【Article】

Cellular Automaton Model of Earthquake and Renormalization Method

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Abstract Earthquakes are phenomena associated with rupture of a part of the earth crust and the subsequent propagation of breakup front or energy front over wider regions of the crust or a plate. Primitive Cellular Automaton Model treats this phenomenon by dividing the crust into many small square cells and introducing interactions among (nearest) neighbours. The state of each cell is specified by the energy (or, equivalently, the number of particles just like sand pile model) stored in it. When the stored energy exceeds a critical value, the cell gives the energy to the neighbouring cells. This model belongs to a conventional cellular automaton model in that the same rule for redistributing energy is adopted. The major differences of our model from the preceding ones lay on the choice of the initial condition and the cells to be revised : All cells are arranged initially to be below but near critical state in order to simulate large earthquakes. In addition, all the cells are subjected to the revision of state at every time step. We employ the renormalization method in statistically analyzing the outcomes of the simulations to show that this model is capable to describe the causal correlations of main and aftershocks of a single strong earthquake. In particular, the data obey relations analogous to the Omori law for aftershocks and the Gutenberg-Richter-Ishimoto-Iida law for seismic magnitudes.

Key Words : earthquake, Omori law, Gutenberg-Richter-Ishimoto-Iida law, cellular automaton, renormalization

1. Introduction

Earthquakes are seismic phenomena associated with a partial rupture of the terrestrial crust. The driving forces are the friction between two plates in relative motion (∼a few cm/year) and the restitution energy accumulated in the deformed crust either at or beneath the palate interface. For the latter case, see Hasegawa et al. (1978). (The terminology ‘stress’ is more common in literature since the rupture is believed to be of crucial process in earthquake. We instead use the term ‘energy’ throughout the paper since it is scalar and will be conceptually easier to handle.) Either a breakup or restitution will take place at the plate interface when the accumulated energy in a part of the plates overwhelms the interplate frictional energy. In some case, the rupture takes place at deep beneath of the

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plate interface. Thus the long-term occurrences of earthquakes are expected to be governed by the dynamics of the plates in relative motions.

The prevailing block model simulates earthquakes of interpolate origin by sudden and intermittent motions of blocks on a plate that were at rest owing to the frictional forces between the blocks and the plate. The sudden motion of a block is caused by the elastic energy stored and accumulated between neighbouring blocks. The frictional energy is assumed to be a function of relative velocity and the state of the block, which is described usually by a single phenomenological parameter (Ruina 1983). Nature of the frictional force as well as the elastic energy of rock is extracted from experiments in laboratory (Dietrich 1979). Once the nature of the frictional and elastic forces are specified, the block model can dynamically determine the motion of the blocks by way of the Newtonian dynamics and consequently the seismic size by the number of blocks that move simultaneously. Supplemented by the data of past earthquakes, the block model has been used even to foresee coming earthquake at given districts. For the precise structure of the model and its application for prediction of earthquakes, see, e.g., Kato and Hirasawa (1999) and references cited therein.

Earthquake is a super-macroscopic phenomenon consisting of a huge number of individual motions of 'elements' in the crust. The block model aims to understand this super-macroscopic phenomenon in terms of macroscopic and microscopic mechanisms. On the other hand, there exists a possibility to view earthquake as a critical phenomenon in which a small change of the state indefinitely grows under some appropriate conditions. In terminology of statistical physics, the correlation length becomes large or infinite when critical phenomenon occurs. The phase transitions of the second kind observed in various matter as temperature, pressure, magnetic field and so on are varied are understood in this way.

Critical phenomena are characterized by divergence of fluctuations and scaling laws. Concerning earthquakes, the former corresponds to the growths of breakup and deformation in a crust which were initiated at a local region. The latter is corroborated by the following rules of thumb :

1. The Gutenberg-Richter formula for the seismic magnitudes : Frequency $n(M)$ of earthquakes with magnitude M is proportional to 10^{-bM}, where b is positive.
2. The Gutenberg-Richter formula for the energies of seismic waves : Energy of the seismic wave is related to the seismic magnitude by $\log E_{\text{in unit of J}}=4.8+1.5M$.

The rules 1 and 2 imply the scaling law $n(M) \propto E^{-b/1.5}$ (Gutenberg and Richter 1956).

3. Ishimoto-Iida law : Frequency of seismic records with an amplitude A recorded at fixed place is proportional to A^{-m}, where m is positive (Ishimoto and Iida 1939).

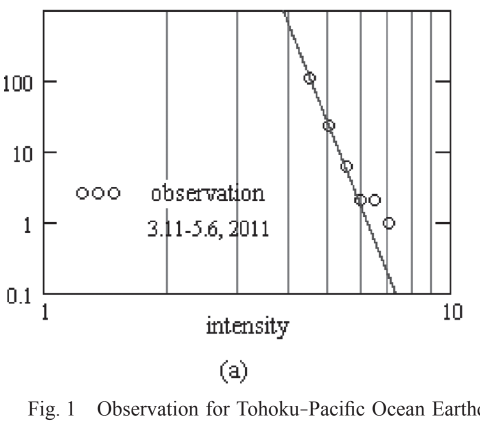
4. The Omori law for aftershocks : Frequency of aftershocks at time t after the main shock is proportional to $(t+c)^{-\nu}$, where $\nu=1$ (Omori 1894) or $\nu\cong 1$ for modified Omori formula (Utsu 1970, Yamashita and Knopoff 1987).

Assuming that E and A are related by $E \propto A^{\alpha}$ and noting that the number of seismic events are invariant under any transformation of variables, i.e., $10^{-bM}\, dM=A^{-m}dA$, the laws 1, 2 and 3 implies b and m are related by $b=(1.5/\alpha)(m-1)$. Hereafter, we consolidate 1, 2 and 3 above to the Gutenberg-Richter-Ishimoto-Iida (GRII) law.

In Fig. 1(a), the seismic intensity-frequency relation during March 11-May 6, 2011 of Tohoku Pacific-Ocean Earthquake is shown. For the aftershocks with smaller intensities, the GRII law seems to fit the observation. The power-law relation between energy and frequency is very likely to hold in this earthquake. Shown in Fig. 1(b) is the number $N(d)$ of aftershocks of the same earthquake as a function of day after the main shock. The Omori law fits the observed data quite well.

If earthquakes are really critical phenomena, it will be possible to understand them in terms of a simple rule of propagation of fluctuations in an ensemble of small domains without referring the details of microdynamics. Then, viewing earthquakes as critical phenomena means we ask under what condition and how the geophysical fluctuations develop and then cease. We may also ask what universality class does earthquake belong.

The purpose of this paper is to answer the first question addressed above by analyzing a primitive cellular automaton model (PCAM) of earthquake (cf. Bak and Tang 1989, Gould and Tobochnik 1996), although our model differs from the prevalent ones in two respects. First, the initial state is



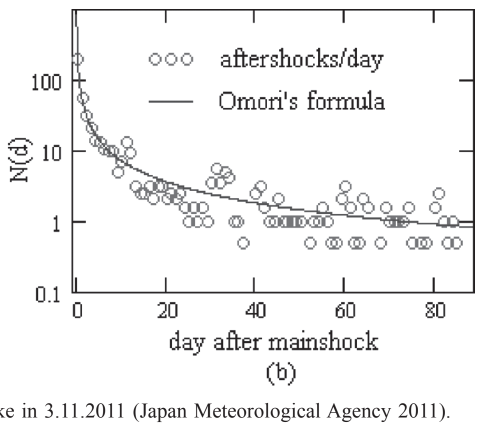


Fig. 1 Observation for Tohoku-Pacific Ocean Earthquake in 3.11.2011 (Japan Meteorological Agency 2011). (a) Seismic intensity and frequency during 3.11-5.6, 2011. (b) Temporal dependence of the number of aftershocks with magnitude $\geq$ 5 per day. The main shock was recorded at 14 : 46, 3.11, so that the original data, $N_0(d)$, released by Japan Meteorological Agency have been modified in accordance with the prescription : $N(1)=N_0(1)+N_0(2)/2$, $N(d)=[N_0(d)+N_0(d+1)]/2$ for $d\geq 2$. This means that 'day' in Fig. 1(b) is approximately equal to 24 hours.

prepared randomly. Second, all the cells are subjected to revision in the same time step.

One may wonder whether this model, so simple that it is quite suitable for beginners' study of deterministic system in discrete space and time, is applicable to real physics. On the contrary, we will see that the PCAM reproduces the very characteristic aspects of earthquakes, including theGRII law and the Omori law.

There are several variants in the cellular automaton model of earthquake. Barriere and Turcotte (1993) took fractal distributions of cell sizes into account as a reflection of complex distribution of actual faults. The GRII law was reproduced but the Omori law was not. Steacy and McCloskey (1998) studied a heterogeneous system in which the cells' strengths are not constant. In addition, the energy of the critical cell is redistributed only to unbroken neighbouring cells. They found, in their model, that there is no special correlation between large and small earthquakes, which renders prediction of large earthquakes from small ones impossible. Nakanishi (1991) studied a cellular automaton version of the block model and derived the GRII law with some variation of the exponent *b*.

This paper is organized as follows. The structure of the model is given in sect. 2. The direct outcomes of the model are presented in sect. 3. In sect. 4, we explain the renormalization method employed in our model. In sect. 5, we present the result of applying the method elaborated in sect. 4. Section 5 is devoted to conclusion and outlook.

2. Model

We consider a square lattice sectioned into $n \times n$ cells. Each cell is specified by two integers i and j, both run from 1 to n. Each cell has its own internal state at every discrete time. The state of a cell (i, j) is specified by a number $E_{i,j}$ that represents the 'stress energy' stored in the cell. Their initial values are given randomly. At every time, $E_{i,j}$ is accumulated by a small positive quantity ΔE for all i and j. When the value of some $E_{i,j}$ exceeds the critical value, E_c, the internal state is changed to $E_{i,j} - E_c$. At the same time, the energies of the neighbouring four cells are respectively increased by $E_c/4$. This last process is the smallest earthquake in the PCAM, which we call the unit event. The unit event around one cell may trigger other unit events through the interaction among nearest cells in case these cells were near to the critical state. In other words, the rate of unit events may remain a constant level or proliferate under some appropriate conditions. The time scale of unit events and their proliferations is very short in reality as compared to the time scale of energy accumulation. In the simulation, however, we do not discriminate these two lengths for matter of convenience. Whenever the critical cells completely disappear, the state of every cell is increased by ΔE. We regard a collection of the unit events at a certain time interval as the seismic phenomenon we usually call earthquake. The Decimal BASIC program of the model is given in Appendix.

3. Results of simulation

We performed simulations for the lattice size 32×32. The parameters are $E_c=4$, $\Delta E=0.0002$. The initial state of each cell is chosen randomly as $E_{i,j}=3.96+0.005r_{i,j}$, where $r_{i,j}$ is a uniform random number in (0, 1) for the site (i, j). One unit event is caused by a cell with the energy greater than E_c, which we shall call a critical cell. The critical cell is a cell that is releasing the energy E_c. We first observed how the number of the critical cells, N, varies with time. The result of simulation is shown in Fig. 2.

The earthquake characterized by a sequence of peaks in N starts at t=175 and ceases at t=1925. In between, many small peaks are observed and the heights gradually decrease on average. The initial large peak is the main shock and subsequent small peaks are aftershocks. The average temporal variation of N seems to be approximated by $N_0/(t+c)$.

The active period, i.e., the temporal length T_a of the sequence of peaks, in Fig. 2(a) is 1750. Outside of this time interval, the system goes into a resting period. As shown in Fig. 2(b), the active period and the resting period appear almost periodically. We note that the length T_r of the resting

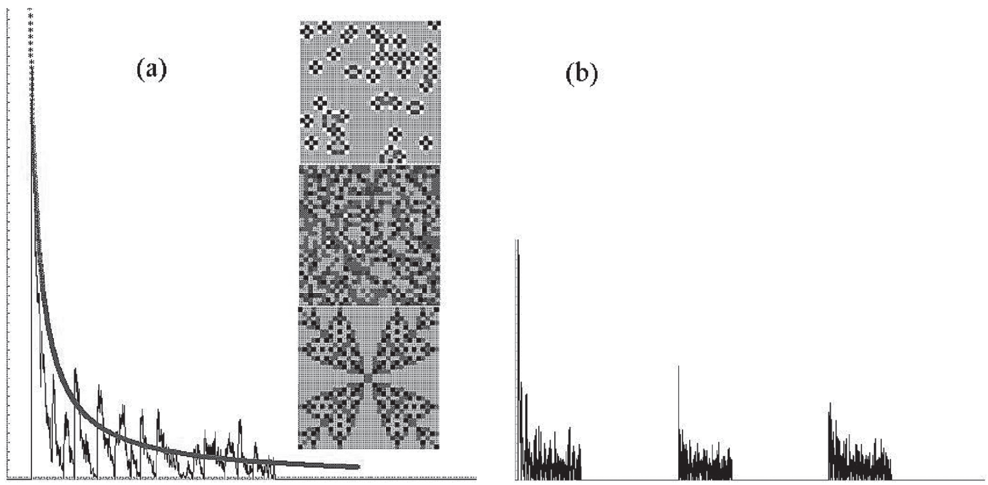
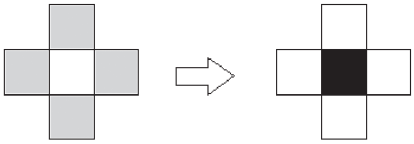


Fig. 2 Temporal behaviour of the number of critical cells. (a) The bold curve is an Omori function drawn up to 2500 time step as a guide. The inset is a collection of snap shots of the state of the lattice. From top to bottom : first unit events, intermediate state and final state. Cells are colored by the rule : black for $0\leq E<1$, dark gray for $1\leq E<2$, intermediate gray for $2\leq E<3$, light gray for $3\leq E<4$, white for $4<E$. (b) Long term variation of the number of critical cells.



(a)　　　(b)

Fig. 3　Example of correlated unit events.　(a) One critical cell is surrounded by four cells whose states are just below the critical one.　(b) The central cell changes the state to the lowest one by releasing energy. The surrounding four cells receive the part of the energy and become critical simultaneously.

period is only about 50% longer than T_a in the present simulation.　Note that, for convenience of calculations, Δt_r, the unit time for the accumulation of energy in the rest period was taken equal to Δt_a, the unit time for the energy release in the active period.　(In the actual simulation, $\Delta t_r = \Delta t_a = 1$)　In order for our model to be applicable to a real long-term seismic history of a certain district, therefore, the unit time-steps in the active and resting periods should be appropriately reinterpreted.　For instance, if the simulation history Fig. 2(b) were to be compare with Tohoku Pacific-Ocean Earthquake, which is guessed to occur with the resting period of $\sim 1000\ yr$, then we would have $T_a/T_r \sim 0.1\ yr/1000\ yr \sim 1750\Delta t_a/(1.5 \cdot 1750\Delta t_r)$ or $\Delta t_r/\Delta t_a \sim 7000$.

Taking the GRII law and the Omori law as established laws, the result mentioned above may be a preferable one if one unit event of the model is interpreted as one earthquake with the seismic intensity greater than a chosen value.　On the other hand, some unit events occur simultaneously with direct correlation.　One example is shown in Fig. 3.　There, the four unit events in (b) occurred at the same time by a common cause.　If the four critical cells in (b) are in contact with other cells that are just below criticality, then those four cells in (b) will cause the next unit events.　It is appropriate to regard these sets of unit events as single earthquakes with greater magnitudes.　Therefore, it will be interesting if one can identify a single earthquake consisting of many unit events together with its magnitude and reanalyze the statistical distribution of seismic magnitudes.　We turn to this problem in the next section.

4.　Coarse graining

One critical cell describes one unit event.　Therefore, in order to extract individual earthquakes from a given pattern of the distribution of cells, we have to identify a certain type of cluster of critical cells as a single earthquake.　This is particularly necessary if one critical cell implies a release of energy of the same order of magnitude. In this case, the difference in seismic magnitude must be attributed to the difference in the number of critical cells in single clusters.

Unfortunately, this is a problem of a little troublesome in two points. First, what is a cluster ? If by a cluster we mean a set of connected critical cells, then we confront the same problem as in percolation. Identifying a connected set is generally not a simple process. Second, in our problem, a connected set do not always imply a single earthquake. See Fig. 4, where a fictitious snapshot of a lattice is given. There are three clusters of critical cells. Upper two clusters are disconnected and may represent two distinct earthquakes. The third cluster at the bottom is also connected. Is it a single earthquake ? The answer will generally depend on the history of cluster evolution. It may be a single earthquake. However, it is also possible that, at previous times, the cluster was disconnected and later two clusters became connected as a result of the propagations of shock fronts generated elsewhere. Discriminating these situations in a given snap shot is a matter of probability.

Recalling that the earthquake may be a critical phenomenon, we shall employ the method of coarse graining, or renormalization group method, for our end. The renormalization group method is quite useful in the field of critical phenomena observed in the systems of atoms, molecules, spins etc. The idea was developed by Kadanoff (1966) and was formulated and successfully applied by Fisher and Willson (1972) and others to quantum field theory to study infrared or ultraviolet properties of Green's functions. For related references, see, e.g., Wilson and Kogut (1974).

Kadanoff et al. (1989) applied the method to the model of avalanches and explored the dependence of the universality class on the rule of automaton. Bak and Tang (1989) suggested that the method would be applicable to earthquakes. Turcotte (1999) performed renormalization in the forest-fire model with an asymmetric coarse graining rule.

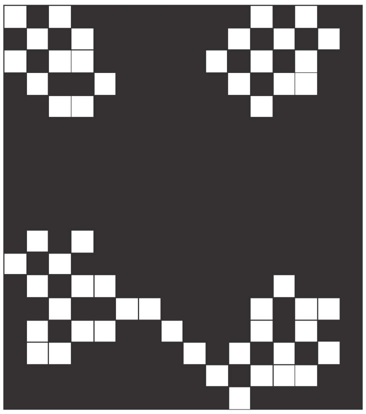


Fig. 4 Examples of connected clusters of critical cells.

Coarse graining is to repeat fusing nearby cells into one larger cell with a state determined by a certain rule. There is some arbitrariness in the choice of the rule except that it should keep the global tendency of the whole lattice unaltered. The rule we adopt for the present problem of earthquake is shown in Fig. 5. As is described below, by one coarse graining, the energy transferred to neighbouring cells gets about four times greater, which brings about the renormalization of the interaction strength.

At the first step of coarse graining, the lattice size reduces from 32×32 to 16×16. We count n_c, the number of the critical (or white) cells that disappeared at this stage. Not that each of them were one unit event releasing the energy E_c so that we regard it as an earthquakes with ‘magnitude’ 1 or M=1. At the second step, the lattice size reduces from 16×16 to 8×8. The critical cells that disappeared at this stage released on average the energy of the order of $4E_c$ per renormalized cell, so that they correspond to the earthquakes with M=2. We repeat this procedure until the lattice size becomes 1×1. In this way, at each time, we count and sum-up all the number $n_M(t)$ of earthquakes up to M=5 in our simulation.

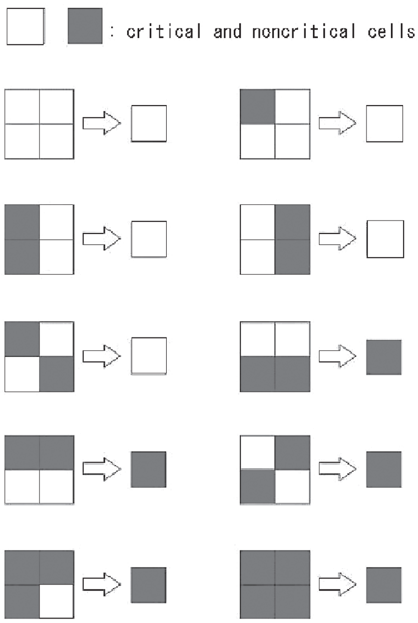


Fig. 5 The rule for coarse graining. Critical cells are designated by white, others by gray. Four cells that are simultaneously in contact via lines or points are fused into one cell. The resultant colour is determined by the majority of the state of the four cells. In case the number of the white cells is two, the final colour is either white or gray with equal probability.

5. GRII law and Omori law

We performed simulations for five times by changing initial condition and took an average. The dependence of the number of aftershocks on magnitude $n(M)$ are obtained by summing $n_M(t)$ over the period the aftershocks continue. The result is shown in Fig. 6.

$n(M)$ is well approximated by

$$\log_{10}\frac{n(M)}{2\times10^5}\approx 1-M.$$

This is the GRII law of our model.

The raw plot of $n(t)$ obtained by summing $n_M(t)$ from M=1 to 5 is shown in Fig. 7(a). Since logarithm of $n(t)$ changes very slowly for small t, we summed $n(t)$ for every fifty t's. Namely, we defined a function $n(d)$ by

$$n(d)=\frac{1}{100}\sum_{t=50d-49}^{50d} n(t),\ d=1,2,3,\cdots$$

and plotted $n(d)$ against d in Fig. 7(b). The simulation data as a whole are on the curve implied by the Omori law, although the number of aftershocks at d=4～8 are less than the ones expected from the

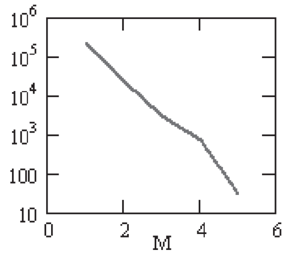


Fig. 6 Behaviour of $n(M)$.

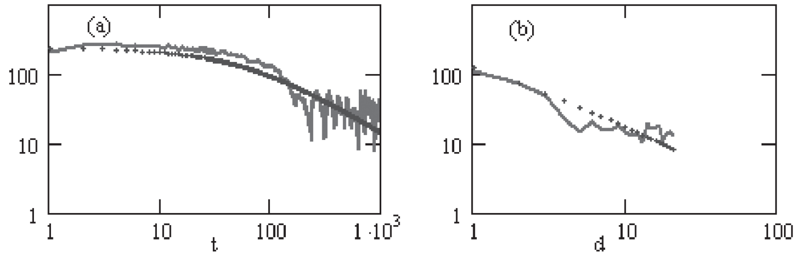


Fig. 7 (a) Solid curve : $n(t)$. Dotted curve : $1500/(t+64)$. (b) Solid curve : $n(d)$. Dotted curve : $18000/(d+0.5)$. For the definition of $n(d)$, see the text.

formula.

Taking a close look at Figs. 6 and 7, we notice some deviations of our results from the idealized GRII law and Omori law. Concerning the former, the number for M=4 is larger, and, as a reflection of this increase, concerning the latter, the frequency of mid-term earthquakes is reduced as compared with the expected ones. This implies that over-clustering to higher magnitudes might have taken place in our renormalization procedure. By altering the clustering rule slightly, these two discrepancies will be improved simultaneously.

We modified the coarse graining rule by setting the coarse graining probability in such a way that, when n_c=2 in 2×2 cells, we form the coarse grained critical cell with the probability p_{coarse}. The result is shown in Fig. 8. Frequency for M=4 is reduced and the time dependence of aftershocks seems to modified toward a linear relation. However, the number of the events with M=5 is deviated from a ideal the GRII law.

One may wonder whether our counting of $n(M)$ gives correct the GRII relation since all of $n_M(t)$ at every time t are summed thereby resulting in overestimations due to double or triple countings of events. Instead, $n(M)$ may be defined as the sum of nonzero $n_M(t)$ with maximum M. The result of counting due to this definition, which corresponds to Fig. 8(a), is given in Fig. 9. We again see that the scaling law holds on average, with the parameter $b\approx 1$.

We finally present an example of the pattern of energy distribution on the lattice with size 64×64 in Fig. 10. It reveals an example of pattern in a relatively quiet term corresponding to the right tail of the sawlike curve in Fig. 2(a) after the main shock. The high-energy domains, $3<E_{i,j}\leq 4$, with various sizes bounded by low-energy cells distribute over the lattice. The cells with energy less than 2 tend to align diagonally, while the cells with $2<E_{i,j}\leq 3$ tend to form either vertical or horizontal boundaries. The former is attributed to the fact that the shock front tends to align diagonally owing to our

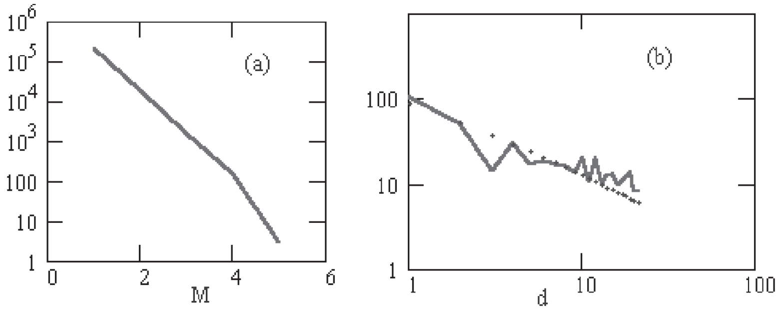


Fig. 8 Results of simulation for the coarse graining probability p_{coarse}=0.39. (a) Frequency vs. magnitude (M). (b) Frequency vs. time (d).

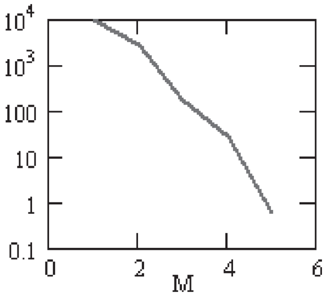


Fig. 9　Number of aftershocks with magnitude M obtained by the counting that avoids double or triple countings.

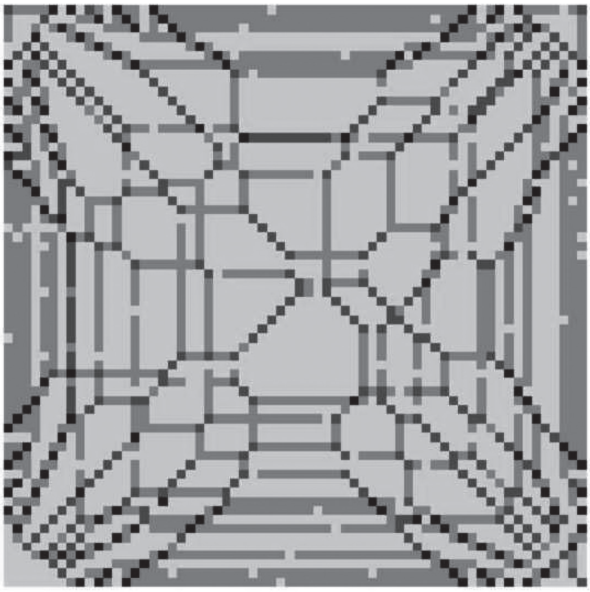


Fig. 10　Example of the pattern of the distribution of energies over the 64×64 lattice.　The domains colloured by lightest gray are ensemble of cells　with the energy $3<E<4$.　They are separated by boundaries that consist of low energy cells.　The fractal dimension of boundaries is about 1.7.

automaton rule.　These boundaries, which we may regard as the faults, form a connected network with the fractal dimension $D\approx 1.7$.　Note that this fractality, which was expected when we obtained the GRII law and the Omori law in our simulation, is not a condition but an outcome of our model.

6.　Conclusion and outlooks

We examined the two-dimensional PCAM of earthquakes.　This model, ignoring all the details of the crustal and interplate interactions, takes the following four rules as essential ingredients into account :

1. energy conservation in unit event
2. nearest neighbour interaction
3. constant rate of energy accumulation
4. common critical energy above which cell's energy is released

We started the simulations with the initial condition that all the cells are near the critical one. This condition suites with exploring the time evolution of large earthquakes. Amazingly enough, in spite of all the above simplification, the model can reproduce the most prominent feature of the earthquakes in reality, i.e., the GRII law and the Omori law, semi-quantitatively. Furthermore, we saw that the exponent of the GRII law does not depend on the details of the method of renormalization and that the fractal pattern of the energy distribution results in after the main shock. These facts strongly indicate that the system of the crust and the mantle is in a critical state and that earthquakes are self-organized and self-similar critical phenomena.

The most significant conclusion derived from that the earthquake is a critical phenomenon is the 'unpredictability' of the occurrence of earthquake (Bak and Tang 1989, Steacy and McCloskey 1998). Since events of small scales have no essential distinction from large-scale events, so that small precursors will not help predict to the coming 'main' shock. This view confronts with the optimism that prediction will be possible by proper accumulations of data that is achieved by putting selected target area under close surveillance (Mogi 1982). There can be various types of prephenomena, if any, of large earthquake, depending on the crustal properties. Mikumo and Miyataka (1983) discussed the classification of the prephenomea.

That earthquake is very likely to be a critical phenomenon means that the distinction between precursors and the main shock is a matter of naming and is possible only ex post facto. However, this does not imply the unpredictability of the phenomena *in any sense*. We should notice that, in real earthquakes, such precursors (or, more appropriately, pre-events) as small shocks, subsidence or upheaval of land in fact frequently accompanied with seismic events with large scales (Mogi 1982, Kikuchi et al. 2002). We should also notice that earthquakes at a given local area occurred so far with a gross periodicity (Terada 1917, Shimazaki 2002). The prediction of earthquake in geophysical time scale is arguably possible with geophysical uncertainties. The important point we should remember is that the time scale in human life is extremely shorter than the characteristic geophysical time scale.

There are several things remained to be done to refine our PCAM. Introducing nonuniformity and anisotropy of initial condition and interaction are particularly interesting for comparison with real seismic phenomena. In order for these improvements of the model to be meaningful, enlarging the lattice size will also be necessary.

What determine the correlation length ? This is the most important question on our model of critical phenomena. The answer must be the distribution of energies in the cells. If the parameter that characterizes this distribution below criticality is identified, then we will be able to scrutinize further the property of phase transition in our model.

Appendix

Here, the main part of Decimal BASIC program (Shiraishi 2010) used for simulation in the text is presented (cf. Gould and Tobochnik 1996). The critical energy and the rate of energy accumulation are set to be 4 and 0.0002, respectively.

```
PROGRAM earthquake
! 2-dimensional cellular automaton model of earthquake
DIM energy (0 TO 41, 0 TO 41)
DIM ir (1681), jr (1681)
DIM nrel (0 TO 20000)

LET  l=32            ! linear lattice size
LET    ncell=l*l
CALL initial (energy, l)
LET nquake=0
LET    ec=4            ! critical energy
LET    de=0.0002    ! energy given to cell in unit time
LET    tmax=ec/de
LET    t=0
LET    ts=0
DO WHILE t <= tmax
```

```
    LET  nrelease=0   ! number of critical cells
    FOR i=1 TO l
        FOR j=1 TO l
            LET  energy (i, j)=energy (i, j)+de
            IF energy (i, j)>ec THEN
                LET  nrelease=nrelease+1
                LET  ir (nrelease)=i
                LET  jr (nrelease)=j
            END IF
        NEXT j
    NEXT i
    LET  nrel (t)=nrelease    ! store the number of critical cells
    IF nrelease>0 THEN
        CALL release (t, energy, ir, jr, nrelease, l, ec)
    END IF
    LET  t=t+1
    WAIT DELAY 0.01
LOOP
END

EXTERNAL    SUB initial (energy (,), l)
! initial distribution of energies
DIM ifor (0 TO 41, 0 TO 41)
FOR i=0 TO l+1
    FOR j=0 TO l+1
        LET  energy (i, j)=0.01*rnd
    NEXT j
NEXT i

SET WINDOW 0, l+1, 0, l+1
FOR i=1 TO l
    FOR j=1 TO l
```

```
        LET    ifor (i, j)=int (energy (i, j))
    NEXT j
NEXT i
MAT PLOT CELLS,IN 6, 6 ;  l+7, l+7 :  ifor
END SUB

EXTERNAL  SUB release (t, energy (,), ir (), jr (), nre, l, ec)
DIM jfor (0 TO 41, 0 TO 41)
LET    p=ec/4
DO WHILE nre>0
    FOR k=1 TO nre
        LET    i=ir (k)
        LET    j=jr (k)
        LET    energy (i, j)=energy (i, j)–ec
        LET    energy (i, j–1)=energy (i, j–1)+p
        LET    energy (i, j+1)=energy (i, j+1)+p
        LET    energy (i–1, j)=energy (i–1, j)+p
        LET    energy (i+1, j)=energy (i+1, j)+p
    NEXT k
    LET    nre = 0
    FOR i=1 TO l
        FOR j=1 TO l
            IF energy (i, j)>ec THEN
                LET    nre=nre+1
                LET    ir (nre)=i
                LET    jr (nre)=j
            END IF
        NEXT j
    NEXT i

    FOR i=1 TO l
        FOR j=1 TO l
```

```
            LET  jfor (i, j)=int (energy (i, j))
        NEXT j
    NEXT i
    MAT PLOT CELLS,IN 6, 6 ;  l+7, l+7 :  jfor
LOOP
END SUB
```

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