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THEORETICAL UNCERTAINTIES IN THE SUBGIANT MASS-AGE RELATION AND THE ABSOLUTE AGE OF ω CENTAURI

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ABSTRACT

The theoretical uncertainties in the calibration of the relationship between the subgiant mass and age in metal-poor stars are investigated using a Monte Carlo approach. Assuming that the mass and iron abundance of a subgiant star are known exactly, uncertainties in the input physics used to construct stellar evolution models and isochrones lead to a Gaussian 1 σ uncertainty of $\pm 2.9\%$ in the derived ages. The theoretical error budget is dominated by the uncertainties in the calculated opacities. Observations by Kałużny et al. of detached double-lined eclipsing binary OGLEGC 17 in the globular cluster ω Centauri have found that the primary is on the subgiant branch with a mass of $M = 0.809 \pm 0.012\ M_{\odot}$ and $[\mathrm{Fe/H}] = -2.29 \pm 0.15$. Combining the theoretical uncertainties with the observational errors leads to an age for OGLEGC 17 of 11.10 ± 0.67 Gyr. The one-sided, 95% lower limit to the age of OGLEGC 17 is 10.06 Gyr, while the one-sided, 95% upper limit is 12.27 Gyr.

Subject headings: cosmology: theory — globular clusters: general — globular clusters: individual (ω Centauri) — stars: evolution — stars: interiors — stars: Population II

1. INTRODUCTION

Traditionally, absolute globular cluster (GC) ages have been determined using the absolute magnitude of the main-sequence turnoff (TO) or subgiant branch (SGB), as this minimizes the theoretical uncertainties associated with stellar evolution models (e.g., Renzini 1991; Chaboyer et al. 1996b). This age determination method requires that the distance to the GC be known. There is considerable uncertainty regarding the distance scale to GCs, and this translates into a significant uncertainty in the absolute-age estimates of GC (Krauss & Chaboyer 2002). To avoid this error, Paczyński (1996) has advocated the use of detached eclipsing double-line spectroscopic binaries to determine the age of GCs. In these binary systems, it is possible to determine the mass of the individual stars. These mass estimates are derived in a fundamental manner and are likely to be free from systematic errors (Paczyński 1996). If one of the members of the binary is at the TO or on the SGB, then the age of the cluster may be determined from the TO/SGB mass-age relation.

In principle, the relation between the TO/SGB mass and age is a robust prediction of stellar evolution theory—it simply depends on the amount of hydrogen fuel available for nuclear burning in the core of the star and the luminosity of the star during its main-sequence lifetime. Thus, the TO/SGB mass-age relation should be insensitive to the details of what occurs near the surface of stars and will not depend on the treatment of convection for the low-mass stars in GCs (Paczyński 1996). For these reasons, one might expect that ages derived from the masses of TO/SGB stars will be relatively insensitive to various significant uncertainties that might otherwise be important in stellar structure calculations.

This Letter explores how the uncertainties in stellar structure and evolution calculations (§ 2) translate into errors in ages derived from SGB masses in GCs (§ 3). This work is motivated by the high-precision mass estimate for the detached eclipsing double-line spectroscopic binary OGLEGC 17 in ω Centauri by Kałużny et al. (2002). The primary in OGLEGC 17 is on SGB (Thompson et al. 2001). The age of this star is derived in § 4, and this Letter concludes with a general discussion of the implications of this age determination in § 5.

2. UNCERTAINTIES IN STELLAR EVOLUTION MODELS

The basic equations of stellar structure are simple: hydrostatic equilibrium, conservation of mass and energy, and an equation for energy transfer. However, the solution of these equations requires a considerable amount of additional information—the composition of the star must be specified, and one needs to know opacities, nuclear reaction rates, surface boundary conditions, etc. There are uncertainties associated with all of these parameters, and these uncertainties in the input physics lead to uncertainties in the calculated structure and evolution of a star. Furthermore, there are uncertainties associated with the modeling of convection in stars and, indeed, with the inclusion of additional physical processes, such as diffusion. Given that the equations of structure must be solved numerically, it is easiest to evaluate the uncertainties associated with stellar structure and evolution calculations using a Monte Carlo (MC) procedure (Chaboyer et al. 1996a). Once the distribution of each input parameter is specified, one randomly selects a specific value for each of the input parameters and constructs stellar evolution models for a variety of masses. These stellar evolution models are then used to construct an isochrone that can be used to derive the age of a GC. This procedure is repeated numerous times, and the result is a set of isochrones that can be used to determine the error associated with stellar age estimates.

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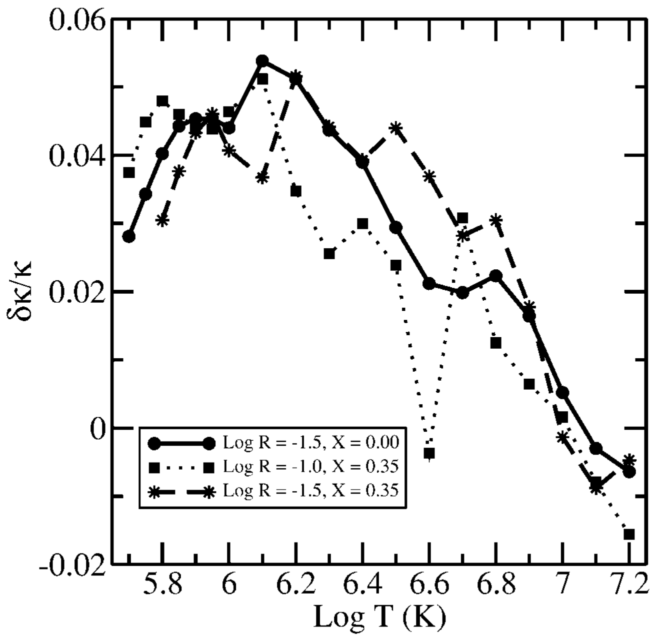

FIG. 1.—Comparison between the OPAL and LEDCOP opacities. Fractional difference in opacity plotted on the y-axis is defined to be $\delta\kappa/\kappa = (\kappa_{\rm OPAL} - \kappa_{\rm LEDCOP})/\kappa_{\rm OPAL}$. Differences in the opacities have been calculated for a variety of hydrogen mass fractions, X, and values of $\log R$ appropriate for the deep interior of a metal-poor $M = 0.80\ M_\odot$ star at the middle and end of its main-sequence lifetime. In the stellar model, the density/temperature parameter $\log R = \log (\rho/T_6^3)$ (where T_6 is the temperature in units of 10^6 K) is in the range of -1.5 to -1.0 for the temperatures plotted.

Full details on our choice of parameters can be found in our previous papers (Chaboyer et al. 1996a, 1998; Krauss & Chaboyer 2002). In brief, the following input parameter distributions were used: mixing length of 1.85 ± 0.25, helium diffusion coefficients multiplied by 0.2–0.8 (flat distribution), high-temperature ($T > 10^4$ K) opacities multiplied by 1 ± 0.02, low-temperature opacities multiplied by 0.7–1.3 (flat), and α-capture abundances [α/Fe] = 0.2–0.7 (flat); surface boundary conditions were either gray or from Krishna-Swamy (1966), color tables were from Green, Demarque, & King (1987) or Kurucz (1992), and nuclear reaction rates were mean values from Adelberger et al. (1998) with errors from Chaboyer et al. (1998). The primordial helium abundance is constrained to be in the narrow range $Y_p = 0.245$–0.25, motivated by recent advances in our ability to estimate this quantity. Observations of deuterium in high-redshift QSO absorption systems, coupled with big bang nucleosynthesis (BBN), allow a reliable estimate of the cosmic baryon fraction, $\Omega_B h^2 = 0.020 \pm 0.001$, where h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1} (Burles, Nollett, & Turner 2001). The value determined by cosmic microwave background (CMB) experiments is $\Omega_B h^2 = 0.022 \pm 0.003$ (de Bernardis et al. 2002). The agreement between these two independent estimates is compelling and allows us to interpret the bound on the baryon fraction, using BBN, in terms of a new allowed range for Y_p. Measurements in extragalactic H II regions yield similar values of $Y_p = 0.245 \pm 0.004$ (Izotov, Chaffee, & Green 2001), although Peimbert, Peimbert, & Luridiana (2002) find $Y_p = 0.238 \pm 0.003$. Using a value of $Y_p = 0.238$ results in an SGB mass age that is 5% higher than that found using $Y_p = 0.245$. However, given the excellent agreement in determinations of $\Omega_b\, h^2$ from the deuterium and CMB observations, we believe that it is very unlikely that the true primordial helium abundance is as low as $Y = 0.238$.

The equation of state was not varied in the MC approach, as it is not thought to be a significant source of error in stellar models. To check this, the SGB mass age of an $M = 0.809\ M_\odot$, [Fe/H] = -2.25 star was calculated using stellar models calculated with the OPAL equation of state (Rogers 1994) and a simple equation of state that uses the Debye-Hückel correction (Guenther et al. 1992). The two sets of isochrones yielded ages that agreed with each other to within 0.5%.

The only differences between the input parameter distributions in this Letter and in Krauss & Chaboyer (2002) are in those for the nuclear reaction rates and the opacity. As we discuss in § 3, the uncertainty in the TO/SGB mass-age relation is dominated by the uncertainty in opacity, leading us to critically reexamine the possible error in modern opacity calculations. There have been two recent studies that have addressed the accuracy of opacity calculations for conditions appropriate in the Sun. Rose (2001) examined the uncertainty in calculating the opacity at the solar core (a temperature of $T = 1.6 \times 10^6$ K) by comparing the results of seven different opacity codes. Rose (2001) found a standard deviation of 5% about the average. Neuforge-Verheecke et al. (2001) performed a detailed comparison of the OPAL (Iglesias & Rogers 1996) and LEDCOP[2] (Magee et al. 1995) opacities throughout the Sun. They found that the OPAL and LEDCOP opacities differ by ~6% at the base of the convection zone and by ~3% at the solar core.

The conditions in GC stars differ from the Sun in that there are significantly fewer heavy elements. This simplifies the opacity calculations, and presumably the errors in low-metallicity opacity calculations will be smaller than in the solar case. Figure 1 shows differences between the OPAL and LEDCOP opacities for conditions appropriate for an $M = 0.80\ M_\odot$ metal-poor star at the middle and end of its main-sequence lifetime. The OPAL and LEDCOP opacity calculations differ by ~4% at $\log T = 6.2$ and by ~1% around $\log T = 7$.

An independent estimate of the opacity for the conditions appropriate for the core of a main-sequence, metal-poor star ($X = 0.35$, $Z = 0.0003$, $\log T = 7.2$, $\rho = 157$ gm cm^{-3}, $\log R = -1.4$) was calculated using the CASSANDRA opacity code (Crowley & Harris 2001). For the same conditions, the CASSANDRA opacity was 0.5% higher than the OPAL opacities and 0.4% lower than the LEDCOP opacities. The OPAL and LEDCOP opacities for this data point were determined via a simple linear interpolation (in $\log T$ and $\log R$) in the public opacity tables.

When the OPAL opacity was calculated using the interpolation routines provided by the OPAL group, it was found to be 1.7% lower than the CASSANDRA opacity. This suggests that the interpolation routine introduced additional errors on the order of 1% into the opacities used in the stellar evolution code.

It is impressive that three different opacity codes yield opacities that agree to within 1% for the conditions appropriate for the core of a metal-poor star. As stressed by Neuforge-Verheecke et al. (2001), the true opacity could be different from the calculations, and N. Magee (2001, private communication) estimates a maximum uncertainty for these conditions of 5%.

From Figure 1 it is clear that there is a systematic difference between the OPAL and LEDCOP opacities and that this difference is a function of temperature. At lower temperatures, the OPAL opacities are always higher than the LEDCOP opacities. To take into account the systematic differences between

[2] See http://www.t4.lanl.gov.

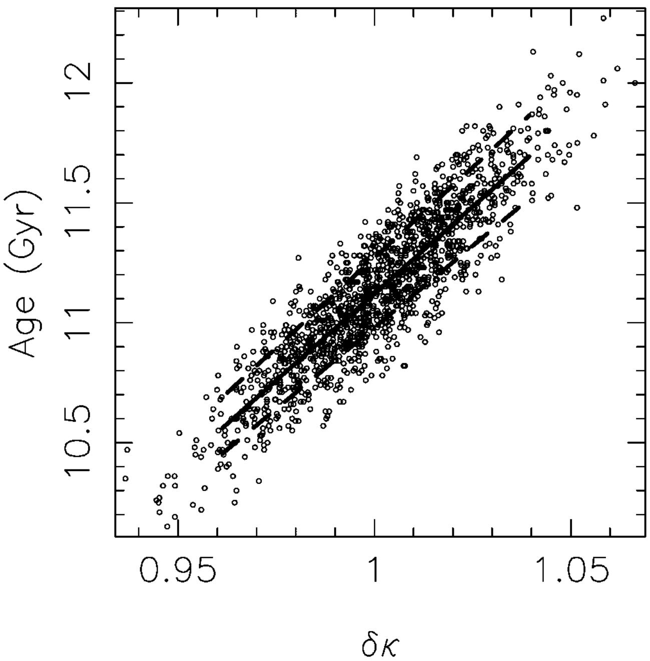

FIG. 2.—Dependence of the derived age of an SGB star on the high-temperature ($T > 10^4$ K) opacities. The *x*-axis $\delta\kappa$ is the coefficient that is multiplying the opacities for $T \geq 10^6$ K. Solid line is the best fit to the median age and has the equation $t_9 = -3.38 + (14.50 \pm 0.37)\,\delta\kappa$, where t_9 is the age in gigayears. Dotted lines are fits to the median $\pm 1\ \sigma$ points with the equations $t_9 = -2.62 + 13.60\delta\kappa$ ($-1\ \sigma$) and $-3.77 + 15.04\delta\kappa$ ($+1\ \sigma$).

the two opacity calculations, the OPAL opacities (which are used in the stellar evolution code) are multiplied by 0.98 for $T \leq 10^6$ K and used at their tabulated values for $T \geq 10^7$ K. Between 10^6 and 10^7 K, the multiplicative factor changes linearly. From the opacity comparisons discussed previously, it is clear that the uncertainty in the opacity calculations increases with decreasing temperatures. As a result, we have taken the uncertainty in the opacities to be Gaussian, with $\sigma = 4\%$ for $T \leq 10^6$ K and $\sigma = 2\%$ for $T \geq 10^7$ K. In between these two temperatures, the Gaussian σ changes linearly with temperature.

3. MONTE CARLO RESULTS

In total, 1500 different sets of input parameters were generated and used to construct isochrones. For each set of input parameters in the MC approach, two isochrones were calculated with differing metallicities, [Fe/H] $= -2.5$ and -2.0. The set of 1500 MC isochrones was used to determine the age of an SGB star, chosen to have properties similar to OGLEGC 17: $M = 0.809\ M_\odot$, [Fe/H] $= -2.25$. Furthermore, we fix the SGB star to be located 0.05 mag (in $B-V$) from the TO. The resulting distribution of ages has a narrow range with a Gaussian $1\ \sigma$ uncertainty of $\pm 2.9\%$. This confirms expectations that the SGB mass-age relation can be a robust prediction of theoretical stellar evolution models (Paczyński 1996).

The set of theoretical MC ages was analyzed to determine which input parameters had a significant effect on the derived age. The dominant source of error in deriving the age of a star of fixed mass on the SGB using its mass are the high-temperature ($T > 10^4$ K) opacities. The relationship between the derived age and the opacity is shown in Figure 2. The solid line is the best fit to the median age as a function of the opacity, and its slope implies that for every 1% increase in the opacities at 10^7 K, the age will increase by 0.14 Gyr, or 1.3%. Given that the SGB age at a given mass is essentially the main-sequence lifetime of a given stellar model, the relationship between age and opacity can be easily understood given the equation of radiative transfer for a star that implies $L \propto 1/\kappa$, where L is the luminosity. Hence, a higher opacity leads to a decrease in the luminosity, which in turn results in an increase in the main-sequence lifetime of a star of a given mass.

TABLE 1
SENSITIVITY OF AGE TO PARAMETER VARIATIONS

Parameter	δ-Parameter	δ-Age (%)
High-temperature opacities (% at 10^7 K)	2	+2.6
Helium mass fraction	0.003	−1.4
[α/Fe] (dex)	0.2	+1.0
Helium diffusion coefficient (%)	30	−1.0

The other input parameters in the MC approach had much smaller effects on the derived age. This can be readily seen in Figure 2, where the width of the age distribution at a given value of $\delta\kappa$ gives an indication of the total uncertainty associated with all of the other input parameters. This width is a factor of about 2 smaller than the range of ages in Figure 2. If the error in the opacity was zero, then from the $\pm 1\ \sigma$ fits shown in Figure 2 the total theoretical uncertainty in the derived ages would have a Gaussian $\sigma = 1.3\%$. This is somewhat more than a factor of 2 smaller than the uncertainty found when including the uncertainty in the opacities. Besides opacity, the only parameters that lead to a significant change in the derived age were the helium mass fraction (Y), the abundance of α-capture elements, and the coefficient of helium diffusion. The effect that increasing each of these parameters has on the age is summarized in Table 1.

4. THE ABSOLUTE AGE OF ω CENTAURI

Thompson et al. (2001) identified a number of detached eclipsing double-line spectroscopic binaries in the GC ω Cen and found that the primary in OGLEGC 17 was on the SGB, ideally situated for an age determination. Kałużny et al. (2002) report improved observations of OGLEGC 17 that yield a primary mass of $M = 0.809 \pm 0.012\ M_\odot$ and a metallicity of [Fe/H] $= -2.29 \pm 0.15$. In order to determine an accurate age for this star, one must determine its location relative to the metal-poor TO of ω Cen. An inspection of the color-magnitude diagram presented by Thompson et al. (2001) leads us to conclude that OGLEGC 17 is located between 0.03 and 0.07 mag redward (in $B-V$) of the metal-poor TO. To determine the uncertainty in the age of OGLEGC 17, the following procedure was performed: We (1) randomly picked an isochrone (out of our set of 1500 MC isochrones), (2) randomly picked a mass from the distribution $M = 0.809 \pm 0.012\ M_\odot$, (3) randomly picked a metallicity using the distribution [Fe/H] $= -2.29 \pm 0.15$, (4) randomly picked a location on the SGB by using a flat distribution that varied from 0.03 to 0.07 mag redward of the TO point, and (5) determined the age of OGLEGC 17 for this particular isochrone, mass, [Fe/H], and location on the SGB. This procedure was repeated 10,000 times. The results are shown in Figure 3.

The age of OGLEGC 17 determined in this way is 11.10 ± 0.67 Gyr; i.e., the total uncertainty in the age of this star is $\pm 6\%$. The one-sided, 95% lower limit to the age of OGLEGC 17 is 10.06 Gyr, while the one-sided, 95% upper limit is 12.27 Gyr. The derived uncertainty is dominated by the uncertainty of the mass determination. If the mass were known exactly, then the $1\ \sigma$ uncertainty in the derived age would be

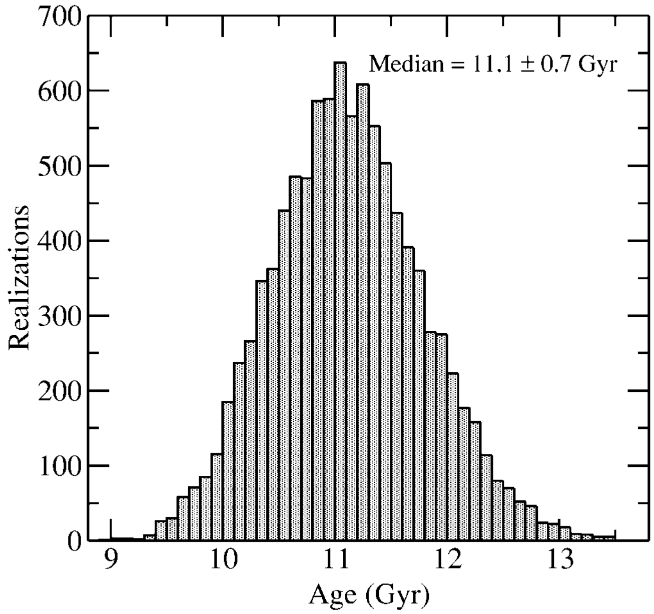

FIG. 3.—Derived age of the metal-poor SGB in OGLEGC 17 in ω Cen, whose mass and metallicity were determined by Kałużny et al. (2002). Histogram incorporates all known theoretical and observational errors and reflects the total uncertainty in the age of OGLEGC 17.

reduced to $\pm 3\%$. Our derived age is fairly similar to that determined by Kałużny et al. (2002), who found $t = 11.8 \pm 0.6$ Gyr, assuming no error in the isochrones of Girardi et al. (2000).

5. DISCUSSION

The age of OGLEGC 17 may be compared to our estimate of the mean age of the 17 metal-poor GCs that used the luminosity of the TO as an age indicator (Krauss & Chaboyer 2002). For the same set of input parameters, we found a median age of 12.5 Gyr and one-sided, 95% confidence level ages of 10.2 and 15.9 Gyr. The non-Gaussian distribution has a lower 1 σ age of 11.0 Gyr, implying that the age of OGLEGC 17 and the mean age of the 17 metal-poor GCs agree at the 1 σ level. The one-sided, 95% confidence level lower limits to the two age determinations are quite similar (10.1 and 10.2 Gyr). This supports our conclusion that the ages of the oldest stars and recent measurements of the Hubble constant require that the cosmic equation of state has $w \equiv$ pressure/density < -0.3 (Krauss & Chaboyer 2002).

The age of OGLEGC 17 was determined assuming that the error in the mass determination was Gaussian. As discussion of the error in the mass determination of OGLEGC 17 has not been published, it is not clear whether this assumption is valid. If it is, then the age of OGLEGC 17 is known much more accurately than the mean age of the metal-poor GCs determined from their TO luminosity. The upper limit to the mean age (12.3 Gyr) is much smaller than that determined in the GC study (15.9 Gyr). The upper limit to the age of OGLEGC 17 can be compared to the age of the universe determined from the CMB of 14.0 ± 0.5 Gyr (Knox, Christensen, & Skordis 2001). Their 2 σ lower limit of 13.0 Gyr is 0.7 Gyr older than our upper limit, implying at least 0.7 Gyr of galaxy evolution before OGLEGC 17 formed. This corresponds to a redshift of globular cluster formation of $z \lesssim 7$ (see eq. [1] in Krauss & Chaboyer 2002). It is worth remarking that when more old GC ages are constrained in this way, a comparison with strict upper limits that one might derive on their ages with the Hubble age may provide the strongest constraints on cosmological models with exotic forms of dark energy, such that $w \leqq -1$.

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