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# <sup>1</sup> Mass density at geostationary orbit and apparent <sup>2</sup> mass refilling

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**Abstract.**

We used the inferred equatorial mass density  $\rho_{m,eq}$  based on measurements of Alfvén wave frequencies measured by the GOES satellites during 1980–1991 in order to construct a number of different models of varying complexity for the equatorial mass density at geostationary orbit. The most complicated models are able to account for 66% of the variance with a typical variation from actual values of a factor of 1.56. The factors that influenced  $\rho_{m,eq}$  in the models were, in order of decreasing importance, the F10.7 EUV index, magnetic local time, MLT, the solar wind dynamic pressure  $P_{dyn}$ , the phase of the year, and the solar wind  $B_Z$  (GSM Z direction). During some intervals, some of which were especially geomagnetically quiet,  $\rho_{m,eq}$  rose to

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14 values that were significantly higher than those predicted by our models. For  
15 10 especially quiet intervals, we examined long-term ( $> 1$  day) apparent re-  
16 filling, the increase in  $\rho_{m,eq}$  at a fixed location. We found that the behavior  
17 of  $\rho_{m,eq}$  varies for different events. In some cases, there is significant appar-  
18 ent refilling, whereas in other cases  $\rho_{m,eq}$  stays the same or even decreases  
19 slightly. Nevertheless, we showed that on average  $\rho_{m,eq}$  increases exponen-  
20 tially during quiet intervals. There is variation of apparent refilling with re-  
21 spect to the phase of the solar cycle. On the third day of apparent refilling,  
22  $\rho_{m,eq}$  has on average a similar value at solar maximum or solar minimum,  
23 but at solar maximum,  $\rho_{m,eq}$  begins with a larger value and rises relatively  
24 less than at solar minimum.

## 1. Introduction

25 Mass density controls the time rate of change of magnetohydrodynamic (MHD) pro-  
26 cesses. It also provides a constraint on composition, that can significantly change the  
27 properties of certain waves such as electromagnetic ion cyclotron (EMIC) waves [*Denton*  
28 *et al.*, 2014a]. It also provides a clue about transport of heavy ions, especially O+.

29 Bulk particle density is difficult to measure using particle instruments because spacecraft  
30 charging can shield low energy particles from reaching the particle detector. Because of  
31 this, waves are often used to infer the particle density. Plasma wave frequencies can be  
32 used to infer the electron density [*Persoon et al.*, 1983; *Benson et al.*, 2004]. And Alfvén  
33 wave frequencies are often used to infer the mass density  $\rho_m$  [*Waters et al.*, 2006; *Denton*,  
34 2006; *Denton et al.*, 2015].

35 While ideally direct measurements of Alfvén wave frequencies can be used to infer  $\rho_m$ ,  
36 often such measurements are lacking, and in that case models are useful to describe the  
37 density. *Takahashi et al.* [2010] showed that the single most important parameter predict-  
38 ing magnetospheric mass density is the solar EUV F10.7 index. Greater EUV radiation,  
39 as characterized by larger F10.7, leads to larger  $\rho_m$ . *Denton et al.* [2011] combined this  
40 dependence with the variation in ion density measured by the Los Alamos National Lab  
41 (LANL) Magnetospheric Plasma Analyzer (MPA) instruments [*Bame et al.*, 1993; *Den-*  
42 *ton et al.*, 2005] to show that there is a variation in composition over the solar cycle,  
43 with significant concentrations of O+ at solar maximum, but low concentrations of O+  
44 at solar minimum. Greater EUV radiation at solar maximum increases the ionospheric  
45 temperature, increasing the ionospheric O+ scale height. This combined with greater

46 wave activity at solar maximum may explain why larger amounts of O+ are able to reach  
47 the magnetic equator at geostationary orbit at solar maximum.

48 Here we will extend the modeling effort of *Takahashi et al.* [2010] and *Denton et al.*  
49 [2011] to consider more parameters. This will lead to a model that is more accurate at  
50 the expense of being more complicated. In addition we will consider the apparent refilling  
51 of  $\rho_m$  during geomagnetic quiet periods following active periods [e.g., *Denton et al.*, 2012].  
52 *Denton et al.* [2014b] showed that the evolution of the mass density could be very different  
53 from that of ion density during these times.

54 In Section 2, we describe the data used in the study; in Section 3, we present a new  
55 model for  $\rho_m$ ; in Section 4, we examine the evolution of  $\rho_m$  during several quiet events;  
56 and in Section 5 we discuss and summarize our results.

## 2. Data

57 The set of Alfvén wave frequencies is the same as that used by *Denton et al.* [2015].  
58 These frequencies were measured by magnetometers on Geostationary Operational En-  
59 vironmental Satellites (GOES) at geostationary orbit between 1980 and 1991. For a  
60 description of the method to get the mass density, see that of *Takahashi et al.* [2010]. In  
61 brief, the wave equation of *Singer et al.* [1981] is solved for the theoretical eigenfrequency  
62 given an equatorial value of  $\rho_m$ ,  $\rho_{m,eq}$ , equal to 1 amu, and the inferred equatorial mass  
63 density is found by comparing the observed and theoretical eigenfrequencies using the fact  
64 that the frequencies are proportional to the Alfvén speed  $\propto 1/\sqrt{\rho_m}$ . The TS05 magnetic  
65 field model [*Tsyganenko and Sitnov*, 2005] is used with an assumed field line dependence  
66 for  $\rho_m$  as discussed below.

67 An example of 36 hours of data is shown in Figure 1. The roughly horizontal bands  
 68 of wave power result from the Alfvén wave harmonics. Note that data gaps occur when  
 69 Alfvén waves do not occur or where they are difficult to identify because of sporadic (non-  
 70 banded) occurrence or because of the simultaneous occurrence of broad band wave power  
 71 such as results from impulsive signals (e.g., at 0500 UT on 11 Feb 1990 in Figure 1).

One difference in method from that of *Takahashi et al.* [2010] is that we use a different model for the field line distribution of  $\rho_m$ . A power law distribution is assumed for  $\rho_m$ ,

$$\rho_m = \rho_{m,eq} \left( \frac{LR_E}{R} \right)^\alpha, \quad (1)$$

72 as has been used by many researchers [*Waters et al.*, 2006; *Denton*, 2006]. Here  $L$  is  
 73 the L shell parameter defined as the maximum geocentric distance to any point on the  
 74 field line using the TS05 magnetic field model [*Tsyganenko and Sitnov*, 2005] divided by  
 75 the Earth's radius  $R_E$ , and  $\alpha$  is the power law index. We use a formula for  $\alpha$  that is  
 76 substantially the same as that of *Denton et al.* [2015],

$$\begin{aligned} \alpha = & 2.06 + 1.24 \cdot \cos((\text{MLT} - .15) \cdot 15^\circ) \\ & + 0.0026 \cdot \text{AE} \cdot \cos((\text{MLT} - 0.73) \cdot 15^\circ) \\ & + 2.1 \cdot 10^{-5} \cdot \text{AE} \cdot \text{F10.7} - 0.010 \cdot \text{F10.7}. \end{aligned} \quad (2)$$

77 Because this formula depends on F10.7, MLT, and AE, our model results have some small  
 78 additional dependence on these variables. But this additional dependence is small. As  
 79 *Denton et al.* [2015] discuss, errors in  $\alpha$  could lead to errors in individual  $\rho_m$  of order 25%.  
 80 At any rate, use of the statistical model (2) based on data should improve the statistical  
 81 results for our  $\rho_m$  model. And hopefully the effect of errors due to incorrect  $\alpha$  values will

cancel out in the averaging. The possible effect of field line dependence not described by  
 (1) is more complicated; see the discussion by *Denton et al.* [2006].

A second difference in method is that for each Alfvén frequency, we find  $\rho_{m,eq}$  from the  
 log average value of the value calculated using the observed frequency minus its standard  
 error and that calculated using the observed frequency plus its standard error. Since  
 reduction in frequency leads to a greater proportional change, this shifts the resulting  
 mass densities to slightly higher values than if the peak frequency values were used. The  
 mean value of the difference in the logarithm of the mass density calculated using the  
 measured frequency minus its standard error and that calculated using the measured  
 frequency plus its standard error was 0.20 (corresponding to a factor of  $10^{0.20} = 1.6$ ), and  
 the median value was 0.14 (corresponding to a factor of 1.38).

### 3. Mass density model

Our process of choosing parameters went through several stages. First we used linear  
 regression and plots of binned quantities using many different solar wind parameters and  
 geomagnetic indices. We eliminated many of these and narrowed down the parameters to  
 the following: the remainder of the fractional year, dYr, indicating season (DOY minus  
 one divided by the number of days in that year); the magnetic local time, MLT, measured  
 in hours; the F10.7 index measured in  $10^{-22} \text{ W m}^{-2} \text{ Hz}^{-1}$ , referred to as the solar flux  
 units (sfu, hereinafter); the logarithmic Kp index; the Dst and AE indices measured in  
 nT; the solar wind dynamic pressure  $P_{dyn}$  measured in nPa; the solar wind electric field  
 value measured in the GSM Y direction allowing only positive values,  $E_{Ys}$  measured in  
 mV/m; the GSM Z component of the interplanetary magnetic field,  $B_Z$ , measured in  
 nT; and the reconnection coupling parameter  $d\Phi_{MP}/dt$  of *Newell et al.* [2007] in units of



104  $(mV \cdot km / (m \cdot s))^{2/3}$ . In addition to the instantaneous value of these quantities, we con-  
 105 sidered averages and extrema of F10.7, Kp, Dst, Ae,  $P_{\text{dyn}}$ ,  $E_{Ys}$ ,  $B_Z$ , and  $d\Phi_{\text{MP}}/dt$ . The  
 106 averages were calculated over the previous 3 hours, 6 hours, 12 hours, 24 hours, 48 hours,  
 107 96 hours, and 192 hours. The extrema were calculated during the same previous intervals.  
 108 For Dst, the most negative value was found, while for all other quantities the most positive  
 109 value was found.

110 In order to ensure that periodic functions would result from dYr and MLT, we considered  
 111 dependencies on  $\sin(dYr \cdot 360^\circ)$ ,  $\cos(dYr \cdot 360^\circ)$ ,  $\sin(MLT \cdot 15^\circ)$ ,  $\cos(MLT \cdot 15^\circ)$ , rather  
 112 than directly on dYr and MLT.

113 Solar wind parameters were taken from the *Kondrashov et al.* [2014] database, which is  
 114 an improvement over the Qin Denton database [*Qin et al.*, 2007]. The database includes  
 115 quality factors for  $P_{\text{dyn}}$  and  $B_Z$ , which range from a value of 0 for a parameter that is far  
 116 from a measured value to 2 for a parameter that is directly measured. A value of at least  
 117 1 means that the quantity is not far from measurements and is significantly better than  
 118 an average value. But even the 0 quality factor values are improved due to Kondrashov  
 119 et al.'s technique. To get quality factors for the averages and extrema, we averaged the  
 120 quality factors over the corresponding interval. For  $E_Y$  and  $d\Phi_{\text{MP}}/dt$ , that are calculated  
 121 from other quantities, the minimum quality factor of the individual quantities was used.  
 122 But note that  $E_Y$  and  $d\Phi_{\text{MP}}/dt$  did not end up in any of our formulas; the other quantities  
 123 were sufficient to account for the amount of variation that could be explained.

124 Then we used the Eureqa nonlinear genetic regression software [*Schmidt and Lipson,*  
 125 2009] to find potential mathematical models for  $\log_{10} \rho_{m,eq}$ , minimizing the squared de-  
 126 viation from the observed values. Each data point was weighted by the inverse of the

127 difference in the logarithm of the mass density calculated using the measured frequency  
128 minus its standard error and that calculated using the measured frequency plus its stan-  
129 dard error; but this weight was limited to a value of 2.5 (corresponding to a  $\log_{10}$  difference  
130 of 0.4). (We might have weighted the data using the square of this quantity. We made  
131 this choice as a compromise between weighted and non-weighted least-squares.) For this  
132 stage of the modeling, we required that the solar wind parameters used in the model, and  
133 some selected averages that commonly occur in models, have at least a quality factor of  
134 1.

135 Eureka gives a family of formulas of different complexity. For each level of complexity,  
136 it gives the formula that best fits the data. We will present several different models of  
137 increasing complexity. After finding the form of a particular model from Eureka, we  
138 tuned the parameters using linear or nonlinear minimization for the weighted squared  
139 error. This procedure was used because Eureka often included only the sine or cosine of  
140 dYr or MLT in the formula, and we consider the formula no more complicated to use both  
141 the sine and cosine, that is, a general phase. Also we used a slightly different data set for  
142 this stage of the process; we did not screen the data for high quality values for quantities  
143 not used in the modeling. We estimated the error of the formula in the following way. We  
144 split the data into intervals of 2 weeks and divided the data in these intervals into five  
145 groups. For each group, we calculated the parameters of the model using the other four  
146 groups of data, and found the standard error of the resulting model for predicting the  
147 observed values of  $\log_{10} \rho_{m,eq}$  for that group. Then we averaged the squared deviations  
148 for the five groups of data and took the square root to get the final standard error for the  
149 model. Thus the error is calculated using data other than that used for the model. While

150 this procedure is the best for getting an estimate of the error, the results were not greatly  
 151 different from using the entire data set, probably because we had a very large amount of  
 152 data.

The simplest possible model is just the average. The weighted average value of  $\log_{10} \rho_{m,eq}$  yields

$$\log_{10} \rho_{m,eq} = 1.02, \quad (3)$$

153 corresponding to  $\rho_{m,eq} = 10^{1.02} = 10.5 \text{ amu/cm}^3$ , and the unbiased weighted standard  
 154 error calculated in the manner described above is 0.34 corresponding to a variation of a  
 155 factor of  $10^{0.34} = 2.17$ . This result is itself interesting. The typical variation from the  
 156 mean is not large.

157 For  $1.7 < L < 3.1$ , *Berube et al.* [2005] found  $\log_{10} \rho_{m,eq} = -0.65L + 5.1$  for  $-9 \text{ nT}$   
 158  $< Dst < -3 \text{ nT}$  and  $\log_{10} \rho_{m,eq} = -0.74L + 5.5$  for  $Dst < -100 \text{ nT}$ . Extrapolation of this  
 159 formula to  $L = 6.8$ , a typical value for GOES spacecraft that are slightly off the magnetic  
 160 equator, yields  $\log_{10} \rho_{m,eq} = 0.68$  for  $-9 \text{ nT} < Dst < -3 \text{ nT}$  and  $\log_{10} \rho_{m,eq} = 0.47$  for  
 161  $Dst < -100 \text{ nT}$ . These values are higher than that in (3), so not surprisingly the unbiased  
 162 weighted standard error using these formulas is larger, 0.48. *Berube et al.*'s average  
 163  $\log_{10} \rho_{m,eq}$  value might be lower due to a steep  $L$  dependence within  $1.7 < L < 3.1$  caused  
 164 by mass loading at the low  $L$  shells owing to their close proximity to the ionosphere.

The next simplest model involves just F10.7.

$$\log_{10} \rho_{m,eq} = 0.088\sqrt{F10.7_{96}}, \quad (4)$$

165 where  $F10.7_{96}$  is the average of the F10.7 index over the previous 96 hours. The unbiased  
 166 weighted standard error is 0.25 corresponding to a variation of a factor of  $10^{0.25} = 1.77$ .

167 This formula shows that  $\rho_{m,eq}$  increases with respect to F10.7 as expected from previous  
 168 studies [*Takahashi et al.*, 2010]. The formula using the square root is slightly more accurate  
 169 than one using a linear term.

170 *Takahashi et al.* [2010] found  $\log_{10} \rho_{m,eq} = 0.42 + 0.0039F10.7$  using 27 day median  
 171 values and *Denton et al.* [2011] found  $\log_{10} \rho_{m,eq} = 0.51 + 0.0036F10.7$  for the yearly  
 172 median  $\rho_{m,eq}$  using the yearly average of F10.7. Using these formulas with  $F10.7_{96}$  (the  
 173 preferred average for our instantaneous  $\rho_{m,eq}$  values), we find for our data set unbiased  
 174 weighted standard errors of 0.26 and 0.25, respectively, which are essentially the same as  
 175 the value 0.25 for (4).

The simplest formula that includes MLT dependence is

$$\log_{10} \rho_{m,eq} = 0.088\sqrt{F10.7_{96}} + 0.17 \cos((MLT - 15.6) \cdot 15^\circ). \quad (5)$$

176 The unbiased weighted standard error is 0.22 corresponding to a variation of a factor of  
 177  $10^{0.22} = 1.66$ . The MLT dependence peaks at mid afternoon local time.

The simplest formula that includes explicit solar wind forcing is

$$\log_{10} \rho_{m,eq} = 0.27 + 0.0042F10.7_{96} + 0.18 \cos((MLT - 15.5) \cdot 15^\circ) + 0.059P_{dyn,12}, \quad (6)$$

178 where  $P_{dyn,12}$  is the average of the dynamic pressure over the previous 12 hours. The  
 179 unbiased weighted standard error is 0.21 corresponding to a variation of a factor of  $10^{0.21} =$   
 180 1.61. This shows that recently higher dynamic pressure leads to increased mass density.

181 The most complicated formula that we found “recommended as a solution” by Eureka  
 182 (after running the program for several days with 14 processors) is

$$\begin{aligned} \log_{10} \rho_{m,eq} = & 0.32 + 0.0038F10.7_{96} + 0.14 \cos((MLT - 13.0) \cdot 15^\circ) \\ & + 0.054P_{dyn,12} + 0.07 \cos((dYr - 0.053) \cdot 360^\circ) + 0.016B_{z,3} \end{aligned}$$

$$+13 \cos((\text{MLT} - 18.4) \cdot 15^\circ) / \text{F10.7}_{192}, \quad (7)$$

183 where  $dYr$  is remainder of the fractional year,  $B_{z,3}$  is  $B_z$  averaged over the previous 3  
 184 hours, and  $\text{F10.7}_{192}$  is the average of the F10.7 index over the previous 192 hours. The  
 185 terms are ordered roughly in order of their importance. The unbiased weighted standard  
 186 error is 0.19 corresponding to a variation of a factor of  $10^{0.21} = 1.56$ . The  $dYr$  dependence  
 187 peaks at about January 20, that is, at the winter solstice. The mass density increases for  
 188 positive  $B_{z,3}$ . The simpler MLT dependence in (5) peaking at  $\text{MLT} = 15.6$  is now divided  
 189 into two terms, one peaking at  $\text{MLT} = 13.1$ , and a second F10.7 dependent term peaking  
 190 at  $\text{MLT} = 18.1$ . So the peak in  $\rho_{m,eq}$  is weaker and shifts from dusk toward noon local  
 191 time at large F10.7, which is characteristic of solar maximum.

192 There are diminishing returns as one goes to a more complicated model. Using just  
 193 F10.7, we can decrease the standard error of  $\log_{10} \rho_{m,eq}$  from 0.34 to 0.25. Adding MLT and  
 194  $P_{dyn,12}$  gets us down to 0.21. Adding  $dYr$ ,  $B_{z,3}$ , and  $\text{F10.7}_{192}$  in the most complicated model  
 195 only decreases the standard error of  $\log_{10} \rho_{m,eq}$  from 0.208 for (6) to 0.197. Nevertheless,  
 196 we do not consider even (7) to be excessively difficult to implement. Using just F10.7, we  
 197 can account for 45% of the variance (square of the standard error) of  $\log_{10} \rho_{m,eq}$ . Using  
 198 the most complicated formula (7), we can account for 66% of the variance.

199 In a not totally successful effort to model apparent refilling (to be described below), we  
 200 added dependence on the average of  $Kp$  during the preceding 12 and 48 hours,  $Kp_{12}$  and  
 201  $Kp_{48}$ , respectively. Figure 2a shows as a blue curve the binned values of  $\rho_{m,eq}$  divided  
 202 by the weighted log average of  $\rho_{m,eq}$ ,  $\rho_{m,eq,av}$ , versus  $Kp_{12}$ , the average of  $Kp$  over the  
 203 preceding 12 hours. The total weight of data points in each bin of width 0.2 is shown in  
 204 Figure 2b. For the vast majority of data points with  $Kp_{12}$  values near 2 (Figure 2b), the

205 dependence of  $\rho_{m,eq}/\rho_{m,eq,av}$  on  $Kp_{12}$  is very small. Because of this, including dependence  
 206 on  $Kp_{12}$  does not greatly affect our model for  $\log_{10} \rho_{m,eq}$  in a statistical sense. But we  
 207 hoped that it would affect the small number of data points with small  $Kp_{12}$ , for which  
 208  $\rho_{m,eq}/\rho_{m,eq,av}$  departs significantly from unity. We modeled the average dependence of  
 209  $\log_{10} \rho_{m,eq}$  using a polynomial of order 3,  $P_{12}(Kp_{12})$ , yielding the red curve in Figure 2a.  
 210 Similarly Figure 2c and d shows the same quantities but using  $Kp_{48}$ . More quantities were  
 211 tried, but these two quantities ended up having the largest coefficients in the modeling  
 212 expansion. (Values of maximum  $Kp$  over the preceding time period yielded a similar  
 213 dependence to that shown in Figure 2a and c.)

214 Our formula including  $P_{12}(Kp_{12})$  and  $P_{48}(Kp_{48})$  is

$$\begin{aligned}
 \log_{10} \rho_{m,eq} = & 0.32 + 0.0038F_{10.7_{96}} + 0.14 \cos((MLT - 12.7) \cdot 15^\circ) \\
 & + 0.055P_{dyn,12} + 0.07 \cos((dYr - 0.050) \cdot 360^\circ) + 0.015B_{z,3} \\
 & + 13 \cos((MLT - 18.5) \cdot 15^\circ) / F_{10.7_{192}} \\
 & + 0.50P_{12}(Kp_{12}) + 0.20P_{48}(Kp_{48}), \tag{8}
 \end{aligned}$$

215 where the polynomials

$$P_{12}(x) = -0.00853x^3 + 0.119x^2 - 0.444x + 0.45 \tag{9}$$

$$P_{48}(x) = -0.0122x^3 + 0.177x^2 - 0.719x + 0.82. \tag{10}$$

216 The unbiased weighted standard error for (8) is 0.19 corresponding to a variation of a  
 217 factor of  $10^{0.19} = 1.55$ . This is not significantly different statistically from that of (8), but  
 218 includes Kp dependence.

219 As an example, we show in Figure 3  $\rho_{m,eq}$  inferred from Alfvén waves measured by GOES  
 220 7 (thick blue curves) and that given by the most detailed model without Kp dependence

221 (7) (solid red curves) and the model with Kp dependence (8) (dotted red curves) during  
222 1991 versus day of year (DOY). This year was at solar maximum and was geomagnetically  
223 very active. The model describes well the daily MLT dependence and captures some of  
224 the longer timescale variation. Note, for instance the variation in  $\rho_m$  between day of year  
225 (DOY) 100 and 130. In this case, there is not much difference between the two models  
226 (solid and dotted red curves).

227 To better understand the causes of the variation between DOY 100 and 130, we plot  
228 in Figure 4 the mass density along with the instantaneous values of the geomagnetic  
229 indices and solar wind parameters described above for this time period. Between about  
230 DOY 106 and DOY 130, there is a roughly sinoidal oscillation in  $\rho_{m,eq}$ . This variation  
231 is caused mainly by an oscillation in F10.7 measured at the Earth's surface (Figure 4b)  
232 with a very small contribution from a similar oscillation in  $B_Z$  (Figure 4i). The period  
233 of this oscillation is roughly the period of a solar rotation (27 days as observed), and the  
234 variation is probably due to rotation of coronal hole structure on the Sun. This shows  
235 that relatively low F10.7 is not necessarily confined to solar minimum. The smaller peak  
236 in  $\rho_{m,eq}$  between DOY 115 and 117 is caused mainly by the peak in the dynamic pressure  
237  $P_{dyn}$  (Figure 4f) with a smaller contribution from the peak in  $B_Z$  (Figure 4i).

238 Figure 5 is similar to Figure 3, but showing the variation of  $\rho_{m,eq}$  during 1988. This is a  
239 quieter year and there is not as much variation other than the daily MLT variation. The  
240 model describes most of the variation in the observed  $\rho_{m,eq}$ , but there are some deficiencies.  
241 Note for instance the large inferred values of  $\rho_{m,eq}$  between DOY 25 and 40 (blue curves)  
242 that are not reproduced by the model (red curves).

243 In order to examine the causes of the evolution of  $\rho_{m,eq}$  during this time, we plot in  
 244 Figure 6 the mass density and geomagnetic indices and solar wind parameters between  
 245 DOY 25 and 40 in the same format as Figure 4. The large densities appear to be correlated  
 246 with low geomagnetic activity as indicated by low Kp, AE, and  $P_{dyn}$ . (Low values of  
 247  $E_{Ys}$  and  $d\Phi_{MP}/dt$  also occur at the time of the large  $\rho_{m,eq}$  values, but low values of  
 248 these quantities also occur when  $\rho_{m,eq}$  is relatively small, such as at DOY  $\sim 20$ .) These  
 249 conditions appear to be what we would normally associate with refilling. And note the  
 250 gradual increase in the inferred value of  $\rho_{m,eq}$  between DOY 22 and 26.

251 The model with Kp dependence does yield larger values of  $\rho_{m,eq}$  than does the model  
 252 without Kp dependence (comparing the dotted and solid red curves in Figure 6a), but the  
 253 Kp dependence is not strong enough to bring the Kp-dependent model (dotted red curve)  
 254 up to the level of the inferred mass density (blue curves). We tried arbitrarily increasing  
 255 the coefficients of the polynomial terms in (8), but in that case the model mass density  
 256 is too high in other regions. Perhaps a more sophisticated technique incorporating the  
 257 historical record of geomagnetic activity could be used [e.g., *Kondrashov et al., 2014*] to  
 258 get better agreement.

#### 4. Mass density refilling

259 Here we examine the apparent refilling of  $\rho_{m,eq}$  in more detail. Note that what we are  
 260 calling refilling may not be refilling of a particular flux tube. Rather it is the observed  
 261 change in  $\rho_{m,eq}$  at the location of the spacecraft versus time. Because the plasma does  
 262 not necessarily co-rotate with the Earth, we may at different times be sampling plasma  
 263 on different drift paths. Our best measure of apparent refilling will be the variation from  
 264 day to day at the same MLT location. Even in that case, the convection may evolve from



265 day to day so that the observed plasma is not on the same drift path, but we are more  
266 likely to be sampling similar plasma if we examine the variation from day to day.

267 We looked for events with low geomagnetic activity as indicated by Kp of no more than  
268 1.33 (when interpolated to an hourly value) for at least 2 days. We further required that  
269 in the hour preceding this quiet intervals, the average of Kp during the previous 12 hours  
270 had to be at least 1.75. This second criterion was so that we would have a shift from a  
271 more active time to a very quiet time. We found 10 intervals during 1980 to 1991 meeting  
272 these criteria and with inferred  $\rho_{m,eq}$  data extending at least 2 days. Figure 7 shows the  
273 Kp values for these events versus hour after the start of the low Kp period for the 10 events  
274 ordered with respect to F10.7 so that the event with lowest F10.7 (corresponding to solar  
275 minimum) is at the top of the figure in Figure 7a, while the event with the highest value  
276 of F10.7 (corresponding to solar maximum) is at the bottom of the figure in Figure 7k.

277 Figure 8 shows the inferred equatorial mass density at GOES (colored symbols and  
278 curves) and model mass density using (7) (solid light gray curves) and (8) (dotted light  
279 gray curves) versus hours after onset of low Kp for each of the events shown in Figure 7.  
280 The elaborate system of symbols (described in the figure caption) enables one to know  
281 the location of the spacecraft in MLT and to compare the mass density at a particular  
282 location to that at the same location on following days. For instance, the red squares  
283 show  $\rho_{m,eq}$  at MLT = 12 hr. By comparing the consecutive red squares from day to day,  
284 we can observe the apparent refilling at MLT = 12 hr. First, note that  $\rho_{m,eq}$  is generally  
285 higher at solar maximum (bottom panels in Figure 8) than at solar minimum (top panels  
286 in Figure 8) due to the F10.7 dependence of  $\rho_{m,eq}$  (in (4), for instance).

287 Some of the events in Figure 8 exhibit what appears to be refilling. Most notable among  
288 these are the ones shown in Figure 8b, c, f, and i. Apparent refilling can lead to  $\rho_{m,eq}$   
289 values significantly above that of our model (light gray curves in Figure 8). On the other  
290 hand some of the events do not seem to exhibit refilling at all. These include the events  
291 shown in Figure 8a, g, and j. In the other three events, there is only slight evidence of  
292 refilling. Thus it appears that apparent refilling is not as common for mass density as for  
293 electron density. (In the case of electron density, there are also quiet periods when the  
294 electron density does not appear to refill [*Denton et al.*, 2012], but such cases appear to  
295 be more frequent for  $\rho_{m,eq}$ .)

296 Based on these results, it is clear that  $\rho_{m,eq}$  does not behave the same for all quiet  
297 intervals. However, in order to develop some intuition about the average behavior, we  
298 take the log average of all data in four time intervals, the 24 hr interval preceding the  
299 onset of low Kp and the first, second, and third 24 hr intervals following the onset of  
300 low Kp. The results are shown in Figure 9. The black curve with circles shows the log  
301 average of all the data. For the day preceding the period of low Kp and the first day  
302 after the onset of low Kp (first two data points in Figure 9), these values are very close to  
303 the model values using (7) for the average parameters (solid light gray curve with squares  
304 in Figure 9) or (8) (dotted light gray curve with squares in Figure 9). But during the  
305 second and third days after the onset of low Kp (third and fourth data points in Figure 9),  
306 the log average  $\rho_{m,eq}$  based on all the data (black curve) rises significantly above that of  
307 the models (light gray curves). This indicates that on average there is apparent refilling  
308 during quiet intervals. Note that the Kp-dependent model (dotted light gray curve with

309 squares in Figure 9) does predict some apparent refilling, but not enough to explain the  
 310 data.

311 Considering that  $\rho_{m,eq}$  is greater at solar maximum than at solar minimum (e.g., com-  
 312 paring  $\rho_{m,eq}$  in the lower panels of Figure 8 to  $\rho_{m,eq}$  in the upper panels), it would not  
 313 be surprising if the apparent refilling is different at solar maximum from that at solar  
 314 minimum, and this is the case. The red curve with upward pointing triangles in Figure 9  
 315 shows the log average of  $\rho_{m,eq}$  during the same four daily intervals, but computing the  
 316 average only of the data with  $F10.7 > 150$  sfu, characteristic of solar maximum. On the  
 317 other hand, the blue curve with downward pointing triangles in Figure 9 is calculated  
 318 only using data with  $F10.7 < 100$  sfu, characteristic of solar minimum. In Figure 9, the  
 319 red curve starts out at higher values of  $\rho_{m,eq}$  and rises relatively less than the average of  
 320 all data (black curve), while the blue curve starts out at lower values of  $\rho_{m,eq}$  and rises  
 321 relatively more than the average of all data (black curve).

322 The three black or blue data points within the interval of low Kp (three data points  
 323 to the right of the vertical gray line in Figure 9) lie almost along a straight line using a  
 324 log scale (Figure 9b). This suggests exponential growth. Despite the fact that the three  
 325 red points do not lie on a straight line, we will characterize all three curves by the slope  
 326 between the first and third data points. We find then

$$\frac{d \log_{10}(\rho_{m,eq})}{dt} = 0.27 \text{ day}^{-1}, \text{ for all data,} \quad (11)$$

$$= 0.16 \text{ day}^{-1}, \text{ for } F10.7 > 150 \text{ sfu,} \quad (12)$$

$$= 0.35 \text{ day}^{-1}, \text{ for } F10.7 < 100 \text{ sfu,} \quad (13)$$

## 5. Discussion and Summary

327 For this study, we used the inferred equatorial mass density  $\rho_{m,eq}$  based on measurements  
328 of Alfvén wave frequencies measured by the GOES satellites during 1980–1991 along with  
329 a model for the field line dependence based on the same data set [*Denton et al.*, 2015].  
330 Using this data, we constructed a number of different models for the equatorial mass  
331 density at geostationary orbit (Section 3). The most complicated model with or without  
332 Kp dependence, (7) or (8), respectively, is able to account for 66% of the variance with a  
333 typical variation from actual values of a factor of 1.56. We also described some simpler  
334 models.

335 Of the factors influencing  $\rho_{m,eq}$  that we considered, the most important factor is the  
336 F10.7 EUV index. This presumably acts by increasing the ionospheric temperature and  
337 raising the scale height of the ions, making it easier for ions to overcome gravity and rise  
338 to the magnetic equator, especially for O+ that disproportionately affects  $\rho_m$  because of  
339 its high ion mass. Other factors may also be involved in getting O+ up to the equatorial  
340 magnetosphere, but increased ionospheric temperature certainly facilitates the process.

341 Mass accumulates as flux tubes convect eastward from midnight local time toward the  
342 afternoon local time sector, apparently because of continued refilling along the drift paths  
343 that extend eastward from the nightside magnetosphere to the afternoon local time sector.  
344 A drop in  $\rho_{m,eq}$  after dusk may occur because the high mass plasma is convected on open  
345 drift paths out toward the magnetopause [*Denton et al.*, 2014b].

346 The mass density is larger for larger solar wind dynamic pressure  $P_{dyn}$ . While we don't  
347 have a detailed explanation for this process, certainly increasing  $P_{dyn}$  leads to greater  
348 geomagnetic activity that could possibly lead to more mass.

349 There is a small dependence of  $\rho_{m,eq}$  on the phase of the year, indicating a seasonal effect.  
350 The mass density is greatest at a fraction of about 0.052 into the year, corresponding  
351 approximately to January 20, that is, the winter solstice. We don't currently have any  
352 explanation of this dependence. It is at most a factor of  $10^{0.08} = 1.20$  (equation (7)).

353 There is also a small dependence of  $\rho_{m,eq}$  on the solar wind  $B_Z$ . Positive  $B_Z$  is more  
354 likely to lead to a closed magnetosphere in which refilling can more easily occur.

355 Our model accounts for much of the variation in  $\rho_{m,eq}$ , but even the Kp-dependent  
356 model does not account well for refilling during extended geomagnetically quiet intervals.  
357 We need a better understanding of the factors that contribute to large  $\rho_{m,eq}$ .

358 For 10 especially quiet intervals, we considered long-term ( $> 1$  day) apparent refilling.  
359 We emphasize that apparent refilling is not necessarily refilling of the same flux tube.  
360 We found that the behavior of  $\rho_{m,eq}$  varies for different events. In some cases, there is  
361 significant apparent refilling, whereas in other cases  $\rho_{m,eq}$  stays the same or even decreases  
362 slightly.

363 Nevertheless, we showed that on average  $\rho_{m,eq}$  increases exponentially during quiet in-  
364 tervals. At solar maximum, the value of  $\rho_{m,eq}$  is larger at the beginning of the quiet  
365 interval, and the subsequent apparent refilling rate is less than that of all the data com-  
366 bined. On the other hand, at solar minimum, the value of  $\rho_{m,eq}$  is lower at the beginning  
367 of the quiet interval, and the subsequent apparent refilling rate is greater than that of all  
368 the data combined. On the third day of apparent refilling, the difference in  $\rho_{m,eq}$  at solar  
369 maximum or solar minimum is small compared to the difference in  $\rho_{m,eq}$  at the beginning  
370 of the quiet interval.

371 Global MHD models are only now starting to incorporate plasmaspheric plasma into  
372 simulations. When the only source of plasma comes from the solar wind, the simulation  
373  $\rho_{m,eq}$  is much lower than realistic. The models and refilling rates that we have described  
374 here are a starting point toward developing radially dependent models for  $\rho_{m,eq}$  that can  
375 be used to construct more realistic plasmasphere models for use in MHD codes. A study  
376 like this one, but incorporating radial variation, would help to achieve this goal.

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381 *drashov et al.* [2014] database was used to fill in missing values during data gaps  
382 (<http://onlinelibrary.wiley.com/doi/10.1002/2014GL059741/full>). Values of F10.7 come  
383 originally from NOAA's National Geophysical Data Center. Numerical data shown in this  
384 paper are available from the lead author upon request.

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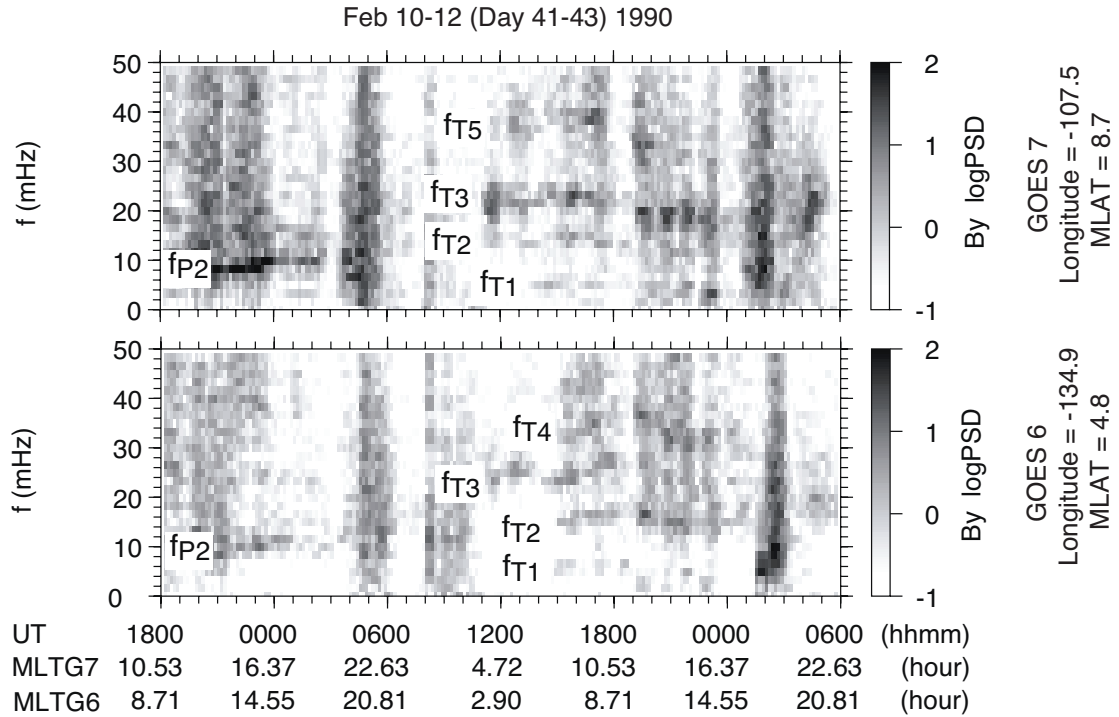


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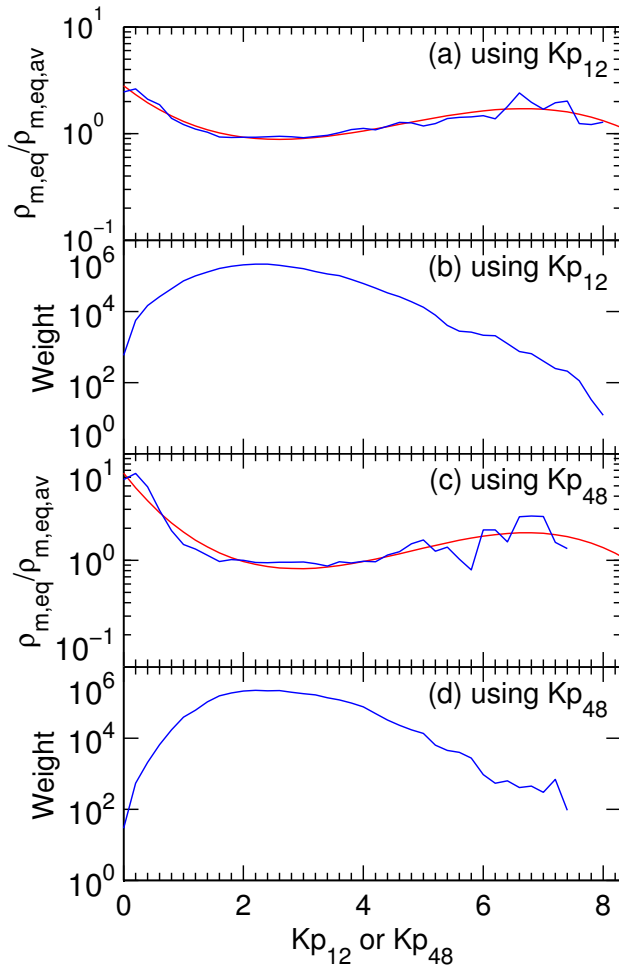
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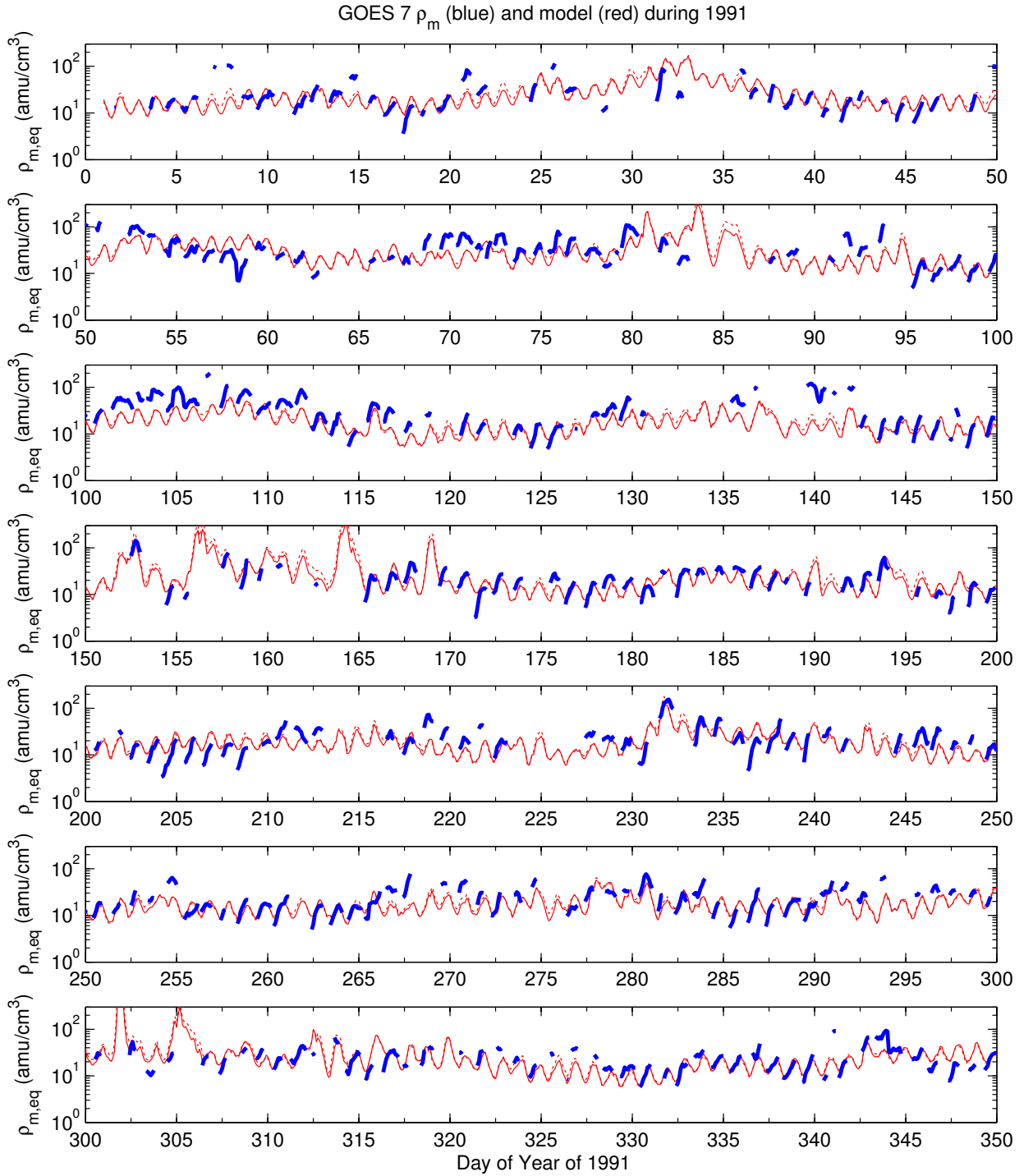
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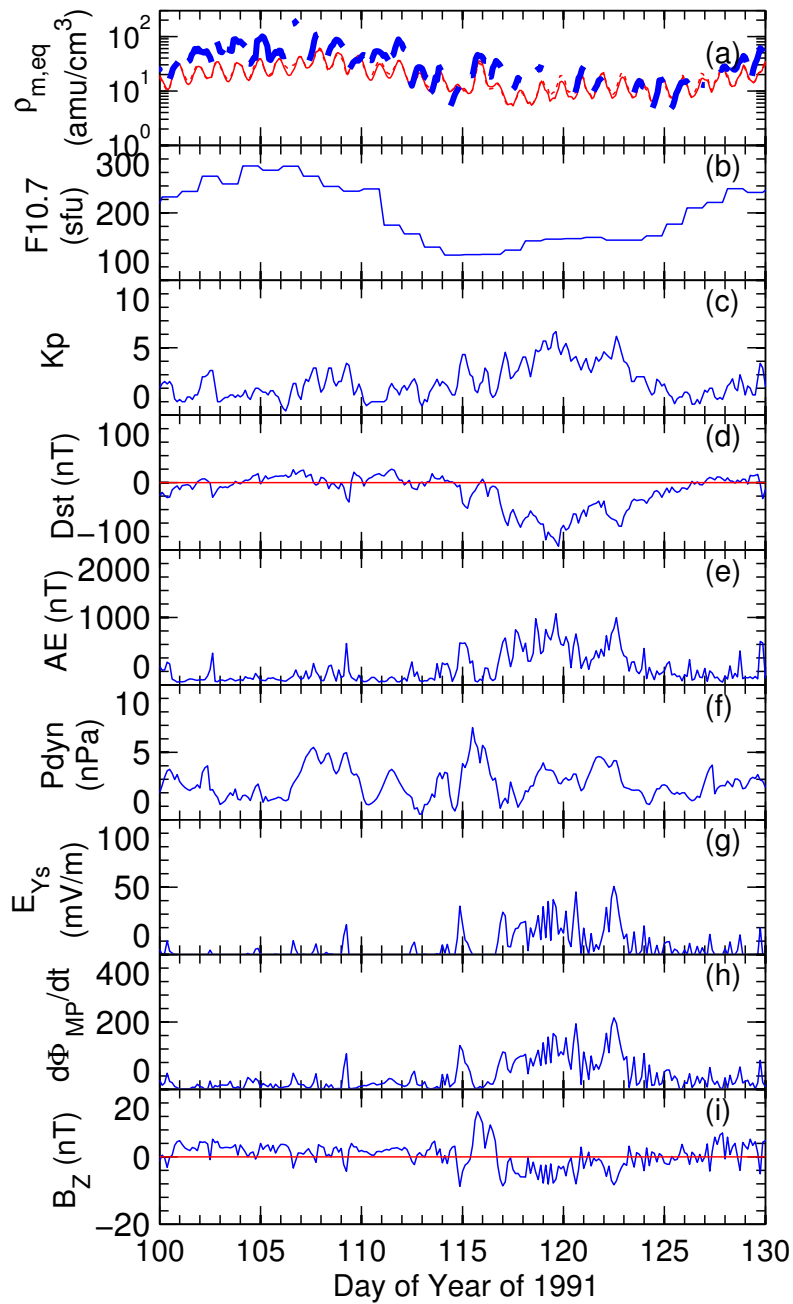
**Figure 1.** Dynamic spectra of the azimuthal component of the magnetic field,  $B_y$  observed by (bottom) GOES 6 and (top) GOES 7 for a 36 h period centered on 1200 UT of day of year (DOY) 42 (11 February 1990). Visible toroidal harmonics are labeled “fT1” through “fT5”. The isolated strong spectral line labeled “fP2” is attributed to the second harmonic poloidal wave [Cummings et al., 1969], based on the even stronger power in the  $B_x$  component (not shown). The MLT value for each satellite is shown at the bottom. (Reproduced from Figure 3 of *Takahashi et al.* [2010])



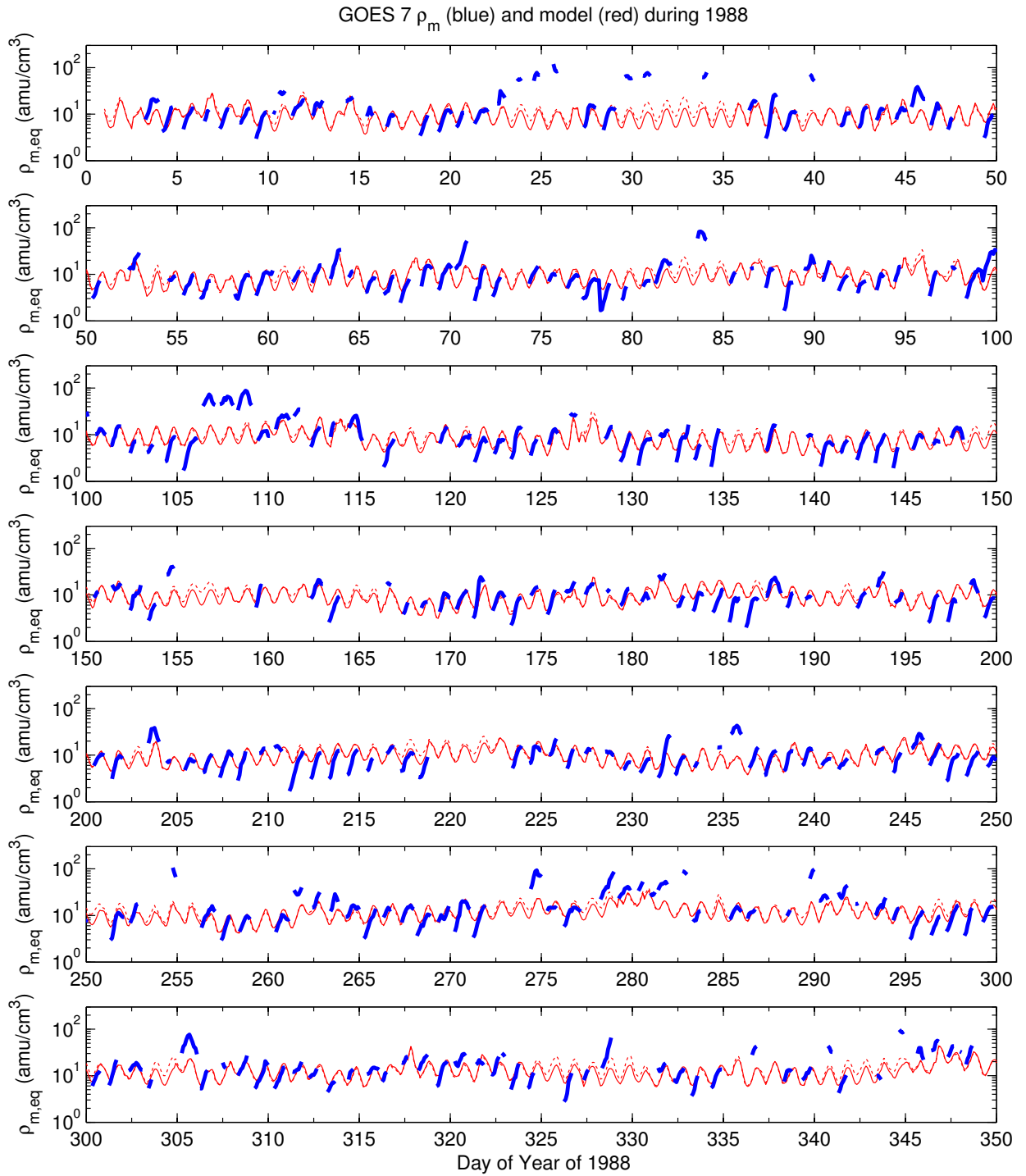
**Figure 2.** (a) Binned values of  $\rho_{m,eq}$  divided by the weighted log average of  $\rho_{m,eq}$ ,  $\rho_{m,eq,av}$ , and (b) weight in bins of width 0.2 versus  $Kp_{12}$ ; (c) and (d) are the same as (a) and (b), except using  $Kp_{48}$ .



**Figure 3.** Mass density inferred from Alfvén waves measured by GOES 7 (thick blue curves) and that given by the most detailed model without  $K_p$  dependence (7) (solid red curves) and the model with  $K_p$  dependence (8) (dotted red curves) during 1991 versus day of year (DOY).



**Figure 4.** (a) Mass density inferred from Alfvén waves measured by GOES 7 (thick blue curves) and that given by the most detailed model without Kp dependence (7) (solid red curves) and the model with Kp dependence (8) (dotted red curves) versus day of year (DOY) 100 to 130 during 1991. Panels (b) through (i) show instantaneous values of various geomagnetic indices and solar wind parameters as described in the text (blue curves). The red horizontal lines are at a value of zero.



**Figure 5.** Similar to Figure 3, but for 1988.

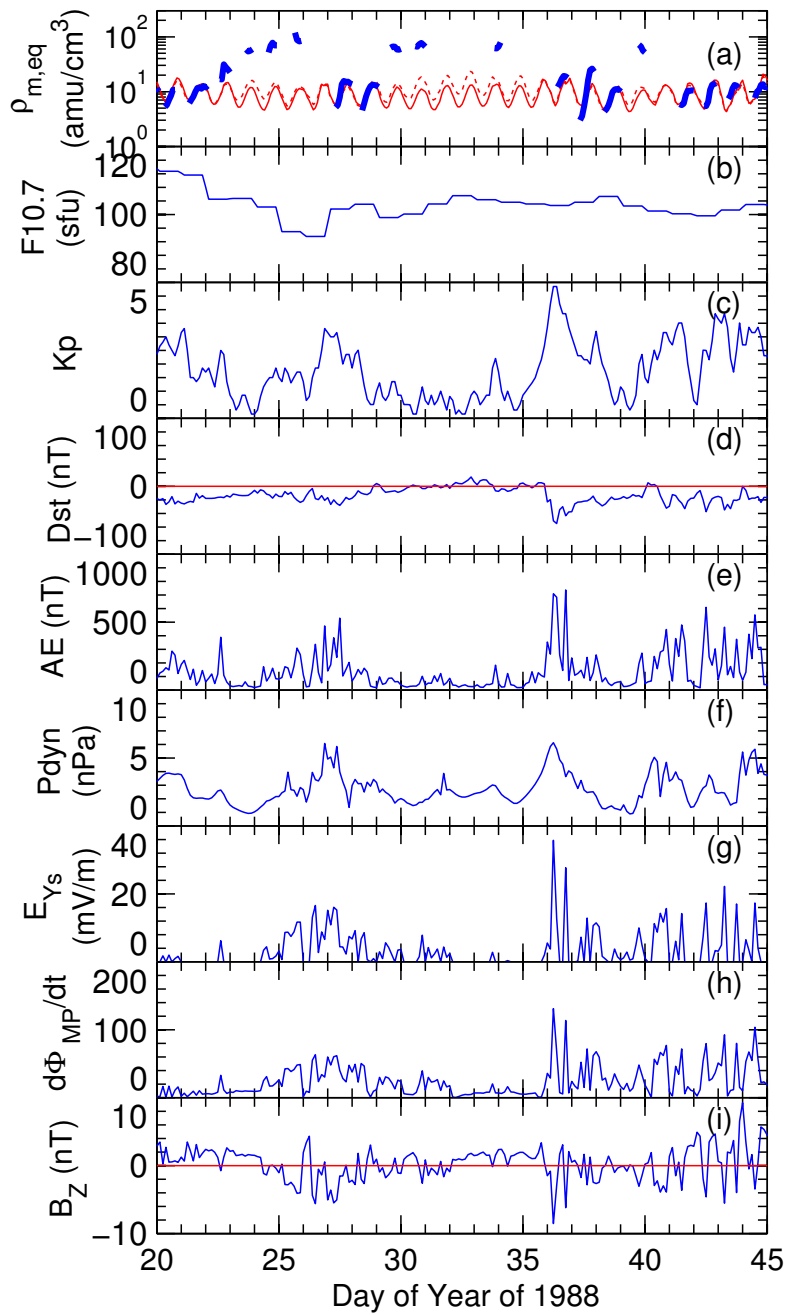
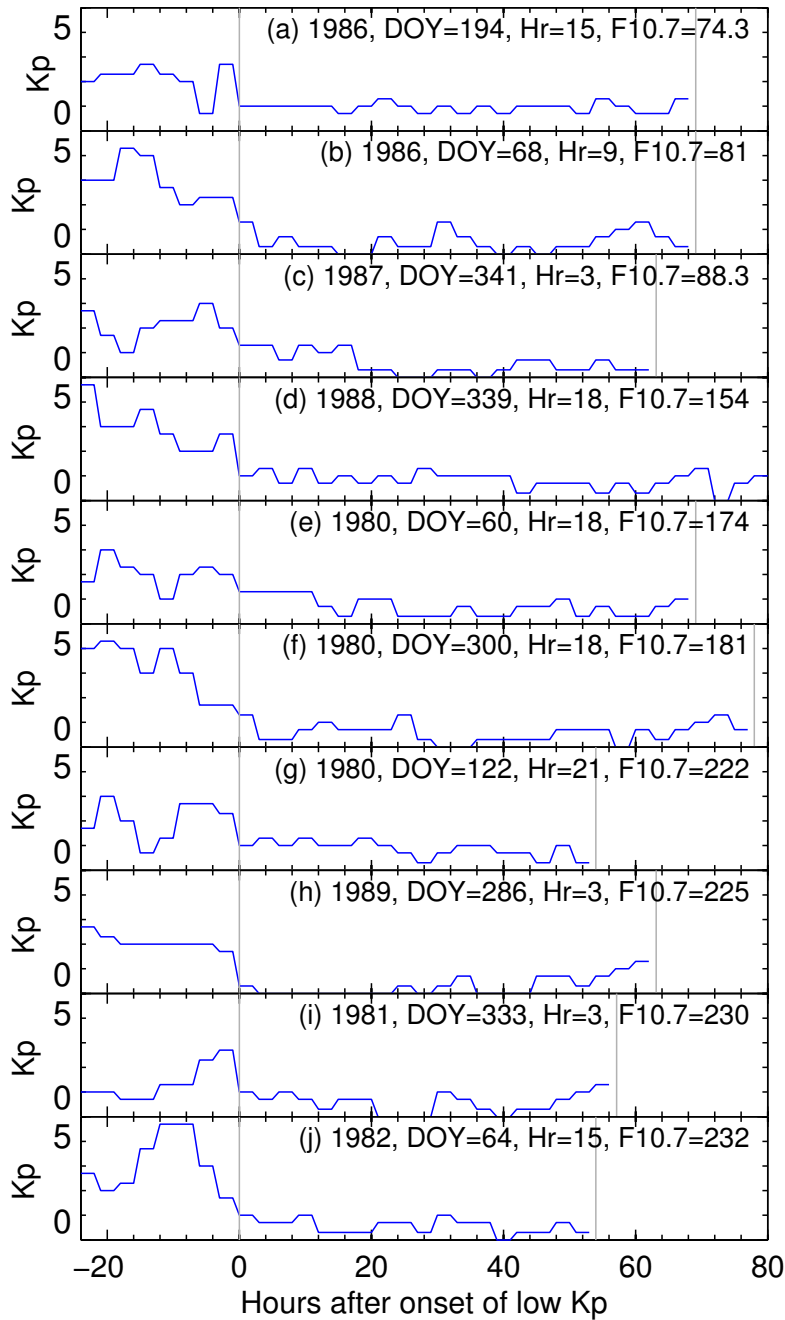


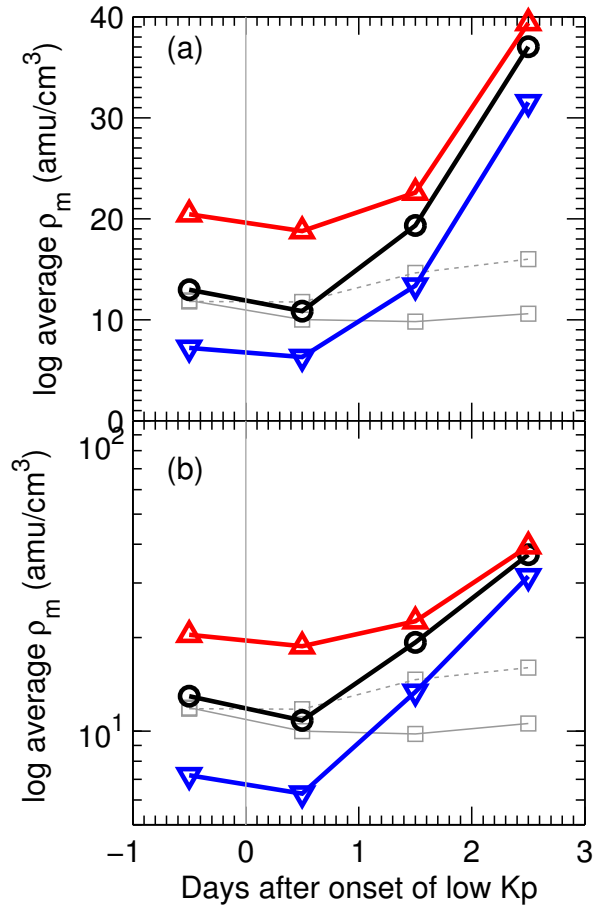
Figure 6. Similar to Figure 4, but for DOY 20 to 45 during year 1988.



**Figure 7.** Kp (interpolated to hourly values) versus hours after the beginning of the period of low Kp for the 10 events, as described in the text. The two vertical gray lines mark the beginning of the period of low Kp (leftmost gray vertical line) and the end of that period (rightmost gray vertical line).







**Figure 9.** Log average daily mass density versus days after onset of low Kp (vertical gray line) using a (a) linear or (b) log scale. The black curve with circles shows the log average of all the data, the red curve with upward pointing triangles shows the average of the data with  $F10.7 > 150$  sfu (solar max), and the blue curve with downward pointing triangles shows the average of the data with  $F10.7 < 100$  sfu (solar min). The light gray curves with squares shows the model values using (7) (solid light gray curve) or (8) (dotted light gray curve) for the average parameters.