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Erin M. O'Malley
Dartmouth College

Christina Gilligan
Dartmouth College

Brian Chaboyer
Dartmouth College

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ABSOLUTE AGES AND DISTANCES OF 22 GCS USING MONTE CARLO MAIN-SEQUENCE FITTING

ERIN M. O'MALLEY

Department of Physics and Astronomy
Dartmouth College, Hanover, NH 03784

CHRISTINA GILLIGAN

Department of Physics and Astronomy
Dartmouth College, Hanover, NH 03784

BRIAN CHABOYER

Department of Physics and Astronomy
Dartmouth College, Hanover, NH 03784
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ABSTRACT

The recent *Gaia* Data Release 1 of stellar parallaxes provides ample opportunity to find metal-poor main-sequence stars with precise parallaxes. We select 21 such stars with parallax uncertainties better than $\sigma_\pi/\pi \leq 0.10$ and accurate abundance determinations suitable for testing metal-poor stellar evolution models and determining the distance to Galactic globular clusters. A Monte Carlo analysis was used, taking into account uncertainties in the model construction parameters, to generate stellar models and isochrones to fit to the calibration stars. The isochrones which fit the calibration stars best were then used to determine the distances and ages of 22 globular clusters with metallicities ranging from -2.4 dex to -0.7 dex. We find distances with an average uncertainty of 0.15 mag and absolute ages ranging from 10.8 – 13.6 Gyr with an average uncertainty of 1.6 Gyr. Using literature proper motion data we calculate orbits for the clusters finding six that reside within the Galactic disk/bulge while the rest are considered halo clusters. We find no strong evidence for a relationship between age and Galactocentric distance, but we do find a decreasing age-[Fe/H] relation.

Subject headings: Globular Clusters; Distances; Ages; Kinematics

1. INTRODUCTION

Globular clusters (GCs) are among the oldest objects in our Galaxy that can be dated with a high level of precision; their ages provide a strict lower limit on the age of the Universe. Although significant progress has been made in recent years, these estimates are still afflicted by uncertainty as high as ~ 1.5 Gyr (Gratton et al. 1997; Krauss & Chaboyer 2003). The largest uncertainty stems from the dependence of GC distance determinations on the Population II distance scale with improvements possible only with more accurate parallax measurements.

The multitude of methods used to determine GC ages assume some comparison to stellar evolution models: horizontal branch morphology, white dwarf cooling sequence, or main-sequence turn-off (MSTO) location. The technical details of each of these methods, along with their individual strengths and weaknesses, can be found in the review by Soderblom (2010). In general, the stellar evolution models, upon which these methods rely, are inherently uncertain due to uncertainties in the physical processes which occur in stars, such as incomplete equations of state and approximate treatment of convection by an arbitrary mixing length parameter, among others.

As the MS is the most well-understood phase of stellar evolution, MS-fitting can provide robust ages for GCs. Calibration of this method at low metallicity requires accurate distances and abundances for very low metallicity

stars. The *Hipparcos* catalog provides accurate parallaxes of over 115,000 stars that could be used to study any number of astrophysical problems. However, only a few hundred of these stars are metal-poor dwarfs, of which only a handful are suitable for identifying the location of the MS in our stellar isochrones. Nevertheless, when the Hipparcos catalog was first released to the community many groups attempted to use the parallax data available to calibrate stellar evolution models and study the distances and ages of GCs (e.g. Gratton et al. 1997; Chaboyer et al. 1998; Carretta et al. 2000; Grundahl et al. 2002; Gratton et al. 2003). The stringent constraints used in these studies (typically $\sigma_\pi/\pi < 0.12$, $M_V > 5.5$) limited the available sample to 10 – 15 stars in most cases (with $-1.6 < [\text{Fe}/\text{H}] < -1.0$), with the exception of a Grundahl et al. (2002) who were interested in a higher metallicity range ($-0.95 < [\text{Fe}/\text{H}] < -0.65$) and therefore had a larger sample of about 20 stars and Gratton et al. (1997), Carretta et al. (2000) and Gratton et al. (2003) who extend their metallicity ranges to include higher metallicities and find samples of about 30 suitable stars for their calibrations.

The studies with larger samples of calibration stars find GC distance uncertainties of ~ 0.06 mag compared to those studies with fewer calibration stars which find GC distance uncertainties of ~ 0.1 mag. However, it should be noted that this improvement in GC distance estimates was not solely due to the increased number of calibration stars, but in large part to the use of im-

proved and consistent metallicity scales and reddening estimates. In general, these studies found that the *Hipparcos* parallaxes were actually larger than the more uncertain ground-based measurements, leading to a determination of GC distance moduli that were on average 15% larger than previously determined (Gratton et al. 2003) and smaller GC ages as a result.

Recently, Chaboyer et al. (2017) present *HST* parallaxes of eight metal-poor stars (with $-2.6 < [Fe/H] < -1.6$) with uncertainties of $\sim 1\%$. The authors use updated stellar models and improved observations to provide robust distances and ages for nine very metal-poor GCs with the lowest overall uncertainty (Chaboyer et al. 2017), ultimately demonstrating the improvements one can expect from the use of precise parallaxes of metal-poor stars.

In this paper, we use the methods of Chaboyer et al. (2017) and present the results of MS-fitting distances and ages to 22 GC in the *HST* ACS GC Treasury Project (Sarajedini et al. 2007) based on the recent Tycho-Gaia astrometric solution (TGAS) (Michalik, Lindgren & Hobbs 2015; Gaia Collaboration 2016; Lindegren et al. 2016) parallaxes and the Chaboyer et al. (2017) parallaxes for nearby subdwarfs with $-2.6 \leq [Fe/H] \leq -0.4$. In §2 we present the sample of subdwarfs used in the calibration of our stellar evolution models and the results of this calibration. We then present in §3 the GCs for which distances and ages will be determined using these calibrated stellar evolution models along with the weighted mean distance modulus and age for each cluster. We discuss the significance of our findings in §4 and §5 where we present updated orbital calculation for these clusters based on the newly derived cluster distances and literature proper motions along with age-galactocentric distance and age-[Fe/H] relations. Lastly, we provide a summary of our work and future endeavors in §6.

2. CALIBRATION OF STELLAR EVOLUTION MODELS WITH LOCAL SUBDWARFS

In order to calibrate stellar evolution models at metallicities less than solar, low metallicity MS stars with precise parallax measurements and abundance determinations are needed, as the physical properties of MS stars are well-known and remain quite stable over time. The recent release of the TGAS contains parallaxes for millions of stars with a sufficient number of metal-poor MS stars with precise parallaxes to allow a detailed calibration of metal-poor isochrones.

2.1. Selection of Calibration Subdwarfs

Chaboyer et al. (2017) demonstrated the calibration of stellar evolution models is possible with the use of accurate parallaxes for metal-poor stars. Here, we extend the range of metallicities over which a similar calibration can be performed by utilizing the TGAS parallaxes available as a result of the first *Gaia* data release. The first *Gaia* data release includes stars from the Tycho-2 Catalog (Hog et al. 2000) which provides positions and proper motions of the 2.5 million brightest stars. In order to select stars with reliable metallicities we cross-correlated the stars included in the Tycho-2 survey with the PASTEL catalog (Soubiran et al. 2016) of high resolution spectroscopy, finding 24,183 stars included in both catalogs. Oftentimes, the PASTEL catalog includes multiple

[Fe/H] measurements for a given star, including very old references; therefore, we take the star's [Fe/H] as the average of only those measurements more recent than the year 2000. As the GCs we are examining are characteristically metal-poor, we are specifically looking to calibrate the location of the MS using metal-poor subdwarfs, $[Fe/H] \leq -0.5$ dex, a requirement that leaves us with 4,643 stars in the initial sample.

Among the criteria for selecting MS stars is the need for the star to be cool and have a high surface gravity. In order to define the location of the MS we assume a MS magnitude of $M_V \geq 5.5$ mag and use isochrones from the Dartmouth Stellar Evolution Database (Dotter et al. 2008, DSEP) in a range of metallicities, $-3.0 \leq [Fe/H] \leq -0.5$ dex, $[\alpha/Fe] = +0.4$ dex and ages of both 10 and 13 Gyr to find the color of the MS as a function of metallicity. We find the color of each isochrone at an absolute magnitude of $M_V = 5.5$ mag and find the best-fitting polynomial relation for V-I color as a function of metallicity given in Equation 1.

$$V - I = 0.895 + 0.258[Fe/H] + 0.104[Fe/H]^2 + 0.015[Fe/H]^3. \quad (1)$$

As expected, we find this relation holds for both the 10 Gyr and 13 Gyr isochrones as the location of the MS is independent of age.

Initial UBVR photometry were tabulated using SIMBAD and stars with observed V-I colors less than the theoretical color given by Equation 1 were removed from our sample, leaving the remaining 415 stars as MS candidates. Since our calibration stars need to have very accurate parameters in order to provide the best calibration of theoretical models, we remove from our sample the 29 stars that have photometry solely from photographic plates.

We estimated the reddening of our calibration stars by cross-correlating our sample with the Stromgren-Crawford uvby β catalog (Pauzen 2015). Stars that are within 80 pc of us do not exhibit much reddening (Reis et al. 2011; Lallement et al. 2014) and are included in our final sample even if they do not have Stromgren photometry. For stars with Stromgren photometry, we used the methods of both Schuster & Nissen (1989) and Karatas & Schuster (2010) to determine the reddening. Both studies define the reddening, $E(b - y) = (b - y) - (b - y)_0$, as a function of three color indices: $m_1 = (v - b) - (b - y)$ used to measure line blanketing from metal lines, $c_1 = (u - v) - (v - b)$ to measure the strength of the Balmer discontinuity, and $\beta = \beta_w - \beta_n$ which is sensitive to the Hydrogen β line and stellar surface temperature. For Schuster & Nissen (1989),

$$(b - y)_0 = 0.579 + 1.541m_0 - 1.066c_0 - 2.965(\Delta\beta) + 9.64(\Delta\beta)^2(-4.383m_0(\Delta\beta) - 3.821m_0c_0 + 6.695c_0(\Delta\beta) + 7.763m_0c_0^2). \quad (2)$$

For Karatas & Schuster (2010),

$$(b - y)_0 = 0.492 - 0.976c_0 + 2.239(\Delta\beta) - 8.77(\Delta\beta)^2 + 6.26m_0c_0 - 16.15c_0(\Delta\beta) - 4.720m_0c_0^2 + 53.24c_0(\Delta\beta)^2 + 0.39(\Delta\beta)^2 + 27.526c_0^2(\Delta\beta) - 26.757m_0c_0(\Delta\beta). \quad (3)$$

where, in both cases, $m_0 = m_1 + 0.3E(b - y)$, $c_0 = c_1 - 0.2E(b - y)$ and $\Delta\beta = 2.720 - \beta$.

We calculate $E(b - y)$ for our sample stars using both methods and find similar results for each; however, the Karatas & Schuster (2010) results gave lower residuals. Therefore, the reddening estimates were calculated using the newer method, except in cases where high resolution spectroscopy found no evidence for interstellar Na I lines (O'Malley et al. 2017), implying that the reddening was negligible. The reddening was converted from $E(b - y)$ to $E(V - I)$ using the relations of Karatas & Schuster (2010) and Winkler (1997).

Finally, we test for the Lutz-Kelker bias (Lutz & Kelker 1973) in our π_{10} sample. We provide a detailed description of our Lutz-Kelker analysis in the appendix for the interested reader, but ultimately find no appreciable correction in either absolute magnitude or metallicity.

Table 1 provides the observational stellar parameters for our final sample of 24 subdwarfs with accurate TGAS and/or *HST* parallaxes ($\sigma_\pi/\pi \leq 10\%$) along with references for the photometric and parallax measurements. Three additional stars are included in this table that do not have TGAS or *HST* parallaxes but which have accurate parallaxes determined by previous studies. The first four columns give the star's ID, V magnitude, reddening listed as $E(V-I)$ and the dereddened V-I color. The last four columns give previously determined parallax measurements, the current TGAS parallaxes, the absolute magnitude and finally the metallicity of the star. The absolute magnitudes are calculated using the TGAS parallaxes where available except for HIP 87788, HIP 54639, HIP 46120 and HIP 103269 which have *HST* parallaxes that are both more accurate and more reliable than the TGAS parallaxes.

To directly show the impact the TGAS parallaxes have on our calibration of metal-poor stellar evolution models we construct two sub-sample groups: π_{10} and π_{12} . The π_{10} group contains all of the stars in our sample that have either TGAS parallaxes, *HST* parallaxes, or both with $\sigma_\pi/\pi \leq 0.10$. For the four stars that have both, we use the *HST* parallaxes in the analysis as they have smaller uncertainties than the TGAS parallaxes. The π_{12} group contains all of the stars with previous *Hipparcos* or *HST* parallaxes with $\sigma_\pi/\pi < 0.12$.

2.2. Comparison to Theoretical Isochrones

We test the reliability of the DSEP isochrones (Dotter et al. 2008) using the same Monte Carlo analysis as in Chaboyer et al. (2017). We will only briefly describe the process here. The reliability of each model depends on the specific choices for the various physical parameters used to construct the stellar models; therefore, we vary the input parameters within probability distributions that are based on their intrinsic uncertainties. Table 2 provides the probability density distributions of the stellar evolution parameters that are varied in this study. As an example, updated nuclear reaction rates

of Adelberger et al. (2011) give *pp*-chain reaction rates uncertain at the 1% level. In constructing the stellar evolution models for this study, the calculated nuclear reaction rate for the *pp*-chain reaction is multiplied by a number drawn randomly from a Gaussian distribution with a mean and standard deviation of 1.00. A total of 2000 independent isochrones are constructed in this manner to allow for the total theoretical uncertainty to be estimated via this MC analysis.

Our choice of mixing length has been updated based on recent studies of metal-poor stars. A uniform probability density distribution from 1.0 - 1.7 was chosen for the mixing length in Chaboyer et al. (2017) based on recent studies by Bonaca et al. (2012), Tanner et al. (2014) and Creevey et al. (2015) which show lower metallicity stars requiring a mixing length parameter much lower than the solar mixing length ($\alpha_\odot = 1.9$). In this paper the range of subdwarf and GC metallicities is extended towards higher metallicities; therefore, for models with $[\text{Fe}/\text{H}] > -1.00$ dex we shift the mixing length density distribution closer to solar, sampling a uniform density distribution from 1.2 - 1.9.

For the majority of the subdwarfs in the sample we construct a suite of theoretical isochrones with a binary $[\alpha/\text{Fe}]$ of +0.2 or +0.4 dex based on the findings of Seden (2004, and references therein) that both metal-poor field stars and GC stars show Ca overabundance of +0.2 - +0.4 dex for metallicities ranging from $[\text{Fe}/\text{H}] = -2.4$ dex to $[\text{Fe}/\text{H}] = -0.8$ dex. For the most metal-poor star, HIP 87788, O'Malley et al. (2017) find $[\alpha/\text{Fe}] > +0.6$ and so for this star we construct a separate suite of models with a distribution of $[\alpha/\text{Fe}]$ as follows: 0.40 dex (25%), 0.60 dex (50%), 0.80 dex (25%).

The stellar evolution tracks, which cover a stellar mass range of 0.3 - 1.0 M_\odot , are converted into isochrones using both the color-temperature relations of Hauschildt et al. (1999) (ISO-P) and Vandenberg & Clem (2003) (ISO-VC). The goodness of fit of these isochrones is then determined using a reduced χ^2 analysis with N degrees of freedom, where N is the number of stars in a given metallicity group. It is well-known that the location of the zero-age main sequence is expected to be a function of metallicity based on theoretical stellar evolution models. In this study we do not attempt to define this location as a function of metallicity, but instead perform our calibration on groups of subdwarfs spanning a narrow metallicity range of ~ 0.4 dex.

The median deviation of the isochrones from the π_{10} calibration stars is 1.86σ for ISO-P and 1.92σ for ISO-VC, with the median of the individual metallicity groups ranging from 0.63 - 3.65 σ . Table 3 provides the average metallicity of each bin, the corresponding median standard deviations determined for both the ISO-P and ISO-VC models and the number of stars used in the determination. The median deviation of the isochrones from the π_{12} calibration stars is similar to or slightly smaller than that of the π_{10} sample for four out of the six metallicity. For the two metallicity bins in which the π_{10} group uncertainty is smaller than that of the π_{12} group, we attribute the larger uncertainty to the fact that although the parallax uncertainty in the π_{12} group is allowed extend to $\sim 12\%$, the average uncertainty is 3.6% which is only slightly higher than that of the π_{10} with an aver-

TABLE 1
CALIBRATION STAR OBSERVATIONAL PROPERTIES

ID	V	E(V-I)	(V - I) ₀	π_{prev} (mas)	π_{TGAS} (mas)	M_V	[Fe/H]
HIP 87788	11.30 ^a	0.00	0.85	10.83 ± 0.13 ^a	10.97 ± 0.26	6.47 ± 0.04	-2.66
Wolf 1137	12.01 ^h	0.08	0.85	8.96 ± 4.39 ⁱ	7.56 ± 0.28	6.20 ± 0.03	-2.53
HIP 54639	11.38 ^a	0.00	0.91	11.12 ± 0.11 ^a	12.26 ± 0.23	6.61 ± 0.04	-2.50
HIP 106924	10.36 ^a	0.00	0.80	14.47 ± 0.10 ^a		6.16 ± 0.04	-2.23
HIP 46120	10.12 ^a	0.00	0.74	15.01 ± 0.12 ^a	14.94 ± 0.21	6.00 ± 0.04	-2.22
HD 321320	10.24 ^e	0.00	0.79	17.36 ± 2.47 ⁱ	16.65 ± 0.25	6.35 ± 0.03	-1.98
HIP 103269	10.27 ^a	0.00	0.77	14.12 ± 0.13 ^a	13.76 ± 0.22	6.02 ± 0.05	-1.83
HIP 108200	10.99 ^a	0.02	0.81	12.40 ± 0.09 ^a		6.41 ± 0.04	-1.83
HD 25329	8.50 ^{d, f}	0.05	1.22	54.12 ± 1.08 ⁱ		7.05 ± 0.07	-1.80
HD 188510	8.84 ^{b, e, h}	0.00	0.74	26.71 ± 1.08 ⁱ	26.20 ± 0.22	5.93 ± 0.03	-1.56
BD+511696	9.92 ^{f, g}	0.00	0.72	12.85 ± 1.33 ⁱ	13.93 ± 0.25	5.64 ± 0.06	-1.51
HD 134439	9.09 ^{d, e}	0.04	1.10	34.14 ± 1.36 ⁱ		6.66 ± 0.07	-1.44
HD 134440	9.46 ^{d, e}	0.06	1.20	33.68 ± 1.67 ⁱ		6.95 ± 0.07	-1.42
HD 97214	9.22 ^c	0.00	1.27	49.38 ± 0.96 ⁱ	50.46 ± 0.40	7.73 ± 0.05	-1.38
HD 145417	7.53 ^{c, d, e}	0.03	0.94	72.01 ± 0.68 ⁱ	73.65 ± 0.30	6.79 ± 0.03	-1.27
Ross 484	10.84 ^d	0.00	1.17	28.8 ± 2.9 ⁱ	21.48 ± 0.22	7.42 ± 0.07	-1.25
BD-033746	9.85 ^f	0.00	1.31	37.04 ± 1.75 ⁱ	38.87 ± 0.23	7.80 ± 0.03	-1.22
CD-35360	10.24 ^{d, e}	0.00	0.85	15.49 ± 1.54 ⁱ	16.79 ± 0.25	6.37 ± 0.03	-1.15
HD 126681	9.29 ^d	0.00	0.72	21.04 ± 1.12 ⁱ	18.04 ± 0.25	5.57 ± 0.03	-1.14
HD 108564	9.46 ^{d, e}	0.00	1.14	36.78 ± 1.01 ⁱ	36.65 ± 0.31	7.32 ± 0.07	-1.14
BD+080335	10.67 ^e	0.00	0.84		11.75 ± 0.26 ⁱ	6.01 ± 0.07	-0.98
BD+022541	10.84 ^j	0.00	0.76	12.21 ± 1.65 ⁱ	12.39 ± 0.23	5.95 ± 0.03	-0.88
HD 230409	10.07 ^{d, e}	0.00	0.79	14.41 ± 1.77 ⁱ	14.51 ± 0.24	5.89 ± 0.03	-0.86
HD 092786	8.02 ^{d, g}	0.00	0.81	37.55 ± 0.76 ⁱ	36.97 ± 0.30	5.86 ± 0.05	-0.81
HD 144579	6.67 ^{d, f}	0.00	0.80	68.87 ± 0.33 ⁱ	69.56 ± 0.22	5.86 ± 0.03	-0.68
HD 073667	7.58 ^c	0.01	0.92	55.13 ± 0.71 ⁱ	54.00 ± 0.33	6.22 ± 0.03	-0.59
HD 216259	8.28 ^{c, d}	0.01	0.95	46.99 ± 0.10 ⁱ	44.81 ± 0.30	6.49 ± 0.03	-0.59

^a Chaboyer et al. (2017), ^b Marshall (2007), ^c Koen et al. (2010), ^d Casagrande et al. (2010), ^e Ryan (1989), ^f Ducati (2002), ^g Carney & Latham (1987), ^h Pancino et al. (2012), ⁱ Van Leeuwen (2007), ^j ESA (1997)

age uncertainty of 2.5%. Additionally, we find that the largest uncertainties stem from the three stars without TGAS or *HST* parallaxes.

We show in Figure 1 the location of the π_{10} calibration stars in a color-magnitude diagram (CMD) compared to the average metallicity isochrone for their respective group, where the median ISO-P isochrone is shown as a dashed line and the median ISO-VC isochrone is solid. From this figure one can see a noticeable offset between the ISO-P and ISO-VC isochrones in the most metal-poor regimes. It is expected then that each set of isochrones in these regimes may give a slightly different distance modulus for a GC, but that weighting the GC distances based on the goodness-of-fit of the isochrones to the calibrations stars will allow us to find the true distance modulus of the GC.

Even within a given isochrone set, either ISO-P or ISO-VC, different stellar model construction parameters will produce isochrones with varying degrees of goodness of fit. In Figure 2, we show the distribution of reduced χ^2 values for the isochrones in each metallicity bin for both the ISO-P (black) and ISO-VC (red) sets of isochrones. As expected, the isochrones in the most metal-rich bin are clustered toward small χ^2_{red} as stellar evolution models already do a good job reproducing observations in this metallicity regime. However, this is not the case for the other five metallicity bins in which a larger spread in the goodness of fit is readily noticeable. Another obvious feature highlighted in the most metal-poor bin is difference in goodness of fits produced by the ISO-P versus the ISO-VC isochrones.

3. DISTANCE AND AGE DETERMINATION OF MILKY WAY GCS

3.1. Selection of Sample Clusters

The GCs used in Chaboyer et al. (2017) were limited to those from the *HST* ACS GC Treasury Project (Sarajedini et al. 2007) with $[\text{Fe}/\text{H}] < -1.4$ dex and $E(\text{B}-\text{V}) < 0.10$ from the Harris (1996; version 2010) GC catalog. We expand upon that study with the inclusion of additional clusters from the *HST* ACS GC Treasury Project (Sarajedini et al. 2007) with $[\text{Fe}/\text{H}] < -0.6$ dex and $E(\text{B}-\text{V}) \leq 0.15$ mag.

It is well known that the selection of color filters is important in GC studies as the effects of multiple stellar populations are prominent in both B-V and B-I where the blue filter is particularly sensitive to light element abundance variations and the long color baseline of B-I is capable of discerning He enhanced populations (Piotto et al. 2007; Marino et al. 2008; Sbordone et al. 2011). Therefore, it was expected that our use of V and I filters would be advantageous in determining the distances and ages to GCs by minimizing the effects of multiple stellar populations. Although this is still true in general, recent studies have shown that the He enhanced populations can be identified in V,V-I CMDs in some GCs (Milone et al. 2012a,b, 2013; Milone et al 2015a,b) and that multiple sub-giant branches may be apparent (Milone et al. 2012a; Piotto et al. 2012).

We cross-reference the ACS GCs that fit our criteria with the $m_{275} - m_{814}, m_{275}$ CMDs of Piotto et al. (2015) to find a suitable sample of clusters for this study. If multiple stellar populations are not present in the long

TABLE 2
MONTE CARLO STELLAR EVOLUTION PARAMETER DENSITY DISTRIBUTIONS

Parameter	Distribution	Standard	Type
He mass fraction (Y) .	0.24725 - 0.24757	PLANCK Collaboration XVI (2014)	Uniform
Mixing length	1.00 - 1.70 ($[\text{Fe}/\text{H}] < -1.00$)	N/A	Uniform
	1.20 - 1.90 ($[\text{Fe}/\text{H}] \geq -1.00$)	N/A	Uniform
Convective overshoot .	$0.0H_p - 0.2H_p$	N/A	Uniform
Atmospheric $T(\tau)$	33.3/33.3/33.3	Eddington (1926, p. 322) or Krishna Swamy (1966) or Hauschildt et al. (1999)	Trinary
Low- T opacities	0.7 - 1.3	Ferguson et al. (2005)	Uniform
High- T opacities	$1.00\% \pm 3\%$ ($T \geq 10^7$ K)	Iglesias & Rogers (1996)	Gaussian
Diffusion coefficients . .	0.5 - 1.3	Thoul et al. (1994)	Uniform
$p + p \rightarrow \text{H} + e^+ + \nu_e^2$.	$1\% \pm 1\%$	Adelberger et al. (2011)	Gaussian
$^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p$	$1\% \pm 5\%$	Adelberger et al. (2011)	Gaussian
$^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma$.	$1\% \pm 2\%$	deBoer et al. (2014)	Gaussian
$^{12}\text{C} + p \rightarrow ^{13}\text{N} + \gamma$. . .	$1\% \pm 36\%$	Xu et al. (2013)	Gaussian
$^{13}\text{C} + p \rightarrow ^{14}\text{N} + \gamma$. . .	$1\% \pm 15\%$	Chakraborty et al. (2015)	Gaussian
$^{14}\text{N} + p \rightarrow ^{15}\text{O} + \gamma$. . .	$1\% \pm 7\%$	Adelberger et al. (2011)	Gaussian
$^{16}\text{O} + p \rightarrow ^{17}\text{F} + \gamma$. . .	$1\% \pm 16\%$	Adelberger et al. (1998)	Gaussian
Triple- α reaction rate .	$1\% \pm 15\%$	Angulo et al. (1999)	Gaussian
Neutrino cooling rate . .	$1\% \pm 5\%$	Haft et al. (1994)	Gaussian
Conductive opacities . .	$1\% \pm 20\%$	Hubbard & Lampe (1969) plus Canuto (1970)	Gaussian

NOTE - As in Bjork & Chaboyer (2006), parameters below atmospheric $T(\tau)$ are treated as multiplicative factors applied to standard tables and formulas.

TABLE 3
MEDIAN STANDARD DEVIATION OF ISOCHRONE FITS TO CALIBRATION STARS

π_{10}				π_{12}			
[Fe/H]	N	σ_P	σ_{VC}	[Fe/H]	N	σ_P	σ_{VC}
-0.75	6	0.74	0.73	-0.71	6	0.91	0.97
-1.20	6	3.10	3.14	-1.20	6	2.21	2.40
-1.37	6	3.57	3.63	-1.46	5	4.75	4.47
-1.74	5	0.95	0.75	-1.68	5	1.46	1.14
-2.02	5	1.10	0.75	-2.02	4	1.19	0.83
-2.43	5	1.80	2.63	-2.40	4	1.23	2.25

color baseline of $m_{275} - m_{814}$, then it is unlikely for them to be visible in V, V-I allowing us to confidently choose the cluster for our analysis. On our inspection of the Piotto et al. (2015) CMDs we remove NGC 362, NGC 1261, NGC 1851, NGC 6715, NGC 6934 and NGC 7089 due to noticeable multiple populations along the MS or SGB. We are encouraged by the agreement with Piotto et al. (2012) which finds multiple distinct SGBs in V-I (F606W-F814W) in 47 Tuc and NGC 1851 as well as broadened SGBs in NGC 362, NGC 6715 and NGC 7089. We also remove from our ACS sample those clusters that do not have CMDs provided by Piotto et al. (2015): NGC 4147, NGC 5139, NGC 7006, and ARP 2. Our final sample contains 22 GCs.

Three clusters in our original ACS sample (47 Tuc, NGC 6752, and NGC 7089) have published population percentages, helium enhancements, and MS ridgelines in F606W-F814W. Thus, it is possible to use the published data to determine the location of the primordial MS for comparison to our isochrones.

- *47 Tuc* - Milone et al. (2012a) use m_{F275W} and m_{F336W} data to identify two populations of stars on the MS. They find the primordial population makes up only 18% of the stars, the second population is enhanced in helium by $\Delta Y = 0.04$

dex, and they provide MS ridgelines in $m_{F606W} - m_{F814W}$, m_{F814W} for both populations. We find a zero-point offset between our CMD in $m_{F606W} - m_{F814W}$, m_{F606W} and that of Milone et al. (2012a); however, the data in Anderson et al. (2009) agrees with that of Milone et al. (2012a). We find the difference between the primordial ridgeline in Milone et al. (2012a) and the median ridgeline in Anderson et al. (2009) to be -0.05 mag in F606W at MS colors of $0.60 \leq m_{F606W} - m_{F814W} \leq 0.67$. We use the Sirianni et al. (2005) relations to transform to an offset of -0.06 mag in V which will be applied to our distance modulus calculation for 47 Tuc. Piotto et al. (2012) find two SGBs in 47 Tuc offset by ~ 0.05 mag in m_{F814W} which we transform to 0.09 mag in V via the Sirianni et al. (2005) relations. We will apply this to the SGB location of 47 Tuc in determining the age of the cluster.

- *NG 6752* - Milone et al. (2013) find three stellar populations in NGC 6752. Pop A is primordial and contains 25% of the stars, Pop B is enhanced in helium by $\Delta Y = 0.01$ dex and contains 45% of the stars, and Pop C is enhanced in helium by $\Delta Y = 0.03$ dex. Like Milone et al. (2012a), m_{F275W} and m_{F336W} data are used to

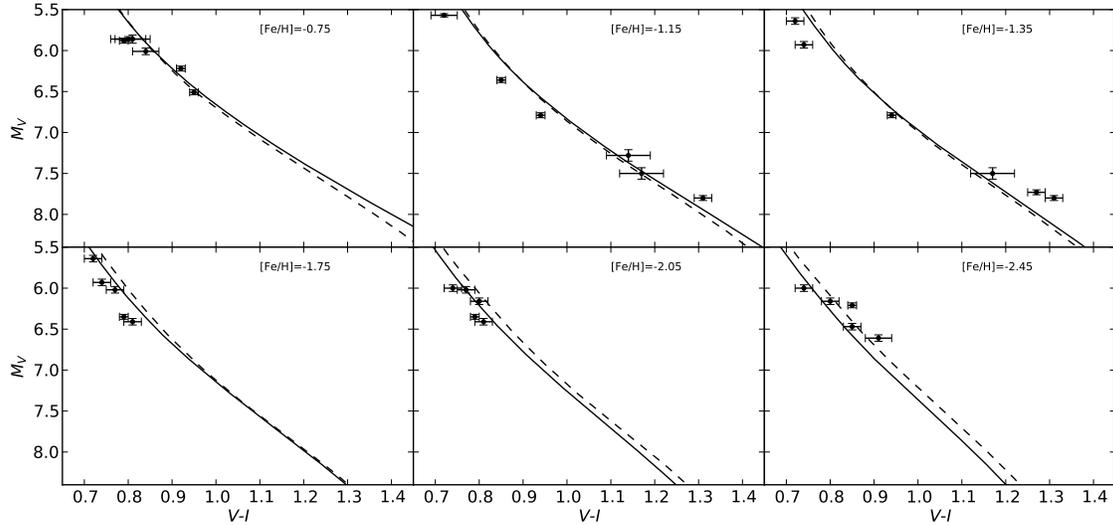


FIG. 1.— Comparison of π_{10} calibration star locations in a CMD to Dartmouth stellar evolution isochrones of mean group metallicity. Shown here are the ISO-P isochrones (dashed) and ISO-VC (solid) isochrones. A distinct offset between the ISO-P and ISO-VC isochrones can be seen in the most metal-poor groups.

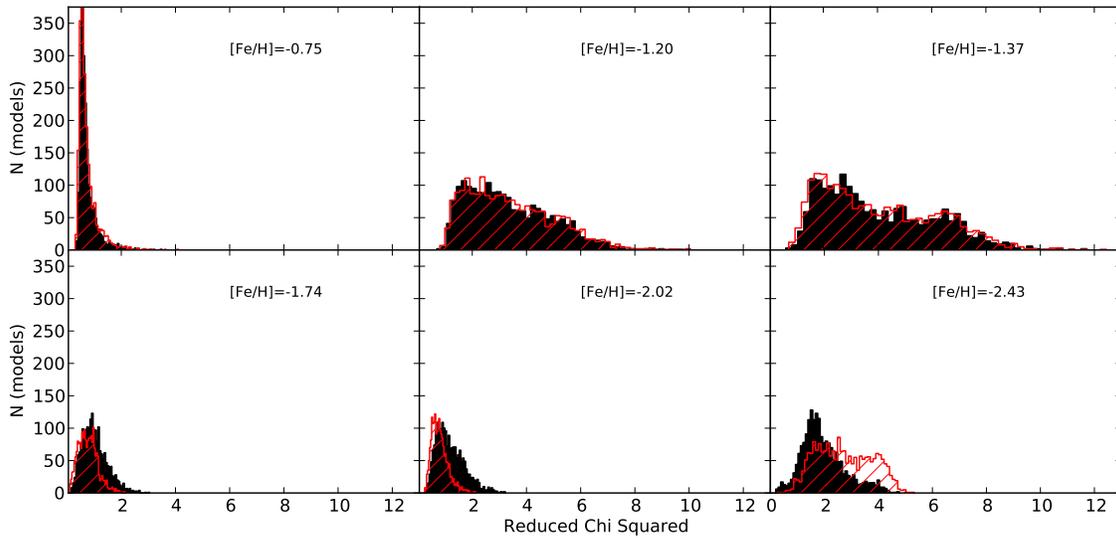


FIG. 2.— The distribution of χ_{red}^2 is shown for both ISO-P (black) and ISO-VC (red dashed) in each metallicity bin. The isochrones fit well the most metal-rich stars within a narrow range of χ_{red}^2 values. The distribution of goodness of fit is more broad for the other metallicity bins. In the most metal-poor bin, there is a noticeable difference in the goodness of fit produced by ISO-P versus ISO-VC.

identify the different populations and MS ridgelines in $m_{F606W} - m_{F814W}$, m_{F814W} are provided as well. They find no distinction between the Pop A and Pop B ridgelines while there is an offset from Pop C. However, because Pop A and Pop B make up 70% of the stars, our median MS ridgeline should follow this combined population along the MS.

- *NGC 7089* – Milone et al (2015a) find seven stellar populations in NGC 7089 using m_{F275W} and m_{F814W} data with helium enhancements of up to $\Delta Y = 0.07$ dex. The authors provide ridgelines

of the primordial population along with combinations of $[\alpha/\text{Fe}]$ enhancements and helium enhancements up to 0.4 dex and 0.084 dex, respectively. Additionally, Piotto et al. (2012) show a spread of 0.15 mag in the SGB of NGC 7089. Although MS ridgelines are available for NGC 7089 like 47 Tuc and NGC 6752, we conservatively remove NGC 7089 from our sample due to the highly complex nature of the multiple stellar populations.

We separate the full set of 22 clusters into 6 metallicity groups ($-2.40 \leq [\text{Fe}/\text{H}] \leq -2.30$; $-2.30 < [\text{Fe}/\text{H}] \leq -1.80$; $-1.80 < [\text{Fe}/\text{H}] \leq -1.50$; $-1.50 < [\text{Fe}/\text{H}] \leq -1.20$; -

$1.20 < [\text{Fe}/\text{H}] \leq -0.90$; $-0.90 < [\text{Fe}/\text{H}] \leq -0.60$) so we can perform MS-fitting using calibration stars covering narrow metallicity ranges. We construct models using the same input parameters as those used in the comparison to the calibration subdwarfs but with $[\text{Fe}/\text{H}]$, $[\alpha/\text{Fe}]$, and helium abundance sampled in the appropriate distribution for GCs. The galactic coordinates and Galactocentric distance from the Harris (1996; version 2010) GC catalog for each cluster along with Carretta et al. (2009) metallicities and Dutra & Bica (2000) far-infrared reddening values are provided in Table 4.

It was found in Chaboyer et al. (2017) that Dutra & Bica (2000) provide more accurate estimates of cluster reddening than Harris (1996; version 2010) based on the red giant branch color. The GCs in this study are sufficiently far away that the use of the Dutra & Bica (2000) far-infrared reddening values can be used with the reddening typically being enhanced over Harris (1996; version 2010) by 0.01 to 0.02 mag, but in some cases by as much as 0.08 mag. These reddening values are converted to $E(V-I)$ in this analysis with the Winkler (1997) relations.

3.2. MS-Fitting Distances and Ages

We perform MS-fitting on *HST* photometry of each cluster, converted into V and I magnitudes via the Sirianni et al. (2005) transformations, and find distances to the clusters given by a fit of the each theoretical isochrone to the median MS ridgeline. We improve upon the method used in Chaboyer et al. (2017) by defining the median ridgeline using the more robust method of rotated histograms described in Marin-Franch et al. (2009).

It was established in Chaboyer et al. (2017) that equally well-fitting distance moduli were obtained by shifting the median ridgeline in both color and magnitude. Therefore, the cluster distance modulus may be calculated using the average reddening and the uncertainty may be propagated using standard techniques. Given that $A_V = 3.1 \times E(B-V)$, the reddening uncertainty used in this study, $\sigma_{E(B-V)} \pm 0.01$ mag corresponds to ± 0.03 mag uncertainty in the distance modulus.

For a given set of isochrones (ISO-P or ISO-VC) the distance modulus typically spans a range of 0.6 mag. However, not all isochrones fit the calibrating parallax stars with the same level of accuracy; therefore, the distance modulus for each cluster is weighted based on the goodness of fit of the isochrone to the calibration stars.

We use the distance modulus for each cluster and isochrone combination to find the absolute magnitude of the cluster SGB which we compare to the SGB magnitude of isochrones ranging in age from 8 – 15 Gyr. Chaboyer et al. (1996a) demonstrate that the SGB magnitude at the location 0.05 mag redder and more luminous than the MS turn-off is an excellent age indicator which minimizes the uncertainty in the derived ages. We apply the same weighting scheme to the ages as we did to the distances to derive ages that incorporate the goodness of fit measure to the calibration stars. The age for each GC is provided along with the distance modulus in Tables 5 for each isochrone set, ISO-P and ISO-VC, along with final, combined results.

One might be interested in the improvements gained using the π_{10} sample of calibration stars versus the π_{12} sample. We perform the same MS-fitting and age dat-

ing analysis weighting the resulting distances and ages by the isochrone goodness of fit to the π_{12} calibrating subdwarf sample and provide the results in Table 6. The distances found using π_{12} calibration stars are on average 0.02 mag larger than when calibrating with our sample of π_{10} subdwarfs, while the ages are 0.2 Gyr lower. It should be noted that the most metal-poor calibration stars are those from Chaboyer et al. (2017) and the *HST* parallaxes were used both in this analysis and the previous analysis with the π_{10} calibration sample as they are less uncertain. Therefore, the difference between the GC distances and ages that are weighted by the fit to these calibration stars will be smaller. If we remove these GCs from the comparison we find the π_{12} distances and ages to be 0.04 mag larger and 0.3 Gyr younger than the π_{10} weighted distances and ages, respectively.

We find the overall uncertainties in distance modulus and age are the same when weighted by the π_{10} and π_{12} calibrating stars, as we are limited observationally by the reddening and in our models by the treatment of convection and diffusion along with uncertainties in the stellar opacities and nuclear reaction rates (Chaboyer et al. 1996b; Gratton et al. 2000; Gratton et al. 2003). As mentioned previously, the 0.01 mag reddening uncertainty is propagated through our calculations using standard techniques and results in 0.03 mag uncertainty in GC distance modulus. Additional photometric errors contribute 0.01 mag uncertainty as only cluster data with $\sigma_V < 0.01$ were used to define the MS ridgeline. The average total error we find for the GC distance modulus is 0.15 mag with the majority being contributed from the intrinsic uncertainties of the theoretical isochrones. Similarly, for a given isochrone, we find the photometric errors and reddening uncertainty contribute 0.3 Gyr uncertainty in age. Given that we find an average total age uncertainty of 1.6 Gyr, the majority of the uncertainty again stems from intrinsic uncertainties of the theoretical isochrones.

For many decades now, it has been known that the derived age for a given cluster depends on the prescribed α -Fe ratio, mainly in the importance of oxygen and its role in the CNO-cycle (Simoda & Iben 1968; Salaris et al. 1993). Many groups have since attempted to quantify $[\alpha/\text{Fe}]$ in both halo field and GC stars and typically find ranges near ~ 0.2 – 0.5 dex (Gratton & Ortolani 1986; Barbuy 1988; Sneden 2004). As the DSEP isochrones cover $[\alpha/\text{Fe}]$ in steps of 0.2 dex, we chose a binary distribution of $[\alpha/\text{Fe}] = +0.2$ and $+0.4$ dex in our suite of models. As expected, we find that the derived ages for each isochrone in the suite depend on the value of the $[\alpha/\text{Fe}]$ construction parameter.

Although we do not find a large difference in the GC distance modulus determined using $[\alpha/\text{Fe}] = +0.20$ dex isochrones versus those with $[\alpha/\text{Fe}] = +0.40$ dex, we do find a noticeable difference in age. Specifically, we find that isochrones constructed using $[\alpha/\text{Fe}] = +0.20$ dex give ages which are, on average, 0.6 Gyr older than the mean GC age while isochrones constructed using $[\alpha/\text{Fe}] = +0.40$ dex give ages which are ~ 0.4 Gyr younger. It is important to note here that because we are calibrating these models along the MS where no difference is found between $[\alpha/\text{Fe}] = +0.2$ and $+0.4$ dex models, both $[\alpha/\text{Fe}]$ ratios give the same distributions of goodness of fit.

We show this relationship Figure 3 where in each panel

TABLE 4
MS-FITTING HIGH METALLICITY CLUSTER DATA

Cluster ID	Name	l(°)	b(°)	R _{GC}	[Fe/H]	E(B-V) _{FIR}
NGC 104	47 Tuc	305.89	-44.89	7.4	-0.76	0.03
NGC 288		152.30	-89.38	12.0	-1.32	0.01
NGC 2298		245.63	-16.00	15.8	-1.96	0.22
NGC 4590	M 68	299.63	36.05	10.2	-2.27	0.06
NGC 5024	M 53	332.96	79.76	18.4	-2.06	0.03
NGC 5053		335.70	78.95	17.8	-2.30	0.02
NGC 5272	M 3	42.22	78.71	12.0	-1.50	0.01
NGC 5466		42.15	73.59	16.3	-2.31	0.02
NGC 5904	M 5	3.86	46.80	6.2	-1.33	0.04
NGC 6101		317.74	-15.82	11.2	-1.98	0.10
NGC 6205	M 13	59.01	40.91	8.4	-1.58	0.02
NGC 6341	M 92	68.34	34.86	9.6	-2.35	0.02
NGC 6362		325.55	-17.57	5.1	-1.07	0.07
NGC 6541		349.29	-11.19	2.1	-1.82	0.16
NGC 6584		342.14	-16.41	7.0	-1.50	0.11
NGC 6652		1.53	-11.38	2.7	-0.76	0.11
NGC 6681	M 70	2.85	-12.51	2.2	-1.62	0.11
NGC 6723		0.07	-17.30	2.6	-1.10	0.16
NGC 6752		336.49	-25.63	5.2	-1.55	0.06
NGC 6809	M 55	8.79	-23.27	3.9	-1.93	0.14
NGC 7078	M 15	65.01	-27.31	10.4	-2.33	0.11
NGC 7099	M 30	27.18	-46.84	7.1	-2.33	0.05

TABLE 5
 π_{10} WEIGHTED GC DISTANCE MODULUS AND AGE

Cluster	ISO-P		ISO-VC		Combined	
	(m - M) _V	Age (Gyr)	(m - M) _V	Age (Gyr)	(m - M) _V	Age (Gyr)
NGC 104	13.57 ± 0.20	11.5 ± 2.0	13.56 ± 0.20	11.6 ± 2.1	13.56 ± 0.20	11.6 ± 2.0
NGC 288	14.91 ± 0.15	11.4 ± 2.0	14.90 ± 0.15	11.6 ± 2.0	14.91 ± 0.15	11.5 ± 2.0
NGC 2298	15.65 ± 0.12	12.1 ± 1.4	15.55 ± 0.11	13.6 ± 1.5	15.61 ± 0.11	12.9 ± 1.5
NGC 4590	15.36 ± 0.10	12.7 ± 1.2	15.40 ± 0.10	12.4 ± 1.3	15.38 ± 0.10	12.5 ± 1.3
NGC 5024	16.56 ± 0.11	13.3 ± 1.3	16.59 ± 0.11	13.0 ± 1.3	16.58 ± 0.11	13.1 ± 1.3
NGC 5053	16.29 ± 0.11	13.3 ± 1.2	16.32 ± 0.10	13.1 ± 1.3	16.31 ± 0.10	13.2 ± 1.3
NGC 5272	15.19 ± 0.13	11.2 ± 1.7	15.17 ± 0.14	11.4 ± 1.8	15.18 ± 0.14	11.3 ± 1.8
NGC 5466	16.15 ± 0.11	13.5 ± 1.2	16.19 ± 0.10	13.2 ± 1.3	16.16 ± 0.10	13.4 ± 1.3
NGC 5904	14.58 ± 0.15	10.7 ± 2.0	14.56 ± 0.15	10.9 ± 2.0	14.57 ± 0.15	10.8 ± 2.0
NGC 6101	16.03 ± 0.11	13.7 ± 1.3	16.07 ± 0.11	13.5 ± 1.4	16.06 ± 0.11	13.6 ± 1.5
NGC 6205	14.54 ± 0.12	12.3 ± 1.5	14.55 ± 0.13	12.1 ± 1.7	14.55 ± 0.13	12.2 ± 1.7
NGC 6341	14.85 ± 0.11	13.0 ± 1.2	14.83 ± 0.12	13.3 ± 1.3	14.84 ± 0.12	13.1 ± 1.5
NGC 6362	14.76 ± 0.19	11.3 ± 2.0	14.76 ± 0.20	11.4 ± 2.1	14.76 ± 0.19	11.4 ± 2.0
NGC 6541	14.98 ± 0.12	12.6 ± 1.4	15.02 ± 0.12	12.3 ± 1.5	15.00 ± 0.12	12.4 ± 1.5
NGC 6584	16.16 ± 0.13	11.6 ± 1.5	16.16 ± 0.13	11.7 ± 1.7	16.16 ± 0.13	11.6 ± 1.7
NGC 6652	15.28 ± 0.19	11.4 ± 2.0	15.27 ± 0.19	11.5 ± 2.1	15.27 ± 0.19	11.4 ± 2.0
NGC 6681	15.26 ± 0.13	12.7 ± 1.5	15.28 ± 0.13	12.6 ± 1.7	15.27 ± 0.13	12.7 ± 1.7
NGC 6723	14.87 ± 0.20	11.9 ± 2.0	14.86 ± 0.20	12.0 ± 2.1	14.86 ± 0.20	11.9 ± 2.0
NGC 6752	13.36 ± 0.13	12.7 ± 1.6	13.37 ± 0.13	12.6 ± 1.7	13.37 ± 0.13	12.6 ± 1.7
NGC 6809	14.17 ± 0.12	11.6 ± 1.3	14.20 ± 0.11	11.3 ± 1.4	14.19 ± 0.11	11.4 ± 1.4
NGC 7078	15.68 ± 0.11	12.7 ± 1.2	15.71 ± 0.10	12.4 ± 1.3	15.69 ± 0.10	12.6 ± 1.3
NGC 7099	14.87 ± 0.11	12.8 ± 1.2	14.89 ± 0.10	12.5 ± 1.3	14.88 ± 0.11	12.7 ± 1.3

the $[\alpha/\text{Fe}] = +0.2$ dex ISO-VC models are in black while the $[\alpha/\text{Fe}] = +0.40$ dex ISO-VC models are in red. The left panel shows the distribution of χ^2_{Red} based on the fit to the calibration stars. The average χ^2_{Red} for $[\alpha/\text{Fe}] = +0.2$ and $+0.4$ dex are comparable, 1.87 and 1.79, respectively. In the center panel we show the distribution of distance moduli of M92 obtained with the different $[\alpha/\text{Fe}]$. As one can see, the distance modulus is relatively unaffected with average distance modulus of 14.78 and 14.80 mag for $[\alpha/\text{Fe}] = +0.2$ and $+0.4$. On the other hand, the last panel shows how $[\alpha/\text{Fe}]$ does impact the age of M92. The $[\alpha/\text{Fe}] = +0.2$ dex models give an aver-

age age of 13.9 Gyr, while the $[\alpha/\text{Fe}] = +0.4$ dex models give an average age of 13.1 Gyr.

The difference in age predicted with the $[\alpha/\text{Fe}] = +0.2$ and $+0.4$ dex models has been ascribed to the oxygen abundance and its importance in the CNO cycle. Therefore, by having a higher $[\text{O}/\text{Fe}]$ abundance, the CNO cycle is able to progress easily with a lower temperature and power output, as evinced from the lower turn-off luminosity (VandenBerg 1992, Figure 1). It is understandable then that the lower turn-off luminosity of an $[\alpha/\text{Fe}] = +0.4$ dex model would in turn predict a lower age for a given GC. It is also understandable that, given

TABLE 6
 π_{12} WEIGHTED DISTANCE MODULUS AND AGE

Cluster	ISO-P		ISO-VC		Combined	
	$(m - M)_V$	Age (Gyr)	$(m - M)_V$	Age (Gyr)	$(m - M)_V$	Age (Gyr)
NGC 104	13.55 ± 0.20	11.7 ± 2.0	13.53 ± 0.20	11.6 ± 2.1	13.54 ± 0.20	11.6 ± 2.0
NGC 288	14.97 ± 0.15	11.5 ± 2.0	15.01 ± 0.15	11.9 ± 2.0	15.00 ± 0.15	11.7 ± 2.0
NGC 2298	15.55 ± 0.12	13.3 ± 1.4	15.59 ± 0.11	13.6 ± 1.5	15.57 ± 0.11	13.4 ± 1.5
NGC 4590	15.40 ± 0.10	12.4 ± 1.2	15.43 ± 0.10	12.3 ± 1.3	15.41 ± 0.10	12.4 ± 1.3
NGC 5024	16.59 ± 0.11	13.0 ± 1.3	16.63 ± 0.11	12.6 ± 1.3	16.60 ± 0.11	12.8 ± 1.3
NGC 5053	16.32 ± 0.11	13.1 ± 1.2	16.36 ± 0.10	12.8 ± 1.3	16.34 ± 0.10	13.0 ± 1.3
NGC 5272	15.20 ± 0.13	10.9 ± 1.7	15.26 ± 0.14	10.2 ± 1.8	15.23 ± 0.14	10.5 ± 1.8
NGC 5466	16.15 ± 0.11	13.5 ± 1.2	16.18 ± 0.10	13.3 ± 1.3	16.16 ± 0.10	13.4 ± 1.3
NGC 5904	14.64 ± 0.15	9.8 ± 2.0	14.62 ± 0.15	10.2 ± 2.0	14.61 ± 0.15	10.0 ± 2.0
NGC 6101	16.07 ± 0.11	13.4 ± 1.3	16.07 ± 0.11	13.4 ± 1.4	16.07 ± 0.11	13.4 ± 1.5
NGC 6205	14.56 ± 0.12	12.0 ± 1.5	14.57 ± 0.13	11.9 ± 1.7	14.56 ± 0.13	12.0 ± 1.7
NGC 6341	14.79 ± 0.11	13.5 ± 1.2	14.82 ± 0.12	13.3 ± 1.3	14.80 ± 0.12	13.4 ± 1.5
NGC 6362	14.75 ± 0.19	11.5 ± 2.0	14.78 ± 0.20	11.2 ± 2.1	14.77 ± 0.19	11.3 ± 2.0
NGC 6541	15.02 ± 0.12	12.1 ± 1.4	15.02 ± 0.12	12.2 ± 1.5	15.02 ± 0.12	12.1 ± 1.5
NGC 6584	16.18 ± 0.13	11.4 ± 1.5	16.17 ± 0.13	11.5 ± 1.7	16.18 ± 0.13	11.4 ± 1.7
NGC 6652	15.26 ± 0.19	11.6 ± 2.0	15.32 ± 0.19	10.5 ± 2.1	15.30 ± 0.19	11.0 ± 2.0
NGC 6681	15.28 ± 0.13	12.5 ± 1.5	15.29 ± 0.13	12.4 ± 1.7	15.28 ± 0.13	12.5 ± 1.7
NGC 6723	14.85 ± 0.20	12.1 ± 2.0	14.91 ± 0.20	11.2 ± 2.1	14.88 ± 0.20	11.6 ± 2.0
NGC 6752	13.38 ± 0.13	12.5 ± 1.6	13.40 ± 0.13	12.3 ± 1.7	13.39 ± 0.13	12.4 ± 1.7
NGC 6809	14.20 ± 0.12	11.3 ± 1.3	14.20 ± 0.11	11.5 ± 1.4	14.20 ± 0.11	11.4 ± 1.4
NGC 7078	15.68 ± 0.11	12.6 ± 1.2	15.70 ± 0.10	12.4 ± 1.3	15.69 ± 0.10	12.5 ± 1.3
NGC 7099	14.86 ± 0.11	12.7 ± 1.2	14.89 ± 0.10	12.5 ± 1.3	14.87 ± 0.11	12.6 ± 1.3

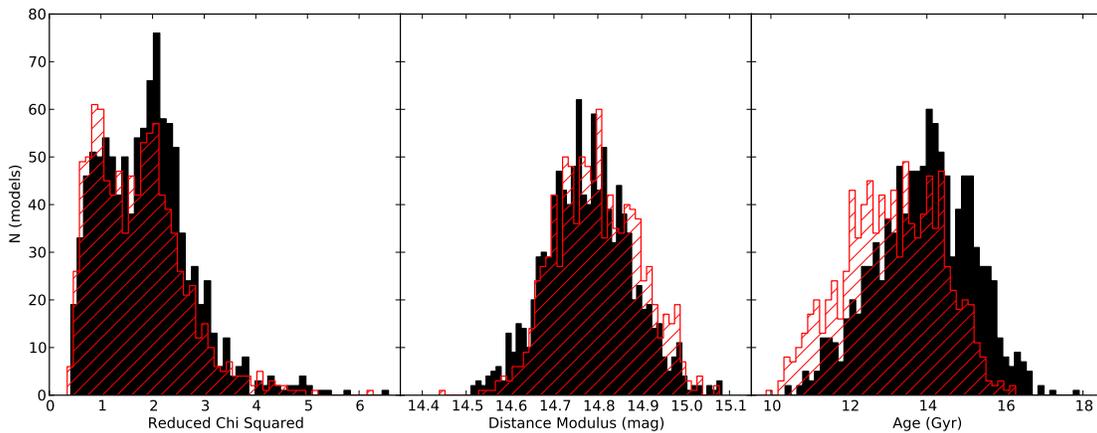


FIG. 3.— Distributions of χ_{Red}^2 , M92 cluster distance and age for $[\alpha/\text{Fe}] = +0.2$ dex (black) and $+0.4$ dex (red) ISO-VC models. *Left* - Distribution of χ_{Red}^2 shows the overall goodness of fit to the calibration subdwarfs is relatively unaffected by $[\alpha/\text{Fe}]$. *Center* - Distribution of M92 distances also shows that mean distance is unaffected by $[\alpha/\text{Fe}]$. *Right* - However, the α -Fe ratio has a significant impact on the cluster age, shifting the mean cluster age by 0.8 Gyr in the case of M92.

that the CNO-cycle dominates H-burning only from the turn-off onwards, we do not see a significant effect on the MS-fitting distances of the GCs between the $[\alpha/\text{Fe}] = +0.2$ and $+0.4$ dex models.

By initially assuming that it is equally likely to find $[\alpha/\text{Fe}] = +0.2$ or $+0.4$ dex in globular clusters, the quoted uncertainties in our derived ages take into account a systematic uncertainty of 0.2 dex in $[\alpha/\text{Fe}]$. If one would like to extend the range of $[\alpha/\text{Fe}]$ to include models of $[\alpha/\text{Fe}] = +0.6$ dex, then the mean age for the most metal-poor clusters would decrease by 0.4 Gyr, while the estimated uncertainty would increase by 0.2 Gyr.

Absolute proper motions are available for 17 GCs in our sample from the compilations and measurements of D. Casetti (2013)¹. We supplemented this collection of proper motions with measurements for NGC 5053, NGC 6101, NGC 6541 and NGC 6652 from Kharchenko et al. (2013) and measurements of NGC 6681 from Masari et al. (2013) thus allowing for the calculation of full space velocities and Galactic orbits of the 22 clusters in our sample. The cluster proper motions and radial velocities are provided in Table 7 along with the cluster distance as found in this study. These data will be combined with the cluster RA and DEC (Table 4) to be used as initial conditions in the orbit integration.

We model the Galactic orbits of our sample GCs using the *galpy: A Python Library for Galactic Dynamics* (Bovy 2015), specifically the orbit calculator with a Milky Way (MW)-like potential. We use the built-in `MWPotential2014` for this analysis which is fit to dynamical data of the MW as described in Bovy (2015) in order to provide a realistic model on both small and large scales. Briefly, this model is based on that of Bovy & Rix (2013) and combines an exponentially cutoff power-law density bulge (power-law exponent of -1.8 and cutoff radius of 1.9 kpc), a Miyamoto-Nagai disk, and a Navarro-Frenk-White dark matter halo. We choose to use an updated solar distance from the Galactic center of $R_0 = 8.20 \pm 0.09$ kpc and circular velocity of $v_0 = 232.0 \pm 3.0$ km s⁻¹ from McMillan (2016) and a total back integration time of 13 Gyr.

We provide the resulting orbital parameters in columns 7 – 11 of Table 7 with successive columns listing the perigalactic and apogalactic distance, the maximum distance from the Galactic plane, the orbital eccentricity and orbital energy. In order to determine the uncertainty associated with each orbital parameter, we perform a systematic error analysis in which we vary each of the input variables ($\mu_\alpha \cos \delta$, μ_δ , v_r , v_0 , and R_0) independently and calculate a new orbit for each cluster. The uncertainties in the proper motions contribute the most to the uncertainties in the orbital parameters, typically changing the apogalactic and perigalactic distances by a few percent to upwards of a factor of two. On the other hand, the uncertainties associated with v_0 and R_0 contribute less than 5% in most cases and, because the cluster radial velocities are known with such precision, they contribute less than 1% to the uncertainty in r_{apo} and r_{peri} for the majority of the clusters. The final uncertainty listed in Table 7 is the combination of the individual effects added in quadrature.

We show the meridional orbits for the clusters in Figures 4 and 5 and find several different classes of GCs based on their orbital shapes. Although the majority of the clusters in this sample reach sufficiently far distances from both the Galactic center and the Galactic plane to be considered outer halo GCs, we do find some that are associated with the Galactic disk and bulge. The two clusters with the most confined orbits, NGC 6541 and NGC 6723, are characterized here as bulge GCs not extending farther than 4 kpc from the Galactic center either radially or vertically. We find four additional disk clusters (47 Tuc, NGC 6362, NGC 6681, NGC 6752 and NGC 6809) with slightly larger orbits, extending out to 8 kpc from the Galactic center radially, but confined to ~ 5 kpc from the plane.

Previous studies have performed orbit integration using GC proper motion data (Dinescu et al. 1999; Allen et al. 2006, 2008); however, due to the differences in data sets and Galactic potential models, along with our updated GC distances it would be impractical to perform a detailed quantitative comparison of our resulting orbital parameters. Nevertheless, we can still qualitatively consider how our results fit within the framework established by previous studies.

In the study by Allen et al. (2006), the authors perform a detailed orbital integration in both an axisymmetric MW-like potential and a non-axisymmetric barred potential. By comparing the orbits of their 48 GCs

in the axisymmetric potential versus those in the non-axisymmetric potential they found two groups of GCs, those whose pericenter distances are less than 4 kpc and whose orbits are affected by the presence of the bar (inner GCs) and those whose orbits are not (outer GCs) because they reside further than 4 kpc from the Galactic center. Qualitatively, our results are in agreement as Allen et al. (2006) find the three clusters (NGC 6362, NGC 6723 and NGC 6809) we identified as Disk/Bulge clusters to be inner GCs with orbits that are noticeably altered by the presence of the bar. The clusters we characterize as halo clusters in this study are found to be outer GCs in Allen et al. (2006).

Allen et al. (2006) also provide images of their meridional galactic orbits for 16 of their GCs, seven of which are in common with this study. We find that five of the seven in common have the same general characteristics, covering the same range both radially and vertically. The two discrepant GCs are NGC 6362 which has an asymmetric shape in Allen et al. (2006) and NGC 7078 which extends twice as far in z and almost four times as far in R in this study than in Allen et al. (2006). There are many factors that could contribute to these particular discrepancies including differences in the proper motions, positions, and distances to the clusters along with the choice of galactic potential. For both NGC 6362 and NGC 7078 we use the same proper motion data and Galactic coordinates as in Allen et al. (2006), therefore, we eliminate these as contributing factors. Allen et al. (2006) do use a slightly larger solar Galactocentric distance, $R_0 = 8.5$ kpc, and a slightly lower circular velocity, $v_0 = 220$ km s⁻¹; however, we find no noticeable difference in the orbits of NGC 6362 and NGC 7078 when adopting these values. In the case of NGC 6362, the distance derived in this study ($d = 8.10 \pm 0.88$ kpc) is consistent with that of Allen et al. (2006) who found a distance of 7.6 ± 0.8 kpc. On the other hand, we find a distance for NGC 7078 ($d = 11.74 \pm 0.76$ kpc) that is 30% larger than that of Allen et al. (2006). In adopting the Allen et al. (2006) distance to NGC 7078, we find the apogalactic distance to be significantly reduced, a mere 4 kpc larger than that of Allen et al. (2006) rather than four times the distance. As is the case with the other clusters in the sample, the remaining small differences can be attributed to differences in the Galactic potential.

Dinescu et al. (1999) perform a similar orbital analysis with 16 of the 22 clusters in our sample in common. Although the authors do not provide meridional orbits for comparison, we can examine their orbital parameters. If we want to characterize the location of each GC in its orbit we can look at how near the cluster's R_{GC} is to its R_{apo} . In this case we will define “near apogalactic” as $R_{GC}/R_{apo} \geq 0.70$, “near perigalactic” as $R_{GC}/R_{apo} \leq 0.30$ and “mid-orbit” as anything else. Of the 16 clusters in common with Dinescu et al. (1999) we find 8 to be near apogalactic, 3 near perigalactic, and 5 in mid-orbit. On the other hand, the results from Dinescu et al. (1999) suggest that 9 clusters are near apogalactic, 5 in mid-orbit and only 2 near perigalactic. The clusters that are in disagreement in this comparison are NGC 4590, NGC 6809 and NGC 7078. In the characterization of NGC 4590 using the results of Dinescu et al. (1999) the cluster was considered to be mid-orbit with $R_{GC}/R_{apo} = 0.38$ whereas we classified the cluster as

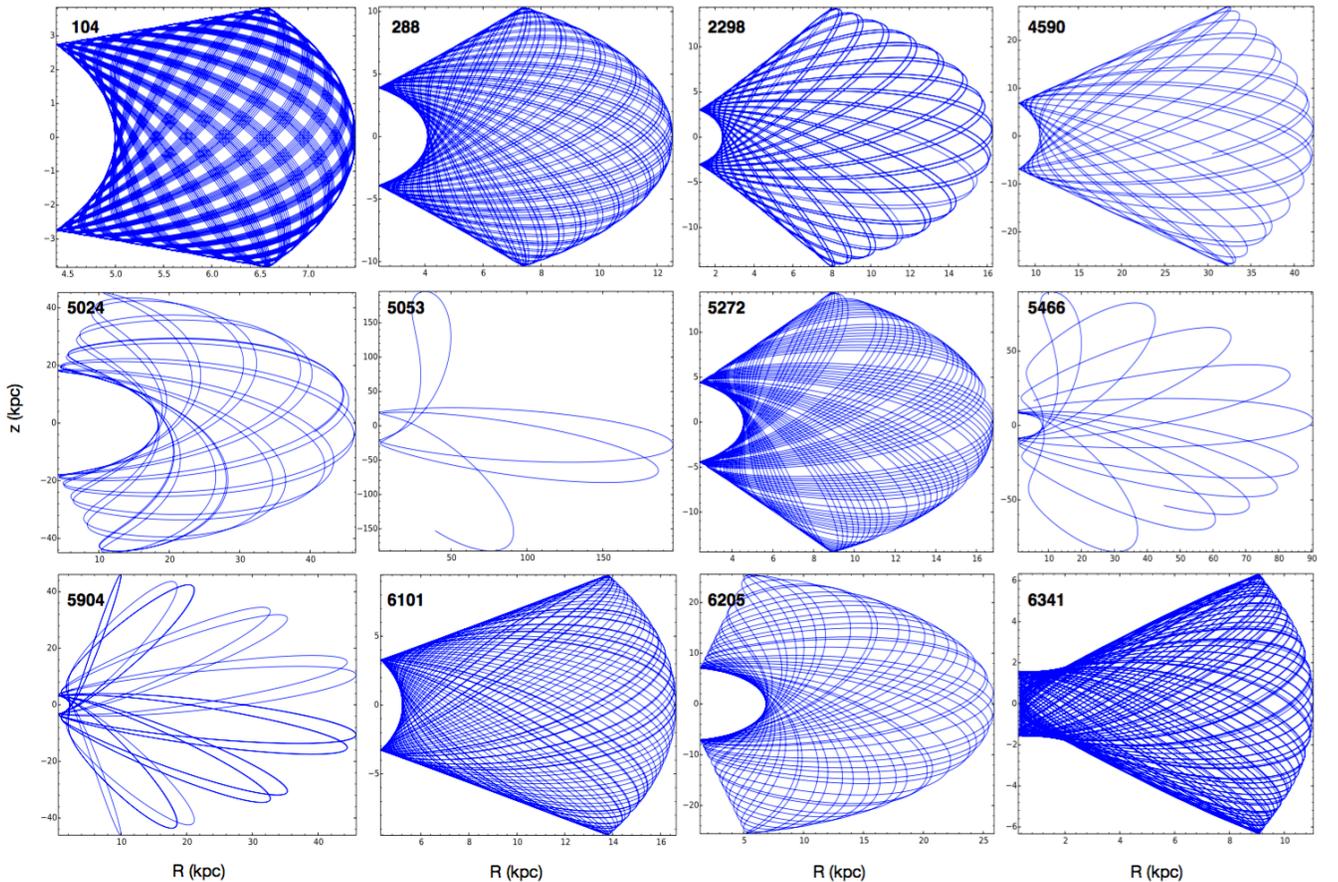


FIG. 4.— We show the meridional galactic orbits of twelve GCs integrated over the lifetime of the galaxy. NGC 104 (47 Tuc) is radially confined to within 8 kpc from the galactic center and vertically to only ± 4 kpc from the galactic plane and can therefore be considered an inner GC. The other GCs have much more extended orbits and are therefore outer GCs.

being near perigalactic with R_{GC}/R_{apo} at 0.20. This is similar to the difference seen for NGC 6809 in which Dinescu et al. (1999) finds $R_{GC}/R_{apo} = 0.68$ where we find $R_{GC}/R_{apo} = 0.78$. The discrepancies can be attributed to the difference in galactic potentials and underlying uncertainty in the galactic orbit parameters. The discrepancy was significantly larger for NGC 7078 where the results of Dinescu et al. (1999) suggest the cluster is near apogalactic with $R_{GC}/R_{apo} = 0.95$ and yet we find it to be near perigalactic with $R_{GC}/R_{apo} = 0.30$. This difference is largely due to the difference in apogalactic distance which, as we saw in our comparison to Allen et al. (2006), is a direct result of a larger derived distance and differences in the Galactic potential. However, it is important to note that given the large uncertainty associated with the apogalactic distance, it is hard to say with certainty exactly where this cluster is in its orbital trajectory.

5. AGE-[FE/H] AND AGE- R_{GC} RELATIONS

As the ages of GCs have long been used to glean information about the origins of our Galaxy it is important to consider how the ages we determined here correlate with other important properties and what that means

in terms of our theories of galactic formation and evolution. Calculating the orbits of these GCs allows us to study where they may have originated and better what their origin means for the formation and evolution of our Galaxy. Along with their origins, though, we are also interested to uncover any underlying relationship between the inherent characteristics of GCs such as their age, metallicity and current and/or past locations in the Galaxy.

In the left panel of Figure 6 we show GC age as a function of Galactocentric distance (R_{GC}). Although there is no clear dependence of age on R_{GC} we do find that the most metal-poor GCs ($[Fe/H] < -1.8$ dex) are located over the full range of R_{GC} , but the same cannot be said for the more metal-rich GCs which are clustered within $R_{GC} < 15$ kpc. This agrees with that of VandenBerg et al. (2013) who find no trend in age or age dispersions with R_{GC} for the set of 55 GCs in their study. Marin-Franch et al. (2009) perform a similar analysis using relative ages determined with two different sets of isochrones and in fact find a trend in age with galactocentric distance in both cases. This would seem to be at odds with our finding of no age- R_{GC} trend; however, it is important to note that Marin-Franch et al. (2009) con-

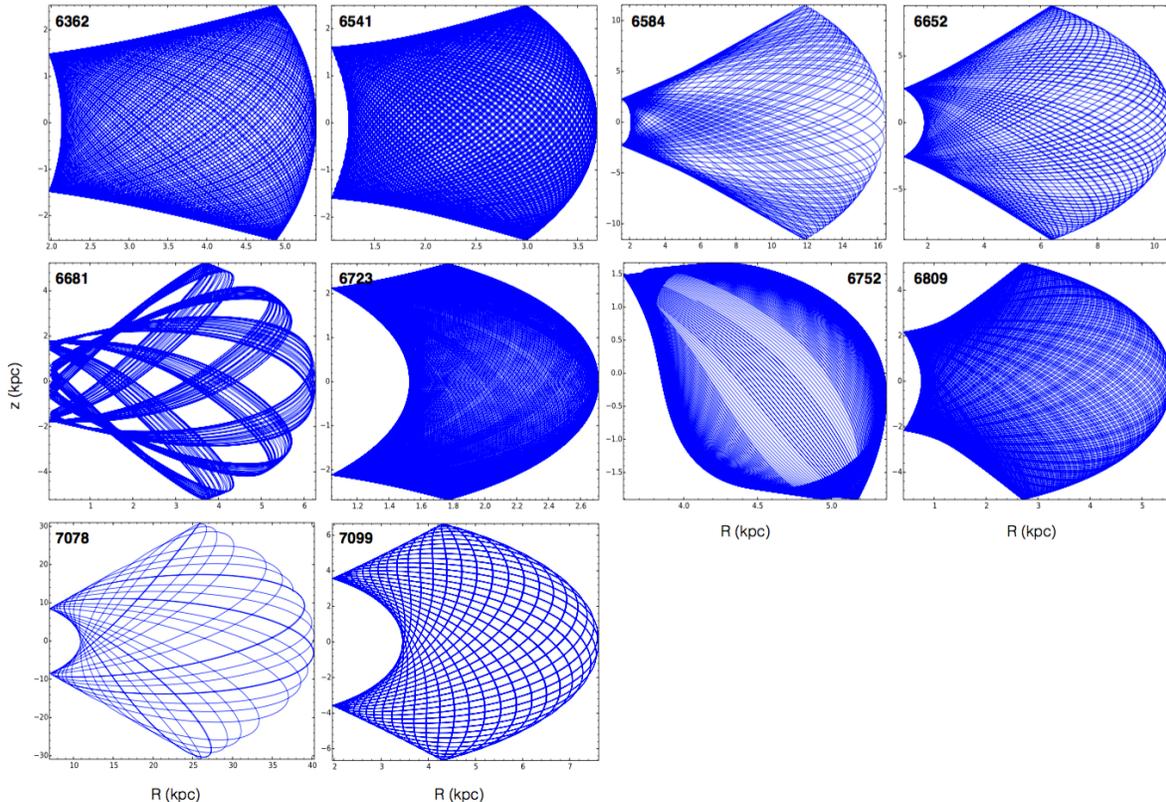


FIG. 5.— As in Figure 4 we show meridional galactic orbits for the remaining 10 GCs in our sample. Here NGC 6362, NGC 6541, NGC 6723, NGC 6752 and NGC 6809 are considered inner GCs being associated with either the Galactic disk or bulge while the rest are outer GCs residing in the halo of the Galaxy.

sider GCs with $[\text{Fe}/\text{H}]$ ranging from -2.02 dex to -0.18 dex, extending to much more metal-rich clusters than we consider here. If we only look at their clusters with $[\text{Fe}/\text{H}] < -0.8$ dex, then their trend becomes negligible and we find agreement in our results.

We also investigate the well-studied age-metallicity relation in the right panel of Figure 6. Previous studies found not only a simple trend of metal-poor GCs being among the oldest (Chaboyer et al. 1996c), but also a visible bifurcation in the age- $[\text{Fe}/\text{H}]$ relation (Marin-Franch et al. 2009; Dotter et al. 2010, 2011; Leaman et al. 2013; VandenBerg et al. 2013). Although we do not find the dramatic bifurcation of Leaman et al. (2013) and VandenBerg et al. (2013), we do find that our results are at least consistent. The results shown here are more compatible with those of Marin-Franch et al. (2009) who find an age- $[\text{Fe}/\text{H}]$ relation with two branches: one having a clear trend of decreasing age with increasing metallicity for the young clusters while the other is a mostly coeval old branch that has a larger dispersion. As the studies of Marin-Franch et al. (2009), Leaman et al. (2013) and VandenBerg et al. (2013) use more than twice as many clusters, they are able to identify the relationship between age and metallicity with much more confidence. Although our results are not in obvious disagreement

with the previous studies, we are limited in the amount of additional evidence our study can supply in support of a bifurcated age-metallicity relation due to the number of clusters studied and the larger scatter in our results.

A bifurcated age-metallicity relation would have strong implications for how we understand the formation of the Milky Way. Leaman et al. (2013) suggest that the offset seen between the two sequences offers evidence that the higher metallicity disk sequence clusters form in situ within in the disk and that these clusters are the most metal-rich clusters at any given age. They also note that the offset suggests that the lower metallicity clusters are remnants from hosts the size of the SMC, WLM, and maybe even larger satellite galaxies such as the LMC and Sagittarius.

6. SUMMARY

We use the methods of Chaboyer et al. (2017) to determine the MS-fitting distances and ages of 22 Milky Way GCs from the *HST* ACS GC Treasury Project (Sarajedini et al. 2007) spanning a range of metallicities, $-2.4 \leq [\text{Fe}/\text{H}] \leq -0.7$ dex, and reddening, $E(B - V) \leq 0.22$ mag. Both TGAS and *HST* parallaxes of subdwarfs with metallicities between -2.7 dex and -0.6 dex are used to calibrate the stellar evolution models used in the MS-fitting. As in Chaboyer et al. (2017), a MC method is

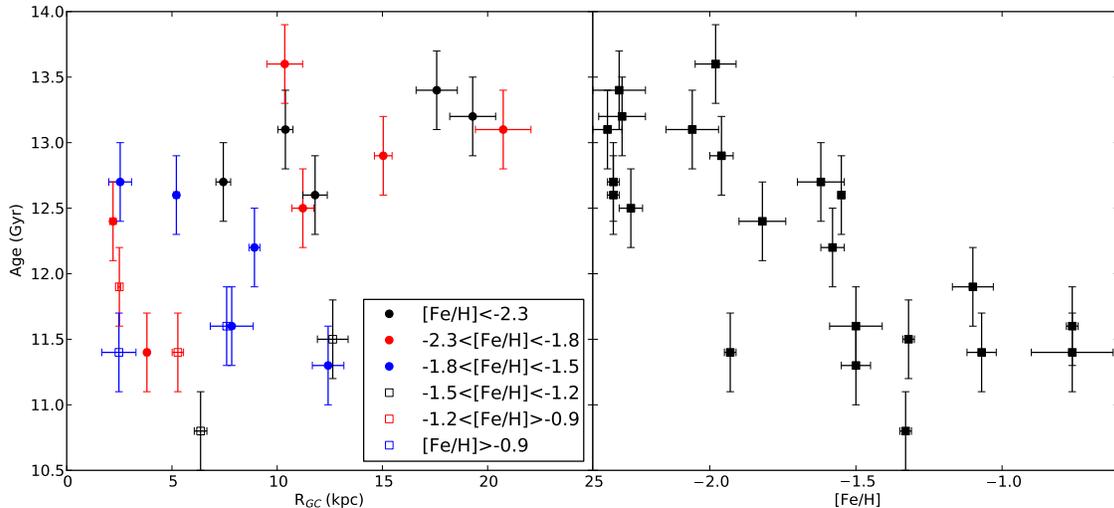


FIG. 6.— *Left* Age is shown as a function of Galactocentric distance with different metallicity GCs marked as shown. We do not find strong evidence for any relation between age and R_{GC} . *Right* – The age-metallicity relation is shown for the 22 Milky Way GCs in this sample. The point markers show the age and metallicity of the individual GCs while the horizontal and vertical errorbars represent the $[Fe/H]$ uncertainty as determined by Carretta et al. (2009) and the 0.3 Gyr random error in the age estimates.

used to construct isochrones with input parameters that cover the parameter space. In doing so, it is possible to not only determine the MS-fitting distances and ages of the GCs, but also the combined observational and theoretical uncertainty.

The distance modulus and corresponding uncertainty for each of the clusters is an average of the distance found using each of the isochrones weighted by the isochrone goodness of fit to the calibration subdwarfs. We find an average distance modulus uncertainty of 0.13 mag along with an offset between the ISO-P and ISO-VC distances that increased with decreasing metallicity and has a direct impact on the ages, resulting in an average offset of ± 0.2 Gyr between ISO-P and ISO-VC ages for a given cluster with ISO-P ages being larger for GC with $[Fe/H] < -1.5$ dex and visa versa with $[Fe/H] > -1.5$ dex.

The uncertainty in our distance and age estimates for these GCs incorporates the observational uncertainty associated with the photometry and reddening estimates while also taking into account the theoretical uncertainty associated with the stellar evolution models. As was shown in Chaboyer et al. (2017), this theoretical uncertainty is most strongly attributed to the choice of mixing length parameter, diffusion coefficients, atmospheric $T(\tau)$ relation, pp-chain reaction rate and high temperature opacity.

With updated distances and ages for these clusters, along with proper motion measurements from previous studies, we are able to calculate orbits for these clusters and infer information about their origins and the formation of our Galaxy. We find the majority of the clusters to be outer halo clusters, with a handful being confined to the Galactic bulge or disk. We do not find any correlation between the GC Galactocentric distance and the cluster’s age; however, we do find an age-metallicity relation that is roughly compatible with the bifurcated relations of Marin-Franch et al. (2009), Leaman et al. (2013) and VandenBerg et al. (2013). As some of the clusters are

very loosely bound with orbits extending to distances at which interactions with nearby dwarf galaxies could occur, it would be interesting to study the GC populations of dwarf galaxies such as the Large and Small Magellanic Clouds to determine if similar patterns exist in those environments, thereby providing further evidence for the origins of MW GCs.

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TABLE 7
CLUSTER ORBITAL PARAMETERS

Cluster	$\mu_{\alpha} \cos \delta$ (mas yr ⁻¹)	μ_{δ} (mas yr ⁻¹)	v_r (km s ⁻¹)	Distance (kpc)	R_{GC} (kpc)	r_{peri} (kpc)	r_{apo} (kpc)	z_{max} (kpc)	e	E (*10 ⁴ km ² /s ²)
NGC 104	5.63 ± 0.21	-2.73 ± 0.29	-18.00 ± 0.10	4.94 ± 0.55	7.14 ± 0.01	4.99 ^{+0.19} _{-0.16}	7.64 ^{+0.07} _{-0.08}	3.91 ^{+0.09} _{-0.08}	0.21 ^{+0.02} _{-0.02}	-5.80 ^{+0.16} _{-0.16}
NGC 288	4.68 ± 0.22	-5.60 ± 0.35	-45.40 ± 0.20	9.46 ± 0.86	12.52 ± 0.73	3.92 ^{+0.72} _{-3.94}	12.67 ^{+0.09} _{-0.08}	10.31 ^{+0.32} _{-0.36}	0.53 ^{+0.06} _{-0.06}	-4.29 ^{+0.21} _{-0.21}
NGC 2298	4.05 ± 1.00	-1.72 ± 0.98	148.90 ± 1.20	9.67 ± 0.66	15.04 ± 0.42	2.25 ^{+3.94} _{-2.06}	16.34 ^{+0.72} _{-0.72}	14.29 ^{+7.69} _{-9.41}	0.76 ^{+0.37} _{-0.37}	-3.56 ^{+0.63} _{-0.63}
NGC 4590	-3.76 ± 0.66	1.79 ± 0.62	-94.70 ± 0.20	10.93 ± 0.71	8.34 ± 0.52	10.05 ^{+6.38} _{-0.51}	42.30 ^{+13.56} _{-8.66}	24.45 ^{+9.41} _{-7.79}	-0.58 ^{+0.08} _{-0.12}	0.06 ^{+0.86} _{-0.86}
NGC 5024	0.50 ± 1.00	-0.10 ± 1.00	-62.90 ± 0.30	19.83 ± 1.40	21.22 ± 1.32	17.87 ^{+1.78} _{-4.63}	44.78 ^{+96.50} _{-24.21}	43.41 ^{+90.86} _{-30.49}	0.43 ^{+0.38} _{-0.23}	0.86 ^{+3.19} _{-1.87}
NGC 5053	-5.81 ± 0.53	-2.76 ± 0.53	44.0 ± 0.4	17.77 ± 1.17	19.29 ± 1.09	15.86 ^{+2.02} _{-0.50}	99.66 ^{+713.35} _{-111.56}	98.13 ^{+125.51} _{-110.23}	0.73 ^{+0.13} _{-0.15}	2.85 ^{+1.96} _{-1.70}
NGC 5272	-0.12 ± 0.61	-2.67 ± 0.40	-147.60 ± 0.20	10.71 ± 0.91	13.24 ± 0.75	4.54 ^{+2.64} _{-2.04}	17.11 ^{+2.09} _{-1.21}	14.60 ^{+1.35} _{-0.96}	0.58 ^{+0.06} _{-0.06}	-3.11 ^{+0.51} _{-0.51}
NGC 5466	-3.90 ± 1.00	1.00 ± 1.00	110.70 ± 0.20	16.58 ± 1.08	17.90 ± 0.98	7.28 ^{+2.63} _{-2.53}	98.79 ^{+211.21} _{-48.05}	85.32 ^{+156.78} _{-47.87}	0.83 ^{+0.07} _{-0.07}	2.01 ^{+2.02} _{-1.61}
NGC 5904	4.27 ± 0.60	-11.30 ± 1.46	53.20 ± 0.40	7.90 ± 0.68	8.23 ± 0.30	2.21 ^{+6.30} _{-0.13}	47.13 ^{+29.06} _{-15.70}	46.07 ^{+15.91} _{-15.80}	0.91 ^{+0.04} _{-0.05}	0.40 ^{+1.34} _{-1.34}
NGC 6101	-2.98 ± 1.62	-0.46 ± 1.62	361.4 ± 1.7	14.12 ± 0.97	7.23 ± 0.85	5.24 ^{+4.19} _{-3.46}	16.06 ^{+9.06} _{-1.10}	8.72 ^{+11.80} _{-10.20}	0.52 ^{+0.31} _{-0.14}	-3.25 ^{+0.35} _{-0.35}
NGC 6205	-0.10 ± 0.80	4.69 ± 0.81	-244.20 ± 0.20	7.90 ± 0.62	7.65 ± 0.26	6.81 ^{+0.40} _{-0.43}	25.86 ^{+8.85} _{-6.20}	25.36 ^{+8.67} _{-6.21}	0.51 ^{+0.10} _{-0.09}	-1.44 ^{+1.01} _{-1.01}
NGC 6341	-3.58 ± 0.89	-0.60 ± 0.60	-120.00 ± 0.10	9.03 ± 0.67	10.39 ± 0.36	0.40 ^{+0.60} _{-0.31}	10.85 ^{+0.48} _{-0.41}	6.04 ^{+3.77} _{-0.13}	0.93 ^{+0.06} _{-0.06}	-5.58 ^{+0.27} _{-0.27}
NGC 6362	-3.09 ± 0.46	-3.83 ± 0.46	-13.10 ± 0.60	8.10 ± 0.88	3.48 ± 0.27	2.14 ^{+0.17} _{-0.17}	4.91 ^{+0.14} _{-0.13}	2.60 ^{+0.06} _{-0.06}	0.45 ^{+0.03} _{-0.03}	-8.16 ^{+0.21} _{-0.21}
NGC 6541	-3.24 ± 1.04	-1.74 ± 1.04	-158.7 ± 2.3	7.96 ± 0.59	2.23 ± 0.15	1.38 ^{+0.17} _{-0.20}	3.81 ^{+0.72} _{-0.53}	2.41 ^{+0.30} _{-0.46}	0.47 ^{+0.09} _{-0.12}	-9.93 ^{+0.73} _{-0.73}
NGC 6584	-0.22 ± 0.62	-5.79 ± 0.67	222.90 ± 15.00	14.58 ± 1.14	7.73 ± 1.02	2.09 ^{+1.00} _{-0.91}	16.62 ^{+5.05} _{-3.03}	11.56 ^{+4.60} _{-3.04}	0.78 ^{+0.07} _{-0.06}	-3.55 ^{+1.17} _{-0.91}
NGC 6652	5.25 ± 0.90	-2.11 ± 0.90	-111.7 ± 5.8	9.68 ± 1.06	2.89 ± 0.81	1.66 ^{+0.10} _{-0.09}	12.95 ^{+5.24} _{-3.24}	8.97 ^{+4.30} _{-2.61}	0.71 ^{+0.09} _{-0.10}	-5.66 ^{+1.65} _{-1.42}
NGC 6681	1.58 ± 0.18	-4.57 ± 0.16	220.30 ± 0.20	9.68 ± 0.76	3.10 ± 0.54	0.05 ^{+0.12} _{-0.06}	6.39 ^{+0.23} _{-0.23}	5.19 ^{+0.18} _{-0.17}	0.98 ^{+0.02} _{-0.03}	-7.88 ^{+0.33} _{-0.30}
NGC 6723	-0.17 ± 0.45	-2.16 ± 0.50	-94.50 ± 3.60	7.46 ± 0.85	3.37 ± 0.07	1.46 ^{+0.11} _{-0.11}	2.64 ^{+0.22} _{-0.22}	2.37 ^{+0.12} _{-0.12}	0.37 ^{+0.05} _{-0.05}	-10.27 ^{+0.34} _{-0.34}
NGC 6752	-0.69 ± 0.42	-2.85 ± 0.45	-26.70 ± 0.20	4.33 ± 0.34	5.32 ± 0.13	3.91 ^{+0.30} _{-0.16}	5.50 ^{+0.63} _{-0.22}	1.91 ^{+0.11} _{-0.01}	0.17 ^{+0.01} _{-0.04}	-7.44 ^{+0.24} _{-0.24}
NGC 6809	-3.31 ± 0.95	-9.70 ± 0.55	174.70 ± 0.30	5.64 ± 0.39	4.58 ± 0.10	0.75 ^{+0.34} _{-0.28}	5.91 ^{+0.34} _{-0.22}	5.06 ^{+0.38} _{-0.32}	0.77 ^{+0.09} _{-0.10}	-8.02 ^{+0.32} _{-0.29}
NGC 7078	-1.23 ± 0.62	-7.57 ± 1.77	-107.00 ± 0.20	11.74 ± 0.76	11.80 ± 0.58	10.27 ^{+0.27} _{-0.58}	39.56 ^{+120.46} _{-23.98}	29.16 ^{+97.30} _{-22.20}	-0.55 ^{+0.29} _{-0.34}	0.23 ^{+3.69} _{-2.62}
NGC 7099	1.42 ± 0.69	-7.71 ± 0.65	-184.20 ± 0.20	8.20 ± 0.62	8.46 ± 0.35	3.16 ^{+0.89} _{-0.76}	7.60 ^{+0.44} _{-0.34}	6.26 ^{+0.39} _{-0.25}	0.41 ^{+0.11} _{-0.10}	-6.31 ^{+0.37} _{-0.31}

APPENDIX

LUTZ-KELKER CORRECTION

One bias that may be present in our sample of calibration subdwarfs is the classical Lutz-Kelker (Lutz & Kelker 1973) correction. When a star has a measured parallax value higher than its true value it is more likely to be included in a sample than if the parallax measured was lower than its true value. Additionally, since we require that $\sigma_\pi/\pi \leq 0.1$, stars with a higher measured parallax will therefore be weighted more strongly.

To test for this bias in our sample we created a Monte Carlo simulation following that of Chaboyer et al. (1998). Each source of possible uncertainty either due to instrumental effects, selection effects, or number statistics were modeled and included in the simulation. Below we provide the steps used to generate synthetic stars that closely resemble our actual data.

1. A non-normalized probability function of $[\text{Fe}/\text{H}]$ was derived for all the stars that were in the Tycho-2 and Stromgren catalogs using only references more recent than the year 2000.

$$P([\text{Fe}/\text{H}]) = \exp(5.19227 + 1.26595[\text{Fe}/\text{H}])$$

2. The $[\text{Fe}/\text{H}]$ probability function is normalized and used to compute a random list of true $[\text{Fe}/\text{H}]$ values, $[\text{Fe}/\text{H}]_t$, for the stars.
3. Gaussian uncertainties in $[\text{Fe}/\text{H}]$ are randomly selected with $\sigma_{[\text{Fe}/\text{H}]}=0.10$ and 0.15 . This yields an observed $[\text{Fe}/\text{H}]$, $[\text{Fe}/\text{H}]_o$.
4. Random distances are chosen assuming a sphere of constant stellar density out to 400 pc in steps of 10 pc. Using the computed distances, an actual parallax, π_t , is created for each star.
5. Masses are included using a Salpeter initial mass function from $0.4 - 0.9 M_\odot$.
6. A mass-luminosity function for MS stars was found using DSEP isochrones. The following fits were used, interpolating between functions based on $[\text{Fe}/\text{H}]$ values.

$$M_V = 15.011 - 11.8925 m \quad ([\text{Fe}/\text{H}] > -0.5)$$

$$M_V = 14.6153 - 12.5644 m \quad (-0.5 > [\text{Fe}/\text{H}] > -1.0)$$

$$M_V = 14.2013 - 12.6459 m \quad (-1.0 > [\text{Fe}/\text{H}] > -1.5)$$

$$M_V = 14.0212 - 12.7825 m \quad (-1.5 > [\text{Fe}/\text{H}] > -2.0)$$

$$M_V = 13.5781 - 12.1817 m \quad (-2.0 > [\text{Fe}/\text{H}] > -2.5)$$

$$M_V = 16.6547 - 16.5498 m \quad (-2.5 > [\text{Fe}/\text{H}] > -3.0)$$

$$M_V = 16.7498 - 16.6888 m \quad (-3.0 > [\text{Fe}/\text{H}] > -3.5)$$

$$M_V = 16.0395 - 15.6256 m \quad (-3.5 > [\text{Fe}/\text{H}])$$

7. Using M_V and π_t , a true apparent magnitude, V_t , was found.
8. A random Gaussian uncertainty in absolute magnitude, $\sigma_{m_V} = 0.02$, was added to the actual magnitude to create an observed apparent magnitude V_o .
9. *Gaia* completeness – the TGAS stars were compared to the stars included in *Gaia* Data Release 1 as a function of magnitude, giving a probability of inclusion as a function of magnitude.
10. Uncertainties for the TGAS parallax were handled two different ways. Both methods produced the same Lutz-Kelker corrections.
 - Gaussian uncertainties for parallax are computed using a fit of parallax uncertainties versus magnitude from Michalik, Lindgren & Hobbs (2015) and used to find the observed parallax, π_o .

$$\sigma_\pi = 7.56106 - 2.3672V_{to} + 0.248459V_{to}^2 - 0.00822623V_{to}^3 \text{ (mas)}$$
 - Gaussian uncertainties for parallax are computed using Table 1 from Michalik, Lindgren & Hobbs (2015). and used to find the observed parallax, π_o .
11. The observed absolute magnitude, M_{V_o} , is calculated as follows: $M_{V_o} = V_o + 5.0 \log(\pi_o) + 5.0$
12. Synthetic stars are accepted into the sample if the following are true: $M_{V_o} > 5.5$, $\sigma_\pi/\pi_o < 0.10$ and $[\text{Fe}/\text{H}]_o < -0.60$.

TABLE 8
METALLICITY BINNED LUTZ-KELKER BIAS

[Fe/H] bin	$\sigma_{[Fe/H]} = 0.10$		$\sigma_{[Fe/H]} = 0.15$	
	M_V bias (mag)	[Fe/H] bias (dex)	M_V bias (mag)	[Fe/H] bias (dex)
-0.77	0.006	0.001	0.009	0.002
-1.17	-0.003	-0.004	0.008	0.002
-1.33	0.009	-0.001	-0.004	-0.002
-1.81	0.005	-0.003	-0.006	-0.001
-2.00	-0.004	-0.001	-0.008	-0.001
-2.43	0.009	-0.002	0.005	-0.002

A sample of 10^7 synthetic stars was created by the simulation with varying amounts of uncertainty included. A synthetic star is accepted if the simulation finds that could be observed; in this case, 10^4 synthetic stars are accepted. Based on these 10^4 stars, we find no appreciable Lutz-Kelker correction.

The bias we find in absolute magnitude is at most $M_V = +0.009$ mag and in metallicity of at most $[Fe/H] = -0.171$ dex. The $[Fe/H]$ bias may seem quite large at first glance, but it is important to examine the bias in each individual metallicity bin. Table 8 gives the bias in absolute magnitude and $[Fe/H]$ in each metallicity bin. The largest $[Fe/H]$ bias in an individual bin was $[Fe/H] = -0.004$ dex with most bins below $[Fe/H] = -0.001$ dex.

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