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Transverse Dynamics for Super-Rapid Acceleration in Circular High-Energy Particle Accelerators

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Abstract: A new particle acceleration concept had been presented to realize a short acceleration time of μs to produce high-energy muons in circular accelerators. The acceleration method is also expected to accelerate heavy-mass particles with large momentum spread: protons, deuterons, and various heavy ions. It is most important to develop a beam injection method into a main accelerator and to investigate the vertical motion of particles, especially for heavy ion acceleration. The outlines of beam injection method and beam focusing method to solve the vertical divergence problem, which causes the beam loss, are shown in this paper. In addition, a new compact magnet system, which produces magnetic field like a quadrupole field for beam focusing especially in wide aperture accelerators, is also described here.

Keywords: Accelerator, Circular, Hadron, Muon, FFAG, Microtron.

1. INTRODUCTION

The possibility of the production and acceleration of muons by circular particle accelerators has been discussed in fundamental physics studies on high-energy muons and applications of GeV-class hadron particles to new energy production by using nuclear fusion reactors [1-3], and the fundamental principle has been shown in Ref. 4. The hardware components of the particle accelerator is similar to that of a microtron accelerator, except for applying main bending magnets which have magnetic field variation in *x*–direction to satisfy the isochronous condition for particle circulation (Figure **1**). The isochronous condition enables to apply the high-field resonant cavity with fixed frequency, and achieve rapid acceleration. The particle accelerator based on the principle is called as Isochronous Microtron Accelerator (IMA) here. This principle is applicable for acceleration of various particles. This accelerator concept is expected to produce not only muons but also neutrinos and heavy particles with various kinetic energy. The beam handling for transverse motion is important to reduce a beam loss, especially for heavy ion acceleration. There are two main articles on transverse motion to understand: one is beam injection into the accelerator and the other is vertical motion during acceleration. Those two articles are discussed theoretically, and the required beam focusing technique to be developed by using active impulse-Q magnet system is introduced in this paper.

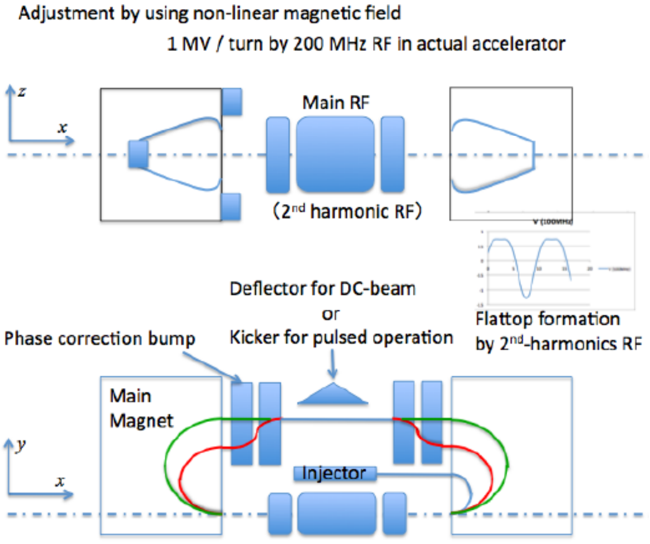

Figure 1: Schematic drawing of the IMA. The top figure shows the cross-section of the IMA, and the bottom figure shows a floor plan.

2. INJECTION METHOD OF BEAMS WITH LARGE MOMENTUM SPREAD

Almost all circular particle accelerators, such as cyclotron, synchrotron and microtron, enable to accelerate only well-directional particles within a momentum error of 1 %, so the beam to be injected should be accelerated or decelerated to a fixed momentum explicitly before injection. But there is a

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case that it is difficult, such as muons, which have broad energy spread. The IMA can accelerate particles with wide-range kinetic energy, if the isochronous condition is satisfied. On the other hand, it is impossible to inject a beam with a large momentum spread by conventional techniques, because the bending angle by the injection field to adjust the beam orbit to the circular main accelerator depends on beam momentum strongly. The application of shift bump handling is one solution against injection problem. Figure **2** shows the simple example of the beam injection scenario by using shift bump system. The transverse beam position is changed by two bending field, according to each beam momentum. The field of the main bending magnet gathers beam orbits. The two bending field should be produced by electric septa with thin plates, because the particles after one circulation pass around the area. The pulsed non-ferric magnet systems can be also applicable, such as a kicker magnet system [5], in the case of short bunched in longitudinal direction, because the circulation time is larger than 100 ns and the time duration of magnetic field can be produced. For example of the case shown in Figure **4**, the motion of the particles to be guided into the acceleration cavity in the IMA is represented by the following equations:

$$L = r\sin\theta \tag{1}$$

$$Y = 2r(1-\cos\theta) + l\tan\theta + 2R \tag{2}$$

$$r = \frac{m_0\gamma\upsilon}{qB} \tag{3}$$

$$R = \frac{m_0\gamma\upsilon}{qB_0} \tag{4}$$

where L, r, θ, Y, l, R, m_0, γ, υ, B and B_0 are the length of a shift bump device, circulation radius in a shift bump, a bending angle by a shift bump device, the distance in the distance in *y*-direction between the injection line and the cavity center, the length between two shift bump devices, circulation radius in a main magnet, the Lorentz factor of the particles, the velocity of the particles, the magnetic flux density in a shift bump device and that in a main magnet. Inserting Eqs. (1)-(3) into Eq. (4), the following equation is obtained:

$$(Y - 2R - 2r)r\cos\theta = -2r^2 + 2L^2 + lL \tag{5}$$

In the case of the following two equations, Eq. (5) is satisfied for any θ, which relates with kinetic energy of the particles directly.

$$\begin{aligned} l &= 2\left(\frac{r^2}{L} - L\right) \\ Y &= 2(R + r) \end{aligned} \tag{6}$$

The acceptable momentum dispersion is increased more than that for conventional circular particle accelerators, although it is limited to the bending angle of $\pi/2$ by a shift bump device. It is possible to increase the acceptance furthermore by combinations of the chicane by the bump orbit.

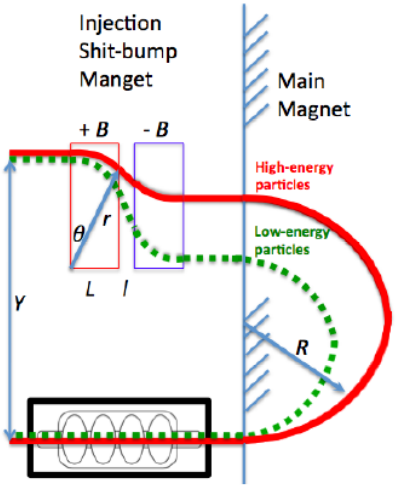

Figure 2: Schematic drawing for beam injection by using shift-bump system.

3. VERTICAL MOTION DURING ACCELERATION AND BEAM FOCUSING

The concept of the IMA is based on the field variation of the main bending magnets. The motion in the midplane is described by the conventional Lorentz force as follows:

$$m_0\gamma\frac{d\upsilon}{dt} = q(\upsilon \times B) \tag{7}$$

$$m_0\gamma\frac{d\upsilon_x}{dt} = q\upsilon_y B_z(x) \tag{8}$$

$$m_0\gamma\frac{d\upsilon_y}{dt} = -q\upsilon_x B_z(x) \tag{9}$$

here r, m_0, γ, q, υ and B indicate the radial position, the invariant mass of the particle, the Lorentz factor of the particle, the particle charge, the velocity of particles and the magnetic flux density, respectively. Integrating Eq. (9), horizontal motion in *y*-direction is obtained:

$$\upsilon_y = -\frac{q}{m_0\gamma}\int_0^x B_z(x)dx \tag{10}$$

The motion is also described by derivative form of Eq. (10) in general as follows:

$$B = \frac{m_0}{q} \nabla \times [\gamma \upsilon] \tag{11}$$

The particles circulating in the accelerator can be permitted to move slightly in the vertical direction, *z*-direction, like as a small oscillation. The field gradient of vertical magnetic flux density means the existence of B_x. The horizontal magnetic flux density B_x can be estimated by using the Ampere-Maxwell equation, and is given by:

$$B_x(z;x) \sim (\frac{\partial B_z}{\partial x})z \tag{12}$$

Inserting Eqs. (10) and (12) into the vertical motion of equation, the differential equation on vertical motion can be obtained.

$$m_0\gamma \frac{d\upsilon_z}{dt} \sim -q\upsilon_y B_x = [\frac{q^2}{m_0\gamma}(\frac{\partial B_z}{\partial x})\int_0^x B_z(x)dx]z$$

$$\begin{gathered} \frac{d^2 z}{dt^2} \sim k^2 z \\ k = \frac{q}{m_0\gamma}\sqrt{(\frac{\partial B_z}{\partial x})\int_0^x B_z(x)dx} \end{gathered} \tag{13}$$

The sign of the field gradient in the main magnets of the IMA is positive, so vertical motion is divergent, which causes beam loss especially for slow/long-time acceleration. Figure **3** shows the growth rate in vertical motion of the standard magnet in the IMA for 300 MeV muons. The maximum value of the growth rate is less than two in this case, so it is possible to compensate by the conventional focusing techniques. The vertical growth rate for heavy particles is less than that for light particles, according to *k* expression.

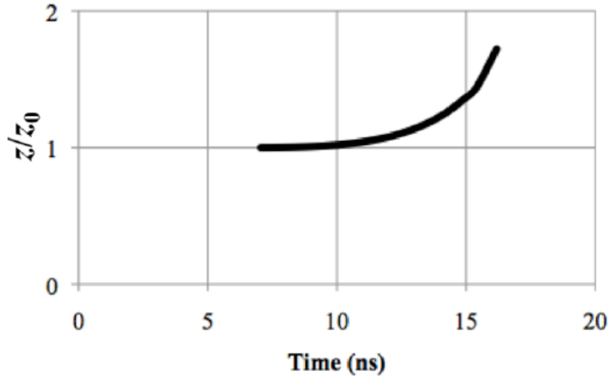

Figure 3: Estimated vertical growth rate throughout a main magnet of the IMA for 300 MeV muons by numerical calculation.

Such a divergent motion can be corrected by a conventional focusing method using conventional quadrupole magnets (Figure **4**).

Wide aperture focusing magnets are required in the straight sections of the particle orbits, except for rf straight sections. And the growth rate changes slightly according to particle momentum (Figure **5**). High acceleration efficiency operation requires some techniques to compensate those requirements. My collaborator team is now developing the active impulse-Q magnet to solve the abovementioned problem and describe in the next section.

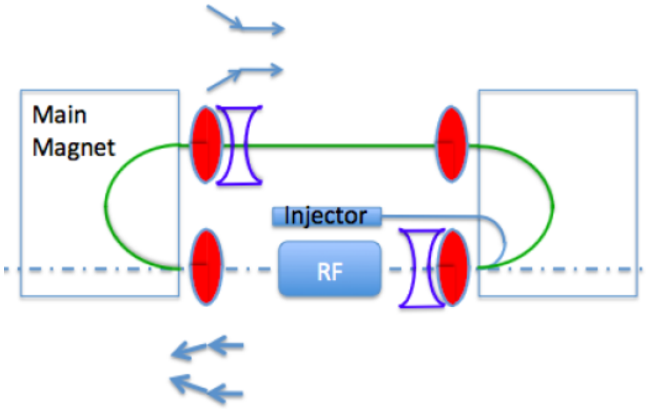

Figure 4: Schematic drawing of the example for focusing. The arrows show the beam envelop of vertical motion, *z*-direction.

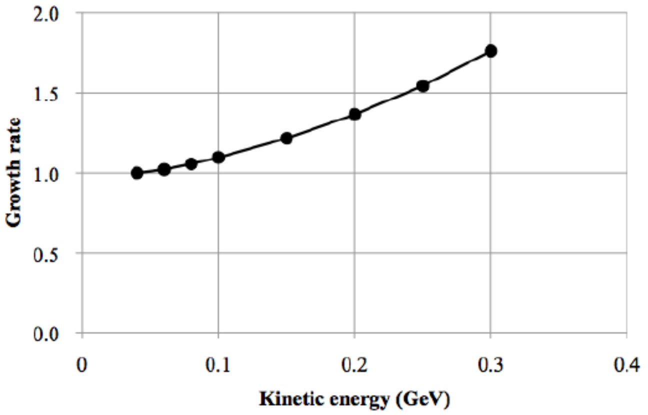

Figure 5: Estimated growth rate for kinetic energy by numerical calculation.

4. COMPACT BEAM FOCUSING MAGNET FOR WIDE APERTURE

A beam focusing system is important to suppress a beam loss due to vertical motion. Quadrupole magnets are often used for such beam focusing, but the size of conventional Q-magnets becomes huge and those require large electric power for excitation. The time response of the magnets is also slow. Twin C-type magnet is discussed here in order to produce focusing magnetic field. The shape of the magnet is similar to a conventional window-frame magnet. Although counter-flow current in the two coils inside magnetic yoke are used to excite uniform vertical magnetic field, the current in same direction can produce Q-field (Figure **6**). Two power supplies feed excitation current to each C-type magnet independently, and various magnetic field configuration can be obtained externally.

Figure **7** shows the demonstration results for short-pulsed magnetic field by using a transmission type of kicker magnet (Table. 1) [6]. The waveform of the excitation current is trapezoid with 0.2 μs length, which is formed by using the high-voltage coaxial cables as a pulse-forming network (PFN). The effective field gradient field dB/dx was obtained, as shown in Figure **8**.

It is possible to produce a time gradient field by the pulse forming devices in power supplies. Figure **9** shows the measured waveform of magnetic field in the case of the capacitor bank of 0.1 μF as PFN. Moreover, it is also possible to produce different field variation. Figure **10** shows the measured field variation in the case of only one-side excitation. The excitation by using different waveform is also applicable because each side is almost separated electrically, which enable time-dependent beam focusing especially for short time in super rapid acceleration. Figure **11** shows the measured results of the example of the time structure handling by which the left side is excited by exponential current and the right side is excited by a trapezoid current.

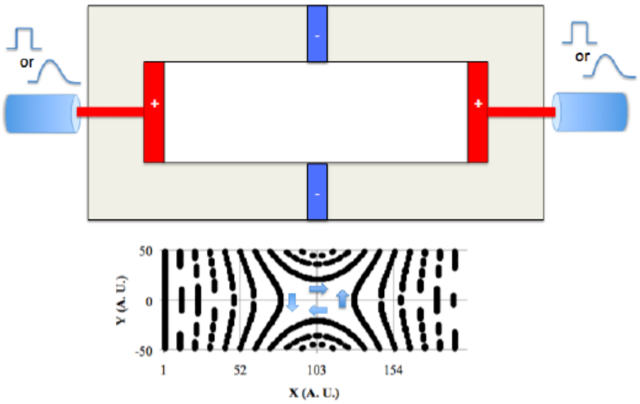

Figure 6: Magnet structure and magnetic flux lines at the center of the impulse-Q magnet shown by equipotential of magnetic field.

Table 1: Parameters of Magnet System for Demonstration

Parameters		
Excitation pulse		
Pulse voltage	5	kV
Pulse current	100	A
Magnet characteristics		
Inductance	0.4	μH
Transmission time	31	ns
Magnet structure		
Gap height	55	mm
Gap width	110	mm
Gap length	0.3	m

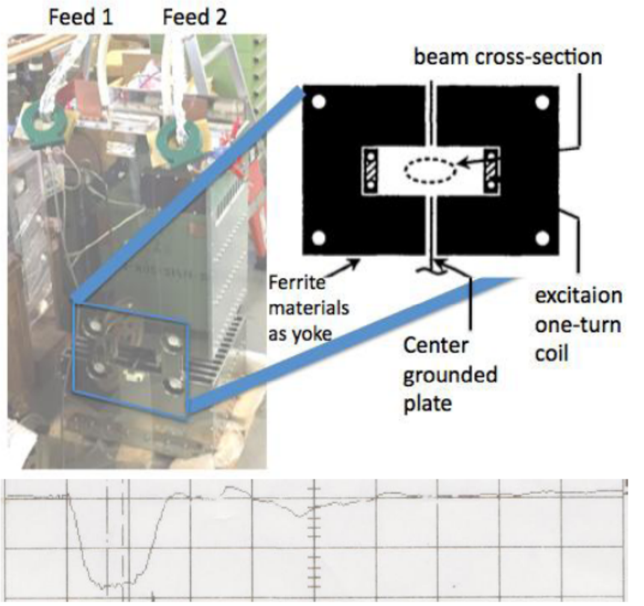

Figure 7: A transmission type of kicker magnet used for demonstration of impulse-Q and magnetic field waveform (100 ns/div.).

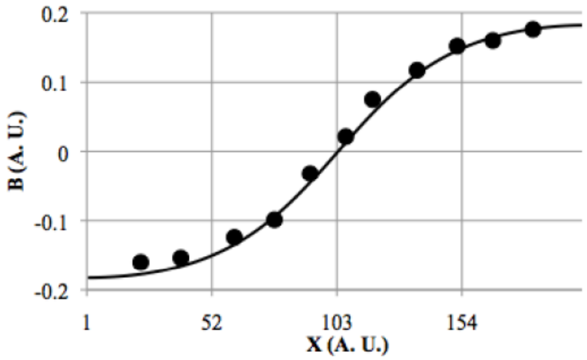

Figure 8: Magnetic field variation in horizontal direction. The solid line indicates the calculated results and the closed circles show the experimental results for short-pulsed excitation by using a transmission type of kicker magnet.

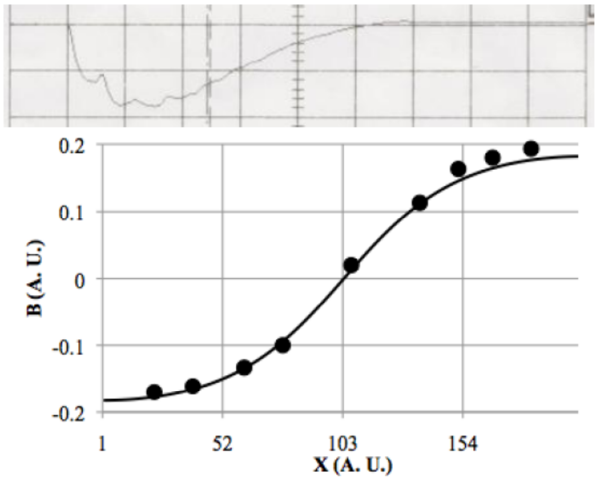

Figure 9: Magnetic field waveform (0.5 μs/div.) and the measured magnetic field variation.

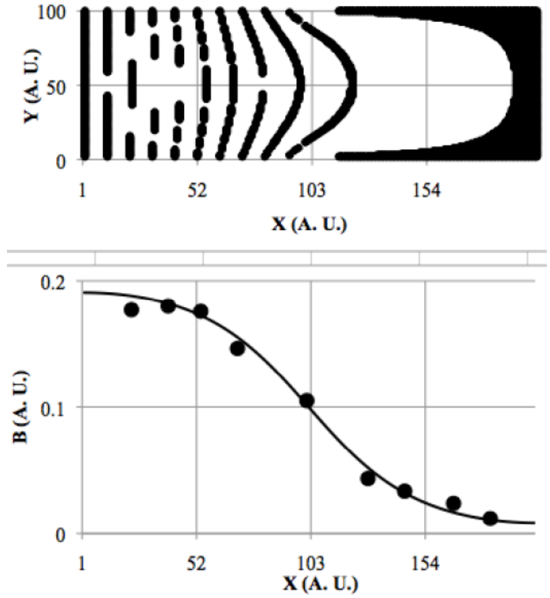

Figure 10: Magnetic structure and magnetic field variation by only one-side excitation.

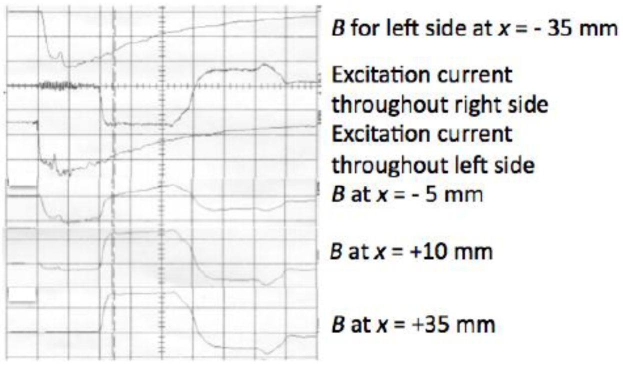

Figure 11: Time structure variation in a magnet gap of the magnet shown in Figure **9**. The unit of the horizontal axis 0.5μ/div., and the unit of each vertical axis is shown in an arbitrary unit. The waveform of the excitation current throughout the left side of the magnet is exponential, and that for the right side is trapezoid.

5. SUMMARY

Transverse motion in the IMA, especially for beam injection and vertical motion during acceleration are discussed theoretically, and the required beam focusing technique to be developed by using active impulse-Q magnet system are introduced in this paper. The IMA is like circular, but the components are line-symmetry. It is difficult to appreciate and design by using conventional Twiss-parameters [7], so the vertical motion related to the beam loss or acceleration efficiency is demonstrated by solving the kinetic equation directly. It is sufficient to design a practical particle accelerator theoretically, and the proof-of-principle experiment by using 400 keV electron beam and the upgrade scenarios for various particle accelerators are now under being planned.

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