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EARLY QUINTESSENCE IN LIGHT OF THE WILKINSON MICROWAVE ANISOTROPY PROBE

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ABSTRACT

We examine the cosmic microwave background (CMB) anisotropy for signatures of early quintessence dark energy—a nonnegligible quintessence energy density during the recombination and structure formation eras. Only very recently does the quintessence overtake the dark matter and push the expansion into overdrive. Because the presence of early quintessence exerts an influence on the clustering of dark matter and the baryon-photon fluid, we may expect to find trace signals in the CMB and the mass fluctuation power spectrum. In detail, we demonstrate that suppressed clustering power on small length scales, as suggested by the combined *Wilkinson Microwave Anisotropy Probe*/CMB/large-scale structure data set, is characteristic of early quintessence. We identify a set of concordant models and map out directions for further investigation of early quintessence.

Subject headings: cosmic microwave background — cosmological parameters — large-scale structure of universe

There exists compelling evidence that the energy density of the universe is dominated by dark energy. The evidence grows stronger with each successive experiment and observation of cosmic evolution and structure, as boldly reinforced by the recent high-precision measurement of the cosmic microwave background (CMB) fluctuations by the *Wilkinson Microwave Anisotropy Probe* (*WMAP*; Bennet et al. 2003; Spergel et al. 2003; Kogut et al. 2003; Hinshaw et al. 2003; Verde et al. 2003; Page et al. 2003). And yet, the nature of the dark energy remains elusive. A cosmological constant (Λ), providing a simple phenomenological fix in the absence of better information, is consistent with current data including the latest *WMAP* results. Lessons from particle physics and cosmology, however, suggest a more attractive solution in the form of a dynamical dark energy (Wetterich 1988; Ratra & Peebles 1988; Peebles & Ratra 1988; Caldwell, Dave, & Steinhardt 1998) that continues to evolve in the present epoch—quintessence.

In the quintessence scenario, the dark energy becomes dominant only at late times, as required for cosmic acceleration. However, the late appearance of the quintessence may not be the whole story. Scalar field models of quintessence with global attractor solutions (Wetterich 1988; Ratra & Peebles 1988; Zlatev, Wang, & Steinhardt 1999; Steinhardt, Wang, & Zlatev 1999) have been shown to “track” the dominant component of the cosmological fluid. One consequence is that just after inflation, the universe may contain a nonnegligible fraction of the cosmic energy density. Through subsequent epochs, the quintessence energy density ρ_q lags behind the dominant component of the cosmological fluid with a slowly varying Ω_q and an equation of state $w_q \equiv p_q/\rho_q$ that is nearly constant. The field energy tracks the background until the current epoch, when the quintessence energy density crosses and overtakes the matter density. A non-negligible fraction of dark energy at last scattering, $\Omega_q^{(ls)}$, and during structure formation, $\Omega_q^{(sf)}$, then arises quite naturally. From the observational viewpoint, detection of any trace of “early quintessence” would give us a tremendous clue as to the physics of dark energy.

In this work, we concentrate on “early quintessence,” characterized by nonnegligible values $\Omega_q^{(ls)}, \Omega_q^{(sf)} \lesssim 0.05$. Typical sca-

lar field models exhibit an exponential form of the scalar potential in the range of the field relevant for early cosmology, with special features in the potential or kinetic term in the range governing the present epoch (Hebecker & Wetterich 2001; Albrecht & Skordis 2000; Armendariz-Picon, Mukhanov, & Steinhardt 2000, 2001; Wetterich 2002, 2003a, 2003b). We describe such models in more detail below. Our attention is drawn toward these models because of the recent claims of suppressed power on small scales in the combined *WMAP*/CMB/large-scale structure data set. We are motivated precisely by the fact that the most prominent influence of a small amount of early dark energy is a suppression of the growth of dark matter fluctuations (Doran, Schwindt, & Wetterich 2001b; Ferreira & Joyce 1998). As we soon discuss, this influence can help to make the fluctuation amplitude extracted from galaxy catalogs or the Ly α forest compatible with a relatively high amplitude CMB anisotropy.

The effect of early quintessence on the mass fluctuation power spectrum can be understood simply as a suppression of the growth function for dark matter and baryonic fluctuations. Just as fluctuation growth is suppressed at late times with the onset of dark energy, so is the growth of linear modes slowed at early times owing to nonnegligible $\Omega_q^{(ls)}, \Omega_q^{(sf)}$. For modes that enter the horizon before equality, $k > k_{eq}$, the effect is an overall suppression. For $k < k_{eq}$, the suppression only takes place after the mode enters the horizon. The consequence is that the smaller scale matter density perturbations are more suppressed, which ultimately appears as a scale-dependent red *tilt* for the $k < k_{eq}$ modes and a flat suppression for the $k > k_{eq}$ modes.

We can directly examine the effect on the mass power spectrum by comparing the σ_8 -values of an early quintessence model with a Λ model having the same amount of present-day dark energy. When we fix the amplitude of the CMB fluctuations over a range of angular multipoles corresponding to $k \approx (8 \text{ Mpc})^{-1}$, then

$$\frac{\sigma_8(Q)}{\sigma_8(\Lambda)} = (a_{eq})^{3\Omega_q^{(sf)}/5} (1 - \Omega_q^{(0)})^{-(1+w)^{-1}/5} \sqrt{\frac{\tau_0(Q)}{\tau_0(\Lambda)}} \quad (1)$$

The dominant effect is the first factor with $a_{\text{eq}} = \Omega_r/\Omega_m \approx 1/3230$. This factor accounts for the slower growth of the cold dark matter fluctuations. The other kinematical factors involve a suitably averaged quintessence equation of state in the recent epoch, \tilde{w} (Doran et al. 2001a; Huey et al. 1999; Perlmutter, Turner, & White 1999a), and the present conformal time τ_0 for the quintessence and Λ models. We emphasize that equation (1) results in a uniform suppression of the cold dark matter amplitudes for all modes that have entered the horizon since z_{eq} .

Now we turn to consider the implications of the CMB for quintessence. The temperature anisotropy power spectrum, from the plateau through the first two peaks, now has been measured with new accuracy. In the context of a spatially flat Λ model, this would tell us the Hubble constant, h , matter and baryon densities, Ω_m and Ω_b , very precisely. For the case of quintessence, a degeneracy exists among these parameters, and the influence of the equation of state can play off the Hubble constant to achieve an otherwise indistinguishable anisotropy pattern out to small angular scales (Huey et al. 1999). Clearly, the CMB sky is consistent with a small amount of early quintessence in addition to $\Omega_q^{(0)}$ insofar as the angular diameter distance to the last scattering surface is preserved. As a means of proof by example, we identify a set of models in Table 1 with observationally indistinguishable CMB patterns, i.e., identical peak positions but differing amounts of $\Omega_q^{(s)}$, $\Omega_q^{(sf)}$, shown as models A and B in Figure 1. Model C is *WMAP*'s best-fit Λ CDM, and model D the best fit for an extended data set with Λ CDM and running spectral index (Spergel et al. 2003). Our methodology, therefore, is to use the CMB data to guide our search for compatible quintessence models rather than to carry out an exhaustive survey of parameter space. We choose models with present-day equation of state $w_q^{(0)} \lesssim -0.9$ so as to focus attention on the early rather than late quintessence behavior, as compared to a Λ model. Because a significant parameter degeneracy between the primordial scalar spectral index n_s and the optical depth to last scattering τ persists in the *WMAP* data, we explore different combinations of n_s , τ .

In Figure 2, we compare the prediction of our models for the matter power spectrum with data extracted from galaxy catalogs (e.g., Two-Degree Field Galaxy Redshift Survey [2dFGRS]; Percival et al. 2001; Verde et al. 2002; Peacock et al. 2001) or the Ly α forest (Gnedin & Hamilton 2002; Croft et al. 2002). In view of the uncertainties from bias and nonlinearities, the agreement is good for all models (A–D).

Increasing the spectral index to $n_s > 1$ induces more power for the fluctuation spectrum on small scales relative to large. We remark that this enhancement of small-scale power can be balanced by an increase in $\Omega_q^{(sf)}$. Typically, for σ_8 to remain constant a 10% increase of n_s is compensated by a 5% increase of $\Omega_q^{(sf)}$. Consequently, we find a degeneracy in the n_s - $\Omega_q^{(sf)}$ plane for σ_8 (see Fig. 3d of Doran et al. 2001b). The degeneracy may be broken once data for much larger k are included, such as the Ly α forest. Whereas $\Omega_q^{(sf)}$ leads to a uniform decrease of all mass fluctuations with $k/h > 0.064 \text{ Mpc}^{-1}$ by a constant factor, the increase of the small-scale matter fluctuations due to n_s depends on a scale proportional to k^{n_s} .

An increase of n_s also influences the detailed CMB spectrum in a number of ways. First, the spectral index influences the precise location of the first peak in angular momentum space l_1 . Parameterizing the location of the peaks as $l_m = l_A(m - \varphi_m)$ (Hu et al. 2001), one observes that the shift φ_1 decreases by 4.7% if n_s increases by 10%. Keeping the well-measured position of l_1 fixed (Page et al. 2003), this results in a decrease of l_A by 5%. As a consequence, the location of the second and

TABLE 1
MODELS AND PARAMETERS

Parameter	A	B	C	D
$\Omega_q^{(sf)}$	0.03	0.05	0	0
$\Omega_q^{(ls)}$	0.03	0.05	0	0
$w_q^{(0)}$	-0.91	-0.95	-1	-1
n_s	0.99	1.05	0.97	0.93
h	0.65	0.70	0.68	0.71
$\Omega_m h^2$	0.15	0.16	0.15	0.136
$\Omega_b h^2$	0.024	0.025	0.023	0.022
τ	0.17	0.26	0.1	0.17
σ_8	0.81	0.87	0.87	0.85
χ^2_{eff}/ν :				
<i>WMAP</i>	1432/1342	1432/1342	1430/1342	1432/1342
CBI-MOSAIC	1.1/3	1.8/3	0.78/3	0.34/3
ACBAR	7.0/7	6.4/7	6.7/7	6.1/7
2dFGRS	29/32	28/32	27/32	29/32

third peaks are shifted by $\Delta l_2 \approx 19$, $\Delta l_3 \approx 38$ toward smaller l . Again, this effect can (partly) be compensated by an increase of $\Omega_q^{(ls)}$ according to $\varphi_1 \approx [1 - 0.466(n_s - 1)](0.2604 + 0.291\Omega_q^{(ls)})$ (Doran & Lilley 2002). Second, increasing n_s lowers the amplitude ratio between the second and first peaks. This can be compensated by a larger fraction of baryons $\Omega_b h^2$. Third, larger n_s adds power to the CMB spectrum at large l , or a lower relative power at low l . To the extent that *WMAP* and *COBE* (Wright et al. 1996; Hinshaw et al. 1996; Banday et al. 1997) observe a lack of power on large scales, $l \lesssim 10$, then a blue tilt is beneficial, as a 10% gain in n_s lowers the quadrupole relative to $l = 40$ by a factor of ~ 1.8 , more in line with observations.

In the extended *WMAP* parameter analysis, combining CMB with non-CMB cosmological constraints, the running k -dependence in n_s is shown to lower the matter power spectrum at σ_8 and smaller scales, as well as reduce the small-angle CMB fluctuation power, without touching the region of l measured by *WMAP* (Spergel et al. 2003). In the case of early quintessence, the CMB power spectrum on small angular scales is only mildly lowered. However, the matter fluctuations turn out to be smaller for a given CMB amplitude. The net effect is a shift of the CMB power extrapolated from structure formation data toward larger values.

The models of early quintessence considered in this article can be described by a scalar field with a nonstandard kinetic term evolving in an exponential potential. The field Lagrangian is

$$L = \frac{1}{2}k^2(\varphi)\partial_\mu\varphi\partial^\mu\varphi + M^4 \exp(-\varphi/M) \quad (2)$$

and corresponds to the case of a cosmon field with a leaping kinetic term (Hebecker & Wetterich 2001). Many more models of early quintessence can be cast into equation (2) by an appropriate rescaling of the scalar field. Typically, the function $k^2(\varphi)$ is nearly constant at early times, leading to the well-known attractor dynamics of quintessence fields with exponential potentials (Wetterich 1988; Ratra & Peebles 1988; Ferreira & Joyce 1998; Copeland, Liddle, & Wands 1998). Hence, in the early universe one has $\Omega_q = \mathcal{N}k^2(\varphi)$ with $\mathcal{N} = 3, 4$ for matter and radiation domination, respectively. No tuning of parameters is required in order to explain why dark energy has been of a similar order of magnitude as radiation and matter in most cosmological epochs (albeit substantially smaller in early times, e.g., $\Omega_q^{(ls)}$ a few percent). For a realistic model, however, $k^2(\varphi)$ must grow in the late universe. The increased weight of the kinetic term in the Lagrangian due to this growth leads to a drain of kinetic

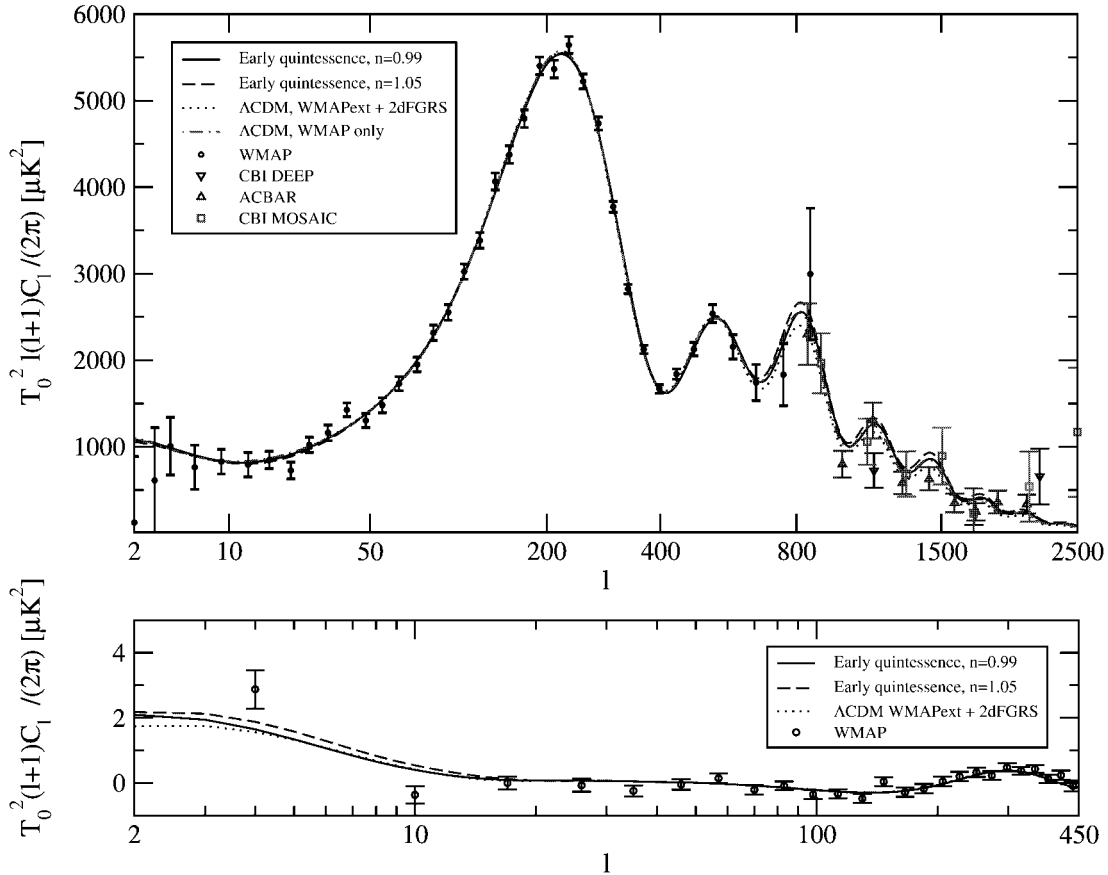


FIG. 1.—Temperature (TT) and polarization (TE) as a function of multipole l . The *WMAP* data (Hinshaw et al. 2003; Kogut et al. 2003) are plotted alongside two early quintessence models with $n_s = 0.99$ and $n_s = 1.05$ (see Table 1 for the other cosmological parameters). For comparison, we plot *WMAP*-normalized spectra for the best-fit Λ CDM model (no $\text{Ly}\alpha$ data) with constant spectral index $n = 0.97$ (Spergel et al. 2003) as well as the best-fit Λ CDM model with running spectral index $n_s = 0.93$, $dn_s/d\ln k = -0.031$. At large l , we plot the Cosmic Background Imager (CBI; Pearson et al. 2003; Mason et al. 2003) and Arcminute Cosmology Bolometer Array Receiver (ACBAR; Kuo et al. 2002) measurements.

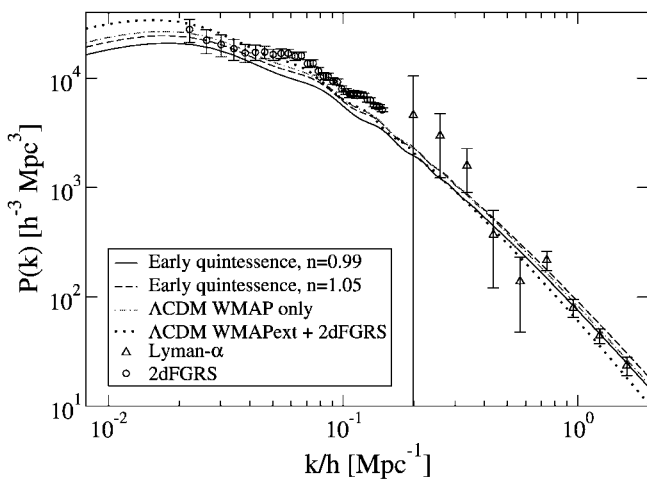


FIG. 2.—Cold dark matter power spectrum today as a function of k/h . We plot the linear spectrum for two early quintessence models with spectral indices $n_s = 0.99$ and $n_s = 1.05$ (see Table 1 for the other cosmological parameters). Also shown is the best-fit Λ model with running spectral index $n_s = 0.93$, $dn_s/d\ln k = -0.031$ (Spergel et al. 2003), normalized to *WMAP* (no $\text{Ly}\alpha$ data). The 2dFGRS (Percival et al. 2001; Peacock et al. 2001; Verde et al. 2002) and $\text{Ly}\alpha$ (Gnedin & Hamilton 2002; Croft et al. 2002) data have been evolved to $z = 0$, although we have not convolved our theoretical data with the experimental window functions. The galaxy power spectrum has a bias compared to theoretical predictions that is not included in the figure.

energy T into potential energy V , effectively “stopping” the field. As the pressure of quintessence is given by $T - V$, this change in $k^2(\varphi)$ leads to negative pressure accelerating the universe today. For a specific example, the recent increase of $k^2(\varphi)$ is related to the renormalization group running of the wave function renormalization of a dilaton-type model (Wetterich 2002, 2003a, 2003b). This needs a mild tuning of a parameter (0.1% level) in order for the crossover to quintessence domination to happen now. Interestingly, this crossover in the dynamical behavior of the scalar field may also be observable as a jump in the rate of variation of the fine-structure constant.

We have computed the spectra in Figures 1 and 2 using CMBEASY (Doran 2003) for a class of “leaping kinetic term quintessence” (Hebecker & Wetterich 2001) models with early quintessence. In these models, $k(\varphi)$ makes a relatively rapid jump from a small to a large value at some approximately chosen value of φ . The main features depend on only two parameters besides the present fraction of dark energy $\Omega_q^{(0)}$ and the present equation of state $w_q^{(0)}$, namely, the fraction of dark energy at last scattering, $\Omega_q^{(ls)}$, and during structure formation, $\Omega_q^{(st)}$. In order to facilitate comparison with other effects of quintessence—for example, the Hubble diagram $H(z)$ for supernovae—we present a useful parameterization of quintessence (Wetterich 2002, 2003a, 2003b) rather than detailed models. For $a > a_{eq}$ and $x \equiv \ln a = -\ln(1+z)$, we consider a

quadratic approximation for the averaged equation of state ($x_{\text{ls}} \approx -\ln 1100$):

$$\begin{aligned} \bar{w}_q(x) &= -\frac{1}{x} \int_x^0 dx' w_q(x') \\ &= w_q^{(0)} + (\bar{w}_q^{(\text{ls})} - w_q^{(0)}) \frac{x}{x_{\text{ls}}} + Ax(x - x_{\text{ls}}). \end{aligned} \quad (3)$$

The time-dependent average equation of state $\bar{w}_q(x)$ is directly connected to the time history of the fraction in dark energy $\Omega_q(x)$ according to

$$\frac{\Omega_q(x)}{1 - \Omega_q(x)} = \frac{\Omega_q^{(0)}(1 + a_{\text{eq}}) \exp[-3x\bar{w}_q(x)]}{1 - \Omega_q^{(0)} \exp(-x)}, \quad (4)$$

which connects $\bar{w}_q^{(\text{ls})}$ to $\Omega_q^{(\text{ls})}$. The parameter A is related to the average fraction of dark energy during structure formation ($a_{\text{tr}} \approx \frac{1}{3}$):

$$\Omega_q^{(\text{sf})} = \int_{\ln a_{\text{eq}}}^{\ln a_{\text{tr}}} \frac{\Omega_q(a) d \ln a}{\ln(a_{\text{tr}}/a_{\text{eq}})}. \quad (5)$$

The parameters describing our models are (A) $\bar{w}_q^{(\text{ls})} = -0.188$, $A = -0.0091$; (B) $\bar{w}_q^{(\text{ls})} = -0.172$, $A = -0.015$. The Hubble expansion has a simple expression in terms of $\bar{w}_q(x)$:

$$\begin{aligned} H^2(z) &= H_0^2 \{ \Omega_q^{(0)}(1+z)^{3[1+\bar{w}_q(x)]} \\ &\quad + \Omega_m^{(0)}[(1+z)^3 + a_{\text{eq}}(1+z)^4] \}. \end{aligned} \quad (6)$$

Our models A and B are consistent with all present bounds for $H(z)$, including Type Ia supernovae (Schmidt et al. 1998; Riess et al. 1998; Garnavich et al. 1998; Perlmutter et al. 1999a, 1999b).

To summarize, we have demonstrated that models of early quintessence are compatible with the presently available data for a constant spectral index of primordial density perturbations. Parameter degeneracies in the angular diameter distance to last scattering are consistent with a small abundance of early quintessence. In turn, the presence of early quintessence results in a differential reduction or scale-dependent tilt in the spectrum of matter fluctuations on scales $k < k_{\text{eq}}$ and a uniform suppression of power for scales $k > k_{\text{eq}}$, which may have significant consequences for the interpretation of combined CMB and large-scale structure data. We note that special care must be taken now to interpret large-scale structure observations in the context of early quintessence models. At the other end of the spectrum, the lack of very large scale power in the CMB can be compensated in part by increasing both the primordial spectral tilt and the amount of early quintessence. We look ahead toward ongoing and future tests that afford tighter measurements of small-scale CMB and matter power spectra. A precision measurement of the position and height of the third peak could be extremely helpful for determining the fraction of early quintessence.

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