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# Results from electrostatic calibrations for measuring the Casimir force in the cylinder-plane geometry

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We report on measurements performed on an apparatus aimed to study the Casimir force in the cylinder-plane configuration. The electrostatic calibrations evidence anomalous behaviors in the dependence of the electrostatic force and the minimizing potential upon distance. We discuss analogies and differences of these anomalies with respect to those already observed in the sphere-plane configuration. At the smallest explored distances we observe frequency shifts of non-Coulombian nature preventing the measurement of the Casimir force in the same range. We also report on measurements performed in the parallel plane configuration, showing that the dependence on distance of the minimizing potential, if present at all, is milder than in the sphere-plane or cylinder-plane geometries. General considerations on the interplay between the distance-dependent minimizing potential and the precision of Casimir force measurements in the range relevant to detect the thermal corrections for all geometries are finally reported.

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## I. INTRODUCTION

Casimir forces [1] have been investigated since their inception as a macroscopic test of the irreducible fluctuations associated to quantum fields. They are geometrical in character, as they originate from the possibility to confine and shape the energy density of quantum fluctuations using proper boundary conditions. So far, the experimental attention has been mainly focused on the original parallel plate configuration [2, 3], and the sphere-plane geometry [4–9], apart from the only experiment performed in a crossed-cylinder configuration [10]. Further motivations to pursue precision studies of the Casimir forces are related to the possibility to discover new forces of strength similar or larger than the Newtonian gravitational coupling but with short-range or different distance scaling and expected to act, according to various models, in the submillimeter range [11]. This can be considered part of a broader program aimed at testing deviations from Newtonian gravitation in the nonrelativistic limit [12, 13]. In the micrometer range the dominant source of background to non-Newtonian gravitational forces is provided by Casimir forces [14, 15] (see also [16, 17] for reviews) and to discover such forces, or at least to provide reliable limits on their existence, one must control at the highest level of accuracy all systematic sources of deviation from the idealized case analyzed by Casimir in his original paper. Major sources of systematic errors that are still considered under partial control are the use of the proximity force approximation (PFA) [18, 19] for curved surfaces, the presence of electric forces not reflected in the purely Coulombian contribution, such as patch effects [20–24], and the combined effect of finite conductivity and finite temperature [25–27].

The use of the PFA has been discussed at length in the literature, with several alternative methods developed to overcome its limitations, and exact solutions have been found in particular curved geometries. It is generally assumed that the PFA for the Casimir force differs from the exact result by an amount smaller than 0.1 %, an assumption compatible with the results of a dedicated experiment [28]. The control on the PFA used to assess limits to Yukawian non-Newtonian gravity has not been addressed until very recently, although it has been used for many years [9, 29–34]. It has been argued that the usual form of PFA cannot be extended unambiguously to volumetric forces [35], and thereafter an alternative form of the PFA has been discussed [36], which, however, has been shown in [37] to coincide with the exact formula for geometries in which one of the two bodies has translational invariance.

The presence of electric forces not incorporated in the Coulombian contribution has been discussed extensively in the literature, in particular in [20]. Anomalies in the electrostatic calibrations of the sphere-plane configuration have been evidenced for large radii of curvature of the sphere and small distances from the planar surface [38, 39]. This has triggered discussions about the nature and the universality of the observed anomalies [35, 40], and the situation is still far from being clarified. The anomalous exponent optimizing the fit of the electrostatic calibrations in [38, 39] has *not* been found in another experiment using spheres of much smaller diameter located at similar distances from the planar surface [41], while the dependence of the minimizing potential on the sphere-plane separation reported in [38] has been confirmed in [41], and for crystalline Ge in [42] (see also [39] for a discussion of some unpublished data

from former experiments). The spatial and temporal variabilities of the minimizing potential has been evidenced in a centimeter-size torsional balance [43], which confirms the necessity for a detailed knowledge of the surfaces and their preparation [44–47].

The finite temperature contribution added to the quantum fluctuations has originated a lengthy debate about the interplay of the thermal contribution with the finite conductivity properties of the surfaces (see for instance [48–59] for the initial steps of the debate). On the experimental side, attempts to evidence the thermal contribution discriminating various models have been reported for the sphere-plane geometry [9], while proposals using torsional balances in the parallel plane configuration [60, 61] are under development. A dedicated experiment in the parallel plane configuration using microresonators [62] and a low-frequency heterodyne technique [63] has been limited so far from patch charges [64].

Recently, the use of a configuration with intermediate features between the parallel plate and the sphere-plane ones, *i.e.* the cylinder-plane geometry, has been proposed, and its experimental feasibility was investigated at gaps of the order of  $20\ \mu\text{m}$ , limited by the roughness of the metallic surfaces [65–67]. This geometry is very relevant from the theoretical viewpoint, since an exact solution for the Casimir force has been found [68, 69], also providing another example of curved geometry in which PFA may be tested against numerical techniques [70, 71]. In this paper we report on the results of electrostatic calibrations for an apparatus using the cylinder-plane geometry in a range of distances of relevance for measuring the Casimir force. We observe a background originating frequency shifts of amplitude large enough to overwhelm the downshift expected from the Casimir force. We also discuss the distance dependence of the contact potential in both the cylinder-plane and parallel plates configurations. The minimizing potential shows no significant distance dependence in the parallel plates configuration with respect to corresponding cases of the cylinder-plane and sphere-plane configurations.

The paper is organized as follows: in Sec. II we briefly recall the Coulomb force in the cylinder-plane geometry, and report on the upgrades to the apparatus with respect to the one described in [66], its overall sensitivity performance, its geometrical characterization, and the parallelization technique. In Sec. III we describe results from electrostatic calibrations, showing that in analogy to the sphere-plane case we observe that (a) the optimal exponent for fitting the dependence of the Coulomb coupling on distance is not the one expected from the idealized situation, at least at the smallest explored distances between the cylinder and the plane and (b) that the minimizing potential depends on distance. We then describe the data analysis leading to force residuals after subtraction of the Coulombian contribution. At the smallest explored gaps we observe residual frequency shifts of amplitude large enough to prevent the measurement of the Casimir force. The presence of shifts neither of Coulomb nor of Casimir origin has been confirmed by implementing a measurement strategy consisting in progressively approaching the two surfaces at constant bias voltage, and measuring the resonator frequency. In Sec. IV we discuss possible explanations for the unexpected

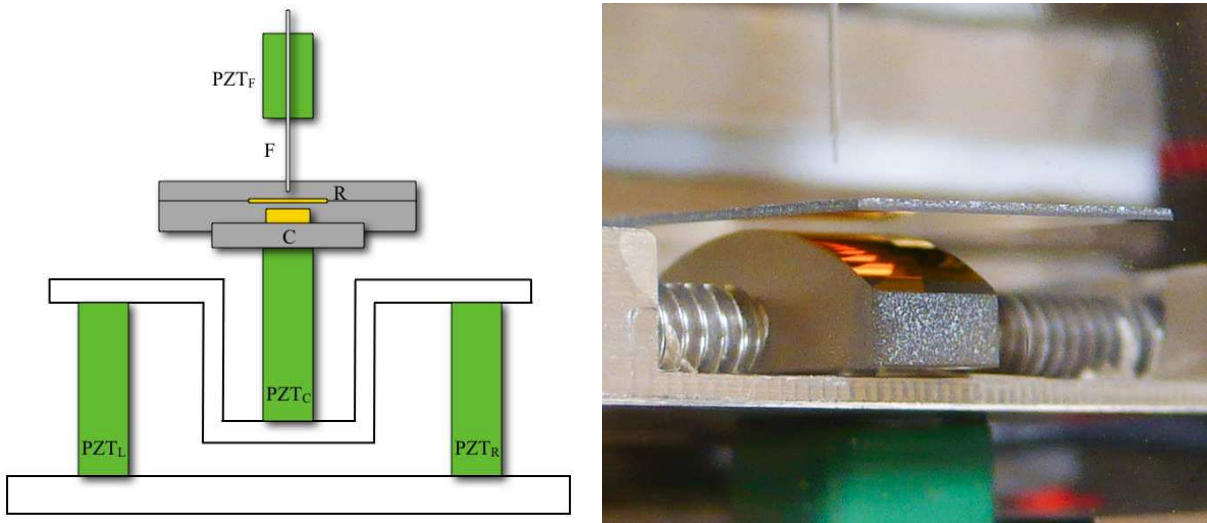


FIG. 1: (Color online) (Left) Schematic view of the experimental set-up with two lateral piezoelectric actuators ( $\text{PZT}_L$  and  $\text{PZT}_R$ ) for fine adjustment of parallelization, and a central piezoelectric actuator ( $\text{PZT}_C$ ) for controlling the cylinder-plane distance. On the top of the central actuator a platform is located with screws to adjust the horizontal position of the Au coated cylindrical lens (C). The resonator (R) is on the top of the cylindrical lens, and below the optical fiber (F) which is attached to a piezoelectric actuator ( $\text{PZT}_F$ ). (Right) Close-up image of the cylindrical lens and resonator region. The cylindrical lens has a radius of curvature of  $a = 12\ \text{mm}$  and a length  $L = 4\ \text{mm}$ , smaller than the width of the resonator of  $10\ \text{mm}$ .

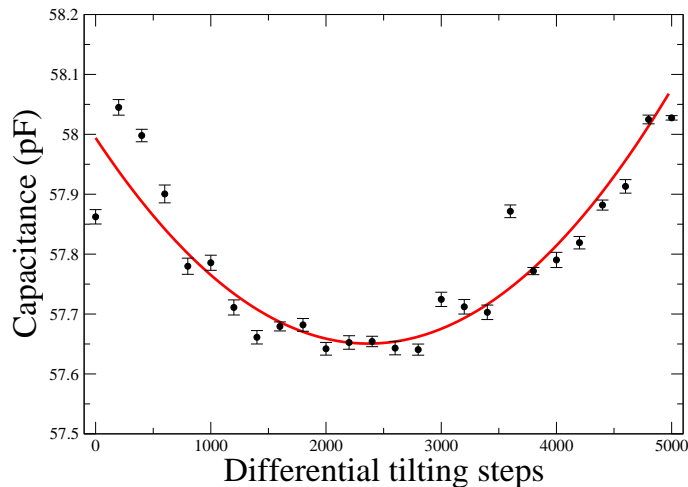


FIG. 2: (Color online) Assessment of the parallelization through capacitance measurements. The cylinder-plane capacitance is shown vs. the difference between the steps traveled by the left and right actuators in a differential mode. Each data point is the average of 20 measurements with an integration time of 1 s. The error bar represents the mean standard deviation on each data set.

scaling law of the Coulomb interaction with distance. We then describe in Sec. V electrostatic calibrations taken in a parallel plane geometry aimed at evidencing the dependence on distance of the minimizing potential also in this configuration, thereby completing the analysis for the three most common geometries of experimental interest. The relevance of measuring and modeling the dependence of the minimizing potential on distance in order to detect the thermal contribution to the Casimir force is discussed in Sec. VI. In the conclusion we put our findings in the more general framework of the recent observations of systematic effects highlighting possible future developments for the cylinder-plane geometry.

## II. CYLINDER-PLANE CONFIGURATION: GEOMETRICAL CONSIDERATIONS

The theory related to the cylinder-plane geometry and a description of the apparatus have been the subject of a former paper [66], while further details of the measurement technique have been reported in [38, 39, 67, 72]. The main difference from the former tests is due to the use of high-quality Au-coated cylindrical lenses, with planarity and roughness comparable to the ones of the resonator, therefore allowing us to reach submicrometer gaps.

The electrostatic force between a conducting cylinder (of length  $L$ , radius  $a$ , with  $L \gg a$ ) parallel to a conducting planar surface, separated by a gap  $d$  and kept at a fixed electrostatic potential difference  $V$  is [73]

$$F_{\text{El}}^{(0)} = \frac{4\pi\epsilon_0 LV^2}{\Delta \ln^2 \left( \frac{h-\Delta}{h+\Delta} \right)} \approx \frac{\pi\epsilon_0 \sqrt{a} LV^2}{2\sqrt{2}d^{3/2}}, \quad (1)$$

where  $\Delta = \sqrt{h^2 - a^2}$  and  $h = d + a$ , with the approximate expression in the right-hand side (r.h.s.) valid in the limit  $d \ll a$ , also coinciding with the result expected from the PFA for electrostatics. Since both the cantilever and the cylinder have finite size, the length  $L$  in the previous expressions should be replaced by an effective length  $L_{\text{eff}}$  that characterizes the relative exposure between the cantilever and the lens, (*i.e.* the minimum between the width of the cantilever and the length of the cylinder). In our initial experimental attempts this corresponded to the cantilever width. However, by visual and optical microscope analysis we noticed the presence of sharp irregularities at the border of the cantilever, most likely originated by the laser cutting process of the Si wafer. We have then chosen cylindrical lenses of length  $L = 4$  mm smaller than the resonator width of 10 mm (see Fig. 1), as the lenses seem to have more regular borders, as visible at the optical microscope. At this point we should note that the curvature of our cylindrical lens is  $a = 12$  mm, larger than its length  $L$ . This implies that border effects in the electrostatic interaction between the cylinder and the cantilever can be important, and the exact logarithmic expression of the electrostatic force in Eq.(1) should not hold in our configuration. However, as long as the conditions for the PFA hold ( $d \ll a$ ) the approximate expression for the force as given by the r.h.s. in Eq.(1) should apply, irrespective of the relative magnitude of  $L$  and  $a$ . Other important geometrical issues are the possible nonperfect parallelization between the

cylinder and the cantilever and the use of a cylindrical lens rather than a full cylinder. In [66], the correction to the PFA expression for the force [r.h.s. of Eq. (1)] due to nonparallelism was computed

$$F_{\text{El}}^{\text{np}} = F_{\text{El}}^{(0)} \frac{1}{\alpha} \left( \frac{1}{\sqrt{1-\alpha}} - \frac{1}{\sqrt{1+\alpha}} \right) \approx F_{\text{El}}^{(0)} \left[ 1 + \frac{5}{8}\alpha^2 + O(\alpha^4) \right], \quad (2)$$

where  $\alpha = L \sin \theta / 2d$ ,  $\theta$  is the deviation angle from ideal parallelism, and in this nonparallel case the distance  $d$  between the cantilever and the cylinder is measured from the midpoint along the axis of the cylinder. The use of a cylindrical lens rather than a full cylinder can be simply evaluated in PFA [72], resulting in a subleading PFA correction (of the order of  $5 \times 10^{-2}$  for a typical separation of  $d/a = 10^{-3}$ ), and will be then discarded in what follows.

To calibrate the apparatus, a controllable electrostatic force is generated by applying bias voltages between the cantilever and the cylindrical lens. At a given separation, the frequency of a resonant mode characterized by an effective mass  $m_{\text{eff}}$  is measured both with ( $\nu$ ) and without ( $\nu_0$ ) the presence of the voltage. This allows for the evaluation of the square frequency difference  $\Delta\nu^2 = \nu^2 - \nu_0^2$ , related to the voltage  $V$  in the PFA ( $d \ll a$ ) as

$$\Delta\nu_{\text{el}}^2 = -\frac{3\epsilon_0\sqrt{a}L_{\text{eff}}}{16\sqrt{2}\pi m_{\text{eff}}} \frac{(V - V_0)^2}{d^{5/2}} \left[ \frac{\alpha^{-1}}{3(1-\alpha)^{3/2}} - \frac{\alpha^{-1}}{3(1+\alpha)^{3/2}} \right], \quad (3)$$

where  $V_0$  is the minimizing potential. For small tilting angles  $\alpha \ll 1$  the correction to the frequency shift due to nonparallelism has a quadratic dependence on  $\alpha$ , given as  $1 + 35\alpha^2/24$ . The parallelization procedure is one-dimensional, thereby simpler than in the case of two flat surfaces. Rather than measuring the frequency shift induced by a constant bias voltage as discussed in [66], we have opted to monitor the cylinder-plane capacitance using a capacitive AC bridge [72]. Using the PFA, the capacitance between the cylinder and the plane is given by

$$C = \frac{2\pi\epsilon_0 L_{\text{eff}} \sqrt{a}}{\sqrt{2d}} \frac{(\sqrt{1+\alpha} - \sqrt{1-\alpha})}{\alpha}. \quad (4)$$

The tilting angle  $\theta$  (and therefore  $\alpha$  in turn) is controlled through two motorized actuators acting in differential mode, with the goal to keep the average distance  $d$  (the separation between the plate and the cylinder as measured from the midpoint of the cylinder) constant while changing the tilting angle (see Fig. 1). The precision of the achieved parallelism is then determined by the quality of the fitting of the capacitance versus differential steps number, propagated to determine the dispersion on the number of steps at minima compatible with the fitting error.

The precision can be improved by minimizing stray capacitance between various contacts since the precision of the capacitance meter is some percentage of the total capacitance in the system, usually 0.05%. An example of parallelization through capacitance measurements is shown in Fig. 2. The use of long-range actuators has been shown to be problematic due to large hysteresis, and therefore we have opted for a fine tuning with the use of two piezoelectric actuators as in the schematics shown in the left plot of Fig. 1. An optical microscope allows to monitor the quality and cleanness of the two surfaces and for a visual, qualitative assessment of the parallelization. An *a posteriori*, off-line check is obtained by fitting the electrostatic curves of the frequency shift using Eq. (3), which results in a value of  $\theta$  compatible with zero within a fitting error of  $\delta\theta = 10^{-3}$  radians.

### III. ELECTROSTATIC CALIBRATIONS AND RESIDUALS ANALYSIS IN THE CYLINDER-PLANE GEOMETRY

An important prerequisite to any Casimir force measurement is the execution of high-quality electrostatic calibrations. Forces are always indirectly measured via their functional relationship to more accessible observables, such as detection of deflection angles, voltages required to keep the apparatus at rest in closed loop schemes, or shifts of the frequency of a mode of a resonator as in our case. It is then crucial to convert such quantities more directly observed into the corresponding force signal by means of well-known and controllable physical signals, for instance by applying external bias voltages and comparing the measured forces with the Coulombian interactions between macroscopic conducting bodies.

The electrostatic calibration starts by finding the best parallelization condition at a given nominal cylinder-plane separation  $d$  using the capacitance technique described previously. Then the parallelization is further fine tuned by adjusting the two lateral piezoelectric actuators  $\text{PZT}_{\text{L,R}}$ . Once the optimal parallel condition is obtained,  $\text{PZT}_{\text{L,R}}$  are left untouched, and the separation  $d$  is changed via the central piezoelectric actuator  $\text{PZT}_{\text{C}}$ . This procedure keeps

the parallelization at the optimal value. As seen previously, in the perfect parallel situation the frequency shift of the cantilever due to pure electrostatics takes the form

$$\Delta\nu_{\text{el}}^2 = -\frac{3\epsilon_0\sqrt{a}L_{\text{eff}}}{16\sqrt{2}\pi m_{\text{eff}}}\frac{(V-V_0)^2}{d^{5/2}} = -K_{\text{el}}(V-V_0)^2. \quad (5)$$

The displacement of the central piezo PZT<sub>C</sub> depends linearly on the voltage applied  $V_{\text{PZT}}$ , and therefore the absolute gap is given by  $d = \beta(V_{\text{PZT}}^0 - V_{\text{PZT}})$ , where  $\beta = (91.9 \pm 0.9)$  nm/V is the actuation coefficient of the piezoelectric transducer, and  $V_{\text{PZT}}^0$  is the PZT voltage required to make contact between the two surfaces. At a given distance, the electrostatic calibration has been performed by measuring the frequency shift induced by a range of electric voltages  $V$  applied between the two surfaces. The curvature coefficient  $K_{\text{el}} = 3\epsilon_0\sqrt{a}L_{\text{eff}}/16\sqrt{2}\pi m_{\text{eff}}d^{5/2}$  and the minimizing potential  $V_0$  can then be obtained by fitting the data with

$$\nu_{\text{el}}^2 = \nu_{\text{el}}^2(V, V_{\text{PZT}}) = \nu_0^2 - K_{\text{el}}(V_{\text{PZT}}) \times (V - V_0)^2, \quad (6)$$

$$K_{\text{el}}(V_{\text{PZT}}) = \gamma(V_{\text{PZT}}^0 - V_{\text{PZT}})^{-5/2}, \quad (7)$$

where  $\gamma \equiv 3\epsilon_0\sqrt{a}L_{\text{eff}}/16\sqrt{2}\pi m_{\text{eff}}\beta^{5/2}$ . This fitting procedure allows the determination of the absolute distance  $d$  once the fitting parameter  $V_{\text{PZT}}^0$  is obtained, and the measurement of the contact potential  $V_0$  as a function of  $d$ . A typical data plot of  $K_{\text{el}}$  vs.  $V_{\text{PZT}}$  is shown in the left plot of Fig. 3. The blue curves are the best fits using Eq. (6), and they deviate significantly from the data points, both including or excluding a curvature offset  $K_{\text{el}}^{(0)}$  representing a hypothetical background electric field. Moreover, the effective mass calculated from the fitting parameter is 30-50 times larger than the physical mass, much larger than the expected value of the effective mass which should be comparable or smaller than the physical mass of the resonator. If instead the power exponent is left as a free parameter, rather than being fixed at 2.5, a new fitting curve with the exponent in the 0.9 to 1.3 range is obtained (the red curves in Fig. 3). This deviation from the expected exponent for the Coulombian force has been confirmed to exist in all our electrostatic calibrations data.

Table I shows the fitting parameters of the electrostatic calibrations for five runs both when the exponent is fixed and left as a free parameter. The effective mass for the latter case is not well defined because of the deviation of the

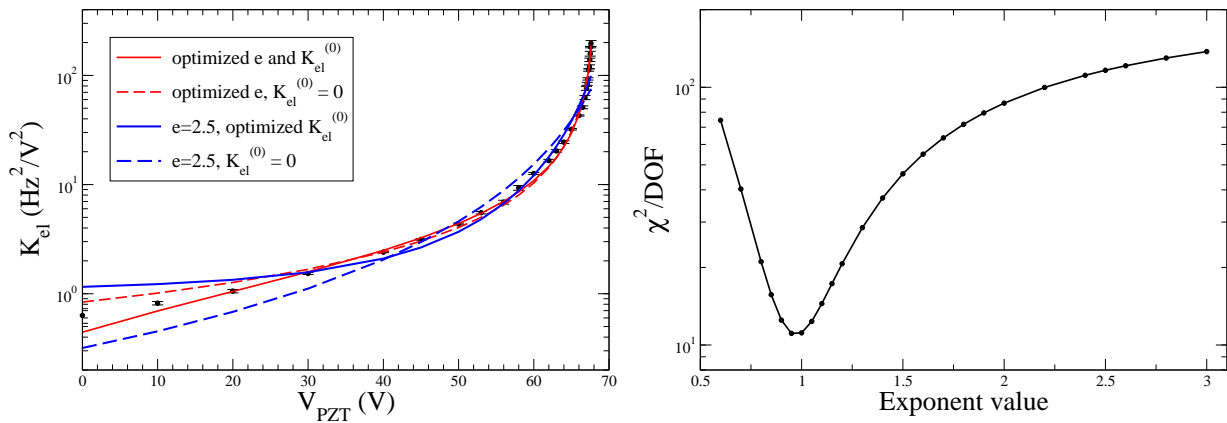


FIG. 3: (Color online) (Left) Plots of curvature coefficient  $K_{\text{el}}$  vs.  $V_{\text{PZT}}$  in cylindrical-plane electrostatic measurements obtained with the curvature technique, and best fits with the expected Coulombian interaction having a 2.5 exponent (blue/dark gray continuous curve), and with a power-law functional dependence in which the exponent is instead a free parameter (red/light gray continuous curve). The dashed curves are obtained by also constraining to zero an offset for  $K_{\text{el}}$ , representing a possible curvature present even at a large distance between the cylinder and the plane, for instance due to stray environmental electric fields. (Right) Plot of the reduced  $\chi^2$  obtained by dividing  $\chi^2$  by the degrees of freedom (DOF) vs. the value of the power exponent in the case of the offset  $K_{\text{el}}^{(0)} = 0$ . Data are taken as described in [38, 39], with a modification of the acquisition code for faster data acquisition and smaller uncertainty from drift effects. In the routine used for data acquisition in [38, 39], the bias voltage always returns to 0 V after each measurement. In the new routine instead, it changes from  $+V_b$  to  $-V_b$  in steps of  $\delta V$  where  $V_b$  and  $\delta V$  (*i.e.* the maximum bias voltage and its minimum step of variation during the calibration) are specified in advance in the code. This allows to double the acquisition speed for a targeted span of voltage values. Although this sacrifices some accuracies in the subsequent fitting of the data, a shorter data acquisition time is preferable considering the relatively large drift experienced during the entire duration of a typical run.

	Run 1	Run 2	Run 3	Run 4	Run 5
fixed exponent	2.5	2.5	2.5	2.5	2.5
$V_{\text{PZT}}^0$ (V)	$79.52 \pm 0.06$	$73.08 \pm 0.06$	$56.76 \pm 0.03$	$71.05 \pm 0.02$	$65.84 \pm 0.02$
$d_{\text{min}}$ (nm)	590	504	491	464	477
$m_{\text{eff}}$ (g)	6.63	9.25	8.87	9.46	9.01
$\chi^2/\text{DOF}$	60	116	283	518	876
free exponent	$1.30 \pm 0.03$	$0.97 \pm 0.01$	$0.93 \pm 0.01$	$1.00 \pm 0.01$	$0.94 \pm 0.01$
$V_{\text{PZT}}^0$ (V)	$74.37 \pm 0.04$	$68.07 \pm 0.02$	$51.78 \pm 0.01$	$66.45 \pm 0.01$	$61.03 \pm 0.01$
$d_{\text{min}}$ (nm)	116	43	33	42	34
$\chi^2/\text{DOF}$	5.3	11	19	42	31

TABLE I: Fitting parameters of the electrostatic calibrations using the curvature technique.

exponent from 2.5, thus it is not listed. As shown in Table I, the exponent, when left as a free parameter, is always smaller than the theoretical value of 2.5, with a reduced  $\chi^2$  smaller by about an order of magnitude with respect to the one expected from the Coulombian scaling. The relatively large value of  $\chi^2$  also indicates that the errors may be underestimated, although this does not affect our conclusions about the relative comparison between Coulombian and optimal exponents. While small deviations from 2.5 are expected considering all the less than ideal conditions such as imperfect parallelization and thermal and mechanical drifts, such a significant difference (an average value of 1.03 versus 2.5) cannot be explained as small deviations from ideality.

Electrostatic calibrations have also been performed with an alternative technique consisting in directly measuring the resonance frequency as the separation gap is decreased maintaining a constant bias voltage. This so-called *fast-approach* measurement technique has the advantage of a faster data acquisition, resulting in a mitigation of the long-term thermal or mechanical drifts, and provides an alternative to check the distance dependence of the electrostatic force. As shown in the left plot of Fig. 4, the optimal exponent obtained from the minimum of the reduced  $\chi^2$  curve is around 0.89. Although this technique seems to be slightly less sensitive to the exponent, as shown from the softer dependence of  $\chi^2$  (the value of  $\chi^2$  for the exponent of 2.5 being only three times larger than the minimum  $\chi^2$  obtained at the exponent of 0.89), the optimal value of the exponent is consistent with the results from the previous technique based upon electrostatic calibrations. The fact that a significant deviation of the exponent from 2.5 is still observed even in *fast-approach* measurements limits or rules out the possibility that systematic effects, such as artifacts from fitting the parabolic dependence in Eq. (5), or long-term drifts such as thermal expansions or relaxation of the PZT actuator, may be responsible for this anomalous behavior.

As can be seen in Table I, the smallest distance achieved is of the order of 500 nm based on the fitting with a

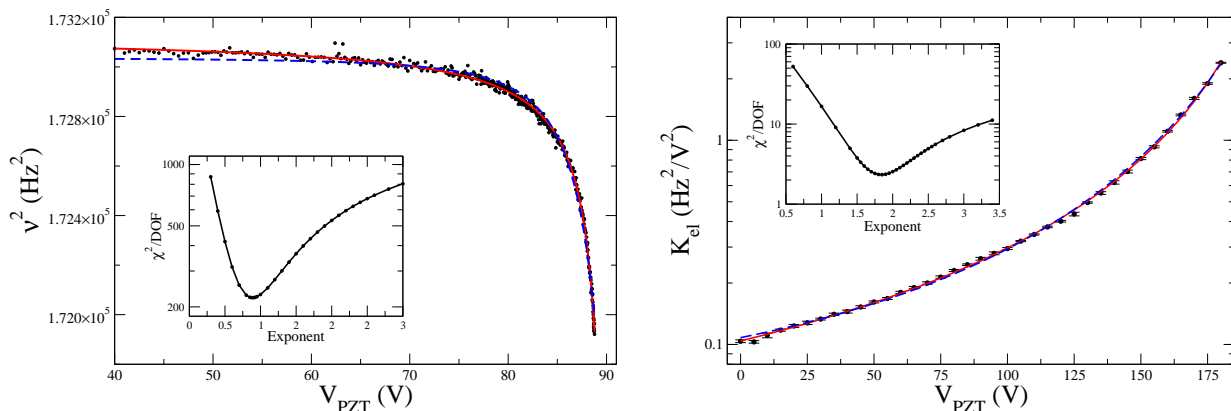


FIG. 4: (Color online) (Left) Squared resonance frequency versus  $V_{\text{PZT}}$  for the fast-approach calibration technique. The data were obtained using a measurement protocol in which a constant bias voltage equal to 4 V was applied across the two surfaces and the resonance frequency was progressively measured from the farthest to the closest distances. The dashed blue curve is the fitting with a 2.5 exponent while the continuous red curve is obtained leaving the exponent as a free parameter, whose optimal value turned out to be about 0.89. In the inset the reduced  $\chi^2$  is plotted versus the value of the free exponent. (Right) Test of the electrostatic scaling law for the cylinder-plane geometry at large distances with the curvature technique. Plot of curvature coefficient  $K_{\text{el}}$  versus  $V_{\text{PZT}}$ . The red curve is the fit with 2.5 exponent, the blue is the fit with the optimal exponent 1.84. In the inset the reduced  $\chi^2$  is plotted versus the value of the free exponent, with a minimum of  $\chi^2$  obtained for an exponent equal to 1.84.

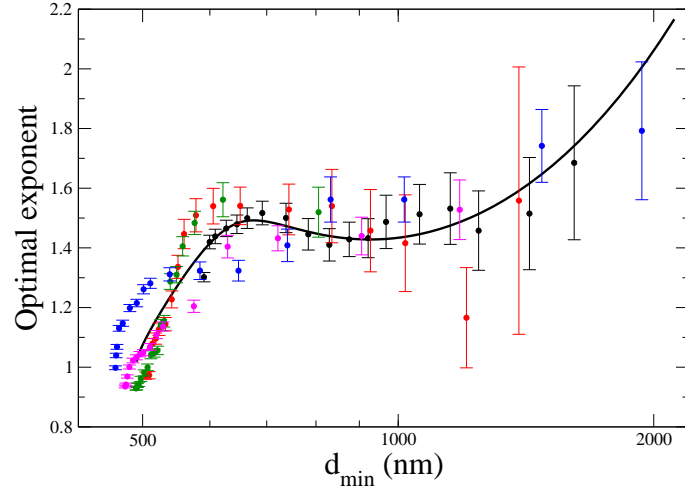


FIG. 5: (Color online) Optimal exponents of five electrostatic calibrations versus the distance of the closest data point used in the fitting, with the continuous line representing a spline curve obtained by considering all data points, weighted by their error bars.

fixed exponent of 2.5. In a previous measurement using wider cylindrical lenses with a larger radius of curvature the smallest distance reached was around  $1 \mu\text{m}$ , and no significant deviation of power exponent from 2.5 has been observed. This suggests that the exceptionally small exponent may be a result of the smaller gaps reachable with the cylindrical lenses of smaller width and radius of curvature. Although the absolute distances obtained from the fitting with exponent 2.5 cannot be fully trusted in light of the relatively inaccurate fitting, they can be still considered as reliable enough to estimate the gap separation.

Electrostatic calibrations with the same lens were performed at relatively large distances, as shown in the right plot of Fig. 4. In the data presented in Table I, the explored distances ranged from about  $500 \text{ nm}$  to  $7.3 \mu\text{m}$ . The new calibrations were instead performed in the range of about  $5.1$  to  $21.6 \mu\text{m}$ . This obviously results in much smaller frequency shifts and larger error bars in the values of  $K_{\text{el}}$ , yet it is evident from the fitting that an exponent of 2.5 is more satisfactory for these large-distance data. Furthermore, the fitting parameters with an exponent fixed at 2.5 gives an effective mass of  $1.2 \text{ g}$  which is much closer to the estimated physical mass of the resonator of  $0.2 \text{ g}$ . If the power exponent is instead left as a free parameter, a value of  $1.84 \pm 0.06$  is obtained. The large-distance data span smaller ranges of  $K_{\text{el}}$  and therefore are less sensitive to the power exponent and as shown in the inset on the left plot of Fig. 4, the difference between the fitting curve of 2.5 exponent and that of 1.84 exponent is not very significant (corresponding to a reduced  $\chi^2$  of 2.3 versus 4.9). Nevertheless, the fitting makes evident that the optimal exponent at large distance is almost two times larger than the one obtained from the small-distance data. We also investigated

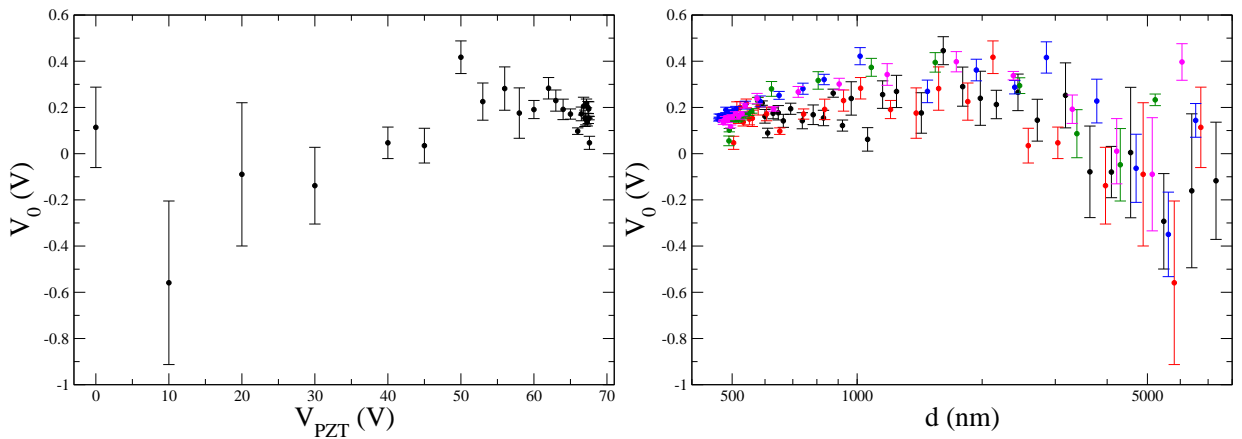


FIG. 6: (Color online) Dependence of the minimizing potential on distance in the cylinder-plane geometry. (Left) Minimizing potential  $V_0$  versus  $V_{\text{PZT}}$  from a typical electrostatic calibration measurement. (Right) Minimizing potential  $V_0$  versus distance  $d$  from various runs.



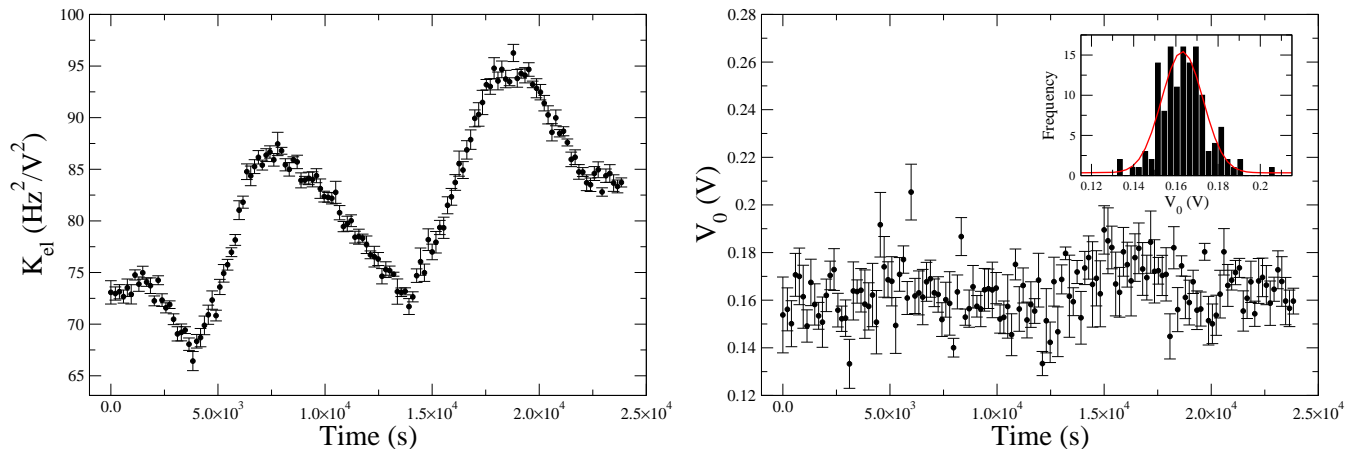


FIG. 7: Plots of the curvature coefficient  $K_{el}$  (left) and the minimizing potential  $V_0$  (right) versus time at constant  $V_{PZT}$ . The inset of the right plot is the histogram of  $V_0$  which shows that  $V_0$  roughly follows a Gaussian distribution centered around 0.163 V with a full width at half maximum of 0.023 V (standard deviation 0.01 V).

how the optimal exponent changes if a subset of data, rather than the entire set, is used in the fitting procedure. In particular, we have fitted subsets of data obtained by progressively removing points at the smallest distances. As shown in Fig. 5, all five runs share the same trend showing that the optimal exponent increases when the number of the removed point of closest distance used in the fitting increases. The absolute distances in the plot were obtained from the fitting with fixed 2.5 exponent. One feature which is noticeable in the figure is a relatively sharp increase in the value of the optimal exponent for a distance range of 500 to 600 nm. This could explain why such a large deviation of the optimal exponent from 2.5 has not been observed in our earlier measurements with a larger radius of curvature cylinders in which we managed to reach minimum distances of only about 1  $\mu\text{m}$ . We have also noticed a strong correlation between the value of the effective mass  $m_{\text{eff}}$  and the exponent. The effective mass obtained from the fitting increases when the distance between the two surfaces decreases. The dependence on distance of the optimal exponent is in contrast to the case of the sphere-plane measurements in which a relatively constant optimal exponent was observed uniformly over the entire range of explored distance [38, 39]. All fits are performed with a weight equal to  $1/\sigma_i^2$ , where  $\sigma_i$  is the standard deviation of  $K_{el}$  from the parabola fitting.

In our experiment we have also studied the possible distance variability of the minimizing potential  $V_0$  (*i.e.* the voltage difference between the cylindrical and the planar conducting surfaces which is minimizing the electrostatic force, as described for instance in [38, 39]). At larger gaps we have observed an approximate linear relationship between the residual potential  $V_0$  and  $V_{PZT}$  in the cylinder-plane configuration. With closer approaching (larger values of  $V_{PZT}$ ),  $V_0$  tends to have a nearly flat dependence on the distance, as is visible in Fig. 6, both on a single run (left plot) and on various runs obtained in different days (right plot).

To estimate the effect of long-term drifts we have studied the time dependence of the electrostatic curvature coefficient  $K_{el}$  and the minimizing potential  $V_0$ , without nominally changing the cylinder-plane gap distance, as shown in Fig. 7. The curvature coefficient shows temporal variations of order 50 %, while the minimizing potential  $V_0$  does not show any evident dependency, rather it fluctuates in a relatively small range of values.

From the electrostatic calibration measurements it is also possible to extract the electrostatic-free frequency  $\nu_0$  as can be seen from Eq. (6). In principle, when the distance between the two surfaces is small enough, one should expect a downward shift of this residual frequency due to attractive Casimir forces. However, as can be seen in Fig. 8, no significant downshift of  $\nu_0$  has been observed, rather a sharp upshift was instead observed in one case. This upshift indicates a very strong repulsive force between the two surfaces at small distances, which could be from either a

$d_{\text{min}} - d_{\text{touch}}$ (nm)	0	92	184	276	368
$V_{\text{bias}} = 3\text{V}$	$-0.91 \pm 0.02$	$-1.52 \pm 0.10$	$-1.56 \pm 0.17$	$-1.91 \pm 0.32$	$-2.05 \pm 0.51$
$V_{\text{bias}} = 4\text{V}$	$-0.89 \pm 0.03$	$-1.20 \pm 0.08$	$-1.73 \pm 0.19$	$-1.89 \pm 0.30$	$-1.83 \pm 0.36$
$V_{\text{bias}} = 5\text{V}$	$-0.98 \pm 0.02$	$-1.70 \pm 0.04$	$-1.60 \pm 0.06$	$-1.61 \pm 0.10$	$-1.62 \pm 0.16$

TABLE II: Optimal exponents for various values of minimum distance used in the fitting of the data from fast approach measurements.

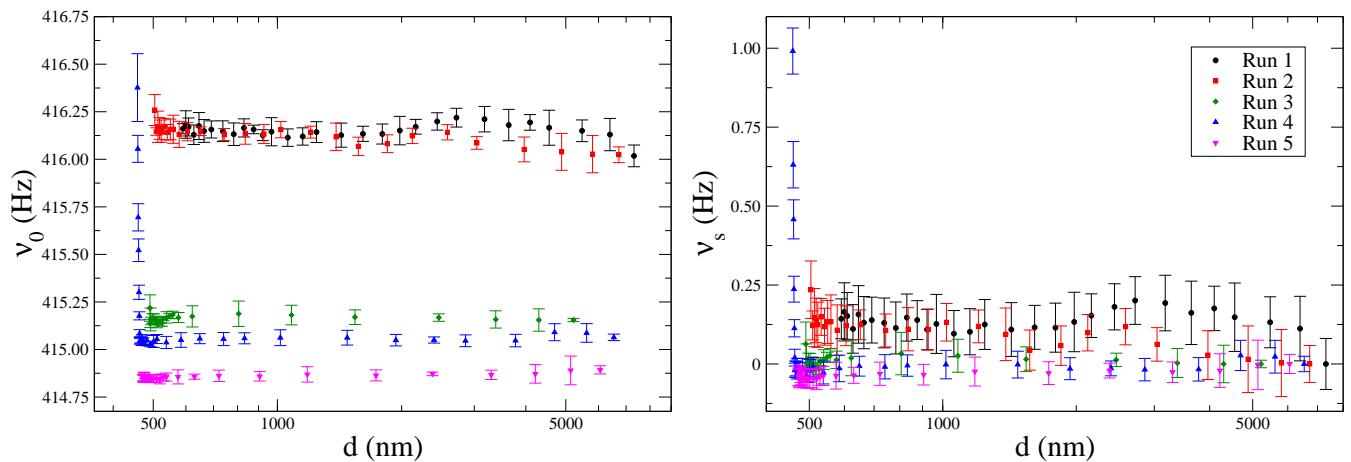


FIG. 8: (Color online) Residuals from electrostatic calibrations with parabola method. (Left) Plot of residual frequencies from various runs of electrostatic calibration measurements *vs.* distance. Because of the dependence of the resonant frequency on temperature, different runs have different frequencies even at the largest gaps at which no residual force is expected. (Right) Same plot but with a common baseline chosen in such a way that the data points at the farthest distance have a common central value, with  $\nu_s = \nu_0 - \nu_0(d_{\max})$ .

repulsive component of the Van der Waals forces or simply a soft contact of the surfaces. To better understand the short-distance behavior and to reduce the effect of thermal drifts, we have performed a series of measurements using the fast-approach technique mentioned previously.

As shown in Fig. 9, with a constant bias voltage  $V_{\text{bias}}=4$  V, the frequency of the resonator decreases as the cylindrical lens approaches the resonator, as qualitatively expected for the attractive Coulomb force, then followed by a sharp increase, most likely from contact of the surfaces. If the Coulomb force is the dominant force, then the downward part of the curve can be fitted using Eq. 5. As mentioned earlier, a fixed exponent of 2.5 produces a marginal fit while the optimal exponent is around 0.9. However, we should keep in mind that the exponent is not supposed to be 2.5 in the first place if at small distances some other forces (either the Casimir force, patch forces, or corrections to the standard Coulomb interaction mentioned in the previous section) become large enough to compete with the Coulomb force. Based on this remark, we further analyze the data by removing data points at the smallest distances, to test the stability of the fit with the Coulomb force. In these conditions the optimal exponent increases, as shown in Table II. In analogy to the previous analysis on electrostatic calibration data, although the optimal exponent is still smaller than 2.5, the value is approaching 2.5 within the relatively large error bars. This indicates once again that the sources of discrepancy between the data and the expected Coulomb force are localized at the smallest distances.

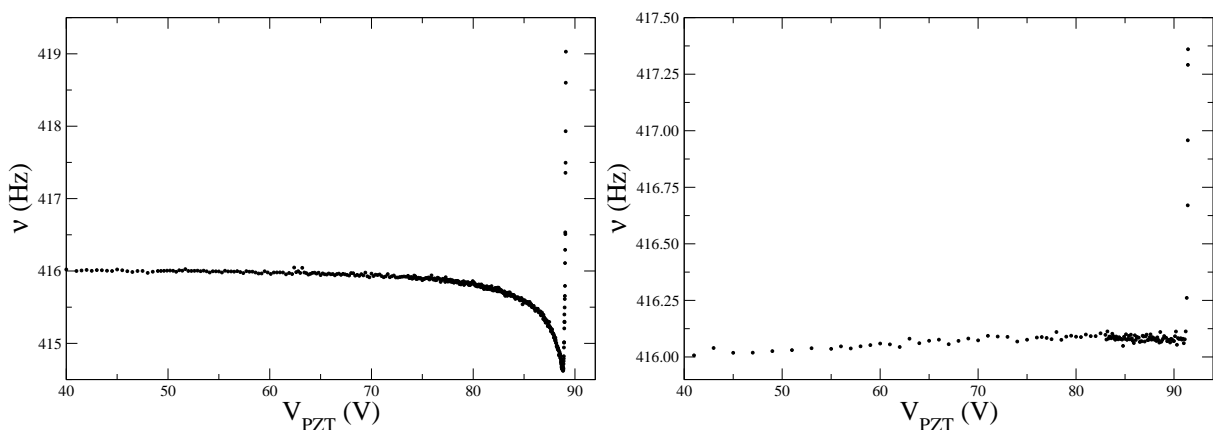


FIG. 9: (Left) Plot of frequency versus  $V_{\text{PZT}}$  with a constant bias voltage  $V_{\text{bias}} = 4$  V obtained using the fast-approach method. (Right) Plot of frequency versus  $V_{\text{PZT}}$  with a constant bias voltage  $V_{\text{bias}} = 0.15$  V approximately compensating the value of the minimizing potential expected at the smaller gaps. No evidence for downshifts attributable to charge-independent forces is visible until the frequency increases due to direct contact between the two surfaces.

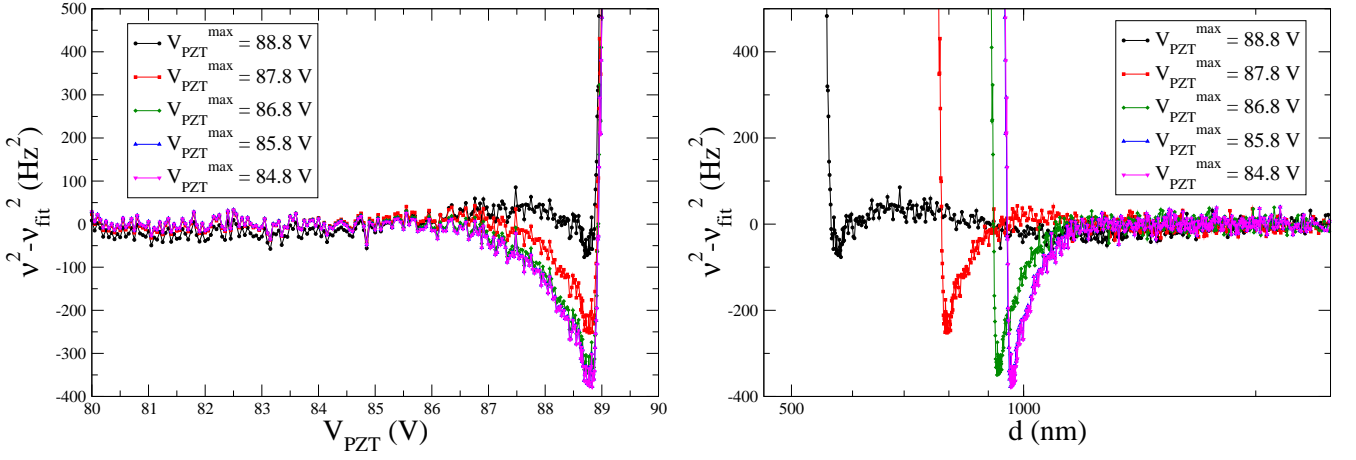


FIG. 10: (Color online) Residuals to the frequency shifts with the fast-approach technique. (Left) Plots of residual frequency square ( $\nu^2 - \nu_{\text{fit}}^2$ ) for a constant bias voltage  $V_{\text{bias}} = 4$  V versus  $V_{\text{PZT}}$  and different values of the maximum value of  $V_{\text{PZT}}$  used for the data analysis. (Right) Plot of the corresponding force versus distance. Both the absolute force and the absolute distance are inferred by the Coulomb fitting at larger distances.

We then can investigate another kind of residual of the data to analyze the effect of data taken at the smallest distances. Instead of relaxing the exponent finding its optimal value, the exponent is kept fixed at 2.5 when fitting the larger distance portion of the data, and the *residuals* at small distances are evaluated. This residual analysis is shown in Fig. 10 for one run, with the soft contact occurring around  $V_{\text{PZT}} = 88.8$  V. If the Coulomb force is the only dominant force for the whole downshifted part before the soft contact, then the fitting curve obtained when excluding a small region of data prior to contact, for instance all the data corresponding to a PZT voltage larger than  $V_{\text{PZT}} = 87.8$  V, should be able to predict the data obtained by excluding distances corresponding to a further Volt of  $V_{\text{PZT}}$  removed, with residuals centered around zero. However, by doing so there is clearly a nonzero downshift residual before the soft contact takes over. In principle this residual could come from fitting artifacts, in particular it could depend on the interval chosen for fitting the data with the Coulomb force. However, when more points were removed, this nonzero residual appears to be stabilized. Considering that the frequency shifts with this fast-approach technique may capture all possible forces acting on the resonator, this electrostatic residual analysis indicates that there are forces other than the expected contribution from the applied constant bias voltage that caused a further downshift in the frequency. To test whether the residual force is correlated to the external bias electric field, measurements were also performed with  $V_{\text{bias}} = 0.15$  V, corresponding to the average value of the residual potential  $V_0$  at small distances as shown in Fig. 6. No noticeable downshift was observed, as seen in the right plot of Fig. 9. Furthermore,

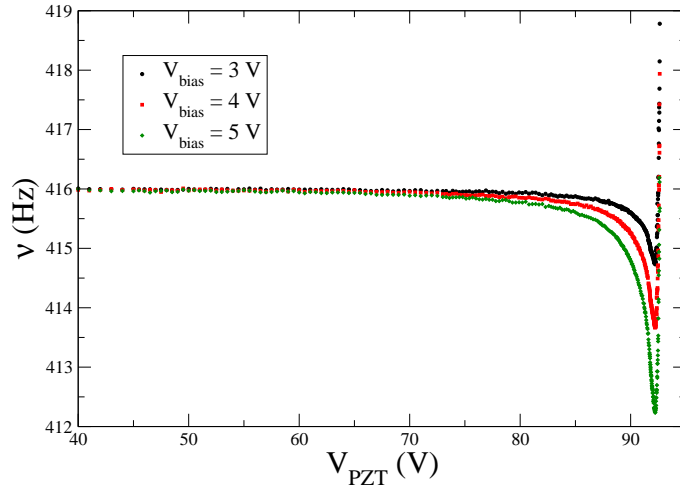


FIG. 11: (Color online) Fast-approach measurements with various values of the bias voltage. Plot of the resonator frequency versus distance with bias voltage  $V_{\text{bias}}$  of 3 V (black circle points), 4 V (red square points), and 5 V (green diamond points).

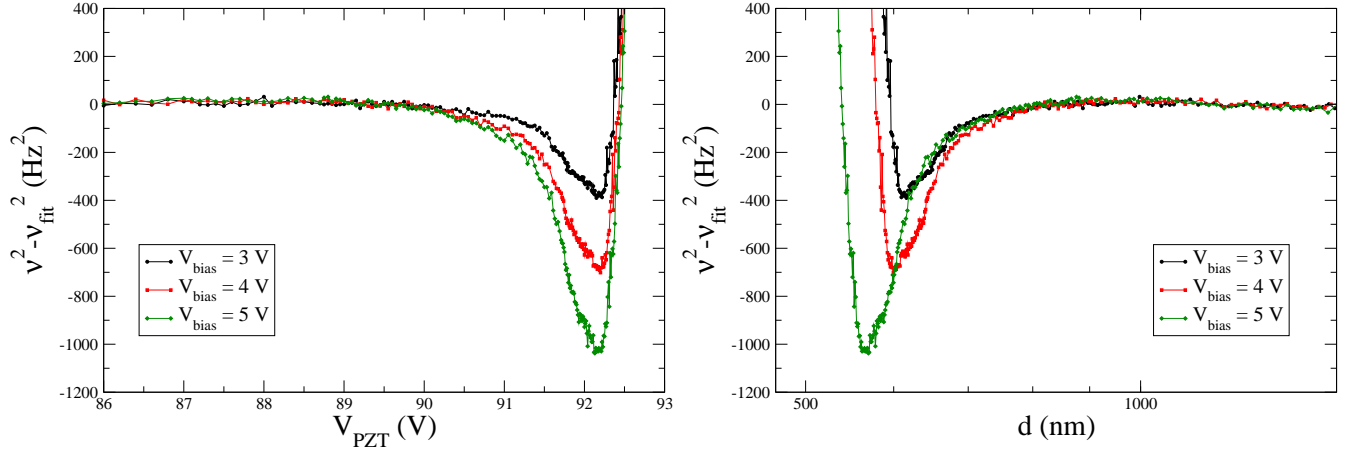


FIG. 12: (Color online) Plots of residual frequency square ( $\nu^2 - \nu_{\text{fit}}^2$ ) from the fast-approach measurement with  $V_{\text{bias}} = 3$  V (black circle points), 4 V (red square points) and 5 V (green diamond points) versus  $V_{\text{PZT}}$  (left) and distance (right). The fittings were done after removing the closest 184 nm data.

to rule out possible changes of configurations (such as parallelism or distance drifts) in runs taken in different days, fast-approach measurements were performed with different  $V_{\text{bias}}$  applied within the same run, (*i.e.*, at each position three values of  $V_{\text{bias}}$  were applied and the respective frequencies measured, as shown in Fig. 11 with bias voltages of 3, 4, and 5 V). This confirms that larger bias voltages result in larger frequency shifts at the same distance. The residual analyses were carried out and the results with the fitting after removing the closest 184 nm data are shown in Fig. 12. The extra downshifts were present in all three curves, with peak value approximately quadratic in the external bias voltage.

Although this residual force  $F_{\text{res}}$  is attractive, it cannot be identified with the sought Casimir force. Apart from the absence of a comparable signal in the residual frequency analysis with the electrostatic calibration technique (Fig. 8),  $F_{\text{res}}$  seems to be dependent on the applied bias voltage as shown in Fig. 12. Moreover, best fits of this residual force with power-law expressions indicate that it is required an exponent which is much larger than that of the Casimir force. The fact that this extra downshifts depends on the applied bias voltage indicates that it may also be present in electrostatic calibrations and could lead to an anomalous exponent.

It should be noticed that within our statistics of runs performed with the fast-approach technique (about 20) we have also observed a couple of runs in which residuals gave rise to short-distance upshifts. This could be explained by the presence of anomalous distance drifts due to external factors such as the environmental temperature. To assess

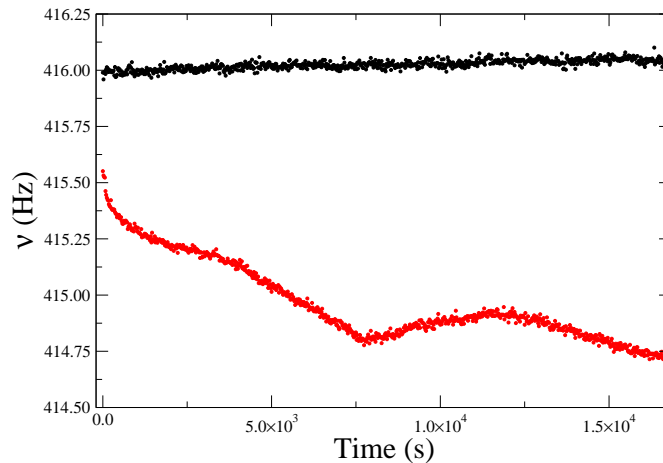


FIG. 13: (Color online) Plots of the resonator frequency versus time in the case of large separation (black upper points), and small separation (red lower points), the latter obtained by imposing an external bias voltage of 4 V at a nominally constant distance. While the first plot reflects the intrinsic resonator change, for instance due to temperature drifts or internal creeps, the second also includes the effect of drifts over time in the cylinder-plane separation distance.

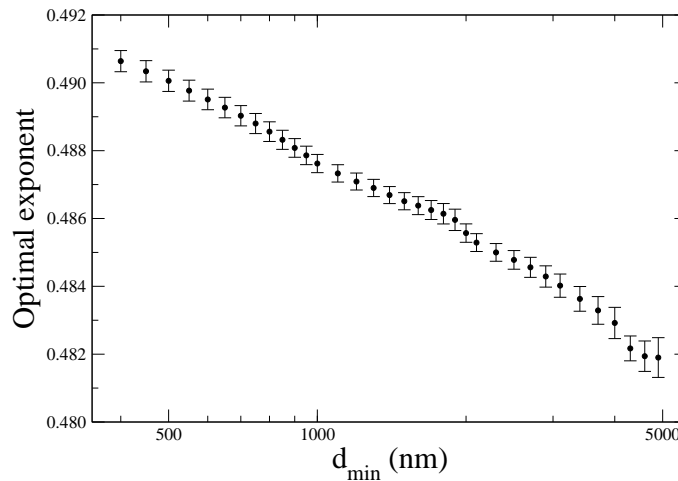


FIG. 14: Optimal exponent *vs.* the distance of the closest data point used in the fitting of the capacitance *vs.* distance curve for the numerical simulation of the finite size cylindrical lens in front of the finite size planar resonator, the geometry corresponding to the actual experimental setup as shown in Fig. 1.

this effect and how much it affects the amplitude of the observed frequency shifts in the residuals, we show in Fig. 13 the typical amplitude of the frequency fluctuations. This allows for disentangling the intrinsic drifts due to changes in the resonator frequency, obtained by monitoring the resonator with a distance from the cylinder large enough to make negligible its influence, and drifts due to changes in the cylinder-plane distance, with a bias voltage of 4 V intermediate between the two extreme values of voltages applied in the fast-approach measurements.

#### IV. POSSIBLE EXPLANATIONS FOR THE ANOMALOUS EXPONENT

In the following section we discuss possible causes of the anomalous exponent obtained in our electrostatic calibrations of the cylinder-plane geometry. In particular, we consider edge effects, local deformations from the idealized geometry, electric forces with steeper distance scaling than the Coulomb force, and electrostatic patch effects.

##### A. Edge effects

One possible reason for a deviation from the ideal Coulomb prediction of 2.5 for the exponent of the frequency shift versus distance is the fact that a finite size cylindrical lens was used in the experiment, instead of a long whole cylinder. In this situation edge effects may not be negligible, and Eq. (1) would not be a good approximation to the actual electrostatic frequency shift. While edge effects have been discussed for Casimir forces with the general world-line approach in [74], we have not found former discussions of this effect in the electrostatic calibrations in Casimir experiments. Using the COMSOL numerical package, we have conducted numerical simulations in which the precise geometry of the measurements was used, and the capacitance between the cylindrical lens and the resonator was evaluated at different distances [75]. By neglecting edge effects, the power exponent for the capacitance versus distance would be 0.5. We repeated the same analysis as shown in Fig. 5 with these data, and Fig. 14 shows how the optimal exponent changes if data points at the smallest distances are progressively removed from the fitting. The optimal exponent obtained for the capacitance from the COMSOL simulation for the finite-size geometry deviates more from the ideal value of 0.5 as the cylinder-plane separation is increased (*i.e.* the regime in which edge effects are more pronounced). In contrast, the optimal exponent obtained experimentally in our electrostatic calibrations deviates more strongly from the ideal value as the separation is decreased. Therefore, border effects cannot explain, either quantitatively or qualitatively, the anomalous exponent observed in our measurements.

##### B. Local geometrical deformations

Another possible reason for the deviation of the exponent with respect to the ideal value 2.5 is the presence of geometrical deformations in the shape of the cylinder. As the experiment is performed with a very large cylinder

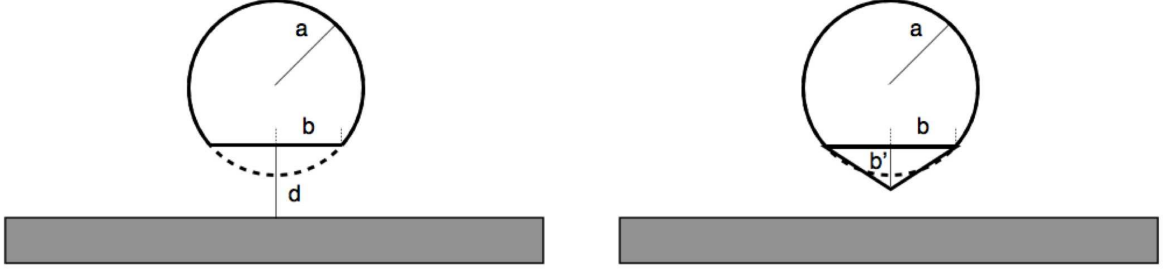


FIG. 15: Two examples of deformations of the cylindrical surfaces. On the left we show a cylindrical lens with a flat deformation, on the right, the cylinder has a tip (not to scale).

( $a=12\text{mm}$ ), the surface may present local deformations at the submillimeter scale, which could induce strong deviations in the exponent. A similar point has been raised for the case of a large sphere in front of a plane in [40], where it was remarked that departures from the ideal spherical surface may noticeably affect the exponent of the electrostatic calibration.

To illustrate this point, we have computed the electrostatic force between a deformed cylinder and a plane using the PFA. Let us first assume that, in the region of minimum distance between surfaces, the cylinder has a flat deformation of width  $2b$  (see left plot in Fig. 15). For simplicity we assume that the same deformation is all along the length  $L$  of the cylinder. In this case it is possible to show that

$$\Delta\nu_{\text{el}}^2 = -\frac{\epsilon_0 L_{\text{eff}}(V - V_0)^2}{4\pi^2 m_{\text{eff}}} \frac{\partial^2}{\partial d^2} \left[ f_{\text{inc}}\left(d - \frac{b^2}{2a}\right) + f_{\text{pp}}(d) \right], \quad (8)$$

where

$$f_{\text{inc}}(d) = \sqrt{\frac{2a}{d}} \arctan \sqrt{\frac{2ad}{b^2}} \quad (9)$$

is the contribution of the (incomplete) cylinder, and  $f_{\text{pp}}(d) = b/d$  is the contribution of the flat deformation. Let us assume that  $\Delta\nu_{\text{el}}^2 = -A/d^B$ . Depending on the relation between the size  $b$  of the deformation and the range of distances  $d$  of the calibration, we expect that the exponent  $B$  will interpolate between the ideal value 2.5 (cylinder-plane) at relatively large distances, and 3 (parallel plates) at short distances (although the interpolation is not necessarily a monotonic function). When  $b = 10^2 \mu\text{m}$ , and the fit is performed for  $d$  between 0.5 and 2  $\mu\text{m}$ , the exponent becomes  $B = 2.8$ , bigger than that of the ideal cylinder-plane geometry. Then this kind of deformation does not help to explain the observed anomalous exponent.

On the other hand, if the cylinder has a deformation with the form of a tip (see right plot in Fig. 15), the exponents are, in general, considerably smaller than the ideal one, and can explain at least part of the anomaly. Indeed, let us assume that the deformation consists of a triangular tip of width  $2b$  and height  $b'$ . In this case, PFA gives

$$\Delta\nu_{\text{el}}^2 = -\frac{\epsilon_0 L_{\text{eff}}(V - V_0)^2}{4\pi^2 m_{\text{eff}}} \frac{\partial^2}{\partial d^2} [f_{\text{inc}}(d + b') + f_{\text{tip}}(d)], \quad (10)$$

where

$$f_{\text{tip}} = \frac{b}{b'} \ln \left( 1 + \frac{b'}{d} \right). \quad (11)$$

If the height of the tip is much larger than  $d$ , the contribution of the incomplete cylinder is almost irrelevant, since the cylinder is shifted upward and its electrostatic energy becomes almost independent of  $d$  in this regime. The main contribution comes from the tip, and the mild logarithmic dependence of the energy with the distance implies an exponent of around  $B \approx 2$ . This can be easily confirmed by performing fits of Eq. (10). For example, for the same parameters as previously, with  $b' = b$ , we obtain an exponent  $B = 2.0$ . Sharper tips may produce even smaller exponents, although the PFA becomes unreliable for very thin tips.

The main conclusion of the PFA estimations is that, as expected, deformations of the cylindrical surface may change appreciably the exponent 2.5 of the electrostatic calibration. Although the previous examples do not explain the full discrepancy between the ideal prediction and the experimental data, this is certainly a crucial point to be taken into account in future experiments.

### C. Additional electric forces

In the residuals analysis of electrostatic calibrations described in the previous section, a residual force which seems to depend on the applied bias voltage was observed. The presence of such a force could lead to an anomalous exponent. Let us consider a hypothetical scenario in which the square of frequency shift  $\Delta\nu_{\text{hs}}^2$  for a cylinder-plane geometry has the following dependence on distance:

$$\Delta\nu_{\text{p}}^2 = - \left( \frac{\alpha_1}{d^{2.5}} + \frac{\alpha_2}{d^p} \right) (V - V_0)^2. \quad (12)$$

in which  $p > 2.5$ . This means that besides the expected electrostatic interaction between the two surfaces, there is another electric force which follows a higher power law upon the cylinder-plane separation. Let us generate a set of pseudo-data following Eq. (12), and let us try to fit the curvature coefficient  $K_{\text{p}} = \alpha_1 d^{-2.5} + \alpha_2 d^{-p}$  with:

$$K_{\text{p}} = \alpha(d - d_0)^{-q} \quad (13)$$

where  $\alpha$ ,  $d_0$  and  $q$  are fitting parameters. In Figs. 16 and 17 we show the results of the fitting using different values of  $\alpha_2/\alpha_1$  versus the distance of the closest point used in the fitting with  $p = 5$ . Not surprisingly, the optimal value of  $q$  from the fitting approaches  $p$  at small distances, and approaches 2.5 at large distances. However, it is not monotonically decreasing from  $p$  to 2.5 when the distance increases, but first goes below 2.5 and then slowly approaches 2.5. The  $\alpha_2/\alpha_1 = 5$  and 10 curves in Fig. 16 resemble the results from our electrostatic calibrations data as shown in Fig. 5. Therefore it is possible that an extra force with a power law dependence on distance steeper than the Coulombian one is responsible for the strong deviation of the optimal exponent from 2.5. The value of  $\alpha$  from the fitting also suffers from similar issues. In the electrostatic calibration measurement, the exponent is fixed at 2.5 and the calibration factor  $\alpha$  is used to calculate the effective mass  $m_{\text{eff}}$  of the resonator. However, when  $\alpha_2 \neq 0$  there is this extra force which is not included in the fitting formula, thus  $\alpha$  obtained from the fitting is not equal to  $\alpha_1$  as can be seen in the top plots of Fig. 17 ( $\alpha_1$  is chosen to be  $10^4$ ), and  $m_{\text{eff}}$  calculated from  $\alpha$  would be incorrect. For example, the  $\alpha_2/\alpha_1 = 5$  and 10 curves show that  $\alpha$  obtained from the fitting is smaller than  $\alpha_1 = 10^4$ . Since  $m_{\text{eff}}$  is inversely proportional to  $\alpha$ , a smaller  $\alpha$  would result in a larger  $m_{\text{eff}}$ . This is in agreement with our observation that the effective mass obtained from electrostatic calibration measurements is larger than expected. The inset of the top right plot in Fig. 17 shows the calibration factor  $\alpha$  versus the distance of the closest point used in the fitting with experimental data from our electrostatic calibration measurements, showing qualitative agreement with the model.

The previous analysis indicates that with a carefully chosen combination of  $p$  and  $\alpha_2/\alpha_1$ , the existence of an extra electric force could well explain the problems we experienced in our electrostatic calibration measurements. Another very important result from this analysis is that unreliable values of  $d_0$  could be obtained from the fitting if there exist forces other than the expected electrostatic force. As shown in the bottom plots of Fig. 17, positive values of  $d_0^{\text{fit}}$  were obtained, both when the exponent  $q$  is left as a free parameter or fixed at 2.5. Based on the way the data were constructed, the correct value for  $d_0$  is 0 nm, and the difference between  $d_0^{\text{fit}}$  and its corresponding null value

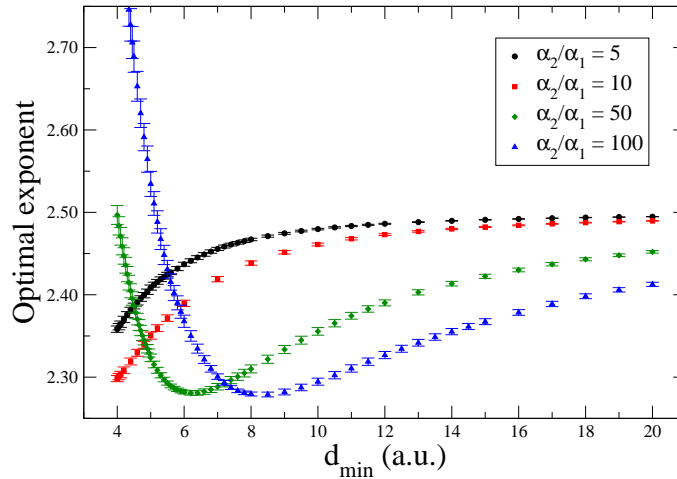


FIG. 16: (Color online) Optimal exponent of the hypothetical scenario data versus the distance of the closest data point used in the fitting. The data are constructed from Eq. (12) with  $p = 5$ ,  $\alpha_1 = 10^4$ , and  $\alpha_2/\alpha_1 = 5, 10, 50, 100$  for the black circle points, red square points, green diamond points and blue triangle points, respectively.

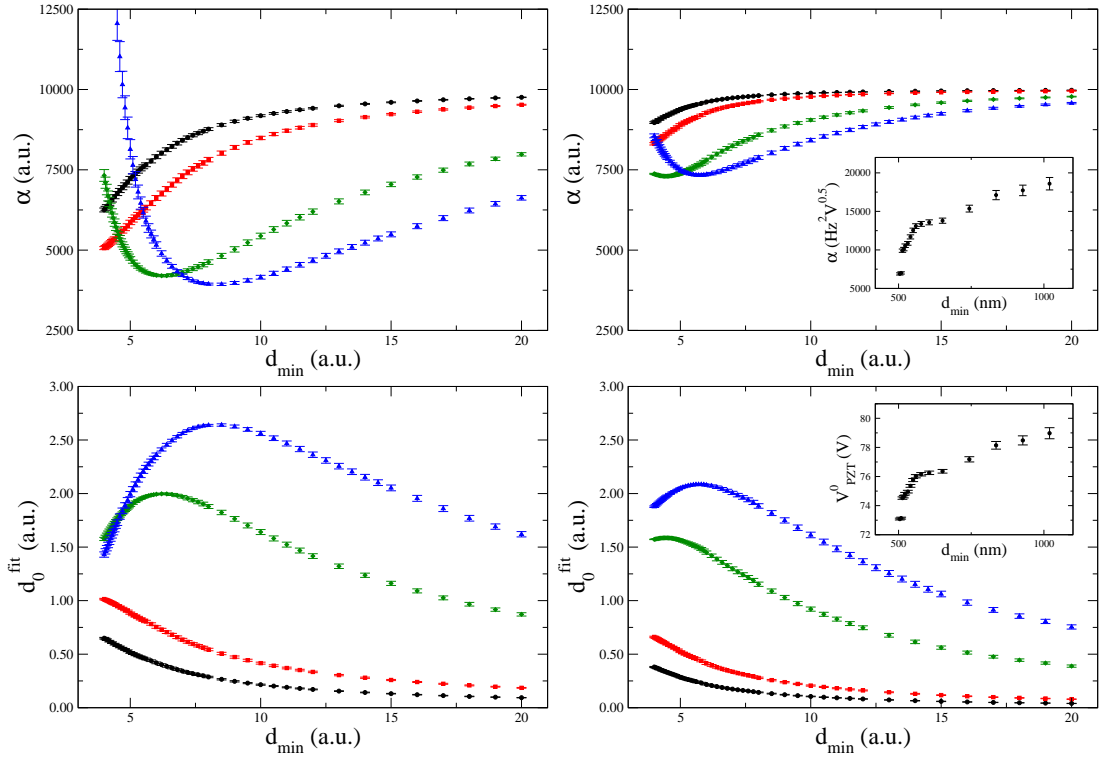


FIG. 17: (Color online) Plots of the calibration factor  $\alpha$  (top) and contact distance  $d_0^{\text{fit}}$  (bottom) of the hypothetical scenario data versus the distance of the closest data point used in the fitting. The plots on the left are the results of the best fit when the exponent  $q$  is a free parameter, and the plots on the right are the results when  $q$  is fixed at 2.5. The data are constructed from Eq. 12 with  $p = 5$ ,  $\alpha_1 = 10000$ , and  $\alpha_2/\alpha_1 = 5, 10, 50, 100$  for the black, red, green and blue curve, respectively. The insets on the right are the corresponding results using experimental data from our electrostatic calibration measurements. The relationship between the distance and the PZT voltage is  $d - d_0 = \beta(V_{\text{PZT}}^0 - V_{\text{PZT}})$  where  $\beta$  is the actuation coefficient of the piezoelectric transducer. The same labeling for the points as in the previous figure is used.

cannot be covered by the fitting uncertainty. This results in a further source of systematic error in the determination of the absolute distance, which adds up to other sources, including the one recently discussed in [76]. It should also be noted that there seems to be a strong correlation between the two fitting parameters  $\alpha$  and  $d_0^{\text{fit}}$ , which is indeed also present in our electrostatic calibration measurements as shown in the insets in the bottom right plot of Fig. 17.

#### D. Electrostatic patch effects

As it is well known, all metallic surfaces in reality are not equipotential surfaces, showing instead voltage variations of order 10-100 mV over micrometer distances. These patch potentials are typically due to local changes in the work function associated to different crystallographic facets of the metal. Electrostatic patches are known to be an important systematic in several precision measurement experiments, including those aiming at detecting the Casimir force. Apart from the static component, patch potentials can also fluctuate in time, a dynamical process that has not been studied in detail to date. It has recently been shown that by cooling a Au sample the electric-field noise above the metal is substantially reduced, a process possibly due to thermal activation barriers in the surface potential [77].

Here we briefly describe the physics of electrostatic patches in the cylinder-plane geometry, and discuss whether they could be partly responsible for the anomalous exponent in our electrostatic calibrations. Our considerations follow closely the model and notation of [23]. A related effect is the fluctuation-induced interaction between monopolar charge disorder within the dielectric slabs [24]. The electrostatic interaction energy between two parallel plates, whose surfaces contain stochastic voltage variations  $V_a(x, y)$  ( $a$  denotes the upper or lower plate) is

$$U_{pp} = \frac{\epsilon_0}{2} \int_0^\infty dk \frac{k^2 e^{-kd}}{\sinh(kd)} S(k). \quad (14)$$

This expression results from assuming zero-average patches, and an isotropic two-point correlation in the transverse



plane-wave basis  $\mathbf{k}$  given by  $\langle V_{a,\mathbf{k}}, V_{b,\mathbf{k}'} \rangle = \delta_{a,b} C_{a,k} \delta^2(\mathbf{k} - \mathbf{k}')$ . Here  $\langle \dots \rangle$  denotes stochastic average,  $k = |\mathbf{k}|$ , and the power spectral density  $S(k)$  is defined as  $\int_0^\infty dk k S(k) \equiv (1/8\pi) \int_0^\infty dk k (C_{1,k} + C_{2,k})$ . The corresponding electrostatic force due to these potential patches is given by  $F_{pp} = -\partial U_{pp}/\partial d$ . To compute the patch effect on the force in the cylinder-plane configuration we make use of the PFA to treat the curvature of the cylinder, which is a good approximation in the limit  $d/a \ll 1$ . We do not impose any restriction on the typical size of the patches (*i.e.* we leave  $kd$  arbitrary). In the limit  $d/a \ll 1$  the electrostatic force due to patches in the cylinder-plane configuration is

$$F_{cp} = \frac{\pi\epsilon_0 L}{2\sqrt{2}} a \left(\frac{d}{a}\right)^{1/2} \int_0^\infty dk \frac{k^3 e^{-2kd}}{\sinh^2(kd)} S(k). \quad (15)$$

Two simple limiting cases can be analyzed. In the large patch limit ( $kd \ll 1$ ) the force is given by

$$F_{cp} \approx \frac{\pi\epsilon_0 L}{2\sqrt{2}} \frac{a^{1/2}}{d^{3/2}} V_{\text{rms}}^2, \quad (16)$$

with  $V_{\text{rms}}^2 = \int_0^\infty dk k S(k)$ . This expression is exactly equivalent to the r.h.s. of Eq. (1) with  $V^2$  replaced by  $V_{\text{rms}}^2$ , as expected for large patches. In the small patch limit ( $kd \gg 1$ ) the force is exponentially suppressed because the patches are small and change sign rapidly, resulting in a vanishing net interaction between the plates. It should be noticed that the patch force depends on distance as an inverse power law (with exponent 1.5) only in the small-patch limit. The smaller the patches, the faster the decay (bigger exponent). More importantly for our purposes is the fact that the electrostatic patch force (16) is *independent* of the applied voltage  $V$  between the cylinder and the plane. Therefore, it cannot explain the anomalous exponent of the electrostatic calibration, that stems from the  $V$ -dependent contribution to the force. The patch force (16) is, instead, a background force that could possibly show up in the analysis of the electrostatic residuals, that is, the force *after* the subtraction of the Coulomb-like  $V$ -dependent terms.

It could be argued that the presence of strong electric fields in the gap between the plates may, in principle, redistribute the spatial configuration of the patches, and then the question is whether the force between the redistributed patches depends on  $V^2$  (note that if this were the case, there might be some hope that the anomalous exponent is partially due to patches). Equation (16) was obtained assuming that the two plates had only stochastic potentials fluctuating around zero. If an external fixed potential difference  $V$  is applied between the plates, the linearity of Laplace's equation implies that the total force will be the sum of the usual  $V^2$  term plus the  $V$ -independent term given in (16). In principle, for sufficiently large external fields in the gap, the power spectrum  $S(k)$  could depend on the external voltage  $V$ , but it is unclear if, and how, such an effect can account for the anomalous exponent in our electrostatic calibrations, at least in the ones performed by maintaining the electric-field approximately constant in the explored distance range.

## V. MINIMIZING POTENTIAL IN THE PARALLEL PLATES GEOMETRY

We have also performed in the same experimental conditions measurements in the plane-plane configuration using flat coated mirrors facing the resonator. Considering that the sphere-plane geometry has been the subject of former work [38, 39], this allows us to complete the picture on the relationship between the distance-dependent minimizing potential and the specific geometrical configuration. Three mirrors with different coatings (Au, Ag and Al) were used to also investigate the possible relationship between  $V_0$  and the nature of the substrate. Although the parallelization is obtainable with a good level of approximation in one direction only, we can introduce an off-line parallelization correction in the data fitting. We analyze separately in the following results on the three substrates. The data of  $V_0$  from four runs were merged in the same plot for each of the three mirrors as shown in Fig. 18 where they appear versus the curvature coefficient  $K_{\text{el}}$  instead of the distance. This is because the range of  $K_{\text{el}}$  achieved is not very large especially in the case of aluminum mirrors due to limited parallelization and smaller conductivity from oxide layers on the surface of the mirror, and consequently the usual fitting procedure to find the absolute distance would produce rather unstable and inaccurate results. Since there is a one to one mapping between  $K_{\text{el}}$  and the distance,  $K_{\text{el}}$  is used here as a fair indicator of the distance for all runs.

In the case of Au mirrors,  $V_0$  fluctuates around 100 mV without any noticeable trend. The fluctuation is bigger at large distance partly because the frequency shifts tend to be small also for relatively larger bias voltage, this being reflected in the error bars in the fitting procedure. As the distance gets smaller,  $V_0$  seems to converge to a constant value. The minimizing potential  $V_0$  shows a similar behavior in all runs, and the values of  $V_0$  are also comparable, with runs 1, 2, and 4 all around 0.1 V, and run 3 slightly lower at around 0.4 V. In Table III, the average value and the standard deviation of  $V_0$  for each run is obtained using the six data points with largest  $K_{\text{el}}$  value from each run. We see that the standard deviation is reasonably small indicating that  $V_0$  is mostly constant.

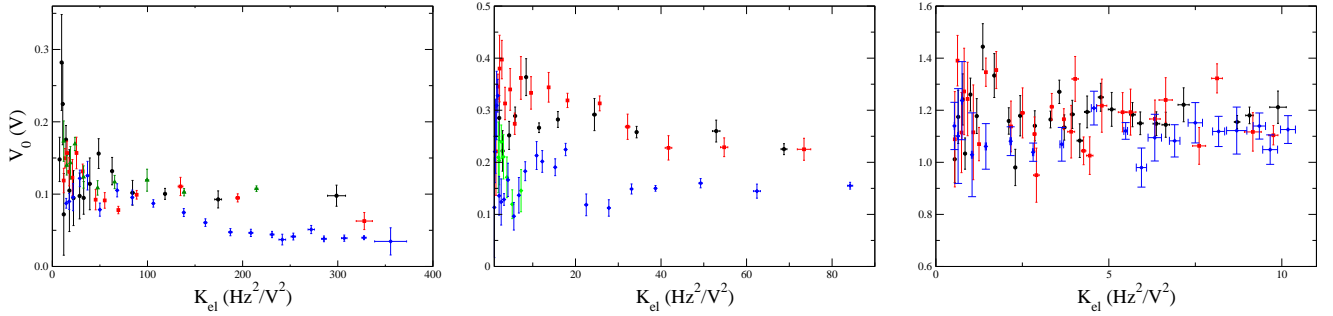


FIG. 18: (Color online) Plots of the minimizing potential  $V_0$  versus curvature coefficient  $K_{el}$  from various runs in plane-plane geometry with three different coatings of the mirror, Au (left), Ag (middle), and Al (right), all facing the Au-coated resonator.

Using Ag mirrors instead, a noticeable difference evident from the electrostatic measurement is that we achieve values of  $K_{el}$  smaller than in the case of Au mirrors. This is possibly due to the fact that Ag is easily oxidized once exposed in air prior to the insertion into the vacuum chamber. Of course, we cannot rule out the possibility that this could also be partly due to slightly different parallelization configurations. In terms of residual potential  $V_0$ , the Ag mirror shares the same behavior as gold mirror, as  $V_0$  fluctuates randomly remaining constant especially at small separation gap. However, in this case the  $V_0$  value is noticeably higher than that of the Au mirror.

Finally, with an Al coating, the lowest value of  $K_{el}$  achieved is much smaller than that of the Au and Ag mirrors. This may be due to the easiness to form oxide layers, and it could also be related to the smaller conductivity with respect to Al and Au. In this case the residual potential  $V_0$  is also constant at small separation gaps as it can be seen in Fig. 18, and it is manifest that  $V_0$  is significantly larger than that for Au and Ag mirrors. A more quantitative analysis is precluded by the strong dependence of the work function of Al on the exposure time in air [78].

A comparison among the various minimizing potentials  $V_0$  and physical parameters of the substrate is shown in the second part of Table III. The average values of  $V_0$  are obtained by averaging three runs from each mirror weighted by their variance. From the table it is clear that  $V_0$  for mirrors coated with Au, Ag, and Al are significantly different. For a rough theoretical comparison, the work function and the Fermi energy as well as their difference are also listed in the table. The work function of a metal is closely related to its Fermi energy, but due to the presence of defects and impurities on the surface these two quantities do not coincide, with their difference largely due to the surface charge distribution and surface dipole distribution. From Table III we can see that Au has the smallest difference between its work function and Fermi energy while Al has the largest. Since this difference indicates the magnitude of the surface charge and dipole distribution which may be directly related to the residual potential  $V_0$ , it could be used to explain the different value of  $V_0$  measured using mirror with different coatings. In fact the average  $V_0$  values for Au, Ag, and Al mirrors is consistent with the order of this difference. Regarding the best fitting exponent for the scaling of the curvature coefficient with distance, we have obtained the expected one from Coulomb force, although it should be noticed that the range of distances was limited to a minimum value of about  $3 \mu\text{m}$ .

	Run	$V_0$ (V)	$\sigma_{V_0}$ (V)	$\langle V_0 \rangle$ (V)	$W$ (eV)	$E_F$ (eV)	$E_F - W$ (eV)
Gold	1	0.113	0.025	0.048	5.1-5.47	5.53	0.06-0.43
	2	0.089	0.017				
	3	0.041	0.006				
	4	0.113	0.008				
Silver	1	0.145	0.017	0.194	4.52-4.74	5.49	0.75-0.97
	2	0.182	0.044				
	3	0.264	0.044				
	4	0.264	0.023				
Aluminum	1	1.177	0.033	1.151	4.06-4.26	11.7	7.44-7.64
	2	1.169	0.097				
	3	1.118	0.036				

TABLE III: Average measured value of  $V_0$  for mirrors made of various substrates, weighted average value of  $V_0$ , and comparison with tabulated data for the work function  $W$  and the Fermi energy  $E_F$ .

## VI. ON THE IMPORTANCE OF THE MEASUREMENT AND THE MODELIZATION OF THE MINIMIZING POTENTIAL IN CASIMIR FORCE MEASUREMENTS

Our emphasis on measuring the minimizing potential at all the explored distances for the sphere-plane configuration [38, 39], and for the cylinder-plane and parallel planes configurations described here, is due to the fact that the understanding of the minimizing potential dependence on distance and time is crucial for the assessment of the accuracy of the Casimir force measurements at small (below  $\simeq 1\mu\text{m}$ ) distances, and for determining the thermal contribution at large (in the 1-5  $\mu\text{m}$  range) distances. In Table IV we report the formulas for the ideal (perfect reflectors, zero temperature) Casimir force and for the Coulomb force in the three different geometries. Following [79], let us define the equivalent Casimir voltage as the external bias voltage that can simulate the ideal Casimir force. Due to the different scalings with distance of Casimir and Coulomb forces, this equivalent Casimir voltage must be specified at each distance. It is, however, important to point out that the difference between the various geometries is just a numerical factor which makes this equivalent voltage for the sphere-plane configuration about half the value of the parallel plane case (with the cylinder-plane, as is customary, in between the two extreme cases even from this point of view). The formula for the equivalent Casimir voltage is

$$V_{\text{Cas}}^{\text{eq}}(d) = \left(\frac{\pi^2}{\xi}\right)^{1/2} \left(\frac{\hbar c}{\epsilon_0}\right)^{1/2} \frac{1}{d}, \quad (17)$$

where  $\xi=360, 192,$  and  $120$  for the sphere-plane, cylinder-plane, and parallel plate configurations, respectively.

In the numerical example presented in the last row of Table IV, at a distance of  $1\mu\text{m}$ , which can be considered the borderline between the short-distance regime and the long-distance regime in which the thermal contribution starts to play a significant role, this equivalent Casimir voltage ranges between 10 and 17 mV depending on the geometry. At  $3\mu\text{m}$ , where the thermal contribution is expected to contribute as 10-20 % of the total force signal, with an absolute value still large enough to be detectable in various apparatus, the equivalent Casimir voltage is three times smaller (*i.e.* between 3 and 6 mV). This voltage is of the same order of magnitude of the variation of the minimizing potential in a range of few micrometers. To take into account this contribution to properly subtract it from the data, it is therefore necessary to model the minimizing potential at the few percent level accuracy. Unfortunately, such stringent theoretical characterization of the minimizing potential is not yet available.

A further degree of uncertainty is also related to the fact that, as pointed out in [39], the minimizing potential may depend on time, a fact that, for instance, could be attributed to temperature drifts. It has been experimentally shown in [80] using a heated atomic force microscope tip that the contact potential depends on temperature, with a slope estimated to be of the order of  $4\text{ mV}/^\circ\text{C}$ . To perform high-precision tests of the Casimir force, one therefore needs a stringent temperature stability of the apparatus during the entire measurement run. In [43], the observation of fast changes in the contact potential have been conjectured as due to the effect of background cosmic rays impinging on the apparatus. For a release of about  $10^{-11}\text{ C}/\text{cm}^2$  through ionization by cosmic rays at sea level, and considering the small values of the capacitances (order of hundreds pF), sudden changes of order 0.1-1 mV could be expected (see [81] for a related discussion). A careful control of the minimizing potential will then require also surrounding the apparatus with lead shields to reduce the radiation background or particle detectors to veto the apparatus during large ionization events, especially for apparatus using microspheres.

	Sphere-Plane	Cylinder-Plane	Parallel Planes
Casimir	$\frac{\pi^3}{360} \hbar c \frac{R}{d^3}$	$\frac{\pi^3}{384\sqrt{2}} \hbar c \frac{La^{1/2}}{d^{7/2}}$	$\frac{\pi^2}{240} \hbar c \frac{S}{d^4}$
Coulomb	$\pi \epsilon_0 \frac{R}{d} V^2$	$\frac{\pi \epsilon_0}{2\sqrt{2}} \frac{La^{1/2}}{d^{3/2}} V^2$	$\frac{\epsilon_0}{2} \frac{S}{d^2} V^2$
$V_{\text{Cas}}^{\text{eq}}(d)$	$\left(\frac{\pi^2}{360}\right)^{1/2} \left(\frac{\hbar c}{\epsilon_0}\right)^{1/2} \frac{1}{d}$	$\left(\frac{\pi^2}{192}\right)^{1/2} \left(\frac{\hbar c}{\epsilon_0}\right)^{1/2} \frac{1}{d}$	$\left(\frac{\pi^2}{120}\right)^{1/2} \left(\frac{\hbar c}{\epsilon_0}\right)^{1/2} \frac{1}{d}$
$V_{\text{Cas}}^{\text{eq}}(1\mu\text{m})$	9.85 mV	13.5 mV	17.1 mV

TABLE IV: Summary of relevant formulas for the ideal Casimir force and the Coulomb force in the cases of the sphere-plane, cylinder-plane, and parallel plane geometries, with both forces in the first two geometries evaluated using PFA. In the third row the equivalent Casimir voltage, *i.e.* the voltage which needs to be applied in order to simulate the Casimir force at a given distance  $d$ , is reported. In the last row the concrete value of the equivalent Casimir voltage is reported in the case of a typical gap distance of  $1\mu\text{m}$ .

## VII. CONCLUSION

In this paper we have summarized the main outcomes from our effort to measure the Casimir force in the cylinder-plane configuration. The presence of uncontrollable frequency shifts of electric origin at the smallest explored gaps, evidenced both from the analysis of the residuals of the electrostatic calibrations and a fast-approach technique, prevent us from identifying a Casimir-like contribution at small distance. At large distances thermal drifts are large, if compared to the expected Casimir force, and a careful control of the dependence of the minimizing potential on distance is also required to extract meaningful information about the Casimir force and its thermal corrections. While unsuccessful, our search for the Casimir force in this geometry, apart from the development of some data taking and analysis techniques applicable elsewhere, has evidenced a number of features which may be of more general interest, as we try to summarize in the following.

First, we have observed anomalous behavior for the best fitting exponent with which the electrostatic coefficient is scaling as a function of the cylinder-plane separation. The exponent is significantly smaller than the Coulombian one at small distances, while it retains its expected value at the largest explored gaps. In the case of the sphere-plane measurements, the exponent was slightly smaller than the expected value but it retained its value in the entire explored range of distances, this last being smaller than in the cylinder-plane case due to the smaller electrostatic signal available in the sphere-plane configuration [38].

Second, we have observed a dependence of the minimizing potential on the cylinder-plane distance, similarly to the sphere-plane case. Although its dependence is milder at small distances, it still retains a strong dependence on distance at larger gaps, and this requires a careful modeling to subtract its contribution when studying forces in the 1-5  $\mu\text{m}$  range of interest to discriminate among the various models proposed to incorporate the thermal contribution.

Finally, we have also explored the case of flat surfaces with a rough parallelization and we have found that in this case no anomalous behavior is observed for both the scaling exponent and the minimizing potential, even using different substrates for the surfaces. The range of explored distances is definitely limited by the approximate two-dimensional parallelism achievable with our setup, and it is therefore unclear if anomalous scaling could be instead observed as in the case of the smaller explored gaps in the sphere-plane and cylinder-plane configurations. Both apparatus built to study the Casimir forces in a parallel plane configuration have not observed dependence of the minimizing potential on distance [3, 64].

Our findings should be then related to recent outcomes from various experiments confirming the presence of nontrivial, formerly unidentified systematic effects in the electrostatic calibrations [38, 39, 41, 42]. Several recent experiments are also showing that the observation of Casimir or Casimir-Polder forces is less trivial than previously stated, for instance, with regards to the dependence on the optical properties of the substrates [82] and the presence of dielectric layers on the substrates [83]. Theoretical arguments have been recently provided for the nontrivial interplay between thermal fluctuations and geometry [84], thermal, conductivity, and roughness corrections [85, 86] and the role of the statistical properties of the conducting surfaces [24]. Deviations from the pure Coulombian contribution and from the hypothesis of a constant minimizing potential have also been observed in atomic force microscopy for sharp tips, for instance, due to capillary forces [87, 88].

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