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The Identity of the Generator in the Problem of Social Cost

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Abstract: One of Coase's central insights is that distinguishing between the generator and recipient of an externality is of limited value because externality problems are reciprocal. We reconsider the relevance of the identity of the generator in a model with non-contractible investment ex ante but frictionless bargaining over the externality ex post. In this framework, a party may distort its investment to worsen the other's threat point in bargaining. We demonstrate that the presence of this distortion depends, among other factors, on whether the investing party is a generator. Social efficiency can sometimes be improved by conditioning property rights on the identity of the generator: for example, assigning damage rights if the rights holder is a generator and injunction rights if the rights holder is a recipient can be more efficient than either unconditional damage or injunction rights.

JEL Codes: K11, D23, C78, H23

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1. Introduction

One of the most important insights in Coase's (1960) classic "Problem of Social Cost" is his emphasis on the reciprocal nature of externality problems:

> The question is commonly thought of as one in which A inflicts harm on B and what has to be decided is: how should we restrain A? But this is wrong. We are dealing with a problem of a reciprocal nature. To avoid the harm to B would inflict harm on A. The real question that has to be decided is: should A be allowed to harm B or should B be allowed to harm A? The problem is to avoid the more serious harm. (Coase 1960, p. 2)

In this paper we seek to understand whether the reciprocal nature of the externality problem obviates the need to distinguish between the generator and recipient of an externality or whether there is still some value in the distinction.

We present a model with frictionless bargaining over the externality and other variables ex post but with transactions cost in the form of non-contractible investment ex ante. (The logic behind the model is that the initial investment decisions made by the first party to locate in an area will be non-contractible if the second party with whom the first will eventually negotiate has not yet shown up.) In this model, there is an asymmetry between the generator and the recipient that makes the distinction between them economically meaningful. The asymmetry arises because the generator's preferred level of the externality is an interior solution which may depend on its ex ante investment. By contrast, in the case of a purely negative externality, the recipient's preferred level is always zero, a corner solution that is independent of its investment. For example, consider the case of an airport generating noise harming a nearby homeowner raised in *Thrasher v. Atlanta*.[1] Allocating an injunction right to the airport essentially gives it a right to produce as much noise as it likes. The airport will have an incentive to distort its ex ante investment—for example expanding the runway to accommodate larger (and noisier) planes or building the runway closer to the homeowner—to increase its interior solution for its preferred externality level. In this way the airport can credibly threaten more harm to the homeowner in the

[1] *Thrasher v. City of Atlanta*, 178 Ga. 514, 173 S.E. 817 (1934). This and the subsequent legal cases we cite were originally cited in Coase (1960).

event that bargaining over the noise level breaks down, thus allowing the airport to extract more bargaining surplus. Allocating an injunction right to the homeowner gives him the right to stop any noise from the airport. He can credibly threaten this same outcome whether his residence is a hovel or a mansion. Regardless of the size of the homeowner's investment in his residence, forbidding the airport to emit noise causes the same harm to it, forcing it to either shut down operations or pay for a device to muffle the noise. Hence an injunction right would not induce the homeowner to distort his ex ante investment as it would the airport.

In order to mitigate the distortion from the identified strategic effect, it may be efficient to weaken rights if the holder is a generator. For example if the airport is the rights holder (say by virtue of its having been in operation before the construction of the nearby residence in a "coming to the nuisance" regime in which the first party to locate obtains the rights), there are cases in which it would have been socially more efficient to have allocated it a damage right rather than an injunction right, that is, the right to collect damages for reducing its noise level to suit the homeowner rather than the right to set the noise level directly. There is no analogous benefit to weakening rights for a recipient since the identified strategic effect does not arise for a recipient. Consequently, we find that allocating an injunction right to the first mover is always more efficient than a damage right if the first mover is a recipient; but whether an injunction or a damage right is more efficient if the first mover is a generator depends on the parameters.

The "generator" label for the party that prefers an interior solution for the externality level and "recipient" for the party that prefers a corner solution are appealing because they continue to be well-defined if the negative-externality problem is translated into the equivalent positive-externality one (mapping, say, pollution into pollution abatement). There is natural maximum that would be preferred by the recipient, namely abating pollution until the environment is returned to the state without any of the generator's pollution (it may prefer an even cleaner environment but the government typically would not enforce such a demand); there is no natural corner for the (negative) amount of abatement that the generator would prefer. Appendix A shows that calling the party that prefers a corner solution for the externality the "generator" and the party preferring a corner solution the "recipient" is consistent with a first-principles definition of the generator as the party that chooses an action affecting the recipient's utility. The appendix outlines the assumptions necessary to make the definitions consistent.

One assumption necessary to make the definitions consistent is that the externality be purely negative. The analysis in Section 4 focuses exclusively on this canonical case. In Section 5.2, we extend the analysis to the case of a mixed externality providing benefits to the recipient at low levels but generating harm at higher levels. In this case, the recipient also has an interior solution for its preferred externality level, blurring the strategic asymmetry between the generator and recipient. Still, we show that there are certain rights regimes under which the distinction between generator and recipient continues to be economically meaningful even for a mixed externality.

Is the identified strategic effect a real-world phenomenon or just a theoretical nicety? One of the more egregious cases of a party's investing to harm another's bargaining threat point is the "spite fence" built by millionaire Charles Crocker in San Francisco during the 1870s, described in Tamony (1952). Crocker offered to buy Nicholas Yung's property, which was surrounded by Crocker's estate. After Yung refused to sell, Crocker built a 40-foot-high wall surrounding Yung's house on three sides, blocking the light and air circulation. Although Yung refused to sell to his dying day (becoming a cause célèbre for the common man struggling against the establishment), the wall succeeded in convincing Yung's heirs to sell out.

The present paper builds on our earlier work in Pitchford and Snyder (2003), which focused on sequential location as a source of transactions costs and on the question of whether it is more efficient for the court to assign property rights to the first mover or the second mover into an area. The efficiency of property rights did not depend on the identity of the generator in our earlier work because of assumptions in the model ensuring the generator's ideal externality level was not a function of its ex ante investment. In particular, we assumed that the externality was constrained to lie in a bounded set $[0, \bar{e}]$; the recipient's ideal externality level was at one corner, zero, and the generator's ideal externality level was at the other corner, $\bar{e}$. In the present paper, we adopt the more natural assumption that the externality level is unbounded above, though it continues to be bounded below by zero. Hence, the generator's ideal externality level is an interior solution that in general depends on its ex ante investment. The assumptions in our earlier work simplified the analysis, but forced us to abstract from the strategic effects that are the focus of the present paper. The model in both Pitchford and Snyder (2003) and the present paper is related to the incomplete-contracts literature begun in Grossman and Hart (1986) and Hart and Moore (1990) to explain ownership in a theory of the firm. As is standard in this literature, in our model there is noncontractible investment ex ante but efficient bargaining ex post. The possibility that injunction rights may lead a party to distort its investment to increase the harm it can threaten another party in externality problems was noted informally by Mumey (1971), and is related to the extensive literature on blackmail (Landes and Posner 1975, Epstein 1983, Lindgren 1984, Helmholz 2001, Posner 2001, Gomez and Ganuza 2002). A by-product of our analysis is a comparison of damages versus injunction rights, the topic of a large literature including Calabresi and Melamed (1972), Ayers and Talley (1995), Kaplow and Shavell (1995, 1996), and Sherwin (1997).

2. Model

The variety of definitions in the economics literature (Cornes and Sandler 1996) suggest how difficult it is to define the concept of an externality, let alone the concepts of generator and recipient that underlie Coase's (1960) debate with Pigou (1912). Thus, we devote considerable attention in Appendix A to a rigorous discussion of general definitions of the generator and recipient. To simplify discussion in the body of the paper, however, most of the analysis will focus on the simple characterization of a unidirectional, purely negative externality provided in this section. Appendix A traces the connection between the general definitions and the simple characterization provided here. Extensions to other cases besides purely negative externalities will be analyzed in Section 5.2.

The model has two periods, an ex ante and an ex post period, two players $i = 1, 2$, and a court, which specifies and enforces a property-rights rule. In the ex ante period, the court specifies a property-rights regime. Then player 1 becomes aware of an opportunity to sink investment expenditure $x_1 \in [0, \infty)$ in a specific location. The land on which player 1 invests is assumed to have been purchased in a competitive market at a price of zero.[2] Player 2 arrives in the ex post period. It has the opportunity to invest $x_2 \in [0, \infty)$ at a location near player 1.[3] Location in the nearby area leads to a negative externality $e \in [0, \infty)$ between the players. We assume the players can engage in frictionless bargaining over x_2 and e, so that they end up choosing the levels which maximize their joint payoff. The sole transaction cost in the model is that players cannot bargain over x_1; this follows directly from our assumption of ex ante anonymity, i.e., that the identity of player 2 is unknown to player 1 when 1 makes its ex ante investment decision.[4]

Let $u_i(x_i, e)$ be the gross surplus function for player $i = 1, 2$. We will maintain a number of assumptions on $u_i(x_i, e)$ throughout the paper (Assumptions 1–5) including differentiability, concavity, and a series of Inada-type conditions ensuring interior solutions for the privately and socially optimal level of x_i and for the generator's optimal stand-alone externality level. Since these assumptions are standard regularity conditions not of central importance to the arguments of the paper, they are relegated to Appendix B. Under these maintained assumptions, the definition of a purely negative externality, which will be the focus of much of the subsequent analysis in

[2]This simplifying assumption can be justified if the second-highest bidder in a second-price private-values auction for the plot is neither a generator or recipient of the externality. If so, the land price will be independent of the property-rights regime and can be netted out of player 1's surplus function without loss of generality. More generally, the land price may depend on the property-rights regime. Abstracting from this complication does not sacrifice much generality because the land price only affects the extensive margin of whether 1 shows up in the location (taken for granted in the model) and not its marginal investment incentives. See White and Wittman (1981, 1982) for a model of externalities with endogenous location.

[3]Player 2's surplus function can be thought of as netting out the price of land following the logic of the previous footnote. For 2 to win the land auction against other bidders that may be less affected by the externality problem, 2 must obtain quasi rents from locating in the area. If not, all externality problems could be solved by the land market and would never be observed in practice. A number of interesting outcomes could emerge from explicitly modeling the land auction. Assuming there are bidders who are affected by the externality but still obtain sufficient quasi rents to outbid those that are less affected, and assuming these bidders are fairly homogeneous, then the land price would be bid up to player 2's equilibrium surplus. The main results of the model would go through unchanged (the sole minor change being that the land price paid by 2 would extract all of its equilibrium bargaining surplus). Assuming instead that bidders are fairly heterogeneous in both their quasi rents and the effect of the externality effect on them, an additional strategic effect would arise in that the player 1's investment would affect the identity of the second mover who wins the auction for the land. We abstract from this selection effect in this paper.

[4]The implicit assumption is that 1 has perfect foresight regarding 2's surplus function but not 2's identity. This assumption simplifies the presentation of the results but is not crucial. The assumption could be dropped by extending the model to allow for a distribution over the second-mover's preferences and having the first mover maximize an expectation over this distribution. There also might be other sources of contractual incompleteness besides ex ante anonymity. As in Grossman and Hart (1986), there may be resolution of uncertainty over time that makes contracting easier ex post than ex ante. Alternatively, the externality may be expected to harm one of a large number of current neighbors but unknown exactly which, and a collective-action problem may prevent efficient ex ante bargaining.

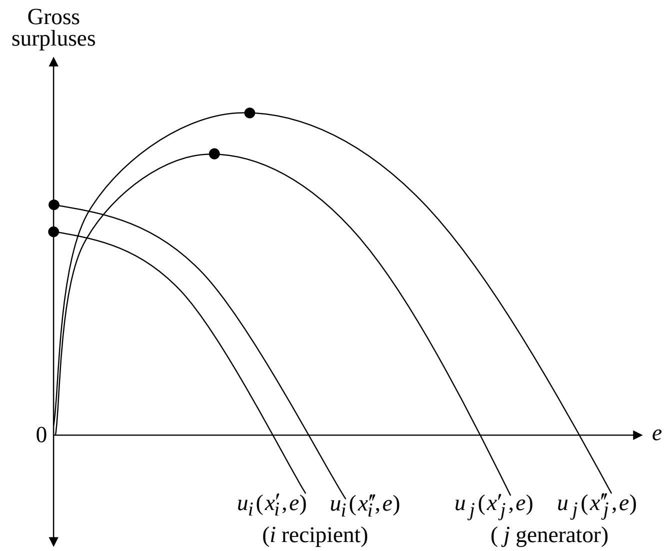

Figure 1: Gross Surplus Functions in the Case of a Purely Negative Externality

the paper, can be made precise.

Definition 1. *Let player i be the recipient of a unilateral externality e. Then e is a* ***purely negative externality*** *if and only if $\partial u_i(x_i, e)/\partial e < 0$ for all $x_i, e \in [0, \infty)$.*

Definition 1 implies that the recipient suffers increasing harm from higher levels of a purely negative externality. Its preferred externality level is thus zero. It is also straightforward to characterize the generator in the case of a purely negative externality. Starting from the general definition of the generator as the party that chooses an action leading to the externality (Definition 6 in Appendix A), under maintained regularity conditions (Assumptions 2, 4, and 5 in Appendix B), the identity of the generator reduces to the identity of the party whose surplus is initially increasing in e, reaching an interior optimum, and then declining for larger e.

With a purely negative externality, the key distinction between the generator and recipient then is that the recipient's preferred externality level, zero, does not vary with its investment, while in general the generator's may. Referring to Figure 1, taking i to the be recipient, an increase in its investment from x_i' to x_i'' does not affect its preferred externality level, which is a corner solution at zero. In contrast, taking j to be the generator, an increase in its investment will affect its desired externality level. This strategic effect depends on the sign of the cross partial derivative $\partial^2 u_j/\partial x_j \partial e$. Figure 1 depicts the case in which $\partial^2 u_j/\partial x_j \partial e > 0$, so that an increase in the generator's investment from x_j' to x_j'' increases its marginal benefit from an additional unit of e, in turn implying that its preferred externality level increases. (This case would arise, for example, if investment increases the size of the generator's facility; the larger the facility, the more pollution generated when the facility is run at optimal capacity.) In this case, by investing more, the generator can increase the harm it can credibly threaten to inflict on the recipient.

The absence of this effect with the recipient, and its presence with the generator, is the fundamental asymmetry that will lead to our main results in Section 4. It is important to point out, however, that this asymmetry is not necessarily present in all cases. As shown in Section 5.2 if the externality is mixed rather than being purely negative, the asymmetry between generator and recipient may be reduced or eliminated.

Let $v_1(x_1, e) = u_1(x_1, e) - x_1$ be 1's surplus net of investment. Let

$$v_2(e) = \max_{x_2 \in [0,\infty)} [u_2(x_2, e) - x_2]$$

be 2's. It turns out to be convenient to specify the second-mover's net surplus as the value function $v_2(e)$ because x_2 is chosen after players bargain and can be set at the private and social optimum, and so does not have an important bearing on the analysis. Define

$$\begin{aligned} e_1^*(x_1) &= \operatorname*{argmax}_{e \in [0,\infty)} v_1(x_1, e) \\ e_2^* &= \operatorname*{argmax}_{e \in [0,\infty)} v_2(e) \\ e^{**}(x_1) &= \operatorname*{argmax}_{e \in [0,\infty)} [v_1(x_1, e) + v_2(e)] . \end{aligned}$$

In words, $e_1^*(x_1)$ and e_2^* are the privately optimal externality levels in the players' stand-alone problems, and $e^{**}(x_1)$ is the joint optimum.

The notation indexes the timing of players' moves independently of the identity of the generator/recipient and the identity of the rights holder. This will facilitate the analysis of a variety of cases including (a) the case in which rights are allocated to the first mover, and the first mover happens to be the recipient of an externality, (b) the case in which rights are allocated to the first mover, and the first mover happens to be the generator of the externality, and (c) analogous cases in which rights are allocated to the second mover. All of these cases will be analyzed below.

Player 1's ex ante choice of x_1 affects both players' equilibrium allocations through the bargain that takes place between players ex post. We assume efficient bargaining, in particular the version of Nash (1950) bargaining in Binmore, Rubinstein, and Wolinsky (1986) involving an exogenous probability of breakdown ex post. Let $\alpha \in (0, 1)$ be player 1's share of the gains from Nash bargaining and $1 - \alpha$ be 2's share. If bargaining breaks down, the default or threat-point outcome is determined by the property-rights regime specified by the court ex ante. That is, a breakdown in bargaining leaves the players to select e according to the property rights specified by the court. Let $t_i(x_1)$ be player i's threat-point payoff. Since the threat points typically involve an inefficient choice of e, players will bargain to the ex-post efficient choice of e. Let $s(x_1)$ denote the resulting maximized joint surplus:

$$s(x_1) = \max_{e \in [0,\infty)} [v_1(x_1, e) + v_2(e)] \tag{1}$$

$$= v_1(x_1, e^{**}(x_1)) + v_2(e^{**}(x_1)). \tag{2}$$

Player 1's equilibrium surplus from Nash bargaining is the sum of its threat point $t_1(x_1)$ and α times the gains from bargaining $s(x_1) - t_1(x_1) - t_2(x_1)$, which upon rearranging equals

$$(1 - \alpha)t_1(x_1) + \alpha s(x_1) - \alpha t_2(x_1). \tag{3}$$

Since it is based on net utility functions, expression (3) already nets out 1's investment expenditure x_1 and thus reflects player 1's surplus from an ex ante perspective. Equation (3) is thus the relevant objective function player 1 maximizes when choosing x_1. We only need to specify player 1's ex ante payoff function because 1's choice of x_1 is the only welfare-relevant one in the model. All other variables (x_2 and e) are chosen optimally ex post conditional on x_1 due to efficient bargaining.

3. First-Mover Property Rights

The court sets the property-rights regime ex ante. Property rights enter the model by determining the threat points $t_i(x_1)$. The threat points enter into 1's objective function (3), and 1's choice in turn determines x_1. In this subsection we will define various property-rights regimes; Section 4 will analyze their efficiency.

Property rights are multidimensional, specifying among other things the variables the holder is allowed to choose, the penalty for infringement, and rules for determining the identity of the holder. For example, property rights can be conditioned on the period in which the players show up. Rights are often allocated to the first mover into a location whether it is a generator or recipient, following the so-called "coming to the nuisance" doctrine. In theory, however, property rights could also be allocated to the second mover. Second-party property rights will be analyzed in Section 5.1. The present subsection and Section 4 will restrict attention to first-party rights because the asymmetry between generator and recipient comes out most strongly in this case.

Besides restricting attention to first-party rights, we restrict attention further to two commonly-studied property-rights regimes, injunctions and damages. An injunction regime gives the holder the right to set e if bargaining breaks down. If player 1 is the injunction-rights holder, it would set e to maximize its stand-alone payoff, i.e., it would choose externality level $e_1^*(x_1)$. The threat-point payoffs corresponding to injunction rights are therefore $t_1(x_1) = v_1(x_1, e_1^*(x_1))$ and $t_2(x_1) = v_2(e_1^*(x_1))$.

We formalize damage rights in the following way. The holder does not have the right to set e—the other player does—but has the right to extract a payment equal to the difference between its surplus if the externality level were set at its preferred level less its realized surplus. More concretely, if player 1 is the damage-rights holder, player 2 has the right to set e but must pay player 1 $u_1(x_1, e_1^*(x_1)) - u_1(x_1, e)$. Player 1's threat-point payoff equals its realized surplus $u_1(x_1, e) - x_1$ plus the damage payment, which upon rearranging, equals $t_1(x_1) = v_1(x_1, e_1^*(x_1))$. To compute $t_2(x_1)$, we need to solve for 2's optimal choice of e if bargaining breaks down. This

Table 1: Threat-Point Payoffs for First-Mover Property-Rights Regimes

First-mover property-rights regime	Abbreviation	Player 1's threat-point payoff $t_1(x_1)$	Player 2's threat-point payoff $t_2(x_1)$
Injunction rights	*FIR*	$v_1(x_1, e_1^*(x_1))$	$v_2(e_1^*(x_1))$
Damage rights	*FDR*	$v_1(x_1, e_1^*(x_1))$	$s(x_1) - v_1(x_1, e_1^*(x_1))$

choice maximizes 2's surplus minus the damage payment, which upon rearranging equals

$$u_1(x_1, e) + v_2(e) - u_1(x_1, e_1^*(x_1)). \tag{4}$$

It is straightforward to see that expression (4) is maximized by setting e to the joint optimum $e^{**}(x_1)$. Substituting $e^{**}(x_1)$ for e in (4) and rearranging, we have $t_2(x_1) = s(x_1) - v_1(x_1, e_1^*(x_1))$.

Table 1 lists the threat points for reference. Note that $t_1(x_1)$ is the same in both injunctions and damages regimes; the rights regimes only differ in the specification of $t_2(x_1)$. Throughout the next section we will refer to the first-mover injunction regime simply as "injunctions" and a first-mover damages regime simply as "damages."

4. Analysis

This section analyzes equilibrium investment and social welfare for the case of first-party rights and for the case of a unidirectional, purely negative externality. We will show that the generator of an externality differs in an economically meaningful way from the recipient in this case. To do this, we determine the social ranking of rights regimes in the case in which player 1 is the recipient (Proposition 1) and compare this ranking with the case in which player 1 is the generator (Proposition 2). Before turning to Propositions 1 and 2, we prove Lemma 1 as a preliminary result. Lemma 1 verifies that the recipient's preferred level of the externality is zero and that the socially preferred level lies strictly between the recipient's and generator's preferred choices. The results will be used in the proofs of the subsequent propositions.

Lemma 1. *Suppose the externality is unidirectional and purely negative and that Assumptions 1 through 5 hold.*

(a) If player 1 is the generator and player 2 is the recipient, then $0 = e_2^* < e^{**}(x_1) < e_1^*(x_1)$.

(b) If player 1 is the recipient and player 2 is the generator, then $0 = e_1^*(x_1) < e^{**}(x_1) < e_2^*$.

The proofs of Lemma 1 and subsequent propositions are contained in Appendix C.

As discussed in Section 2, the efficiency of a rights regime is completely determined in the model by how close ex ante investment x_1 is to the first best since x_1 is the only variable not set by frictionless bargaining. In equilibrium, x_1 may be either too high or too low relative to the first best, depending on the interaction between the recipient's investment and the externality, which in formal terms depends on the sign of the second cross partial $\partial^2 u_1(x_1, e)/\partial x_1 \partial e$.

Definition 2. *Investment* ***increases recipient i's vulnerability*** *if an increase in* x_i *increases* i*'s marginal harm from the externality; i.e.,* $\partial^2 u_i(x_i, e)/\partial x_i \partial e < 0$.

For example, in the *Thrasher v. Atlanta* case, the homeowner, the recipient of the airport's noise externality, could invest in a big house with fine construction details, thus exposing more housing value to the noise externality. Such investments are referred to as increasing the homeowner's vulnerability. Alternatively, the homeowner could build the house farther from the property lines with soundproofed walls. As the following definition states, we refer to such investment as decreasing the homeowner's vulnerability.

Definition 3. *Investment* ***reduces recipient i's vulnerability*** *if an increase in* x_i *reduces* i*'s marginal harm from the externality; i.e.,* $\partial^2 u_i(x_i, e)/\partial x_i \partial e > 0$.

Let x_1^{FIR} be player 1's equilibrium ex ante investment if it holds injunction rights, x_1^{FDR} if it holds damage rights, and x_1^{1ST} first-best investment. (The F in the superscript designates that rights are allocated to the first mover.) The next proposition states that x_1^{FIR} is closer to x_1^{1ST} than x_1^{FDR}—and so injunctions are more efficient than damages—if 1 is the recipient.

Proposition 1. *Suppose the following: the externality is unidirectional and purely negative, Assumptions 1 through 5 hold, player 1 is the recipient and the rights holder, player 2 is the generator, investment either increases 1's vulnerability for all* $x_1, e \in [0, \infty)$ *or decreases 1's vulnerability for all* $x_1, e \in [0, \infty)$*, and* $\alpha \in (0, 1)$.

(a) Social welfare is strictly less than in the first best with both injunctions and damages.

(b) Injunctions are strictly socially more efficient than damages.

(c) If player 1's investment reduces its vulnerability, then there is underinvestment in both regimes relative to the first best, with $x_1^{FDR} < x_1^{FIR} < x_1^{1ST}$.

(d) If player 1's investment increases its vulnerability, then there is overinvestment in both regimes relative to the first best, with $x_1^{1ST} < x_1^{FIR} < x_1^{FDR}$.

The logic behind the main result in Proposition 1—that injunctions are more efficient than damages if the rights-holding player 1 is a recipient—can be seen by comparing the distortion in player 1's investment incentives if it holds injunction rights with that if it holds damage rights. First we will compute the investment distortion under injunction rights, labeled Δ^I. To do this we will subtract player 1's surplus function under injunctions from the first-best surplus, and then differentiate to obtain the distortion in marginal investment incentives.

Player 1's surplus under injunctions is derived by substituting the relevant threat points from Table 1 into the expression for the Nash bargaining surplus (3), yielding

$$(1-\alpha)v_1(x_1, e_1^*(x_1)) + \alpha s(x_1) - \alpha v_2(e_1^*(x_1)). \tag{5}$$

Subtracting (5) from the first-best objective function $s(x_1)$ yields

$$(1-\alpha)[v_1(x_1, e_1^*(x_1)) - s(x_1)] + \alpha v_2(e_1^*(x_1)). \tag{6}$$

Differentiating (6) with respect to x_1 and manipulating the resulting expression (by applying the Envelope Theorem and Fundamental Theorem of Calculus) yields

$$\Delta^I = (1-\alpha)\int_{e_1^*(x_1)}^{e^{**}(x_1)} \frac{\partial^2 v_1(x_1, e)}{\partial x_1 \partial e} de + \alpha v_2'(e_1^*(x_1)) \frac{de_1^*(x_1)}{dx_1} \tag{7}$$

as the distortion for injunctions. Second, we will compute the investment distortion under damages. Player 1's surplus function under damages is derived as equation (5) by substituting the relevant threat points from Table 1 into equation (3), giving

$$v_1(x_1, e_1^*(x_1)) - s(x_1) \tag{8}$$

for damages. Subtracting (8) from $s(x_1)$, differentiating with respect to x_1, and rearranging gives the investment distortion under damages, Δ^D:

$$\Delta^D = \int_{e_1^*(x_1)}^{e^{**}(x_1)} \frac{\partial^2 v_1(x_1, e)}{\partial x_1 \partial e} de. \tag{9}$$

The first term in each of Δ^I and Δ^D is the distortion due to the fact that player 1's surplus depends in part on its threat point rather than solely on social surplus. This distortion is larger in Δ^D than in Δ^I for all $\alpha > 0$ (because Δ^I involves the leading factor $1 - \alpha$). Intuitively, there is no need for bargaining with damage rights because the damage payment induces player 2 to set the externality efficiently. Thus, player 1's equilibrium surplus is solely its threat point, implying that 1 does not internalize any of 2's surplus. An injunction right induces a more encompassing objective function since it allows player 1 to obtain a share of joint surplus in proportion to its bargaining power. On these grounds alone, damages would be more distortionary than injunctions.[5]

However, the second term in Δ^I—reflecting the distortion in player 1's investment to gain a better bargaining position by worsening 2's threat point—must still be accounted for. In principle, this second term further distorts investment from the first best: below we will show

[5]An alternative damage-rights regime, which we will label "perfect" damages, involves a compensation payment that returns 1 to the surplus it would have earned if e were set at the social optimum, $e^{**}(x_1)$ (rather than at its stand-alone optimum, $e_1^*(x_1)$, as with "plain" damages). The first best can be obtained if perfect damages are allocated to player 1. Mathematically, this can be seen in equation (9): moving from plain to perfect damages would cause the lower limit of integration to change from $e_1^*(x_1)$ to $e^{**}(x_1)$ thus causing the entire integral to disappear, eliminating any investment distortion. Perfect damages may be informationally burdensome to implement in practice: whereas the court might be able to determine the actual surplus player 1 earned while it was alone in the area and use this as a basis to compute plain damages, the court would somehow have to estimate two counterfactuals—the first-best externality level $e^{**}(x_1)$ and 1's surplus at this externality level—to implement perfect damages.

that if player 1 is the generator then this second term has the same sign as the first and thus exacerbates the distortion. However, if player 1 is the recipient of a purely negative externality as we are positing here, $e_1^*(x_1) = 0$, implying $de_1^*(x_1)/dx_1 = 0$, in turn implying that the second term in Δ^I disappears. This is precisely where the fact that the recipient's preferred externality level is a corner solution comes into play. For the recipient of a purely negative externality, only the first term of Δ^I remains, which, as argued in the previous paragraph, is less than the first term of Δ^D for all $\alpha > 0$. In other words, for $\alpha > 0$, injunctions distort investment less than damages.

Parts (c) and (d) of Proposition 1 can be summarized together as saying that if player 1 is the recipient of a purely negative externality, its investment will always be distorted in the direction of making it more vulnerable whether it holds injunction or damage rights. (In particular, part (c) says that 1 will underinvest if investment reduces its vulnerability, and part (d) says 1 will overinvest if investment increases its vulnerability.) Player 1 does not fully internalize its vulnerability to the externality because, as the rights holder, it is insulated from harm from the externality in its threat point.

The preceding discussion allows us to characterize some knife-edged cases not covered by Proposition 1. First, if $\alpha = 0$, injunctions and damages are equally socially efficient, though both are strictly less efficient than the first best, as can be seen by substituting $\alpha = 0$ into (7) and (9). Second, if $\alpha = 1$, injunctions yield the first best and are strictly more efficient than damages. This can be seen by substituting $\alpha = 1$ into (7) and noting that $d_1^*(x_1)/dx_1 = 0$ if 1 is the recipient. Third, if $\partial^2 u_1(x_1, e)/\partial x_1 \partial e = 0$ for all $x_1, e \in [0, \infty)$, injunctions and damages both yield the first best, since the first term in both (7) and (9) disappears.

We next turn to the analysis of the case in which player 1 is the generator. Although the investment distortion terms Δ^I and Δ^D were derived in the middle of the analysis of the case in which player 1 is the recipient, they are perfectly general and apply as well to the case in which player 1 is the generator. As mentioned, if player 1 is the generator, the second term in Δ^I typically does not disappear because $e_1^*(x_1)$ is an interior solution with $de_1^*(x_1)/dx_1 \neq 0$. Which is larger, the two terms in Δ^I or the single term in Δ^D, depends on functional forms and parameters. The next proposition provides some cases in which damages are socially more efficient than injunctions if player 1 is a generator, in contrast to the case where player 1 is a recipient. A series of definitions concerning the type of technology that the generator can adopt will allow us to state the proposition succinctly. An investment is said to be "dirty" if higher levels increase the generator's marginal benefit from the externality. For example, in the *Thrasher v. Atlanta* case, if the noise generated by the airport is in proportion to the number of takeoffs and landings, expanding the scale of the airport's operation will naturally increase its benefit from an extra interval of noise. Alternatively, a "clean" investment reduces the marginal benefit from pollution. This could occur, for example, if the airport buys new airplanes that are quieter than the old ones. Formally, we have the following definitions.

Definition 4. *Generator i's investment is **clean** if an increase in x_i reduces i's marginal benefit from the externality; i.e., $\partial^2 u_i(x_i, e)/\partial x_i \partial e < 0$.*

Definition 5. *Generator i's investment is **dirty** if an increase in x_i increases i's marginal benefit from the externality; i.e., $\partial^2 u_i(x_i, e)/\partial x_i \partial e > 0$.*

Proposition 2. *Suppose the following: the externality is unidirectional and purely negative, Assumptions 1 through 5 hold, player 1 is the generator and rights holder, player 2 is the recipient, $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$ for some $\gamma > 0$, and $\alpha \in (0, 1)$.*

(a) Both injunctions and damages are strictly socially inefficient compared to the first best.

(b) If player 1's investment is dirty, there exists γ' such that for all $\gamma < \gamma'$, $x_1^{1ST} < x_1^{FDR} < x_1^{FIR}$, and social welfare is higher with damages than with an injunction.

(c) If player 1's investment is clean, there exists γ'' such that for all $\gamma < \gamma''$, $x_1^{FIR} < x_1^{FDR} < x_1^{1ST}$, and, again, social welfare is higher with damages than with an injunction.

Given the functional form for player 1's surplus, $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$, as γ becomes small, the impact of the externality on its total and marginal payoff becomes negligible, whereas the choice of externality by the generator is unaffected by γ since $e_1^*(x_1)$ solves $\partial h(x_1, e_1^*(x_1)/\partial e \equiv 0$. In other words, player 1 does not benefit much from polluting, but since the benefit is positive, its ideal pollution level can remain relatively high. Under an injunction it can continue credibly to threaten the other party with substantial harm from the externality. Thus, the strategic incentive that the generator has to harm the recipient through its choice of externality does not vanish. Under damages, as γ becomes small, player 1's marginal investment incentives, $\partial g(x_1)/\partial x_1 + \gamma \partial h(x_1, e_1^*(x_1))/\partial x_1$ approach $\partial g(x_1)/\partial x_1$; i.e., marginal investment incentives are negligibly influenced by the externality. In the first best, marginal social investment incentives, $\partial g(x_1)/\partial x_1 + \gamma \partial h(x_1, e^{**}(x_1))/\partial x_1$, also approach $\partial g(x_1)/\partial x_1$. Therefore, social welfare under damages approaches the first best.

The discussion surrounding equations (7) and (9) allows us to immediately characterize some knife-edged cases not covered by Proposition 2. If $\alpha = 0$, social welfare is the same whether the first-party generator is allocated an injunction or damage right, though both are strictly less efficient than the first best, as can be seen by substituting $\alpha = 0$ into (7) and (9). If $\partial^2 h(x_1, e)/\partial x_1 \partial e = 0$ for all $x_1, e \in [0, \infty)$, injunctions and damages both yield the first best. This result holds because the integrand in the first term in both (7) and (9) disappears; the second term in (7) also disappears because $de_1^*(x_1)/dx_1 = 0$ if $\partial^2 h(x_1, e)/\partial x_1 \partial e = 0$. Combined with the result from above that the first best is obtained if either injunction or damage rights are allocated to a first-party recipient if its surplus function satisfies $\partial^2 u_1(x_1, e)/\partial x_1 \partial e = 0$ for all $x_1, e \in [0, \infty)$, we have that the first best can be obtained regardless of the allocation of property rights and the identity of the generator if there is no interaction effect between x_1 and e in 1's surplus function.

Under what conditions can the identified strategic effect, which leads to the asymmetry between the generator and recipient, be expected to lead to substantial social costs? One basic condition is that expenditures on x_1 be substantial, for this is the sole source of distortion in the model. Other conditions can be understood from an examination of the second term in (7), which is the mathematical expression for the asymmetric strategic effect. The distortion is larger if investment and the externality are mainly used for offensive rather than socially productive purposes, i.e., if $e_1^*(x_1)$ has a large effect on the threat point, and x_1 has a large effect on $e_1^*(x_1)$, but x_1 has little effect on social welfare given the efficient externality choice $e^{**}(x_1)$. The distortion is also larger the higher is α. If α is high, the only source of surplus for player 2 is its threat point, and so the only way for 1 to extract surplus from 2 is to distort its investment to harm 2's threat point. The law can target such cases by, for example, taking away property rights if they are abused solely to injure the other party.[6] In practice it may be difficult to condition rights on abuse/intent and simpler to condition rights on the identity of the generator/recipient.

5. Extensions

5.1. Second-Mover Property Rights

The analysis has so far been restricted to property rights allocated to the first mover in the area. We restricted attention to this case for two reasons. One reason is that the greatest asymmetry between generator and recipient is exhibited by this case. The asymmetry between generator and recipient is less apparent with second-mover property rights because, as we will see from the main result proved in this subsection, the same rights regime will be efficient whether the rights holder is a generator or recipient. Another reason is that, while a particular second-mover rights regime will turn out to attain the first best in our simple model, in a richer model second-mover rights can be quite inefficient. In a fully specified dynamic model, second-mover rights would induce players to engage in a war of attrition, delaying until the other player moves in order to be second and win the property rights. The resulting delay may waste a considerable amount of social welfare. In addition, we have abstracted from player 1's decision to show up in the area. Player 2 may extract so much surplus from 1 if 2 holds the rights that 1 decides not to show up, again leading to a substantial loss of social welfare. The remainder of this subsection abstracts from these complexities, but they should be kept in mind as caveats to the result that the first best is attained by a particular second-mover rights regime.

For reference, Table 2 lists the threat points for the two rights regimes that will be considered in this subsection: second-mover injunctions and damage rights. Substituting the threat points into the Nash bargaining formula (3) yields the following objective function determining player 1's equilibrium investment:

$$(1-\alpha)v_1(x_1, e_2^*) + \alpha s(x_1) - \alpha v_2(e_2^*) \tag{10}$$

[6] See Hale (1946) for additional relevant (and interesting) cases of malicious injury in tort law.

Table 2: Threat-Point Payoffs for Second-Mover Property-Rights Regimes

Second-mover property-rights regime	Abbreviation	Player 1's threat-point payoff $t_1(x_1)$	Player 2's threat-point payoff $t_2(x_1)$
Injunction rights	*SIR*	$v_1(x_1, e_2^*)$	$v_2(e_2^*)$
Damage rights	*SDR*	$s(x_1) - v_2(e_2^*)$	$v_2(e_2^*)$

if player 2 holds an injunction right and

$$s(x_1) - v_2(e_2^*) \tag{11}$$

if player 2 holds a damage right. To compute an expression for investment distortion, we subtract the first-best objective function from each of (10) and (11) and differentiate with respect to x_1, giving

$$(1 - \alpha) \int_{e^{**}(x_1)}^{e_2^*} \frac{\partial^2 v_1(x_1, e)}{\partial x_1 \partial e} de \tag{12}$$

for injunctions and zero for damages.

The calculations thus show that there is no distortion with second-mover damage rights. The first best is attained. Intuitively, if player 2 holds damage rights, player 1 internalizes 2's surplus through the damage payment and thus makes the efficient externality choice ex post and the efficient investment choice ex ante. Since no mention has been made regarding the identity the generator and recipient, second-mover damage rights obtain the first best regardless of which player is which.

As equation (12) shows, there is typically a distortion if player 2 holds an injunction right. Player 1 distorts its investment to make itself less vulnerable to expropriation when 2 sets the externality at e_2^* in the threat point. The results are summarized in the following proposition. The proof is straightforward from the discussion in this subsection and is omitted.

Proposition 3. *Suppose the following: the externality is unidirectional and purely negative, Assumptions 1 through 5 hold, player 2 is the rights holder, $\partial^2 v_1(x_1, e)/\partial x_1 \partial e$ is nonzero and has the same sign for all $x_1, e \in [0, \infty)$, and $\alpha \in (0, 1)$.*

(a) The first best is obtained if damage rights are allocated to player 2 regardless of whether it is the generator or recipient.

(b) Social welfare is strictly less than the first best if injunction rights are allocated to player 2.

It is easy to see from (12) that in the knife-edged case of $\alpha = 1$ not covered by Proposition 3, the distortion (12) disappears and the first best is also obtained if injunction rights are allocated to player 2.

5.2. Mixed Externalities

The analysis has so far been restricted to purely negative externalities. Another possibility is that the externality is mixed, providing a marginal benefit to the recipient at low levels but marginal harm at higher levels.[7] For example, consider the *Thrasher v. Atlanta* case cited in the Introduction, in which the plaintiff was a homeowner harmed by the noise from the defendant's nearby municipal airport. Suppose for the sake of argument that the plaintiff obtained some benefits from the airport: increased local economic growth, better transportation, etc. The plaintiff then might prefer a small airport to none, although at higher air-traffic levels the harm from the noise might begin to outweigh the benefits. In this case, the municipal airport would be the generator and the homeowner the recipient of a mixed externality.

Figure 2 depicts the mixed-externality case. The recipient is labeled i and the generator j. The fact that the recipient's surplus function $u_i(x_i, e)$ is initially increasing in e implies that the externality is positive at low levels. The fact that the recipient's optimal stand-alone externality level $e_i^*(x_i)$ is less than the generator's $e_j^*(x_j)$ implies that the externality is marginally harmful

[7] The general definitions of generator and recipient in Appendix A apply to the case of mixed externalities.

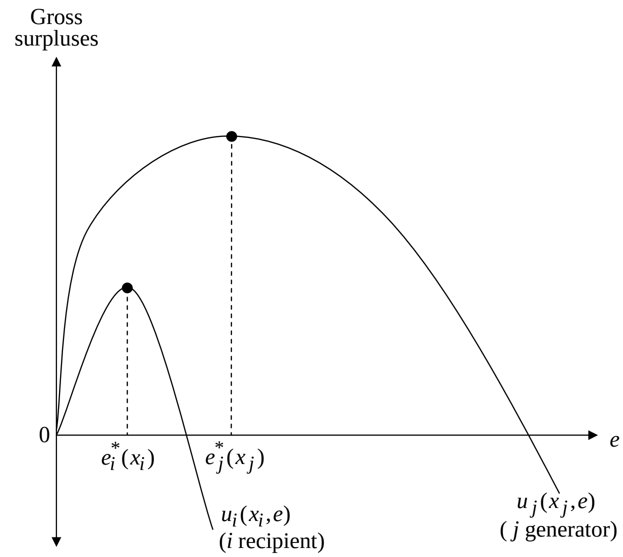

Figure 2: Gross Surplus Functions in the Case of a Mixed Externality

to the recipient in equilibrium, as will be seen.[8,9]

The key difference between the analysis of a mixed and purely negative externality that will emerge is that with a mixed externality, the recipient's stand-alone optimum $e_i^*(x_i)$ is an interior solution, which may depend on its investment, raising the possibility of new strategic effects that are absent with a purely negative externality. Recall that with a purely negative externality the recipient's stand-alone optimum was zero, independent of its investment. The richer set of possibilities with mixed externalities requires care in organizing the relevant subcases. To avoid a proliferation of subcases, we will return to the focus in Section 4 on first-party property-rights regimes, in particular, injunction and damage rights.

First, suppose player 1 is the generator and 2 is the recipient. Analysis of mixed externalities is identical to that for purely negative externalities in this case. Whether player 2's stand-alone optimum e_2^* is zero or an interior solution does not affect the proofs because e_2^* does not show

[8]In most applications with mixed externalities, it is reasonable to suppose the recipient prefers lower levels of the externality than the generator, but it is theoretically possible that the reverse is true. In such cases, the analysis is similar to that for a purely positive externality and is omitted here since we are focusing on the problem of social harm.

[9]Note that $u_i(x_i, e)$ is drawn so that i is better off with $e = 0$ than $e = e_j^*(x_j)$. This implies that the recipient would rather do without the externality than allow the generator to pollute freely, an assumption that will be relied on to eliminate cases in the subsequent analysis.

up directly in player 1's surplus function, as can be seen by substituting the entries in Table 1 in to the bargaining-surplus function (3).

Next, suppose player 1 is the recipient and 2 is the generator. Mixed externalities raise new possibilities in this case. Before proceeding with the analysis, a preliminary issue that must be addressed is that the specification of injunction and damage rights become ambiguous with mixed externalities. Consider the case of first-party injunction rights. One natural specification is that the injunction is all or nothing, requiring the externality to cease if the injunction is enforced ($e = 0$). Another natural specification is that the recipient can constrain the externality to be no greater than its stand-alone optimum ($e \leq e_1^*(x_1)$). We will call the first specification a "corner injunction," since e is forced to a corner at zero, and the second a "peak injunction," since e is constrained to the peak of 1's surplus function. (This ambiguity did not arise with a purely negative externality since the recipient's surplus function peaked at the corner of zero.) The same ambiguity arises with damage rights. One natural specification is that the recipient's compensation should make it as well off as it would be in the absence of the second party/generator, a regime we will call "corner damages," analogous to corner injunctions. Another is that the recipient's compensation make it as well off as it would be had e been set at its stand-alone optimum $e_1^*(x_1)$, a regime we will call "peak damages."

There are two reasons for introducing corner and peak variants of the rights regimes. One is that it is unclear *a priori* which is socially more efficient. Another is that there may be technological barriers preventing the government from implementing a variant even if it would otherwise be more efficient. For example, in the *Thrasher* airport case, it may be prohibitively expensive for the government to monitor the frequency and decibel level of takeoffs and landings as would be required to implement a peak injunction, but straightforward for the government to shut the airport down, all that is required to implement a corner injunction. We expect that the corner variants of both injunctions and damages regimes would generally require less information and monitoring on the part of the government than their peak analogues in most applications and thus would be easier to implement.

Table 3 provides the threat-point payoffs for the corner and peak variants of first-party in-

Table 3: Threat-Point Payoffs for Recipient of a Mixed Externality

First-mover property-rights regime	Abbreviation	Player 1's threat-point payoff $t_1(x_1)$	Player 2's threat-point payoff $t_2(x_1)$
Corner rights			
Injunction rights	*FCIR*	$v_1(x_1, 0)$	$v_2(0)$
Damage rights	*FCDR*	$v_1(x_1, 0)$	$s(x_1) - v_1(x_1, 0)$
Peak rights			
Injunction rights	*FPIR*	$v_1(x_1, e_1^*(x_1))$	$v_2(e_1^*(x_1))$
Damage rights	*FPDR*	$v_1(x_1, e_1^*(x_1))$	$s(x_1) - v_1(x_1, e_1^*(x_1))$

junctions and damage rights. It is immediate that the threat points for the corner-rights regimes are identical to those for the recipient of a purely negative externality. (To see this, compare the entries in Table 3 to those from Table 1 after substituting the stand-alone optimum for the recipient of a purely negative externality, $e_1^*(x_1) = 0$.) Thus, if attention is restricted to corner-rights regimes, the analysis of mixed externalities is identical to the analysis in Section 4. In particular, the conclusions of Proposition 1 continue to hold. Moreover, the asymmetry between generators and recipients persists. Even though both have interior solutions for their optimal stand-alone externality levels, it is the corner solution $e = 0$ that still factors into the recipient's threat points with corner rights.

With peak rights, the threat points for player 1 if it is the recipient of a mixed externality from Table 3 have the same form as they would have if player 1 were the generator of a purely negative externality from Table 1. The recipient of a mixed externality has an incentive to distort its investment to affect its preferred externality level to inflict more harm on the second mover in the threat point, just as does the generator of a purely negative externality. Hence, under the peak variants of rights regimes, any asymmetry between generators and recipients of mixed externalities is only a matter of degree not of kind. The next proposition summarizes these results.

Proposition 4. *Suppose the following: the unidirectional externality is mixed with* $0 < e_1^*(x_1) <$

e_2^*, Assumptions 1 through 5 hold, player 1 is the recipient and rights holder, player 2 is the generator, investment either increases 1's vulnerability for all $x_i, e \in [0, \infty)$ or decreases 1's vulnerability for all $x_i, e \in [0, \infty)$, and $\alpha \in (0, 1)$.

(a) Social welfare is strictly higher under corner injunctions than corner damages.

(b) Let $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$ for $\gamma > 0$. There exists γ''' such that, for all $\gamma < \gamma'''$, social welfare is higher under peak damages than peak injunctions.

The next proposition compares the efficiency of corner-injunction rights with peak-injunction rights. Intuition for the proposition comes from comparing expressions for the investment distortion under the two rights regimes. If player 1 is the recipient and holder of peak-injunction rights, the distortion in its investment is precisely captured by Δ^I in equation (7), repeated here:

$$(1-\alpha)\int_{e_1^*(x_1)}^{e^{**}(x_1)} \frac{\partial^2 v_1(x_1, e)}{\partial x_1 \partial e} de + \alpha v_2'(e_1^*(x_1)) \frac{de_1^*(x_1)}{dx_1}. \tag{13}$$

If player 1 is the holder of a corner-injunction right, the distortion in its investment is similar to (13) except for the substitution of the legally enforced externality level, zero, for $e_1^*(x_1)$, causing the second term to drop out:

$$(1-\alpha)\int_{0}^{e^{**}(x_1)} \frac{\partial^2 v_1(x_1, e)}{\partial x_1 \partial e} de. \tag{14}$$

If $\alpha = 0$, the second term drops out of equation (13) as well, leaving the integral in both. The range of integration in (14), from 0 to $e^{**}(x_1)$, is larger than in (13), from $e_1^*(x_1)$ to $e^{**}(x_1)$. The range of integration reflects the difference between the externality level in player 1's threat point and the efficient level; with a mixed externality, the recipient's preferred externality level is between zero and the efficient level. Since this is the only source of distortion if $\alpha = 0$, the distortion is larger with corner-injunction rights than peak-injunction rights. On the other hand, if $\alpha = 1$, the first term drops out of (13), leaving the second term, but the sole term drops out of (14), leaving no distortion. If $\alpha = 1$, therefore, corner-injunction rights attains the first best and is more efficient than peak-injunction rights.

Proposition 5. *Suppose the following: the unidirectional externality is mixed with $0 < e_1^*(x_1) < e_2^*$, Assumptions 1 through 5 hold, player 1 is the recipient and rights holder, player 2 is the generator, investment either increases 1's vulnerability for all $x_i, e \in [0, \infty)$ or decreases 1's vulnerability for all $x_i, e \in [0, \infty)$, and $\alpha \in (0, 1)$.*

(a) Social welfare is strictly less than the first best with both corner and peak injunctions.

(b) For α sufficiently close to 0, social welfare is strictly higher under peak injunctions than corner injunctions.

(c) For α sufficiently close to 1, social welfare is strictly higher under corner injunctions, which approaches the first best, than under peak injunctions.

Comparing peak- with corner-damage rights reduces to comparing the investment distortion under peak-damage rights,

$$\int_{e_1^*(x_1)}^{e^{**}(x_1)} \frac{\partial^2 v_1(x_1, e)}{\partial x_1 \partial e} de \tag{15}$$

which is identical to Δ^D from equation (9), with the investment distortion under corner-damage rights,

$$\int_{0}^{e^{**}(x_1)} \frac{\partial^2 v_1(x_1, e)}{\partial x_1 \partial e} de, \tag{16}$$

which is similar to equation (15) with 0 substituted for $e_1^*(x_1)$. The integrals on the right-hand side of equations (15) and (16) are the same except for differing ranges of integration. The range is larger in (16) and thus there is unambiguously more distortion in investment with corner damages than peak damages.

Proposition 6. *Suppose the following: the unidirectional externality is mixed with $0 < e_1^*(x_1) < e_2^*$, Assumptions 1 through 5 hold, player 1 is the recipient and rights holder, player 2 is the generator, investment either increases 1's vulnerability for all $x_i, e \in [0, \infty)$ or decreases 1's vulnerability for all $x_i, e \in [0, \infty)$, and $\alpha \in (0, 1)$.*

(a) Social welfare is strictly less than the first best with both corner and peak damages.

(b) Social welfare is strictly higher under peak damages than corner damages.

6. Conclusions

We presented a model with frictionless bargaining over the externality and other variables ex post but with transactions cost in the form of non-contractible investment ex ante. We identified a strategic effect in the model that affects generators and recipients differently in some cases. The generator can distort its ex ante investment to make itself more harmful to the recipient, thus increasing the surplus the generator extracts from the recipient in ex post bargaining. This strategic effect is not always open to the recipient. For example, in the case in which the recipient has an injunction right over a purely negative externality generated by the other party, the harm the recipient can inflict on the generator by exercising the injunction is independent of the recipient's ex ante investment. We showed that property rights over the externality, which optimally should be designed to minimize distortions in ex ante investment, can be made more efficient in some cases by conditioning rights on the identity of the generator and recipient in a way that takes account of the asymmetry in the identified strategic effect. Thus, the distinction between the generator and the recipient can be economically meaningful in externality problems.

The asymmetry between the generator and recipient is less marked in some of the extensions to the model discussed in Section 5. If the externality is mixed, the recipient's preferred externality level may be, like a generator's, an interior solution, raising the possibility that the recipient's investment may be distorted, like a generator's, if it is allocated an injunction right. However, even in this case, there are natural specifications of property rights—corner rights—under which the asymmetry between generator and recipient persists. If rights are given to the second mover rather than the first, the same rights regime—damage rights allocated to player 2—is efficient whether 2 is the generator or recipient. This result does not undermine our earlier results on the asymmetry between the generator and recipient because it is an artifact of some simplifying assumptions. Allocating rights to the second mover quickly becomes quite inefficient if certain dynamic elements of the model are fully specified.

Appendix A: General Definition of Generator and Recipient

Consider two players i and j with utility functions $U_i(x_i, a_i, e_j(a_j))$ and $U_j(x_j, a_j, e_i(a_i))$, respectively. Player i's utility, for example, is a function of its own investment expenditure $x_i \in [0, \infty)$, its own action vector $a_i \in A_i$ (where A_i is i's action space), and the externality $e_j(a_j) \in [0, \infty)$, which is a by-product of the other player's action $a_j \in A_j$. The first two arguments of U_i—x_i and a_i—are choice variables for player i, whereas the third—$e_j(a_j)$—results from a choice by the other firm. U_i and U_j are gross utility functions in that they do not subtract off the cost of the investment expenditure x_i.

We distinguish between i's actions a_i and the externality emanating from those actions $e_i(a_i)$ in order to be clear about subtle issues related to the nature of ownership, control, and other legal rights. The law regarding externalities overlies more basic ownership rights players have over their land and other assets. The law regarding externalities may constrain $e_i(a_i)$. Within that constraint, however, the basic ownership rights that i has over its land and other assets (residual rights of control in Grossman and Hart's 1986 terms) allow it to choose a_i (and x_i) freely. For example, if i is a factory, a_i could include the inputs and technology i uses in production; $e_i(a_i)$ might be the pollution that results from production that flows into neighboring land. A private legal action might end up constraining the factory's pollution or requiring the factory pay for harm caused by its pollution. Constraints on the factory's input or technology choices are more typically the result of direct regulation by statute than private legal action.

A deeper question is why private legal actions would target the externality $e_i(a_i)$ rather than the underlying actions a_i. Presumably, the actor i will have better information and more skill than a court in choosing the means to achieve the target $e_i(a_i)$. Granting a third party, perhaps the other party involved in the externality problem, rights to choose a_i may give too it much bargaining power and lead to excessive "hold up" (Klein, Crawford, and Alchian 1978) of i's investment.

With the specification of utilities in hand, we propose a general definition of generator and recipient. The terms are defined jointly as follows.

Definition 6. *Player j is a **generator** and i is the **recipient** of the externality e_j if there exist a'_j and $a''_j \in A_j$ such that $U_i(x_i, a_i, e_j(a'_j)) \neq U_i(x_i, a_i, e_j(a''_j))$ for some $x_i \in [0, \infty)$ and $a_i \in A_i$.*

In words, the generator is a player that can, by altering actions under its control, affect the other player's utility; and the player so affected is the recipient. A few remarks about Definition 6 are in order. First, the definition allows for the possibility that j generates an externality affecting i and vice versa, in which case i and j would simultaneously be generators and recipients. Second, one can imagine taking any two players and fabricating a generator/recipient relationship between them by allowing extreme enough actions to be open to the generator, for example vandalism or assault. We are focusing here on externalities that typically arise in economic applications such as pollution rather than extreme actions such as vandalism or assault, which one can assume are forbidden by criminal law.

It is instructive to apply Definition 6 to cases in which identifying the generator and recipient might be tricky. Consider the *Bryant v. Lefever* case in which the higher walls of the defendant's renovated house prevented the free circulation of air from the chimneys of the plaintiff's neighboring house, causing the chimneys to smoke [*Bryant v. Lefever*, 4 C.P.D. 172 (1878–1879)]. Both parties contributed to the chimney smoke: the plaintiff by building fires and the defendant by building a higher wall, so it might be difficult to determine which is the generator. Definition 6 can be applied by taking the externality to be the prevention of the free circulation of air. Building a higher wall (action a''_j) rather than a lower wall (action a'_j), the defendant reduces the utility of the plaintiff given the number of fires $a_i > 0$ the plaintiff chooses to light; in formal terms, $U_i(x_i, a_i, e_j(a'_j)) > U_i(x_i, a_i, e_j(a''_j))$. According to Definition 6, the defendant is the generator of the externality and the plaintiff is the recipient. In *Bass v. Gregory*, the plaintiff's pub exhausted brewing gases into a duct that led into an unused well on the defendant's property and from there out into the air [*Bass v. Gregory* 25 Q.B.D. 481 (1890)]. The defendant covered the well, presumably because he disliked the smell of the brewing gases, causing the gases to build up in the pub's cellar and disrupting brewing operations. According to Definition 6, both parties are generators and both are recipients. The plaintiff's action, brewing, creates fumes as a by-product which cause a nuisance on the defendant's property. The defendant can take an action, blocking the well, that makes the plaintiff worse off by disrupting its brewing operations.

The preceding two cases involved purely negative externalities; that is, increases in the generators' actions make the recipients worse off. Our general definitions of generator and recipient also apply to positive and mixed externalities. Suppose that in *Bass v. Gregory* Gregory enjoys beer so much that brewing fumes give him pleasure. Presumably there would be no reason for him to block the well, so the only relevant externality would be the brewing gases entering Gregory's property. The pub would be the generator and Gregory the recipient of this positive externality. Definition 6 also applies to the mixed-externality case discussed in Section 5.2.

We conclude this appendix by establishing the connection between Definition 6 and the simple characterization of a unilateral externality provided in Section 2. Letting i be the generator, if the externality only flows in one direction, i cannot also be a recipient. Hence $e_j(a_j)$ drops out of i's gross utility function, which becomes $U_i(x_i, a_i)$. Letting j be the recipient, its gross utility function remains $U_j(x_j, a_j, e_i(a_i))$. It is notationally convenient to move from the direct utility functions U_i and U_j to the following indirect utility functions. Let u_i be generator i's indirect utility function:

$$u_i(x_i, e) = \begin{cases} \max\limits_{a_i \in A_i} U_i(x_i, a_i) \\ \text{subject to } e_i(a_i) = e. \end{cases} \tag{17}$$

Let u_j be recipient j's indirect utility function:

$$u_j(x_j, e) = \max_{a_j \in A_j} U_j(x_j, a_j, e). \tag{18}$$

Defined in this way, players' indirect utility functions conveniently have the same form (e.g., $u_i(x_i, e)$ for player i) whether they are generators or recipients. Section 2 starts with surplus functions of the form in (17) and (18).

Appendix B: Regularity Conditions on Surplus Functions

We following regularity conditions on players' surplus functions are maintained throughout the paper.

Assumption 1. *$u_i(x_i, e)$ is continuously differentiable in both arguments for all $x_i, e \in [0, \infty)$.*

Assumption 2. *$u_i(x_i, e)$ is strictly concave for all $x_i, e \in [0, \infty)$.*

Assumption 3. *$u_i(x_i, e)$ satisfies an Inada condition in x_i; i.e., $\partial u_i(0, e)/\partial x_i = \infty$ for all $e \in [0, \infty)$.*

Assumption 4. *The net utility function $u_i(x_i, e) - x_i$ is coercive; i.e.,*

$$\lim_{\|x_i, e\| \to \infty} [u_i(x_i, e) - x_i] = -\infty,$$

where $\|x_i, e\| = \sqrt{x_i^2 + e^2}$ *is the distance norm.*

Assumption 5. *If* i *is the generator,* $u_i(x_i, e)$ *satisfies an Inada condition in* e*:* $\partial u_i(x_i, 0)/\partial e = \infty$ *for all* $x_i \in [0, \infty)$.

Assumptions 1 and 2 are standard. Assumptions 3 and 4 ensure that the privately and socially optimal investment levels are in the interior of $[0, \infty)$. This is not essential for the results, but allows us to state our propositions more elegantly with strict inequalities, eliminating a number of economically uninteresting cases. Note that the assumption of coerciveness implies that both players' net surpluses become very negative if either the investment or the externality grow without bound. Assumption 5 ensures that the generator's privately optimal externality level and the socially optimal one are both in the interior of $[0, \infty)$. Again, this assumption is not essential for the results, but allows us to state our propositions more elegantly with strict inequalities.

Appendix C: Proofs of Propositions

Proof of Lemma 1: We will prove part (a); the proof of part (b) is similar and thus omitted. Suppose player 1 is the generator and 2 is the recipient. Then $\partial u_2(x_2, e)/\partial e < 0$ by definition of the recipient, implying $v_2'(e) < 0$. Thus $e_2^* = \operatorname{argmax}_{e \in [0,\infty)} v_2(e) = 0$. Assumption 5 implies $e^{**}(x_1) > 0$. The proof is completed by showing $e^{**}(x_1) < e_1^*(x_1)$. Consider the nested objective function

$$u_1(x_1, e) + v_2(e) - \theta v_2(e), \tag{19}$$

where $\theta = 0$ yields the objective function for $e^{**}(x_1)$ and $\theta = 1$ that for $e_1^*(x_1)$. We proceed by verifying the conditions required for Strict Monotonicity Theorem 1 of Edlin and Shannon (1998) hold for expression (19). Expression (19) is continuously differentiable because the individual terms are continuously differentiable by Assumption 1. Assumptions 4 and 5 imply $e_1^*(x_1)$ is an interior solution. The second cross partial of expression (19) with respect to e and θ equals $-v_2'(e) > 0$. Hence, (19) exhibits increasing marginal returns. Thus, Strict Monotonicity Theorem 1 of Edlin and Shannon (1998) applies, implying $e^{**}(x_1) < e_1^*(x_1)$. $\square$

Proof of Proposition 1: Consider the case in which investment by player 1 (the recipient) reduces its vulnerability. The case in which player 1's investment increases its vulnerability is analyzed similarly and thus omitted.

We will first prove $x_1^{FDR} < x_1^{FIR}$. Substituting the threat points associated with a damages regime from Table 1 into expression (3), player 1's ex ante equilibrium surplus under damages equals

$$v_1(x_1, e_1^*(x_1)). \tag{20}$$

Similarly, it can be shown that player 1's ex ante equilibrium surplus under an injunction equals

$$(1 - \alpha)v_1(x_1, e_1^*(x_1)) + \alpha s(x_1) - \alpha v_2(e_1^*(x_1)). \tag{21}$$

Nesting (20) and (21), player 1's objective function, determining its ex ante investment, can be written

$$v_1(x_1, e_1^*(x_1)) + \theta[s(x_1) - v_1(x_1, e_1^*(x_1)) - v_2(e_1^*(x_1))], \tag{22}$$

where $\theta = 0$ under damages and $\theta = \alpha$ under an injunction. The second cross partial of (22) with respect to x_1 and θ equals

$$s'(x_1) - \frac{dv_1(x_1, e_1^*(x_1))}{dx_1} - \frac{dv_2(e_1^*(x_1))}{dx_1} \tag{23}$$

$$= \quad s'(x_1) - \frac{\partial v_1(x_1, 0)}{\partial x_1} \tag{24}$$

$$= \quad \int_0^{e^{**}(x_1)} \frac{\partial^2 v_1(x_1, e)}{\partial x_1 \partial e} de. \tag{25}$$

Equation (24) holds since player 1 is the recipient, so $e_1^*(x_1) = 0$ by part (b) of Lemma 1. Since $e_1^*(x_1)$ is a constant, the derivative in the third term of (23) is zero. Equation (25) holds by applying the Envelope Theorem to find the derivative of $s(x_1)$ using the definition of $s(x_1)$ in equation (2), and then by applying the Fundamental Theorem of Calculus. To show (25) is positive, note first that $\partial^2 u_1(x_1, e)/\partial x_1 \partial e > 0$ since investment reduces 1's vulnerability, implying $\partial^2 v_1(x_1, e)/\partial x_1 \partial e > 0$, and note second that $e^{**}(x_1) > 0$ by Lemma 1. Hence, expression (22) exhibits increasing marginal returns in x_1 and θ. Steps similar to the proof of Lemma 1 can be used to show that Strict Monotonicity Theorem 1 implies $x_1^{FDR} < x_1^{FIR}$.

Next, we will show $x_1^{FIR} < x_1^{1ST}$. The objective function in the first best is $s(x_1)$. This can be nested with the objective function under an injunction in expression (21) as follows:

$$s(x_1) + \theta \left[s(x_1) - v_1(x_1, e_1^*(x_1)) + \left(\frac{\alpha}{1 - \alpha} \right) v_2(e_1^*(x_1)) \right], \tag{26}$$

where $\theta = -(1 - \alpha)$ under injunctions and $\theta = 0$ in the first best. Arguments paralleling those in the preceding paragraph can be used to show that the Strict Monotonicity Theorem 1 applies to expression (26), implying $x_1^{FIR} < x_1^{1ST}$.

Next, we need to translate the investment ranking into a social-welfare ranking. By Assumption 2, $u_2(x_2, e) - x_2$ is strictly concave. Furthermore it is maximized over a convex set $x_2 \in [0, \infty)$. By the Maximum Theorem under Convexity (see, e.g., Sundaram 1996, Theorem 9.17.3), the associated value function $v_2(e)$ is also strictly concave. By Assumption 2, $u_1(x_1, e)$ is strictly concave, implying $v_1(x_1, e)$ is strictly concave. The sum of strictly concave functions $v_1(x_1, e) + v_2(e)$ is strictly concave. By the Maximum Theorem under Convexity, the associated value function $s(x_1)$ is strictly concave. Therefore, the ranking $x_1^{FDR} < x_1^{FIR} < x_1^{1ST}$ implies damages are strictly less efficient than an injunction, which in turn is less efficient than the first best. □

Proof of Proposition 2: Suppose $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$ for some $g(x_1)$ satisfying Assumptions 1–4; for some $h(x_1, e)$ satisfying Assumptions 1–5; and for $\gamma > 0$. We will prove the proposition for the case in which investment by player 1 (the generator) is dirty. The proof for the case in which its investment is clean is similar and thus omitted.

We will first show $x_1^{1ST} < x_1^{FDR}$ for all $\gamma > 0$. Substituting the functional form for v_1 into the threat points associated with the damage regime listed in Table 1 and then substituting the resulting threat points into expression (3) yields the following expression for player 1's ex ante equilibrium surplus under damages:

$$g(x_1) + \gamma h(x_1, e_1^*(x_1)). \tag{27}$$

Substituting our functional form for v_1 into equation (2) yields the following expression for the social welfare function in the first best:

$$g(x_1) + \gamma h(x_1, e^{**}(x_1)) + v_2(e^{**}(x_1)). \tag{28}$$

Nesting the objective functions (27) and (28),

$$g(x_1) + \gamma h(x_1, e^{**}(x_1)) + v_2(e^{**}(x_1)) + \theta \left[\gamma h(x_1, e_1^*(x_1)) - \gamma h(x_1, e^{**}(x_1)) - v_2(e^{**}(x_1))\right], \tag{29}$$

where $\theta = 0$ for the first best and $\theta = 1$ for the damages regime. The second cross partial of (29) with respect to x_1 and θ equals

$$\frac{d}{dx_1}\left[\gamma h(x_1, e_1^*(x_1))\right] - \frac{d}{dx_1}\left[h(x_1, e^{**}(x_1)) + v_2(e^{**}(x_1))\right] \tag{30}$$

$$= \gamma \left[\frac{\partial h(x_1, e_1^*(x_1))}{\partial x_1}\right] - \gamma \left[\frac{\partial h(x_1, e^{**}(x_1))}{\partial x_1}\right] \tag{31}$$

$$= \gamma \int_{e^{**}(x_1)}^{e_1^*(x_1)} \frac{\partial^2 h(x_1, e)}{\partial x_1 \partial e} de. \tag{32}$$

The first (respectively, second) term of equation (31) comes from differentiating the first (respectively, second) term in square brackets in (30) using the Envelope Theorem. Equation (32) follows from the Fundamental Theorem of Calculus. To see (32) is positive, note first that $\partial^2 h(x_1, e)/\partial x_1 \partial e > 0$ because 1's investment is dirty and note second that $e^{**}(x_1) < e_1^*(x_1)$ by part (a) of Lemma 1. Hence, expression (29) exhibits increasing marginal returns in x_1 and θ. Steps similar to the proof of Lemma 1 can be used to show that Strict Monotonicity Theorem 1 implies $x_1^{1ST} < x_1^{FDR}$.

Next, we shown $x_1^{1ST} < x_1^{FIR}$ for all $\gamma > 0$. This fact, together with the fact from the previous paragraph that $x_1^{1ST} < x_1^{FDR}$, are sufficient to establish that both injunctions and damages are strictly socially inefficient. Nesting the objective functions determining x_1^{1ST} and x_1^{FIR} results in the same expression as in equation (26). The second cross partial of (26) with respect to x_1 and θ equals

$$-\int_{e^{**}(x_1)}^{e_1^*(x_1)} \frac{\partial^2 h(x_1, e)}{\partial x_1 \partial e} de + \left(\frac{\alpha}{1-\alpha}\right) v_2'(e_1^*(x_1)) \frac{de_1^*(x_1)}{dx_1}, \tag{33}$$

derived using steps similar to equations (30) through (32). Both terms in equation (33) are negative. The first term is negative since it has the opposite sign of equation (32), which was shown to be positive. To see that the second term is negative, note first that since player 2 is the recipient of a purely negative externality, $v_2'(e_1^*(x_1)) < 0$. Monotone comparative statics arguments similar to those used in the proof of Lemma 1 can be used to prove that, under the maintained assumption that player 1's investment is dirty, $de_1^*(x_1)/dx_1 > 0$.

Finally, we show that there exists $\gamma' > 0$ such that $x_1^{FDR} < x_1^{FIR}$ for all $\gamma \in (0, \gamma')$. Substituting our functional form for v_1 into player 1's objective function under an injunction, expression (21), and nesting with its objective function under damages, expression (27), yields

$$\begin{aligned} &g(x_1) + \gamma h(x_1, e_1^*(x_1, \gamma)) \\ &\quad + \theta \left[\gamma h(x_1, e^{**}(x_1, \gamma)) + v_2(e^{**}(x_1, \gamma)) - \gamma h(x_1, e_1^*(x_1, \gamma)) - v_2(e_1^*(x_1, \gamma))\right], \end{aligned} \tag{34}$$

where $\theta = 0$ under damages and $\theta = \alpha$ under an injunction. We have added an argument to $e_1^*(x_1, \gamma)$ and $e^{**}(x_1, \gamma)$ to reflect their dependence on γ, which we will vary in the comparative statics exercise to follow. The second cross partial of (34) with respect to x_1 and θ is

$$\frac{d}{dx_1}\left[\gamma h(x_1, e^{**}(x_1, \gamma)) + v_2(e^{**}(x_1, \gamma))\right] - \frac{d}{dx_1}\left[\gamma h(x_1, e_1^*(x_1, \gamma)) + v_2(e_1^*(x_1, \gamma))\right] \quad (35)$$

$$= \gamma \left[\frac{\partial h(x_1, e^{**}(x_1, \gamma))}{\partial x_1}\right] - \gamma \left[\frac{\partial h(x_1, e_1^*(x_1, \gamma))}{\partial x_1}\right] - v_2'(e_1^*(x_1, \gamma))\frac{\partial e_1^*(x_1, \gamma)}{\partial x_1}. \quad (36)$$

The first term in equation (36) comes from applying the Envelope Theorem to compute the derivative in the first term of (35). The second and third terms in equation (36) come from applying the Envelope Theorem to compute the derivative in the second term of (35). In the limit as $\gamma \to 0$, the first and second terms of (36) vanish, implying that the sign of (36) is determined by the sign of the third term. But the third term in equation (36) has the opposite sign of the second term in equation (33), which was shown to be negative. Hence the third term of (36) is positive, implying (36) is positive for sufficiently small $\gamma > 0$. Therefore, (34) exhibits increasing marginal returns in x_1 and θ for sufficiently small $\gamma > 0$. Steps similar to the proof of Lemma 1 can be used to show that $x_1^{FDR} < x_1^{FIR}$ for sufficiently small $\gamma > 0$.

Using arguments paralleling those in the last paragraph of the proof of Proposition 1, we can show that the investment ranking translates into a social welfare ranking, so that damages are socially more efficient than an injunction for sufficiently small $\gamma > 0$. □

Proof of Proposition 4: Comparison of corner-injunction and corner-damage rights held by the recipient in the presence of a mixed externality is identical to a comparison of injunction and damage rights held by the recipient in the presence of a purely negative externality. Therefore, part (a) of the proposition follows from part (b) of Proposition 1. Comparison of peak-injunction and peak-damage rights held by the recipient in the presence of a mixed externality is identical to a comparison of injunction and damage rights held by the generator in the presence of a purely negative externality. Therefore, part (b) of the proposition follows from parts (b) and (c) of Proposition 2. □

Proof of Proposition 5: Analysis of corner-injunction (respectively, peak-injunction) rights held by the recipient of a mixed externality is identical to the analysis of an injunction held by the recipient (respectively, generator) of a purely negative externality. Therefore, part (a) of the proposition follows from part (a) of Propositions 1 and 2.

To prove part (b), consider the case in which investment reduces the recipient's vulnerability. The analysis is similar for the case in which investment increases the recipient's vulnerability and is omitted. Nest the recipient's objective functions for peak and corner injunctions as follows:

$$(1-\alpha)[v_1(x_1, 0) - s(x_1)] + \alpha v_2(0) + \theta \left\{ v_1(x_1, e_1^*(x_1)) - v_1(x_1, 0) + \left(\frac{\alpha}{1-\alpha}\right)[v_2(e_1^*(x_1)) - v_2(0)] \right\}, \quad (37)$$

where $\theta = 0$ for corner injunctions and $\theta = 1 - \alpha$ for peak injunctions. Let x_1^{FCIR} be equilibrium investment under corner injunctions and x_1^{FPIR} that under peak injunctions. Arguments used repeatedly in previous proofs can establish that $x_1^{FCIR} < x_1^{FPIR}$ if the second cross partial of (37) with respect to x_1 and

θ exhibits increasing marginal returns. This second cross partial equals

$$\int_0^{e_1^*(x_1)} \frac{\partial^2 v_1(x_1,e)}{\partial x_1 \partial e} de + \left(\frac{\alpha}{1-\alpha}\right) v_2'(e_1^*(x_1)) \frac{de_1^*(x_1)}{dx_1}. \tag{38}$$

In the limit as $\alpha \to 0$, the second term of (38) drops out. The first term, which remains, is positive because $\partial^2 v_1(x_1, e)/\partial x_1 \partial e > 0$ if investment reduces 1's vulnerability. Hence $x_1^{FCIR} < x_1^{FPIR}$. Arguments along the lines of the proof of Proposition 2 can establish $x_1^{FPIR} < x_1^{1ST}$. Arguments used repeatedly in previous proofs can be used to prove that $x_1^{FCIR} < x_1^{FPIR} < x_1^{1ST}$ implies that peak-injunction rights are more efficient than corner-injunction rights.

To prove part (c), in the limit as $\alpha \to 1$, the second term dominates (38). Arguments from the proof of Proposition 2 can be used to establish that if $\partial^2 v_1(x_1, e)/\partial x_1 \partial e > 0$, then $de_1^*(x_1)/dx_1 > 0$. Further, $0 < e_1^*(x_1) < e_2^*$ implies $v_2'(e_1^*(x_1)) > 0$. Hence, in the limit as $\alpha \to 1$, $x_1^{FPIR} < x_1^{FCIR}$. It is easy to see that $x_1^{FCIR} = x_1^{1ST}$ in the limit as $\alpha \to 1$. Arguments used repeatedly in the previous proofs can be used to establish that $x_1^{FPIR} < x_1^{FCIR} = x_1^{1ST}$ implies that social-welfare in the limit as $\alpha \to 1$ has the ranking given in part (c). □

Proof of Proposition 6: Analysis of corner (respectively peak) damages held by the recipient of a mixed externality is identical to damages held by the recipient (respectively, generator) of a purely negative externality. Therefore, part (a) of the proposition follows from part (a) of Propositions 1 and 2.

To prove part (b), consider the case in which investment reduces 1's vulnerability (the case in which investment increases 1's vulnerability is analyzed similarly). Nesting the objective functions for the recipient of corner and peak damage rights:

$$v_1(x_1, 0) + \theta[v_1(x_1, e_1^*(x_1)) - v_1(x_1, 0)], \tag{39}$$

where $\theta = 0$ for corner damages and $\theta = 1$ for peak damages. Let x_1^{FCDR} be equilibrium investment under corner damages and x_1^{FPDR} be that under peak damages. Arguments used repeatedly in the previous proofs can be used to show that $x_1^{FCDR} < x_1^{FPDR}$ if the second cross partial of (39),

$$\int_0^{e_1^*(x_1)} \frac{\partial^2 v_1(x_1,e)}{\partial x_1 \partial e} de, \tag{40}$$

is positive. But the facts that $e_1^*(x_1) > 0$ and that investment reduces 1's vulnerability imply (40) is positive. It is easy to prove that $x_1^{FPDR} < x_1^{1ST}$. Arguments used repeatedly in the previous proofs can be used to establish that $x_1^{FCDR} < x_1^{FPDR} < x_1^{1ST}$ implies peak damages are more efficient than corner damages. □

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