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# Information-based trading and the bid-ask spread

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# Information-based trading and the bid-ask spread

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## Abstract

We analyze the equilibrium spread when the transaction size of informed traders is elastic in the value of private information ( $\alpha$ ). We show that the pooling equilibrium is likely to be inefficient when trade size is sensitive to  $\alpha$  and the inefficient equilibrium can occur before the market breaks down. The pooling equilibrium spread does not monotonically increase with  $\alpha$ , although it increases with the elasticity of informed trades to  $\alpha$ . The upper bound of the elasticity of informed trades for the market to remain open for the active specialist is higher than the corresponding value for the passive specialist when the specialist has enough leverage over brokers.

*JEL classification:* G18; G19

*Key words:* Spread; Specialist; Passive vs. Active Equilibrium; Informed Trading

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## 1. Introduction

Benveniste, Marcus, and Wilhelm (1992) (BMW hereafter) bring to light an important intermediary role of exchange specialists by showing that their long-term relationships with brokers help mitigate the effects of information asymmetry. The authors show that a specialist who actively differentiates between informed and uninformed traders through his long-term relationship with brokers can achieve equilibria that Pareto-dominate a pooling equilibrium in which he does not differentiate between the two types of traders. BMW provide an important insight that human intermediation is valuable in securities markets and the specialist system is a viable form of exchange markets.

BMW's paper has provided financial economists with theoretical foundations for a variety of empirical issues, including inter-market comparisons of execution quality (Huang and Stoll, 1996; Venkataraman, 2001), specialist behavior (Madhavan and Sofianos, 1998; Kavajecz, 1999; Harris and Panchapagesan, 2005; Anand and Weaver, 2006), the price impact of trades and price improvement (Chung, Chuwonganant, and McCormick, 2004), and the informational efficiency of market quotes (Chung, Chuwonganant, and Jiang, 2008).

BMW assume that the volume of liquidity trades is elastic in the cost of trading. Specifically, they assume that the volume of liquidity trades in each trading round depends on the absolute difference between the transaction price (i.e., the bid or ask price) and the security's intrinsic value. However, BMW assume that informed traders are constrained by an aggregate position limit and thus always trade a fixed size. BMW hold that the latter assumption is necessary to rule out infinite demand for securities that otherwise would result from the assumption of risk neutrality.

In this study, we analyze how informed agents' trading behavior affects market equilibrium by

assuming that the transaction size of informed traders is elastic in the expected profit of trades. Specifically, we assume that the transaction size of informed traders is a positive, linear function of the difference between the value of private information and the spread charged on their trades by the specialist.<sup>1</sup> The trading behavior of informed traders in our model is more in line with theoretical constructs and empirical evidence in prior studies than the one employed in BMW. For example, Easley and O'Hara (1987) examine the effect of trade size on stock prices. They show that trade size introduces an adverse selection problem into trading because informed traders, given that they wish to trade, prefer to trade large sizes at any given price. Consequently, market makers' pricing strategies depend on trade size, with large trades being executed at less favorable prices.

Our assumption on informed trading is also consistent with the one employed in Kyle (1985). In Kyle (1985), the informed traders want to trade aggressively, e.g., buying a large quantity if their information is positive. Although our model is similar in spirit to Kyle's model, it differs from his in that we neither assume that the market maker protects himself by setting a *price* that is increasing in the net order flow nor formulate the profit maximizing behavior of the informed trader *per se*.<sup>2</sup> Our model incorporates only the notion that informed traders submit larger orders when the value of their private information is greater.

Reexamining the BMW model under an alternative, more realistic assumption on informed trading is important for at least two reasons. First, it would be interesting to find out whether the main results of the BMW model (derived under the restriction assumption that informed traders always trade a fixed size) can also be obtained when informed traders trade larger sizes when they have more

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<sup>1</sup> In our model, the transaction size of informed traders is endogenously bounded to ensure that the market remains open.

<sup>2</sup> Although we do not model this behavior, our model predicts that the market maker quotes wider spreads when the elasticity of informed trades to trading profits is higher.

valuable information. Second, it would be interesting to find out whether the alternative assumption yields additional model implications.

As in BMW, we analyze the interaction between the specialist, broker, informed trader, and liquidity trader. We consider two types (i.e., passive and active) of specialists and consequently two types (i.e., pooling and separating) of equilibrium. We show that some of BMW's original results are robust to the behavior of informed traders. For example, we show that the result of BMW that the separating equilibrium Pareto-dominates the pooling equilibrium holds even when the transaction size of informed traders is elastic in the expected profit of trades. The specialist can allow more severe information asymmetry and both the uninformed and informed traders face more favorable trading terms in the separating equilibrium in which the specialist forces brokers to voluntarily disclose the type (i.e., informed vs. uninformed) of traders that they are representing. We also show that the power to sanction those who exploit private information can keep the market open and achieve an efficient equilibrium under conditions of severe adverse selection in which the pooling market would otherwise close if the active specialist has enough leverage over brokers.

More importantly, our model yields a number of new insights that are obscured by BMW's assumption that informed traders always trade a fixed size. First, we show that the pooling equilibrium is likely to be inefficient when trade size is highly sensitive to the value of private information ( $\alpha$ ) and the inefficient equilibrium can occur before the market breaks down. In contrast to BMW, the pooling equilibrium spread does not monotonically increase with  $\alpha$ . The pooling equilibrium spread increases with  $\alpha$  at the low level of  $\alpha$  and then decreases as  $\alpha$  becomes larger. The specialist narrows the spread as  $\alpha$  becomes larger because greater expected losses to informed traders can be offset by greater revenues from liquidity traders. This result differs sharply from the result of

the BMW model that the pooling equilibrium spread increases monotonically with  $\alpha$ . We also show that the pooling spread increases monotonically with  $\alpha$  if liquidity trading is inelastic to trading cost and this positive relation reflects the specialist's need to curve information-based trading given fixed liquidity trading. Finally, we show that the upper bound of the elasticity of informed trades to the value of private information for the market to remain open for the active specialist is higher than the upper bound for the passive specialist if the specialist has enough leverage over brokers.

## 2. The model

We follow the basic setup and nomenclature of BMW. All traders have the same information initially and thus assign an identical intrinsic value,  $p^*$ , to an asset. The only source of uncertainty is the possibility of a random shock to  $p^*$  before a round of trade that becomes public information only after the trading round is completed. The random shock, which occurs with probability  $\pi$ , will lead to a revision of the asset's intrinsic value to either  $p^* + \alpha$  or  $p^* - \alpha$ , with each revision having equal probability.

The volume of liquidity trades is elastic in the cost of trading. Specifically, liquidity trading volume in each round depends on the absolute difference between transaction prices and the asset's intrinsic value (i.e., ask price  $- p^*$  for buyer-initiated trades and  $p^* -$  bid price for seller-initiated trades) or one-half of the bid-ask spread charged on liquidity traders,  $S_t$ . A stable proportion of informed traders learn of a shock to  $p^*$  when it occurs. Thus with probability  $\pi$ , informed traders will participate in a round of trade. We assume that the trade size of informed traders depends on the value of private information. Informed traders learn of a shock to  $p^*$  when it occurs and use its value  $\alpha$  in their trading. Specifically, we assume that the transaction size of informed traders increases with the difference

between  $\alpha$  and the trading cost ( $s_i$ ) imposed on informed traders by the specialist.

Given the above setup, we now define the demand schedules of liquidity and informed traders as well as the revenue and cost functions of the specialist. As in BMW, the transaction size of liquidity traders ( $q_l$ ) is related to the spread ( $s_l$ ) in the following manner:

$$q_l = V(s_l) = q^* - \delta s_l, \quad (1)$$

where  $q^*$  is the transaction size of liquidity traders when the spread is zero and  $\delta$  denotes the sensitivity of liquidity trading to the spread. The transaction size of informed trades ( $q_i$ ) is assumed to be related to the net value of private information in the following manner:

$$q_i = V(s_i) = T(\alpha - s_i), \quad (2)$$

where  $T$  denotes the sensitivity of informed trading to the value of private information ( $\alpha$ ) net of trading cost ( $s_i$ ).

Because each liquidity trade crossed results in a net revenue of  $2s_l$  for the specialist, the specialist's total revenue from liquidity trading,  $R(s_l)$ , is

$$R(s_l) = 2s_l(q^* - \delta s_l). \quad (3)$$

The specialist anticipates informed buy orders for  $T(\alpha - s_i)$  shares when good information is forthcoming and informed sell orders for  $T(\alpha - s_i)$  shares when bad information is forthcoming. The cost per share to the specialist is the mispricing by  $\alpha$  net of the spread charged on the trades ( $s_i$ ). The total cost following a price shock is therefore  $T(\alpha - s_i)(\alpha - s_i)$ . Because the probability that information is forthcoming is  $\pi$ , the expected cost borne by the specialist in each period is

$$C(s_i) = \pi(\alpha - s_i)q_i = \pi T(\alpha - s_i)(\alpha - s_i). \quad (4)$$



### 3. Equilibrium and the specialist

We consider two types of specialists who differ in their ability to distinguish informed traders from liquidity traders.

#### 3.1. Equilibrium with a passive specialist

In this section, we consider a pooling equilibrium under the passive specialist system. This case is characterized by a passive specialist who, by definition, does not differentiate between informed and liquidity traders. The specialist uses an identical spread  $s_p = s_l = s_i$  for the two types of traders. The equilibrium with the passive specialist is defined by a pooling spread,  $s_p$ , for which the specialist's expected loss to informed traders is equal to the trading gain from liquidity traders. Replacing  $s_l$  and  $s_i$  in equations (3) and (4) with  $s_p$ , the zero-profit equilibrium condition becomes

$$R(s_p) - C(s_p) = 0. \quad (5)$$

By substituting equations (3) and (4) into equation (5), we obtain the following quadratic equilibrium equation:

$$2s_p(q^* - \delta s_p) - \pi T(\alpha - s_p)(\alpha - s_p) = 0. \quad (6)$$

Solving the quadratic equation yields two zero-profit spreads. See Figure 1 for a graphical illustration of the specialist's revenue and cost curves, together with two feasible spreads. Competition ensures that the equilibrium spread is the smaller of the two feasible spreads. If the specialist quotes a spread that is greater than  $\alpha$ , the informed traders will not trade. As a result, the cost curve  $C(s)$  coincides with the horizontal axis for spreads greater than  $\alpha$ . Therefore the pooling equilibrium spread is

$$s_p = \frac{q^*}{2\delta + \pi T} + \frac{\pi T \alpha - \sqrt{(\pi T \alpha + q^*)^2 - \pi T \alpha^2 (2\delta + \pi T)}}{2\delta + \pi T}. \quad (7)$$

Equation (7) shows that the pooling spread is zero if  $\pi$ ,  $\alpha$ , or  $T$  is zero, indicating that the non-zero pooling spread is a result of the informed traders' threat on the specialist. If the threat from informed traders is very large (i.e., either  $T$  or  $\alpha$  is large, which means that informed trading is less restricted or the value of private information is large), the term under the square root of equation (7) can be negative. In such a case, the market will breakdown because the specialist cannot compensate himself with the revenue from liquidity traders. We now examine under which conditions the term under the square root is negative, i.e.,

$$(\pi T \alpha + q^*)^2 - \pi T \alpha^2 (2\delta + \pi T) < 0. \quad (8)$$

Solving the above inequality in terms of  $T$  and  $\alpha$ , respectively, yields the following conditions:

$$\alpha > \frac{\pi T q^* + \sqrt{\pi^2 T^2 q^{*2} + 2\pi T \delta q^{*2}}}{2\pi T \delta} \quad \text{or} \quad (9)$$

$$T > \frac{q^{*2}}{2\pi \alpha (q^* - \alpha \delta)}. \quad (10)$$

From (9) and (10), the ranges of  $\alpha$  and  $T$  for which the market remains open are

$$0 \leq \alpha \leq \frac{\pi T q^* + \sqrt{\pi^2 T^2 q^{*2} + 2\pi T \delta q^{*2}}}{2\pi T \delta} \quad \text{and} \quad 0 \leq T \leq \frac{q^{*2}}{2\pi \alpha (q^* - \alpha \delta)}.$$

To shed further light on the equilibrium spread, we show that the equilibrium spread may be efficient or inefficient, where inefficiency is defined as follows.

*Definition.* A pooling spread  $s_p$  that satisfies (6) is inefficient if and only if there exist spreads  $s_l$  and  $s_i$  such that

- (i)  $s_l < s_p$ ,
- (ii)  $s_i < s_p$ , and
- (iii)  $C(s_i) \leq R(s_l)$ .

Lemma 1 in BMW characterizes the condition under which the pooling spread is inefficient,

$s_p > q^*/2\delta$ . By definition, the equilibrium is inefficient if it occurs at a point for which  $\partial s_p/\partial s_l > 0$ .

Applying this condition to  $2s_l(q^* - \delta s_l) - \pi T(\alpha - s_l)(\alpha - s_l) = 0$ , we find that any spread larger

than  $s_{\max} = q^*/2\delta$ , which maximizes the specialist's revenue, is inefficient. Because the specialist's

revenue function is identical between our and BMW models, Lemma 1 applies to our model as well.

Using Lemma 1, we now restate Theorem 1 in BMW, which gives the condition for the pooling

spread to be inefficient. By letting  $s_p > s_{\max} = q^*/2\delta$ , we obtain

$$\frac{q^*}{2\delta + \pi T} + \frac{2\pi T\alpha - \sqrt{(2\pi T\alpha + 2q^*)^2 - 4\pi T\alpha^2(2\delta + \pi T)}}{2(2\delta + \pi T)} > \frac{q^*}{2\delta}. \quad (11)$$

Solving (11) in terms of  $\alpha$  and  $T$  respectively, we obtain the following theorem:

*Theorem 1. The pooling equilibrium is inefficient if and only if*

$$\alpha > s_{\max} + \frac{q^*}{\sqrt{2\delta\pi T}} \quad \text{or} \quad T > \frac{2\delta q^{*2}}{\pi(2\alpha\delta - q^*)^2}.$$
<sup>3</sup>

Theorem 1 shows that the pooling equilibrium is likely to be inefficient either when the value of private information is large relative to the revenue generated by liquidity trading (as measured by  $q^*$ )

or when informed trading is highly sensitive to the value of private information (when  $T$  is large, i.e.,

informed trading is less restricted). This result expands BMW's Theorem 1 by explicitly incorporating

the elasticity of informed trading with respect to the value of private information into model

formulation.

Theorem 2 in BMW states that when there is nonzero probability of trade driven by private

information, there always exist values of  $\alpha$  for which a pooling equilibrium exists and is inefficient.

This implies that as the value of private information increases, the market first reaches an inefficient

equilibrium, and then breaks down. The proof of Theorem 2 requires that the upper bound of  $\alpha$  for the

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<sup>3</sup> Derivation of these results requires lengthy algebraic operations, which are available from the authors upon request.

market to remain open (i.e., the right-hand side of equation (9)) exceeds the upper bound of  $\alpha$  for the efficient pooling equilibrium, i.e.,

$$\frac{q^*}{2\delta} + \frac{\sqrt{\pi^2 T^2 q^{*2} + 2\pi T \delta q^{*2}}}{2\pi T \delta} > \frac{q^*}{2\delta} + \frac{q^*}{\sqrt{2\delta\pi T}}. \quad (12)$$

Since the above inequality always holds,<sup>4</sup> the market will first reach an inefficient equilibrium before it breaks down as the value of the private information increases. Hence, BMW's Theorem 2 holds even when the size of informed trades increases with the value of private information.

We now examine whether the upper bound of  $T$  for the market to remain open (i.e., the right-hand side of (10)) exceeds the upper bound of  $T$  for the inefficient pooling equilibrium, i.e.,

$$\frac{q^{*2}}{2\pi\alpha(q^* - \alpha\delta)} > \frac{2\delta q^{*2}}{\pi(2\alpha\delta - q^*)^2} \quad \text{or, alternatively,} \quad \frac{1}{2\alpha(q^* - \alpha\delta)} > \frac{2\delta}{(2\alpha\delta - q^*)^2}.$$

After rearrangement and simplification, we obtain the following results. If  $q^* - \alpha\delta > 0$ , (1) the market reaches an inefficient equilibrium as  $T$  increases before it breaks down when

$$0 < \alpha < \frac{q^*}{2\delta} - \frac{\sqrt{2}q^*}{4\delta} \quad \text{or} \quad \frac{q^*}{2\delta} + \frac{\sqrt{2}q^*}{4\delta} < \alpha < \frac{q^*}{\delta}; \quad \text{and (2) the market breaks down first as } T$$

increases before it reaches an inefficient equilibrium when  $\frac{q^*}{2\delta} - \frac{\sqrt{2}q^*}{4\delta} < \alpha < \frac{q^*}{2\delta} + \frac{\sqrt{2}q^*}{4\delta}$ .

If  $q^* - \alpha\delta < 0$ , the market breaks down quickly as  $T$  increases because either the specialist cannot take the loss from informed trading or liquidity trading reduces quickly, which in turn leaves the specialist with only informed traders.

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<sup>4</sup> Note that  $\frac{q^*}{2\delta} + \frac{\sqrt{\pi^2 T^2 q^{*2} + 2\pi T \delta q^{*2}}}{2\pi T \delta} > \frac{q^*}{2\delta} + \frac{q^*}{\sqrt{2\delta\pi T}} \Rightarrow \frac{\sqrt{\pi^2 T^2 q^{*2} + 2\pi T \delta q^{*2}}}{2\pi T \delta} > \frac{q^*}{\sqrt{2\delta\pi T}}$   
 $\Rightarrow \frac{\pi^2 T^2 q^{*2} + 2\pi T \delta q^{*2}}{4\pi^2 T^2 \delta^2} > \frac{q^{*2}}{2\delta\pi T} \Rightarrow \frac{\pi^2 T^2 + 2\pi T \delta}{2\delta\pi T} > 1.$

*The effect of model parameters on the pooling equilibrium spread*

In this section we examine how the pooling equilibrium spread is related to model parameters.

The partial derivative of the equilibrium spread with respect to  $\alpha$  is

$$\frac{\partial s_p}{\partial \alpha} = \frac{\pi T}{2\delta + \pi T} \left( 1 - \frac{q^* - 2\alpha\delta}{\sqrt{q^{*2} + 2q^* \pi T \alpha - 2\pi \alpha^2 \delta T}} \right). \quad (13)$$

The sign of (13) alternates from positive to negative at  $\alpha = q^*/\delta$ , showing that the pooling equilibrium spread does not monotonically increase with  $\alpha$  when the trade size of informed traders increases with  $\alpha$ .<sup>5</sup> The specialist widens the spread as  $\alpha$  increases at the low level of  $\alpha$  (i.e.,  $\alpha < q^*/\delta$ ). At  $\alpha = q^*/\delta$ , the specialist charges the maximum possible spread,  $s_p = \frac{\pi T}{2\delta + \pi T} \cdot \alpha$ . Once  $\alpha$  becomes larger than  $q^*/\delta$ , the specialist narrows the spread as  $\alpha$  increases. The specialist narrows the spread as  $\alpha$  increases because larger expected losses to informed traders can be offset by larger trading volume (and thus larger revenues) by (from) liquidity traders resulted from smaller spreads. This result differs sharply from the result of the BMW model. BMW show that the pooling equilibrium spread increases monotonically with  $\alpha$  when informed trading is inelastic to  $\alpha$ .

The partial derivative of the equilibrium spread with respect to  $\alpha$  is simplified to the following expression if  $\delta = 0$ :

$$\frac{\partial s_p}{\partial \alpha} = 1 - \frac{q^*}{\sqrt{q^{*2} + 2\pi T \alpha q^*}} > 0.$$

Hence, the pooling spread increases monotonically with  $\alpha$  if liquidity trading is inelastic to trading cost. Combined with the result in BMW, therefore, we conclude that the pooling spread increases

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<sup>5</sup> The value of  $\alpha$  at which market breaks down is greater than the value of  $\alpha$  at which  $s_p$  reaches its maximum value. To see this point, note that  $\frac{q^*}{2\delta} + \frac{\sqrt{\pi^2 T^2 q^{*2} + 2\pi T \delta q^{*2}}}{2\pi T \delta} > \frac{q^*}{\delta} \Rightarrow \frac{\sqrt{\pi^2 T^2 q^{*2} + 2\pi T \delta q^{*2}}}{2\pi T \delta} > \frac{q^*}{2\delta}$   
 $\Rightarrow \frac{\pi^2 T^2 q^{*2} + 2\pi T \delta q^{*2}}{\pi^2 T^2} > q^{*2} \Rightarrow \frac{\pi^2 T^2 + 2\pi T \delta}{\pi^2 T^2} > 1$  since  $2\pi T \delta > 0$ .

monotonically with  $\alpha$  if either demand for liquidity trades is inelastic to trading costs (i.e., spread) or demand for information-based trading is inelastic to trading profits. The positive relation between  $s_p$  and  $\alpha$  in the former case reflects the specialist's need to curve information-based trading given fixed liquidity trading. The positive relation between  $s_p$  and  $\alpha$  in the latter case reflects the specialist's need to set a wider spread when faced with larger expected losses to informed traders, which will counterbalance the reduced revenue from liquidity based trading.

The partial derivative of the equilibrium spread with respect to  $T$  is

$$\frac{\partial s_p}{\partial T} = \frac{\pi}{(2\delta + \pi T)^2} \left[ (2\alpha\delta - q^*) + \frac{q^{*2} + 4\delta^2\alpha^2 + \pi T\alpha q^* - 2\delta\alpha q^*}{\sqrt{q^{*2} + 2\pi T\alpha q^* - 2\pi T\alpha^2\delta}} \right] > 0.$$

The intuition behind this result is simple. All else being equal, the specialist's loss to informed traders increases with the informed traders' trading sensitivity to the expected trading profit. Therefore, the specialist increases the spread as  $T$  increases.

The partial derivative of the equilibrium spread with respect to  $q^*$  is

$$\frac{\partial s_p}{\partial q^*} = \frac{1}{\pi T + 2\delta} \left[ \frac{\sqrt{(q^* + \pi T\alpha)^2 - 2\pi T\alpha^2\delta - \pi^2 T^2\alpha^2} - (q^* + \pi T\alpha)}{\sqrt{q^{*2} + 2\pi T\alpha q^* - 2\pi T\alpha^2\delta}} \right] < 0.$$

The spread decreases with an increase in the fixed liquidity trading volume. All else being equal, the larger fixed liquidity volume increases the specialist's revenue for any spread and allows the specialist to quote a narrower spread in equilibrium.

### **3.2. Equilibrium with an active specialist**

As in BMW, we assume that an active specialist seeks to distinguish between informed and liquidity-motivated trades. Traders are represented by brokers and the specialist separates informed trades from liquidity-motivated trades to the extent that brokers are able to do so. The active specialist

differs from the passive one in several important ways. The active specialist may observe after the fact whatever information was available to the broker at the time of a trade. The active specialist may sanction brokers who misrepresented an informed trade as a liquidity trade. In addition, the active specialist can increase the value of broker's franchise by offering terms of trade that depend on the broker's representation of trading motive.

The optimization problem for the specialist, acting as a rational agent, is to quote a spread pair  $(s_i, s_l)$  that maximizes the broker's expected profits subject to the zero-profit constraint and to incentive-compatibility constraints that induce brokers to reveal their signals truthfully BMW (p. 75). We identify the range of pairs that is consistent with potential equilibrium.

As in BMW, we call  $\lambda$  ( $0.5 \leq \lambda \leq 1$ ) the probability that the broker's inference about the nature of the trade is correct and assume that the probability of a correct inference is the same whether the trade is liquidity- or information-motivated. We further assume that with probability  $\gamma$  the specialist will observe the noisy signal the broker receives at the time of the trade. It is not relevant to our analysis whether the broker was correct; it matters only that there is some probability that the specialist can eventually infer whether the broker intended to be truthfully.

The specialist gains from liquidity-motivated traders and loses to traders with superior information. Because of the potential misidentification by the brokers, risk-neutral traders make their trading decisions based on the probability-weighted average of the spread pair  $(s_i, s_l)$  quoted by the specialist. Therefore, the expected spread faced by liquidity and informed traders and their respective demand schedules are as follows:

$$s_i^e = \lambda s_l + (1 - \lambda) s_i, \tag{14}$$

$$s_l^e = \lambda s_i + (1 - \lambda) s_l, \tag{15}$$

$$q_l = V(s_l^e) = q^* - \delta s_l^e, \text{ and} \quad (16)$$

$$q_i = V(s_i^e) = T(\alpha - s_i^e). \quad (17)$$

The specialist maps expected spreads  $(s_l^e, s_i^e)$  into actual spreads  $(s_l, s_i)$  for a given value of  $\lambda$  according to the following function:

$$(s_l, s_i) = S(s_l^e, s_i^e, \lambda). \quad (18)$$

The specialist's revenue and cost functions are  $R(s_l^e) = 2s_l^e(q^* - \delta s_l^e)$  and  $C(s_i^e) = \pi T(\alpha - s_i^e)^2$ , respectively. Thus, the zero-profit condition requires that the specialist to set  $(s_l^e, s_i^e)$  to satisfy

$$2s_l^e(q^* - \delta s_l^e) - \pi T(\alpha - s_i^e)^2 = 0. \quad (19)$$

The spread pair quoted by the specialist must also satisfy the following two incentive compatibility conditions. First, for a broker to reveal truthfully whether a trade is liquidity motivated, the liquidity spread cannot exceed the informed spread. This is the incentive compatible constraint for the brokers who are representing liquidity trading,

$$s_i \geq s_l. \quad (IC_l)$$

Second, for the brokers to truthfully disclose the informed trades, the threat by the specialist contingent on detection of intentional misrepresentation must exceed the benefit from doing so. Following BMW (p. 77), we define the incentive compatible constraint for the brokers who are representing informed trading as

$$s_i - s_l \leq \gamma[N(s_i - s_l) + C]. \quad (IC_i)$$

where  $N$  is the number of broker orders over some interval that will be treated by the specialist as information-motivated,  $C$  is the cost imposed by the specialist on brokers who misrepresented trade motives, and all other variables are the same as previously defined.

Figure 2 shows the graphic representation of equilibrium with an active specialist. The vertical



axis represents either the quoted informed spread or the expected informed spread. Likewise, the horizontal axis represents the quoted or expected spread for liquidity traders. Solving for the zero-profit condition (19) for  $s_i^e$  yields

$$s_i^e = \alpha - \sqrt{\frac{2s_i^e(q^* - \delta s_i^e)}{\pi T}}. \quad (20)$$

Incentive compatibility for liquidity traders requires that the equilibrium quoted spread pair lie to the left of  $(IC_1)$ . Incentive compatibility of informed traders requires that the equilibrium quoted spread pair lie below  $(IC_1)$ . Admissible solutions to the specialist's maximization problem for a given  $\lambda$  lie on  $S(\lambda)$ , and thus are bounded by  $\underline{s}$ , which represents the quoted liquidity trade spread for which  $(IC_1)$  is binding, and  $\bar{s}$ , which is equal to the pooling equilibrium spread  $s_p$ .

The minimum point along the zero-profit locus occurs at  $s_{\max}$ , corresponding to the value of  $s_i^e$  that maximizes the expected revenue from liquidity trades. As in the case of the pooling equilibrium,  $s_{\max} = q^*/2\delta$ . Combinations of  $s_i^e$  and  $s_i^e$  to the right of  $s_{\max}$  are strictly inefficient. Both liquidity and informed traders can be made better off in the region by lowering both  $s_i^e$  and  $s_i^e$  along the zero-profit locus. The locus of points labeled  $S(\lambda)$  represents the mapping using the zero-profit locus of expected spreads into the corresponding locus of quoted-spread pairs for a given value of  $\lambda$ . For  $\lambda < 1$ , the locus of zero-profit actual spread pairs will lie above and to the left of the locus of zero-profit expected spread pairs.  $(IC_1)$  constrains the solution to the specialist's maximization problem to  $(s_i^e, s_i^e)$  pairs to the left of  $(IC_1)$ .  $(IC_1)$  constrains the solution to  $(s_i, s_i)$  pairs to the right of the boundary it defines. Thus, the specialist is constrained to quoting spread pairs along the segment  $S(\lambda)$  bounded by  $(IC_1)$  and  $(IC_1)$  in Figure 2 given the value of  $\lambda$  as the informativeness of the broker's signal. This corresponds to expected liquidity spreads between  $\underline{s}$  and  $\bar{s}$ .

*Market breakdown condition*

Substituting (14) and (15) into (19), and letting  $(IC_i)$  binding, the lower bound of  $s_l$  (i.e.,  $\underline{s}$ ) must satisfy the following condition

$$2[\lambda \underline{s} + (1 - \lambda)(\underline{s} + a)][q^* - \delta[\lambda \underline{s} + (1 - \lambda)(\underline{s} + a)]] - \pi T[\alpha - [\lambda(\underline{s} + a) + (1 - \lambda)\underline{s}]]^2 = 0 \quad (21)$$

where  $a = \frac{C\gamma}{1 - \gamma N}$ .

Solving equation (21) for  $\underline{s}$  and taking the smaller root, we obtain:

$$\begin{aligned} \underline{s} = \frac{1}{2\delta + \pi T} [ & 2\delta\lambda a - \pi T\lambda a - 2\delta a + \pi T\alpha + q^* - (q^{*2} + 2\pi T a q^* - 8\delta\lambda^2 a^2 \pi T + 8\delta\lambda a \pi T\alpha \\ & + 8\pi T\lambda a^2 \delta - 4\delta a \pi T\alpha - 4\pi T\lambda a q^* + 2\pi T\alpha q^* - 2\pi T\delta a^2 - 2\delta\pi T\alpha^2)^{\frac{1}{2}} ]. \end{aligned} \quad (22)$$

From the above equation, the condition that must be satisfied for equilibrium to exist with an active specialist is

$$\begin{aligned} q^{*2} + 2\pi T a q^* - 8\delta\lambda^2 a^2 \pi T + 8\delta\lambda a \pi T + 8\pi T\lambda a^2 \delta - 4\delta a \pi T\alpha - 4\pi T\lambda a q^* + 2\pi T\alpha q^* \\ - 2\pi T\delta a^2 - 2\delta\pi T\alpha^2 \geq 0. \end{aligned} \quad (23)$$

Inspection of (23) reveals that the upper bound of  $\alpha$  for the market to remain open is

$$\alpha = \frac{q^*}{2\delta} + \frac{\gamma C}{1 - \gamma N} (2\lambda - 1) + \frac{\sqrt{q^{*2} \pi^2 T^2 + 2\delta\pi T q^{*2}}}{2\pi T\delta}. \quad (24)$$

Comparing (24) with (9) ( $\alpha = \frac{\pi T q^* + \sqrt{\pi^2 T^2 q^{*2} + 2\pi T\delta q^{*2}}}{2\pi T\delta}$ ), we find that the upper bound of  $\alpha$

for the market to remain open for the active specialist is greater than that for the passive specialist by

$\frac{\gamma C}{1 - \gamma N} (2\lambda - 1)$ , where  $\frac{\gamma C}{1 - \gamma N}$  measures the specialist's ability to sanction brokers identified as

having exploited private information. Hence, our result reveals that for a given set of parameters for

which the passive specialist closes the market, the active specialist will keep the market open if he has

enough leverage ( $\frac{C\gamma}{1 - \gamma N}$ ) over brokers.

Inspection of (23) also shows that the upper bound of  $T$  for the market to remain open is

$$T = \frac{q^{*2}}{2\pi} \cdot \frac{1}{(-4\delta\lambda\alpha + 4\delta\lambda^2 a^2 - q^* \alpha - q^* a - 4\delta\lambda a^2 + \alpha^2 \delta + 2\delta\alpha a + \delta a^2 + 2\lambda a q^*)}. \quad (25)$$

Comparing (25) with (10) ( $T > \frac{q^{*2}}{2\pi\alpha(q^* - \alpha\delta)}$ ), it can be shown that the upper bound of  $T$  for the

market to remain open for the active specialist is higher than the upper bound of  $T$  for the passive specialist when

$$\frac{C\gamma}{1 - \gamma N} > \frac{1}{2\delta} \cdot \frac{[-q^* + 2\alpha\delta + (q^{*2} + 4\alpha\delta(q^* - \alpha\delta))^{\frac{1}{2}}]}{(2\lambda - 1)}. \quad (26)$$

Hence our result shows that for a given set of parameters for which the passive specialist closes the market, the active specialist will always keep the market open if he has enough leverage ( $\frac{C\gamma}{1 - \gamma N}$ ) over brokers.<sup>6</sup>

The results in Section 4.2 of BMW's paper are valid in the extended setting. Lemma 2 and Theorem 3 in BMW remain valid in our model because they follow directly from the model setup. Hence, the equilibrium value of  $s_i^e$  is in the set bounded by  $\underline{s} \leq s_i^e \leq \bar{s}$  and the expected spread faced by liquidity traders under an active specialist is always smaller than or equal to the spread set by a passive specialist. The ability to sanction those brokers who attempt to disguise the motives for trading gives the specialist the leverage necessary to weaken the adverse selection problem and thereby improve the terms of trade for liquidity traders.

Likewise, Lemma 3 and Theorem 4 in BMW continue to hold in our model.<sup>7</sup> Hence the

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<sup>6</sup> It can be shown that  $\frac{\partial s}{\partial \alpha} > 0$  if  $a(2\lambda - 1) < \alpha < a(2\lambda - 1) + \frac{q^*}{\delta}$ ,  $\frac{\partial s}{\partial \alpha} \leq 0$  otherwise; and  $\frac{\partial s}{\partial q^*} < 0$  if  $a \neq \frac{\alpha}{(2\lambda - 1)}$ .

<sup>7</sup> The equilibrium is inefficient if it occurs at a point for which  $\frac{\partial s_i^e}{\partial s_i^e} > 0$ . Applying this condition

specialist's ability to improve the terms for liquidity traders is enhanced as the broker's ability to infer the motivation for a trade increases (i.e.,  $\partial \underline{s} / \partial \lambda \leq 0$ ). Also, the specialist's ability to differentiate between liquidity and informed traders improves the terms of trade for both traders. The spread quoted by a passive specialist is more likely to be inefficient when the information asymmetry is higher. Thus, the specialist's leverage over brokers is most likely to lead to better terms for informed traders when their information advantage is greatest. The ability to charge lower spreads to liquidity traders under higher informational asymmetry generates increased revenues that can be used to reduce the spread charged to informed traders.

Lemma 2 shows that the expected-spread pair under the active specialist will always be efficient as long as the pooling equilibrium spread is efficient. Theorem 5 in BMW shows that the active-specialist equilibrium could be efficient even when the pooling equilibrium is not if the specialist has enough leverage over brokers identified as having exploited private information. By setting  $\underline{s} \leq s_{\max}$  and after simplification, it can be shown that market parameters that result in an inefficient equilibrium in a pooling environment will result in an efficient equilibrium in a separating environment if the specialist's ability to sanction brokers identified as having exploited private information satisfies the following condition:

$$\frac{C\gamma}{1-\gamma N} > \frac{K}{2(-4\delta\lambda + 2\lambda^2\delta + 2\delta + \pi T\lambda^2)}, \quad (27)$$

where  $K = -2\pi T\lambda s_{\max} + 2q^* + 4\delta\lambda s_{\max} - 4s_{\max}\delta + 2\pi T\alpha\lambda - 2\lambda q^*$

$$+ 2(-8\pi T\lambda^2 s_{\max}^2 \delta + q^{*2} - 2\pi T\alpha^2 \delta - 2s_{\max}^2 \delta \pi T + 4\pi T\alpha \delta s_{\max} - 2\pi T\lambda q^* s_{\max} + 2q^* \pi T\alpha\lambda + 4\delta\lambda \pi T\alpha^2 - 2\lambda^2 \pi T\alpha^2 + 8\pi T\lambda s_{\max}^2 \delta + 4\pi T\lambda^2 s_{\max} q^* + 8\delta\lambda^2 s_{\max} \pi T\alpha - 12s_{\max} \delta \pi T\alpha\lambda - 2\pi T\alpha\lambda^2 q^* - 2\lambda q^{*2} + \lambda^2 q^{*2})^{\frac{1}{2}}$$

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to  $2s_i^e (q^* - \delta s_i^e) - \pi T(\alpha - s_i^e)^2 = 0$ , we can show that Theorem 4 holds in our model.

Thus the active specialist always produces Pareto-efficient terms of trade if he has enough leverage over brokers even when the volume of informed trades is elastic in trading profits.

#### **4. Conclusion**

Benveniste, Marcus, and Wilhelm (1992) bring to light an important intermediary role of exchange specialists. They show that a specialist who distinguishes informed traders from uninformed traders achieves equilibrium that Pareto-dominates a pooling equilibrium in which he does not differentiate between the two types of traders. We extend their study by replacing its restrictive assumption that informed traders always trade a fixed size with a more reasonable assumption that informed traders buy/sell larger sizes when they have more valuable private information.

Our analyses show that although some of BMW's results remain valid under the alternative assumption, the assumption of inelastic informed trading obscures a number of other valuable implications of the model. By reformulating the model with a more reasonable assumption that the transaction size of informed traders is elastic in the expected profit of trades, we show that the model yields many more valuable insights. For example, we show that the pooling equilibrium spread does not monotonically increase with the value of private information. We also show that the pooling equilibrium spread increases with  $\alpha$  if liquidity trading is inelastic to trading cost. We obtain several other interesting results that shed further light on the effect of informed trading on market liquidity.

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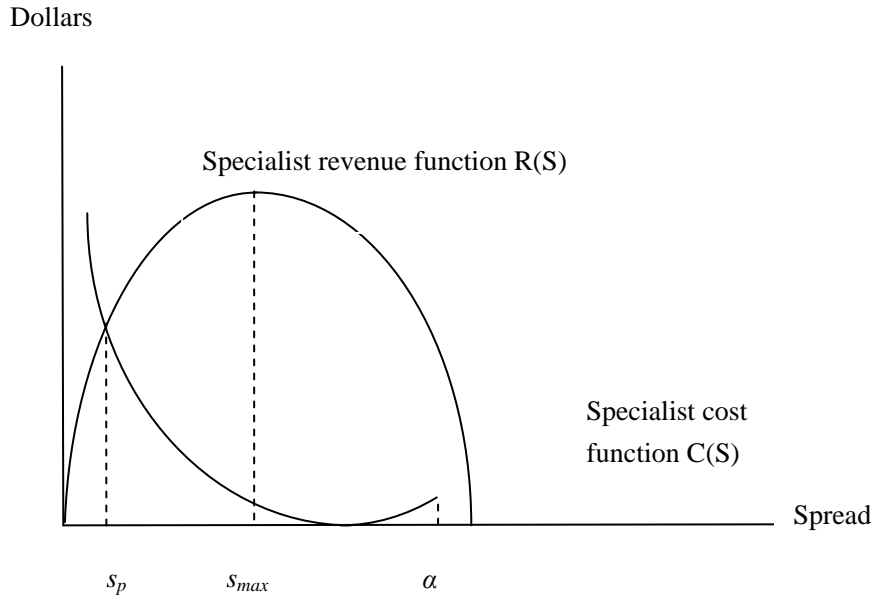


Fig. 1. The passive specialist equilibrium.  $R(S)$  and  $C(S)$  are the revenue and cost curves of the specialist.  $s_{max}$  is revenue maximized spread. Intersections of  $R(S)$  and  $C(S)$  determine solutions to the specialist's problem. However, competition requires that the smaller spread to be the feasible pooling spread. If the specialist quotes a spread that is greater than  $\alpha$ , the informed traders will not trade. As a result, the cost curve  $C(s)$  coincides with the horizontal axis for spreads greater than  $\alpha$ .

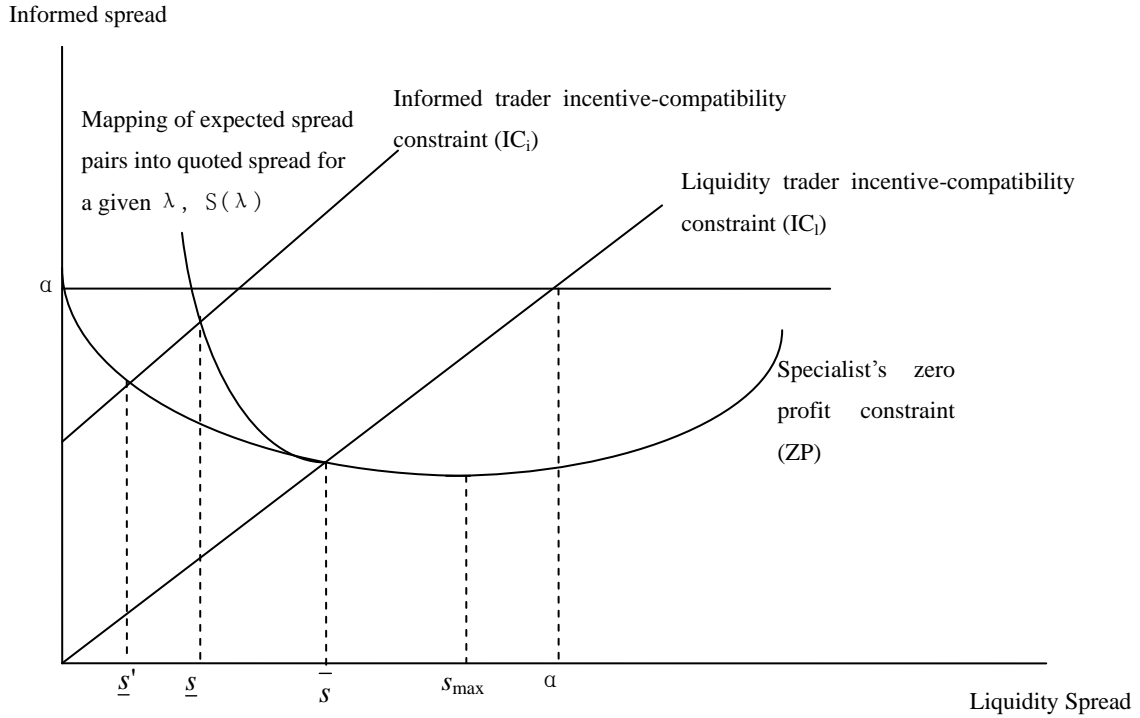


Fig. 2. The active specialist equilibrium. The specialist's zero-profit constraint (ZP) determines the locus of the expected spread pair  $(s_t^e, s_i^e)$ .  $S(\lambda)$  represents the mapping of expected spread pairs into quoted spread pair for a given  $\lambda$ . Incentive compatible for liquidity trader requires that the equilibrium spread pair lie to the left of  $(IC_l)$ . Incentive compatible for informed trader requires that the equilibrium spread pair lie to the right of  $(IC_i)$ . Feasible solutions to the specialist's maximization problem for a given  $\lambda$  lie on  $S(\lambda)$ , and thus are bounded by  $\underline{s}$ , which represents the quoted spread liquidity spread for which  $(IC_i)$  is binding, and  $\bar{s}$ , which is equivalent to the pooling spread.