we could easily reject the null hypothesis of pure randomness even with very conservative tests—conservative in the sense of a low and decidedly lower bound estimate for *n* and for *s* and because we use just the known, reported cases of simultaneous discovery of the essential idea and that number of cases reported will be a lower bound.

Now, whatever that probability of finding the solution is, the probability that the potential inventor finds the solution first – that is, before any other potential inventor – will be less if there are other potential inventors searching for the answer. That fact is the reason that competition will matter, and moreover why it will improve the performance of the invention process, hastening the discovery of the solution. The probability distribution over the potential inventor's relative performance shifts leftward given greater competition (Scott, 2009; Scott and Scott, 2014). Moreover, for the invention process of discovering the initial essential idea (in contrast to the development process bringing the essential idea to commercial reality as an innovation), there are typically many potential inventors, and their reward may have more to do with finding the solution before others and establishing precedence than with financial rewards, and the value that they associate with such precedence will not be diminished by the presence of many competitors who are seeking the same solution. Indeed, the presence of the competitors would probably increase the sense of accomplishment and therefore the reward for the inventor first to find the solution. The results in Scott (2009) and Scott and Scott (2014) establish that the combination—caused by more competition—of the leftward shift in the potential inventor's probability distribution over relative performance and the undiminished and perhaps increased value as a function

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[14] Of course the probability is less rather than more if the steps taken to focus the search exclude some of the essential elements of knowledge.

of relative performance implies that in the presence of greater competition, potential inventors will invest more effort in finding the solution.

Thus, with the focus on the end of the spectrum where invention insights are sought, if inventors get their satisfaction from being part of the pursuit of knowledge and from priority in discovering knowledge, the story about an archetypal competition with freely shared ideas and free entry and exit of potential inventors would speed the process of finding invention insights. Under what circumstances is this story about the pursuit of invention insight plausible? In such a scenario, would the potential inventors have the incentive to incur the costs of pursuing discovery of invention insights? Applying the ideas of Kealey and Ricketts (2014) and Meyer (2003, 2007, 2013, 2015), arguably the answer is yes. Kealey and Ricketts discuss science as a contribution good for which research ideas are shared freely among scientists pursuing new knowledge, and only those scientists who participate in the research can use in their own research the shared ideas of others. The participants have the incentive to share knowledge because each receives more benefit from sharing ideas—and contributing to the production of the contribution good—than would be received without participation in the mutually shared creation of knowledge.[15] The archetypal competing inventors pursuing inventive insights in our foregoing story are analogously participating in the process of discovery that is producing the contribution good of invention insights.

Meyer provides a scenario that explains why incentives support participation in the production of the contribution good by freely sharing one's ideas among a community of participants who all share their ideas. In the pursuit of invention, there is great uncertainty, and the potential inventors do not perceive that they are close enough to the discovery of an invention to have any valuable intellectual property to hide from the observation of others. Moreover, they receive great satisfaction from pursuing the discovery of the invention insight in a community of potential inventors, and they all pursue discovery while sharing freely ideas and interacting with the others.

Meyer supports his story with observations of collective invention from the history of technology. He also observes that eventually, in the process of discovery, the uncertainty

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[15] Perhaps something like this is going on with the sharing of knowledge about genetically engineered mice having positive effects on the entry of new researchers and the diversity of research yet no negative effect on the introduction of new strains of genetically engineered mice (Murray, et al., 2016).

has been resolved sufficiently, and then, when the potential inventors have the invention in sight, they cease to share their ideas. Following Meyer's explanation of collective invention, in our story, potential inventors freely share ideas in an environment of great uncertainty, and by sharing their ideas and interacting in a community of inventors, they improve—by their observations of the ideas and the successes and failures of others—their decision making in the exclusion step, the diversity step, the focus step, and the composition step of the discovery process.

Meyer observes that the sharing of ideas will stop once uncertainty is sufficiently resolved, and so that change in behavior would be expected in the process of discovering invention insight that we have described in the context of the invention-insight sample space. Using the now classic example of the invention of the transistor at Bell Labs, we can see an example of the period of freely shared ideas among a community of inventors' interactive discovery process followed by the incentive for the individual inventors to use others' ideas but withhold their own ideas when they are close to the invention insight.

Even within a single organization with researchers who share ideas as they pursue an invention insight, the problem of incomplete sharing of information among researchers comes into play once the community of researchers gets close to the invention insight. The problem is illustrated in Isaacson's (2014, pp. 132-149) history of the development at Bell Labs of the transistor and the behavior of Shockley who wanted to ensure that he received credit for his unique invention insight of the p-n junction transistor. Shockley developed his insight without sharing it immediately with his colleagues and thus without letting them help in the process of developing it beyond the help that they had provided when he observed their point-contact transistor. He then made his invention insight public when he had the right moment to claim the credit for himself and assert priority in the invention. Isaacson (2014, p. 148) describes the reaction of Shockley's colleagues who nine years later would share with him the Nobel Prize:

> Bardeen and Brattain were taken aback. The fact that Shockley had been so secretive about his new idea—thus violating the code of sharing that was part of the Bell culture—upset them. Yet they could not help but be impressed by the simple beauty of Shockley's approach.

Importantly, Shockley was a direct participant in the research being done at Bell Labs. He participated and benefited from the ideas that his colleagues developed and shared—in particular, Bardeen and Brattain who shared their ideas openly with colleagues as their work on their point-contact transistor progressed. Bardeen and Brattain working openly in the context of the diverse and interactive environment of Bell Labs illustrate the organizational ideal of sharing knowledge as it is created in a diverse research environment. Each individual can have the benefit of a set of s knowledge elements that has benefited from the ideas of colleagues. In contrast, Shockley's behavior illustrates the problem—the temptation to participate sufficiently to receive the benefits of others' insights and then withhold information, free riding on the diverse environment's research process—that is expected to surface among the community of potential inventors once uncertainty has been resolved sufficiently and the invention insight is close at hand.

V. Evidence of the Discovery Process

This section reviews evidence about the discovery process that has been described with the invention-insight sample space and the exclusion, diversity, focus, composition, and recognition steps. In the context of uncertainty, the sample space for a potential inventor's creative act of invention can be summarized with the set $\{p(f); f = 0, 1, . . ., t\}$ as detailed in Section III. In words, the potential inventor's knowledge and experience set may have no overlap with the t knowledge elements necessary for the invention, or it may overlap with one or more of the t necessary elements, and the set $\{p(f); f = 0, 1, . . ., t\}$ gives the set of proportions of the sample space with 0, 1, and so on to t overlaps between the s elements in the potential inventor's knowledge set and the t elements that comprise the essential initial idea. A successful inventor focuses on the portion of the sample space where the desired invention's elements overlap with the inventor's knowledge elements. Strategies and policies designed to improve creative performance need, at bottom, to increase the potential inventor's focus on those parts of the sample space. Strategies and policies to improve the exclusion, diversity, focus, and composition steps of a potential inventor's discovery process, as described in Section III, will result in a higher probability for the success of the recognition step—that is, the discovery of the invention insight by an inventor.

Observing actual organizations, we see examples of behavior that align well with our description of the discovery of invention insights. In addition to examples of collective invention reviewed and developed by Meyer (2003, 2007, 2013, 2015), we find evidence about the discovery process in the observations of multiple invention, organizational strategies, sharing of knowledge across firms, and liberal arts education.

Sharing Knowledge, Diversity, and Multiple Invention. Recall the observations of Arthur and Kelly to the effect that the invention insight always comes to an individual, although (as many writers have observed) many individuals may have that basic insight about the same time. Thus, the idea of using diversity to stimulate invention must incorporate some way to get many of the potentially useful diverse insights into each potential inventor's mind. For a group of brain-storming individuals or for lone inventors each openly sharing their knowledge with the others, several of the inventors, aided by the collective insights, might perceive the answer at more or less the same time. We have explained that the probability of at least one success increases with competition in the discovery process, and in fact we expect the process to produce several successes.

Diversity of knowledge elements for an individual inventor, who has the advantage of knowledge of other potential inventors' key elements of knowledge and insights about how those other insights might be useful for an invention, would be expected to result in the individual inventor making better choices for the subset of the universe of n knowledge elements from which to choose s elements to master, thereby improving exclusion intuition. Moreover, the diversity of insights would improve the choice of the s elements of knowledge from the portion of the n elements perused, thereby informing the inventor's search to discover the invention's essential t elements. Thus, the availability of knowledge about the successes and failures of many other potential inventors improves the inventor's decisions in the exclusion, diversity, focus, and composition steps of the discovery process. The often observed occurrence of simultaneous, independent discoveries—multiple inventors find the essential idea for an invention at about the same time—suggests that the collection of potential inventors at a particular time in history are benefiting from diversity of insight informed by collective understanding and appreciation of the invention-relevant aspects of the universe of n knowledge elements. Sharing information, the collection of potential inventors have used the exclusion, diversity, focus, and composition steps to the point where the vast uncertainty associated with the original problem has been eliminated and the discovery of the invention insight is close at hand, and then, unsurprisingly, many of the potential inventors find the answer at about the same time.

Arthur (2009, pp. 125-126) describes that process with multiple contemporaneous inventors or inventive teams working to establish the inventive insight with the result that typically there will not be a single originator of a new technology. Arthur observes (2009, p. 126):

> [E]ven with a single originator, human interaction and informal networks of communication greatly enhance the process . . . . They steep the originator in the lore that has built up around the problem, offer suggestions of principles at work in other domains, and provide equipment and know-how to bring concepts to physical reality.

Many potential inventors are learning about where to focus among the universe of *n* knowledge elements and also about the important knowledge elements to include in their own set of *s* elements to be understood well. There is general understanding in the community of inventors about those parts of the universe of *n* elements that can be ignored and about the most likely portions of the subset of the remaining *n* elements that should be mastered. Also, through time, the universe of the *n* elements of knowledge is expanding, new elements of knowledge and understanding of their relationships are opening up, and the community of inventors will see and consider the usefulness of the new knowledge. For example, the invention of the transistor by Shockley, Bardeen, and Brattain created new possibilities for invention in computing and information technology as detailed in Isaacson (2014).

With many potential inventors pursuing a particular invention in a community of inventors who are all informed about the universe of knowledge elements and who share the knowledge about the portions of that universe on which to focus and about the most likely knowledge elements that should be mastered, several inventors are expected to take the recognition step and make more or less simultaneous discoveries of the sought after invention. As Johnson (2014, p. 66) observes when discussing the multiple, independent inventors introducing devices to produce artificial cold, the invention provides:

> . . . an example of one of the great curiosities in the history of innovation: what scholars now call “multiple invention.” Inventions and scientific discoveries tend to come in clusters, where a handful of geographically dispersed investigators stumble independently onto the very same discovery. The isolated genius coming

> up with an idea that no one else could even dream of is actually the exception, not the rule. Most discoveries become imaginable at a very specific moment in history, after which point multiple people start to imagine them. The electric battery, the telegraph, the steam engine, and the digital music library were all independently invented by multiple individuals in the space of a few years.

And then how do we understand the process of multiple inventors with the same breakthrough? Johnson explains (2014, p. 64, italics in original):

> How do we explain this breakthrough? It's not just a matter of a solitary genius coming up with a brilliant invention because he or she is smarter than everyone else. And that's because ideas are fundamentally *networks* of other ideas. We take the tools and metaphors and concepts and scientific understanding of our time, and we remix them into something new. But if you don't have the right building blocks, you can't make the breakthrough, however brilliant you might be. The smartest mind in the world couldn't invent a refrigerator in the middle of the seventeenth century. It simply wasn't part of the adjacent possible at that moment. But by 1850, the pieces had come together.

To Johnson's explanation of the historical episodes of multiple invention, we add that the sharing of information as potential inventors carried out the steps of the invention-insight discovery process greatly reduced uncertainty to the point where the discovery of the invention insight was nigh and the recognition step was made by many at more or less the same time.

Organizational strategies. The foregoing discussion of sharing ideas and subsequent multiple invention implies that diversity is addressed by putting together individuals with their *s*-element sets that will differ (not overlap completely), yet preserving the individual inventors and letting each have access to the ideas of the diverse group.

Organizations have recognized the benefits of bringing together individuals with diverse talents and experience to have cross-disciplinary R&D laboratories. Edison's Menlo Park lab is an early example, and its example for cross-disciplinary research was followed by Bell Labs and Xerox PARC and many others (Johnson, 2014, p. 211). Bell Labs in the post-World-War-II era is an outstanding example, and Isaacson (2014, p. 132-133, pp. 139-140) describes how Bell Labs brought together individuals of many and quite diverse talents in close physical proximity to allow frequent face-to-face interactions.

These examples illustrate organizations that had to find ways to facilitate the discovery of invention insights and came up with effective policies to develop teams of individuals with diverse talents who have knowledge elements likely to include those necessary for a particular desired invention. Bell Labs used that strategy to develop the transistor as Isaacson (2014, pp. 132-149) describes. Organizations may develop systematic research processes to make discovery of invention insights more likely. Duggan (2007, pp. 141-143) describes General Electric's what-works matrix which can be seen as a corporate method for choosing the set of *s* knowledge elements with which to work and then then searching for the essential *t* elements that will constitute the invention insight. Firms with research will hope to have the "absorptive capacity" to use ideas generated by researchers in other firms (Cohen and Levinthal, 1989, 1990), thus increasing the quality of the *s* knowledge elements with which they work. Firms hope to benefit from open innovation strategies (Lichtenthaler, 2011) both to acquire knowledge from others' research and to profit from sharing their own research ideas. Firms work with universities—perhaps when they reckon most of the benefits will accrue to them (Link, 2015)—and will at times locate in research parks to assimilate more effectively the benefits of links to universities (Link and Scott, 2003, 2007, 2015), again increasing the quality of their *s* knowledge elements.

Sharing knowledge across organizations. Companies clearly share knowledge by means of spillovers of knowledge from one firm to another as they seek invention insights that enable patents for new technology. Even with controls for other things, including the effects associated with the industry and technology areas in which each firm of a pair of firms operates, each firm is about nine times more likely to use (as reflected in citations of the other's patents) the other's invention insights when the pair meet to a highly significant extent in their sets of innovation markets (Scott, 2000, 2001a, 2001b, 2003). Scott (2003, pp. 249-250) describes the result:

> [A] model of the citations of one firm of the patents of another. . . . shows the incidence rate ratios . . . for citations given insignificant congruence (the probability of more congruence is 1.0 against the null hypothesis of random meetings) of the citing and the cited firms' operations as compared with completely significant congruence (the probability of more congruence is 0.0 given the null hypothesis). . . . The model controls for the firms' numbers of patents, the science linkage of their patents, their product market diversification (as indicated by the industries where they have sales), their innovation market diversification (as indicated by the product categories where they have patents), their locations in

> product and innovation markets, and the significance of the congruence of their product market operations and their innovation market operations. . . . [Sharing of knowledge] is evidenced by the greatly increased frequency of mutual citations apart from the effects associated with particular locations in the product and innovation markets. Imagine two firms that have completely congruent operations in *product* markets. Then even after controlling for the effect of that congruence, and even after sweeping out the effects associated with the particular locations in product and innovation markets, with the closeness to science and size of the patent portfolios, and with the diversification in product and innovation markets, the additional effect of significant congruence in innovation markets increases the expected citations by about nine . . . times. (italics in original)

Thus, companies are in effect—whether through explicit agreements such as licensing agreements (Link and Scott, 2002) or through spillovers of knowledge reflected in the citations of the patents of other firms—sharing knowledge. In the context of our description of the invention-insight sample space, the sharing of knowledge elements improves the quality of the *s* elements of knowledge available to companies' inventors.[16]

Universities and liberal arts education. Recall Poincaré's observation, quoted in Section III, that the most productive combinations for invention often use elements of knowledge taken from areas that are ostensibly very different. Isaacson (2014, pp. 487-488) emphasizes the importance of the intersection of science and the humanities for the digital revolution, indeed beginning his history and analysis of that revolution with the story (Isaacson, 2014, pp. 7-33) of the collaboration of Babbage and Lovelace. He recognizes Lovelace's "appreciation for poetical science" (Isaacson, 2014, p. 33) as the reason she could foresee in Babbage's mechanical, programmable machine for processing numbers the multifaceted uses of computers that only came into existence over a century after their collaboration. Johnson (2014, pp. 241-255) also provides an insightful history of the collaboration between Babbage and Lovelace, emphasizing how far ahead of their time were Babbage's Analytical Engine and Lovelace's vision of applying programmable computers not just to the processing of numbers but to the arts. Johnson (2014, pp. 252-253) observes: "Ada Lovelace could see the aesthetic possibilities of Babbage's Analytical Engine because

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[16] Firms collaborate in many ways, from licensing agreements to research joint ventures, to share knowledge and enhance their R&D performance. A lesser known way that firms collaborate to improve their R&D results, among other things, is in the collaborative development of infrastructure-technology standards (for example, see Leech and Scott, forthcoming) that enable communication within and across organizations. Scott and Scott (2015b) find that such standards increase the value of R&D investments.

her life had been lived at the unique collision point between advanced math and Romantic poetry. . . . [enabling her] to see beyond the surface appearances of things [and] . . . imagine a machine capable of manipulating symbols or composing music . . .”

The history of the collaboration between Babbage and Lovelace suggests that the traditional curricular requirements of universities support invention and innovation in ways consistent with the description of discovery of invention insights in the context of the invention-insight sample space. Disciplinary depth is provided by a student’s major field or fields, but a diversity of knowledge is encouraged by distribution requirements that can increase the specialist’s ability to appreciate and communicate with a specialist in another field of endeavor. The university is a growing repository for knowledge elements, and it provides students with access to the knowledge in particular fields— learning in depth from the students’ majors but also with an appreciation for the breadth of knowledge from their study to satisfy distribution requirements. Universities help train students for successful invention by helping them focus on subsets of knowledge elements (helping with the exclusion step in the discovery process) and helping them achieve better choices for the set of s knowledge elements that are mastered (helping with the diversity, focus, and composition steps) and helping them learn to recognize and appreciate new combinations of knowledge elements (helping with the recognition step). Moreover, a university education can help prepare the students to appreciate other areas of knowledge and be ready to collaborate with experts in those other fields.[17]

Thus, universities help potential inventors in particular technology areas focus by excluding chunks of the n knowledge elements as potential study candidates, and university training also helps them chose s elements to master wisely from among the subset of the n elements that remains. We have overall subject matter in universities and it evolves. The subject matter available in universities is one way of excluding large chunks of the n possible elements of knowledge, and then the various disciplines and allowable majors, minors and distribution requirements to ensure diversity of approaches and areas of learning in the student’s curriculum are ways of helping individuals choose their s elements to study

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[17] The argument developed in this section, using Babbage and Lovelace as an exemplifying case study, is that technical and liberal arts education are complements. In that case, universities that are emphasizing STEM (science, technology, engineering, and mathematics) curricula at the expense of the liberal arts more generally are using a counterproductive policy.

and master. Moreover, research within the university system supports the growth of the knowledge universe of *n* elements.[18]

VI. Conclusion

The description of the creative process of discovering invention insights has implications for the role of government in promoting invention insights and also for the concern that altogether new areas of technology are opening up more slowly.

Government's Role. Regarding the role of government in supporting discovery of invention insights, the straightforward implication of the effect of competition on the process of discovery is that government should support the free exchange of ideas and unfettered competition to discover new ideas. Implementing that role, however, is not straightforward, at least in part because the very process of providing support may rely on the type of expert oversight for administering the support that will limit free entry of potential inventors and channel the selection of promising ideas and hence inhibit the free flow of ideas in shaping the exclusion, diversity, focus, and composition steps that precede the recognition step in the discovery of inventive insights.[19]

The subtlety of the policy problem will surely require new approaches—what Leyden and Link (2015) aptly call "public sector entrepreneurship" and place, moreover, in the context of networks of potential innovators. Their view of public sector entrepreneurship fits well with the context of our discussion of potential inventors freely sharing ideas as they pursue invention insights in the context of great uncertainty. As Leyden and Link (2015, p. 14) explain:

> [P]ublic sector entrepreneurship refers to innovative public policy initiatives that generate greater economic prosperity by transforming a status-quo economic environment into one that is more conducive to economic units engaging in creative activities in the face of uncertainty. Through policy initiatives that are characterized by public sector entrepreneurship, there will be more development of new technology and hence more innovation throughout the economy.

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[18] However, the Bayh-Dole Act (P.L. 96-517, Patent and Trademark Act Amendments of 1980) and universities' assertion of intellectual property rights inhibits the sharing of information and thus to some extent inhibits the development of new invention insights.

[19] For example, Le Fanu (2012, p. 400 and Part III generally) discusses the "collective self-delusion" of the committees of experts and reviewers coordinating publicly-funded research in the life sciences and channeling support away from directions that would threaten the reputations of established scientists.

The transforming of the economic environment comes through diversity in the ideas available to inventors and their sharing of those ideas—a diversity and a sharing that this paper has described in the context of the invention-insight sample space. Leyden and Link (2015, p. 14) observe:

> [T]he dominant method by which public sector entrepreneurship can improve that transformation today is by increasing the effectiveness of social networks, that is, by increasing the heterogeneity of experiential ties among economic units and the ability of those same economic units to exploit (i.e. to learn from) such diversity.

Then to restate the question of government's role given the descriptions of the invention-insight sample space and the process of discovering invention insights, how can public sector entrepreneurship best facilitate the effectiveness of the search for invention insights? The answer that emerges from our descriptions: Support competition and the sharing of ideas among would-be inventors.

It must be emphasized that the desired sharing of ideas is during the period of great uncertainty before potential inventors are close to discovering the invention insight. Once the potential inventors are close to the invention insight, further sharing of ideas freely must be avoided if the inventor wants to protect profitable intellectual property. We shall in the upcoming discussion emphasize the conflict between freely sharing ideas and assertion of intellectual property rights—a conflict that arises once inventors are close to discovering the invention insight. As we discus next, new policy to support the discovery process might reverse the trend of slowing introductions of fundamentally new technologies.

Slowing of Fundamentally New Technologies. Youn, et al. (2015) use patent statistics to demonstrate that although the opening up of altogether new areas of technology is slowing, the appearance of new technologies is not expected to slow because of the vast potential for new technologies based on combinations of existing technologies. Our descriptions of the invention-insight sample space and the process of discovery of invention insights suggest a conjecture about why we might reasonably expect an increase in the pace of the introductions of altogether new areas of technology.

Reflecting on the examples of invention insights in Table 1 and the many others described in the literature about invention, the directly observed t, the number of elements of knowledge combined in an invention insight, appears typically to be quite low—that is, a small number of knowledge elements are combined for an invention insight. The number of elements combined will depend somewhat on how one conceptualizes the invention insight's combination of essential elements, so an observer might count three, or four, or five, et cetera. Yet even allowing such variance across observations, the number of elements combined has been small for the typical invention insight.

Of course, the typical invention insight's combined knowledge elements are extraordinarily numerous in the sense of the underlying knowledge elements—the elements that were combined to create each of the directly-observed elements of the particular invention insight being studied, and then the elements combined for each of those preceding elements, and so on. Each element of an invention insight's t elements is itself a combination of elements of knowledge discovered earlier. Yet, the pace of invention is grounded in the new, typically "low-t" invention insights—i.e., the new invention insights combining small numbers of existing elements in their aggregated sense.

Reflecting on Figure 4, we see the vast potential for speeding up the pace of invention—and hence the pace of technological progress—by opening up the exploration of possibilities for inventions using "higher-t" combinations of elements of knowledge. The opening up of the exploration of "higher-t" invention insights is to be expected for two reasons. First, advances in information technology will make possible the conjunction of human and artificial intelligence needed to examine such insights. Second, the great uncertainty associated with the discovery process for "higher-t" insights will support the free exchange of ideas among potential inventors pursuing those insights.

Kealy and Ricketts (2014, p. 1015) define a pure contribution good ". . . as a good whose benefits are non-rival over contributors but that cannot be accessed by non-contributors." What would be a situation for which the research to develop an invention insight would be a pure contribution good in the sense of Kealy and Ricketts? The research for "higher-t" invention insights would be a likely candidate for such a situation, while research for "lower-t" invention insights might be a less likely candidate. As t, the number of knowledge elements combined in the invention insight, increases, we expect it will become more difficult for an individual to withhold information and yet have a good probability of being the individual who discovers the invention insight. With a higher-t invention insight, the projects become sufficiently complex that the individual needs to share the knowledge as it develops so that others can participate in helping with the further developments until someone has the invention insight. If an individual participating in the research does not share all ideas as they develop, progress toward the invention insight will be much less likely—indeed, unlikely. The insight may not occur or will be developed more slowly. Thus, for research seeking higher-*t* invention insights, both the participation constraint and the incentive constraint hold for each individual researcher to share completely all ideas as they are developed.

For an individual researcher to share ideas fully with colleagues, the participation constraint requires that the benefit (which for many potential inventors may include a substantial amount of nonmonetary satisfaction from the joy of pursuing new knowledge—satisfaction that augments whatever salary they may command for their research services) is greater than the cost of effort, and the incentive constraint requires that the individual's return from sharing information freely exceeds the expected return to the researcher if the researcher does not share all of his or her ideas with other researchers. We expect that as *t*, the number of elements of knowledge in the invention insight increases, the research becomes so difficult that sharing ones' ideas to secure the feedback from others is necessary to make progress, and so the incentive constraint will hold, and each researcher will share all ideas as they are developed.[20] Thus, the research for higher-*t* invention insights would, if our expectation is true, have the circumstances sufficient for what Kealy and Ricketts call a pure contribution good.[21] Hence our conjecture: If the conjunction of human and artificial intelligence opens up the exploration of "higher-t" invention insights, the pace of science and altogether new technology will quicken. Thus, a new opportunity for public sector entrepreneurship in the sense of Leyden and Link (2015) is the initiation of policies to promote pursuit of "higher-t" invention insights—that is, insights that directly combine unusually large numbers of elements of knowledge.

From our discussion of the invention-insight sample space, the government's role regarding discovery of *invention insights* is to promote competition and the free exchange of

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[20] In our discussion of the discovery of the invention insight for the transistor, we observed a case where participants may not have access to 100% of the research produced because some participants (like Shockley) withhold some information even while others (like Bardeen and Brattain) share their information freely.

[21] Kealy and Ricketts (2014), p. 1016) consider the possibility that an individual's participation in science is an either-or decision, and the individual needs to participate—i.e. do science—to have 100% access to the contribution good of the knowledge produced by the community of researchers, with non-participants receiving some proportion of the good, with the proportion being zero in the case of a pure contribution good.

ideas. Given sufficient uncertainty in the search for insights, those pre-invention-insight ideas in themselves should be freed from the restrictions of intellectual property, although—because of the overlapping and intermingling of the pursuit of invention insight and R&D projects and because when potential inventors are getting close to the discovery of an invention insight they will be less willing to share their ideas—the ideal of freely shared ideas among potential inventors who can easily enter the pursuit of invention insight will not coincide with the reality of the process.[22]

In the first sentence of the preceding paragraph, "invention insights" has been italicized to emphasize that the promotion of the free exchange of ideas is in the idealized pre-invention-insight situation. When potential inventors operating outside the protective confines of the large corporation have learned enough to be close to the answer afforded by an invention insight, the practical problem of the conflict between the benefits of sharing information and protecting hoped-for future profit must be recognized, and—if those profits are to be had—sharing of information must cease until an arrangement for a satisfactory return on any resulting invention insight is obtained. While sharing information may have a benefit during that final stage before someone has the complete insight, it will also be likely to have a crushing cost to the inventor if the inventor's chance for intellectual property is lost. There is a huge practical difference between (1) the idealized setting before potential inventors are close to the answer of the invention insight and are all benefiting from sharing knowledge and (2) the setting where, still short of the complete insight, protecting proprietary interests by not sharing must take precedence if an inventor is to have a good chance of profiting from an invention.

Sometimes an inventor—Kearns with his invention insight for the intermittent windshield wiper is an example—will share a completed invention insight before property rights and arrangements for a financial return on the invention are secured, and the results can be devastating for the inventor. Kearns' case (Schudel, 2005; Seabrook, 1993; Vecchio, 2005) illustrates (despite the eventual $30 million in settlements from patent infringement suits) that if one starts sharing a complete invention insight before securing the property rights and the licensing or manufacturing arrangements to generate income therefrom,

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[22] Scherer (1984, p. 26) provides interesting discussion and insights about the expectation that the protection of intellectual property is less important for invention than for investment, development, and innovation.

disaster awaits--especially if dealing with a corporation with an in-house staff of engineers dedicated to developing technology to be used by the corporation. In one of the infringement lawsuits, Ford argued that Kearns' patent was invalid because his windshield system did not use new concepts, while Kearns' view was that his patent was valid because he had conceived a new combination of parts to achieve his intermittent windshield wiper (Vecchio, 2005). The view espoused by Kearns is consistent with our definition of the invention insight that spawns an invention.

The story of Farnsworth (Gladwell, 2002; Schwartz, 2000), like Kearns an inventor attempting to go it alone without the support of a large corporation, and the invention insight for television illustrates the point in the context of an innovation that entailed a much more complicated combination of knowledge elements for invention insights enabling the systems innovation of television. Again, the danger is in sharing technology that is in the development stage, with the invention insight complete, but development not being complete. Again, the case of Farnsworth and television, like the case of Kearns and the intermittent windshield wiper, illustrates the difficulty of a lone inventor attempting the development and commercialization without the support of the large corporation. Ford's engineers looked at Kearns' work and used it; RCA's engineers looked at Farnsworth's work and used it. For Farnsworth, an alternative would have been to work as an inventor in RCA's corporate research group; for Kearns, an alternative path would have been working within the engineering group at Ford if his personal temperament and Ford's corporate expectations for their engineers would have allowed that arrangement. Potential inventors operating outside the support of a large corporation should read the stories of Kearns and Farnsworth and learn well the lessons the stories offer.

Much remains for future research. High on the list is the development of the incentives story in the context of free and open competition in the process of discovery of invention insights and the development of that story in the context of the sample space for the insights.[23] Although the formalism of this paper's description of the invention-insight

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[23] With the example from Bell Labs and the invention of the transistor, we have observed that the incentive to share information is expected to decline once researchers get sufficiently close to the discovery. The researchers share in the first place because of the great uncertainty about how to proceed. After the uncertainty is sufficiently resolved, there is a real free rider problem, where the best solution from a personal perspective would be to have everyone else share what they are doing but to keep one's own knowledge private. How might that issue be addressed? The size of a research system and the characteristics of the researcher will

sample space yields intuitive results, it serves to provide the diagrams showing that until potential inventors are very close to solving the problem of discovering a sought-after invention insight, they are confronted with vast portions of the invention-insight sample space and great uncertainty. The diagrams also show that the "small-*t*" portions of the sample space where invention insights are typically found leave vast portions where there are extraordinarily larger numbers of higher-*t* combinations of elements with potential invention insights that could speed the pace of technological progress. From the description of the invention-insight sample space and the decomposition of the invention-insight discovery process, we have deduced a novel opportunity for public sector entrepreneurship to speed the pace of technological progress by supporting the discovery of invention insights combining large numbers of knowledge elements directly rather than indirectly over time as small numbers of elements of knowledge are accumulated in new technologies.

The challenge of embracing the opportunity for public sector entrepreneurship to develop new policy to stimulate "higher-t" invention insights is considerable. The challenge is heightened because higher-t invention insights can provide the basis for innovations across all the various modes of innovation identified by Martin and Scott (2000, Table 1, p. 439). Higher-t invention insight is possible whether the insight supports innovation of inputs (such as software or instruments) for using industries, the application (as in agriculture or light industry) of such inputs, the development of complex systems (as in aerospace or telecommunications), or the application of high science content technology (as in materials science or pharmaceuticals).

In meeting the challenge of designing new policies to stimulate higher-t invention insights, the description of the discovery of invention insights also provides the basis for the conclusion that the appropriate policies should promote competition in the form of easy entry of potential inventors who freely share ideas. Assignment of intellectual property should not occur for results early in the search for invention insights, and because of the nonlinear, interactive nature of the search for insights and the development of those insights

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affect incentives too. For example, the researcher within a large corporation may have very different characteristics from the lone inventor—as exemplified by the contrasts between the lone inventor Farnsworth and Zworykin, the top television researcher for RCA and previously head of television R&D at Westinghouse (Gladwell, 2002; Schwartz, 2000). In many cases the people that do not find the solution get absolutely nothing beyond the satisfaction of participating in the pursuit of knowledge, while fame and sometimes riches are bestowed on the winner. How does that affect incentives?

into commercially successful innovations, technological progress would probably be more rapid if there were more sharing of pre-invention-insight ideas.

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Table 1. Examples of the Elements of Knowledge Combined for the Invention Insight Underlying an Invention.

Invention	Elements of Knowledge
19th century's electric light bulb	"some kind of filament that glows when an electric current runs through it, some mechanism to keep the filament from burning out too quickly, and a means of supplying electric power to start the reaction . . ." Edison and other inventors used "a carbon filament, suspended in a vacuum to prevent oxidation, thus keeping the filament from burning up too quickly." Edison's lightbulb used a filament of carbonized bamboo and outperformed others.[a]
Wright brothers' 1903 flying machine	"means of control and stability of flight", "wing sections with good lift", "a lightweight propulsion system", and "a high-efficiency propeller" based on the underlying principles of propulsion, lift, and wing-warping control[b]
Birdseye's commercially viable frozen foods	"flash freezing" at very low temperatures with its smaller ice crystals, "scientific knowledge of how to produce temperatures well below freezing", "industrial knowledge of how to build a production line"[c]
Whittle's and von Ohain's jet engine	Ideas of compressed and hence pressurized air, fuel burning in a flow of compressed air, and expulsion of high-velocity gas are brought together in the jet engine using "five main systems: intake, compressor, combustor, turbine, and exhaust nozzle" to "burn fuel in a constant flow of pressurized air and expel the resulting high-velocity gas backward. . . . [producing] [b]y Newton's third law . . . an equal-and-opposite forward force."[d]
Bardeen's and Brattain's December 1947 point-contact transistor	A semiconductor (germanium) doped with impurities to give it extra electrons and make it a better conductor, two very closely placed gold electrodes penetrating the semiconductor's surface, and electric current to positively charged gold foil electrodes; these three essential elements together overcame the surface state shield effect of the semiconductor and allowed the field effect to work and switch on the flow of electrons from the negatively charged semiconductor.[e]
Shockley's February 1948 p-n junction transistor	A middle layer of positively charged (p-type) semiconductor (germanium doped with impurities to provide a deficit of electrons) sandwiched between a top and bottom layer of

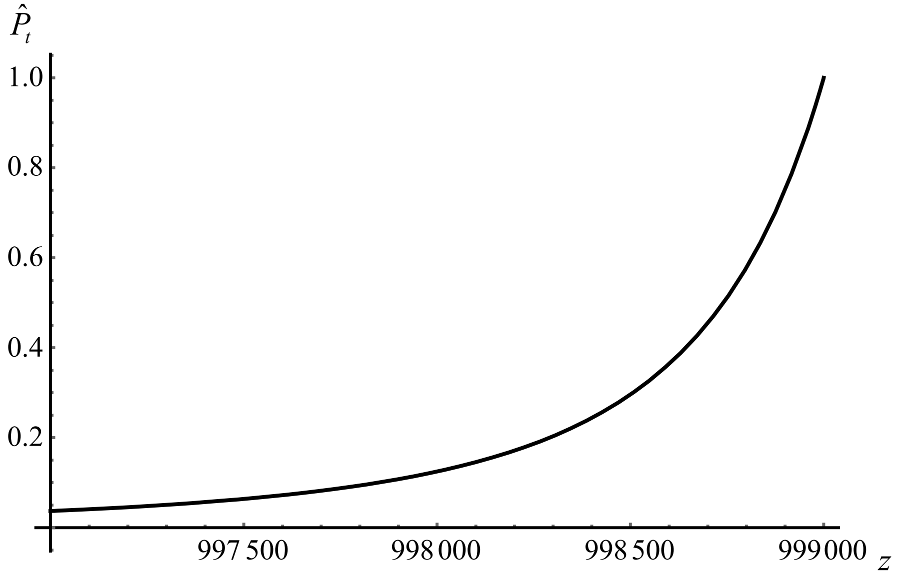
	negatively charged (n-type) semiconductor (germanium doped with impurities to have an excess of electrons), and wires into each of the three layers to regulate voltage to each; a small positive voltage delivered to the middle layer dramatically increased the flow of electrons between the two outside layers, and the greater the positive voltage to the middle layer the greater the flow between the outer layers, and so adjusting the voltage for the middle layer could amplify or switch off current going through the semiconductor.[f]
Gates's and Allen's operating system software for microcomputers (Microsoft)	the computer language BASIC developed by Dartmouth College professors Kemeny and Kurtz, the PDP-8 minicomputer from Digital Equipment Corporation (DEC), the Intel 8080 chip (microprocessor), and Robert's desktop microcomputer Altair from Robert's company MITS[g]
Brin's and Page's commercially viable internet search (Google)	Reverse links to rank Web sites, downloading of the entire Internet to many powerful computers allowing full-text search of the Internet, data mining algorithms—the three elements together allowed the use of the "data mining algorithms to search all the reverse links on the Internet and rank these links like academic citations", and then the fourth knowledge element of "advertisements in simple lists, just like its search results".[h] The ranking of the web pages used ". . . a recursive process with multiple feedback loops: each page was ranked by the number and quality of links coming into it, and the quality of these links was determined by the number and quality of links to the pages that originated them, and so on."[i]

[a] Source: Johnson (2014, p. 206, pp. 209-210)
[b] Source: Arthur (2009, p. 120), McCullough (2015, pp. 38-39)
[c] Source: Johnson (2014, p. 74, pp. 68-76)
[d] Source: Arthur (2009, p. 20, p. 34)
[e] Source: Isaacson (2014, pp. 141-145)
[f] Source: Isaacson (2014, p. 147)
[g] Source: Duggan (2007, pp. 84-92), Isaacson (2014, pp. 313-343)
[h] Source: Duggan (2007, p. 95, p. 98, pp. 93-99)
[i] Source: Isaacson (2014, p. 458)

Table 2. For n = 1,000,000, s = 1,000, t = 3, the effect of the exclusion step.

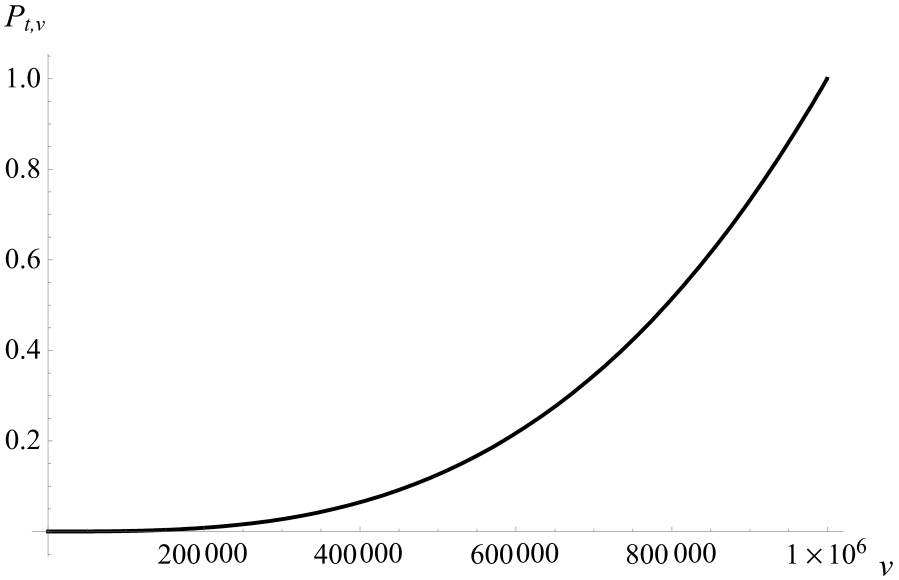
z = number of knowledge elements successfully eliminated from consideration	Proportion, $\hat{P}_t$, of the invention-insight sample space with $f = t$
0	9.97×10^{-10}
100,000	1.37×10^{-9}
500,000	7.98×10^{-9}
800,000	1.25×10^{-7}
900,000	9.97×10^{-7}
950,000	7.98×10^{-6}
990,000	9.97×10^{-4}
995,000	7.98×10^{-3}
996,000	1.56×10^{-2}
997,000	3.70×10^{-2}
998,000	0.125
998,500	0.296
998,600	0.364
998,700	0.455
998,800	0.578
998,900	0.751
998,920	0.794
998,940	0.839
998,960	0.889
998,980	0.942
998,990	0.971
998,995	0.985
999,000	1.000

Figure 1. The exclusion step increases the proportion of the invention-insight sample space for which the inventor's mastered set contains the invention insight.[a]



[a] Illustration for n = 1,000,000, s = 1,000, t = 3.

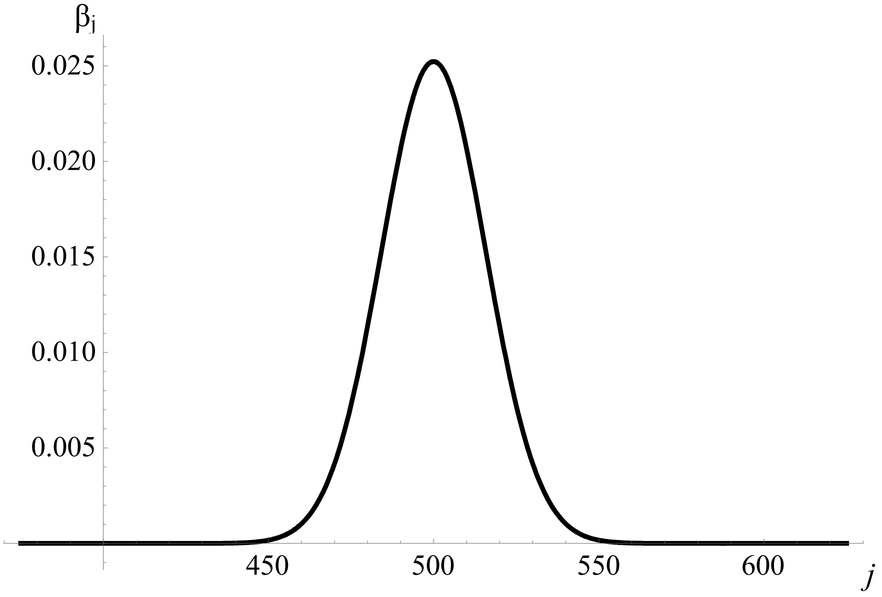
Figure 2. The diversity step increases the proportion of the inventor’s invention-insight sample space that contains the invention insight.[a]



[a] Illustration for n = 1,000,000, s = 1,000, t = 3.

Figure 3. The proportion of the possible knowledge-element combinations eliminated by the third step—the focus step—of successfully reducing *s* by 1.

Figure 4. The proportion of the possible combinations eliminated by the fourth (composition) step eliminating from consideration *j* as a possibility of the number of elements *t* in the essential idea for the invention.[a]



[a] Illustration for s = 1,000.