New Approach for Designing of High-Energy Circular Particle Accelerators

Eiji Nakamura[1-4,*]

[1]*High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801 Japan*

[2]*The Graduate University for Advanced Studies (SOKENDAI), Shonan, Hayama, Kanagawa 240-0193 Japan*

[3]*Tohoku Gakuin University, Faculty of Engineering, 1-13-1 Chuo, Tagajo, Miyagi 985-8537 Japan*

[4]*Tohoku Gakuin University, Research Institute for Engineering and Technology, 1-13-1 Chuo, Tagajo, Miyagi 985-8537 Japan*

Abstract: New particles and new nuclei have been discovered by artificial techniques recently. High-energy particle accelerators work the important roles to bring such discoveries. The applications of the particles are being discussed at various communities as a next step to get new technologies. The life-time of the particles are very short to handle by conventional techniques. It can be extended by the theory of relativity. The compact particle accelerator, which enables rapid acceleration, is required especially for the industrial applications. The possibility to satisfy the isochronous condition in the case of DC and uniform magnetic field is discussed here by getting back to the fundamental theoretical approach.

Keywords: Circular particle accelerator, Design, Nuclei conversion, Nuclear fusion, High-energy particles, New particle production.

1. INTRODUCTION

Linear particle accelerator (LINAC) is most popular to produce high-energy and high-intensity particles, and is capable to accelerate particles rapidly, such in μs. LINAC, however, has demerits to require expensive acceleration devices, to require long space to install whole accelerators, and to accelerate up to TeV due to a practical limitation. Circular accelerators can be constructed at a lower price by reducing the number of the acceleration devices using magnets and the superimposing method of an rf acceleration scheme, including beam stabilities on synchrotron oscillation of longitudinal motion and betatron oscillation of transverse oscillation, and are capable to accelerate over TeV [1], although the acceleration time is very slow. The various developments to improve the acceleration time were performed and confirmed the possibility of 1 Kpps [2, 3], but the acceleration time is still much slower than that of LINAC. Cyclotron [4] can produce high-intensity coasting beam, and is categorized to a circular accelerator, although it should be categorized as a spiral LINAC from the viewpoint of a beam orbit. But the acceleration energy is limited to GeV due to the requirement of huge amount of magnetic materials for yokes, and the acceleration time is still slow because of the requirement of wide aperture for an rf device. The slowness of the acceleration time for circular particle accelerators is caused by the requirement of synchronization to a non-linear frequency pattern for particle motion, so almost all circular particle accelerators cannot apply high-field resonant cavities over 1 MV/m. The fundamental factor of this problem is laid to undeveloped study field on magnet design. One of the study fields is the isochronous acceleration, which enables to apply high-field resonant cavities with a fixed frequency, by finding the optimum condition on magnetic configuration. Conventional methods to design magnets for high-energy particle handlings are based on the polynomial expression of the separated function of magnetic field, *i.e.* for bending, focusing, correction of chromaticity, and so on. The isochronous condition can be derived by conventional methods, but the solution is applicable for a practical use because the solution induces the vertical beam loss inevitably [5]. The transverse emittance of the high-intensity beam accelerated by conventional methods becomes very large [6]. It is, therefore, difficult to find optimum conditions on magnetic field configuration. On the other hand, one of the examples to satisfy the isochronous condition has been shown by using non-linear field concept recently [7]. The application of non-linear magnetic field concept has not been performed in the particle accelerator study field yet, but that for the study field of nuclear fusion has been established [8-10]. The possibility to satisfy the isochronous condition in the case of DC and uniform magnetic field is discussed here by getting

*Address correspondence to this author at the High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan; Tel: +81 29 864 5293; Fax: +81 29 864 3182; E-mail: eiji.nakamura@kek.jp

E-ISSN: 2408-977X/16

back to the fundamental approach at first. In addition, the new expression using curve linear coordinates for magnetic field configuration is introduced in this paper. The transverse beam emittance becomes small by applying the resonant cavities even for the high-intensity beam, and high-quality beam can be expected.

2. ISOCHRONOUS CONDITION BY DC AND UNIFORM MAGNETIC FIELD

It is important to satisfy the isochronous condition for applying high-field resonant cavities. It is difficult for simple-symmetry uniform magnets, but it is possible by adjusting the lengths of the magnetic field and the drift space. One of the simple cases using four bending magnets with $\pi/2$ bending angle is introduced here. The isochronous condition is that particles with any kinetic energy of qK should turn around in same revolution time of T_0. Assuming a drift space length of L_D for installation of acceleration devices, beam extraction devices and so on, the revolution time is described by a drift space length along the other axis y and the time of T_C throughout a magnet for $\pi/2$ bending:

$$\frac{2L_D + 2y}{|\upsilon|} + 4T_C = T_0(const.) \tag{1}$$

$$\gamma = 1/\sqrt{1-(\frac{|\upsilon|}{c})^2} \tag{2}$$

$$m_0\gamma c^2 = m_0 c^2 + qK \tag{3}$$

where γ, m_0 , q, $|\upsilon|$ and c are the Lorentz factor, the rest mass of the particle to be accelerated, the charge of the particle, the absolute value of the particle velocity and the light velocity in vacuum, respectively. The total time throughout four magnets with a magnetic field of B_0 is described by cyclotron radial frequency ω.

$$4T_C = \frac{2\pi}{\omega} = \frac{2\pi m_0\gamma}{qB_0} \tag{4}$$

The relation between y and $|\upsilon|$ is described as follows:

$$y = \frac{|\upsilon| T_0}{2} - \frac{\pi m_0 \gamma |\upsilon|}{qB_0} - L_D\,. \tag{5}$$

The left graph in Figure **1** shows one example of the circular accelerators to energize 400 MeV protons up to 3 GeV by an rf cavity installed at (X, Y) = (0, 0). The structure of the upper bending magnet is more complex than the bottom one. The machining accuracy developed recently and end-shims techniques [10] enable such a complex structure by adjusting the pole gap length of magnetic yoke. The orbits at a top track are different according to particle energy. The orbit difference can be used for beam extraction.

The orbit difference at a top track can be also reduced by changing the values of the magnetic flux density of two magnets. Assuming the distance between a top track and a bottom track H_0 and the magnetic flux density of the top bending magnet B_1 in the ideal case, the revolution time T_0 can be derived as follows:

$$\frac{2L_D + 2x + 2y}{|\upsilon|} + 2T_{CB} + 2T_{CT} = T_0(const.) \tag{6}$$

$$y + R_B + R_T = H_0(const.) \tag{7}$$

where the Larmor radius and the time throughout the magnet for the bottom magnet and the top magnet is represented by R_B, R_T and T_{CB}, T_{CT}, respectively. The horizontal displacement at a top track x is the difference of the values of Larmor radius:

$$x = R_B - R_T = \frac{m_0\gamma|\upsilon|}{qB_0} - \frac{m_0\gamma|\upsilon|}{qB_1}\,. \tag{8}$$

The vertical drift length can be derived by the following equation:

$$\frac{2L_D + 2y}{|\upsilon|} + \{\frac{(\pi+2)m_0\gamma}{qB_0} + \frac{(\pi-2)m_0\gamma}{qB_1}\} = T_0\,. \tag{9}$$

The right graph in Figure **1** shows one of the calculated results. The whole size of the accelerator becomes compact but the acceleration energy is limited to 2.4 GeV. The parameters used for the proton accelerators are listed in Table **1**.

Figure **2** shows the example to accelerate 40 MeV muons up to 300 MeV. The revolution time is 97.013 ns in this case of the parameters listed in Table **2**.

Various magnetic field configurations can be achieved by changing the pole gap width. The isochronous condition for spatial field profile and the related techniques are described in Refs. 5 and 6. There is, of course, feasibility on sharing bending angles. The figure of the accelerators for four π/2 bending magnets described in this paper is like a racetrack. The accelerator assuming 2π/3-π/3 combination is like triangle.

Some of the parameters, a drift length, magnetic flux density, or else, assumed in the calculations are changeable, so the parameters should be optimized at a precise design for a practical accelerator.

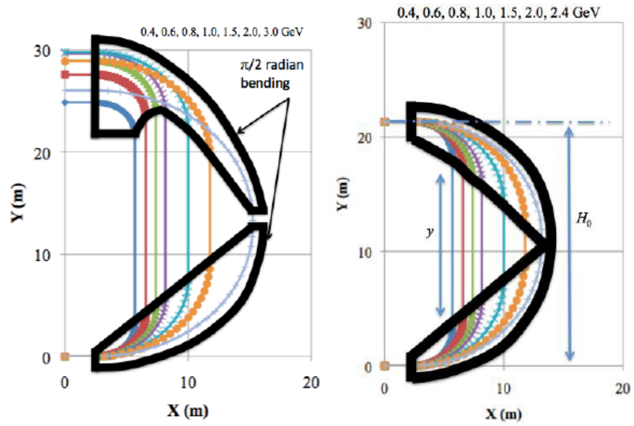

Figure 1: Half parts of each accelerator with structures of DC bending magnets and proton orbits, energized from 0.4 GeV to 2.4 ~ 3 GeV to satisfy an isochronous condition. The calculation result in the case of same magnetic flux density is shown in the left graph. The calculation result in the case for different magnetic flux density is shown in the right graph.

Table 1: Parameters for Proton Acceleration

Drift length, L_0	5	[m]
Magnetic flux density, B_0	1	[T]
Revolution time in the case of same B, T_0	313.42	[ns]
Magnetic flux density, B_1	0.9515	[T]
Vertical width H_0	21.3	[m]
Revolution time in the case of different B, T_1	280.68	[ns]

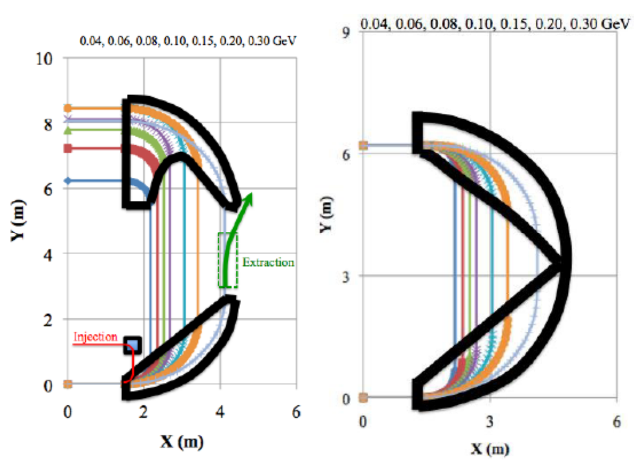

Figure 2: Half parts of each accelerator with structures of DC bending magnets and muon orbits energized from 0.04 GeV to 0.3 GeV to satisfy an isochronous condition. The calculation result in the case of same magnetic flux density is shown in the left graph. The calculation result in the case for different magnetic flux density is shown in the right graph.

Table 2: Parameters for Muon Acceleration

Drift length, L	3	[m]
Magnetic flux density, B_0	0.5	[T]
Revolution time in the case of same B, T_0	97.013	[ns]
Magnetic flux density, B_1	0.5702	[T]
Vertical width H_0	6.2	[m]
Revolution time in the case of different B, T_1	97.013	[ns]

3. STATIC MAGNETIC FIELD EXPRESSION BY PARTICLE MOTION

The first step to design a circular particle accelerator is to imagine the ideal orbit of particles. This procedure does not need the time structure, so the ideal orbit should be described by the functions of position and velocity. The motion of particles can be described by Lorentz force:

$$\frac{dp}{dt} = q(\upsilon \times B) + \mu_m \nabla\left(\frac{\upsilon \cdot B}{|\upsilon|}\right) \tag{10}$$

$$p \equiv m_0 \gamma \upsilon \,. \tag{11}$$

The right term in the right hand of eq. (10), which describes the effect by a magnetic momentum μ_m, is small in the problem discussed in this paper, and is ignored here. The inner product of the equation of motion and particle momentum p becomes zero, so magnetic field does not provide energy to particles, and the parameters γ and $|\upsilon|$ are constant.

In the case of static magnetic field, each component can be described as follows:

$$\begin{aligned} \frac{m_0\gamma}{q}\frac{d\upsilon_x}{dt} &= B_z\upsilon_y - B_y\upsilon_z \\ \frac{m_0\gamma}{q}\frac{d\upsilon_y}{dt} &= B_x\upsilon_z - B_z\upsilon_x \,. \\ \frac{m_0\gamma}{q}\frac{d\upsilon_z}{dt} &= B_y\upsilon_x - B_x\upsilon_y \end{aligned} \tag{12}$$

The following equations integrated by time describe the particle motion in static magnetic field.

$$\begin{aligned} &\frac{m_0\gamma}{q}(\upsilon_x - \upsilon_{x,0}) = \int_0^t B_z\upsilon_y dt - \int_0^t B_y\upsilon_z dt \\ &= \int_{y_0}^{y} B_z dy - \int_{z_0}^{z} B_y dz \\ &\frac{m_0\gamma}{q}(\upsilon_y - \upsilon_{y,0}) = \int_{z_0}^{z} B_x dz - \int_{x_0}^{x} B_z dx \\ &\frac{m_0\gamma}{q}(\upsilon_z - \upsilon_{z,0}) = \int_{x_0}^{x} B_y dx - \int_{y_0}^{y} B_x dy \end{aligned} \tag{13}$$

The mathematical operations of the partial differentiation to the first equation leads to the following equation:

$$\begin{aligned}
&\frac{\partial}{\partial x}[\frac{m_0\gamma}{q}(\upsilon_y-\upsilon_{y,0})]-\frac{\partial}{\partial y}[\frac{m_0\gamma}{q}(\upsilon_x-\upsilon_{x,0})]\\
&=\frac{\partial}{\partial x}[\int_{z_0}^{z}B_x dz-\int_{x_0}^{x}B_z dx]\\
&-\frac{\partial}{\partial y}[\int_{y_0}^{y}B_z dy-\int_{z_0}^{z}B_y dz]\\
&=\int_{z_0}^{z}\frac{\partial B_x}{\partial x}dz-B_z-(B_z-\int_{z_0}^{z}\frac{\partial B_y}{\partial y}dz)\\
&=-2B_z+\int_{z_0}^{z}(\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y})dz
\end{aligned} \quad . \qquad (14)$$

The magnetic flux is conservative.

$$\begin{aligned}
&\frac{\partial}{\partial x}[\frac{m_0\gamma}{q}(\upsilon_y-\upsilon_{y,0})]-\frac{\partial}{\partial y}[\frac{m_0\gamma}{q}(\upsilon_x-\upsilon_{x,0})]\\
&=-2B_z-\int_{z_0}^{z}\frac{\partial B_z}{\partial z}dz=-3B_z+B_z(x,y,z_0)
\end{aligned} \qquad (15)$$

The general covariant expression of static magnetic field using particle orbit can be derived by the same mathematical operations.

$$\begin{aligned}
&B=-\frac{1}{3q}\nabla\times(p-p_0)+\frac{1}{3}B^*\\
&B^*\equiv(B_x(x_0,y,z),B_y(x,y_0,z),B_z(x,y,z_0))
\end{aligned} \qquad (16)$$

This equation can be rewritten by using a vector potential of magnetic field *A* in some mid-plane using an arbitrary function ϕ as follows:

$$\begin{aligned}
&A=\frac{p-p_0}{2q}+\nabla\phi\\
&-2\frac{d[\nabla\phi]}{dt}=\upsilon\times B
\end{aligned} \qquad (17)$$

Inserting eq. (16) into eq. (10), a new equation of motion in a static magnetic field can be obtained. The left term in eq. (18) can be neglected in a static field, so it indicates the relation between the initial momentum and B^*.

$$3m_0\gamma\frac{\partial p}{\partial t}=\{-\frac{1}{2}\nabla p_0{}^2+p\times(B^*-\nabla\times p_0)\} \qquad (18)$$

Equation (16) means that the magnetic flux density is derived directly by the rotation of the ideal motion of a charged particle. It is useful to imagine the ideal magnetic field required for various particle motions. For an example of simple circulation expressed by $p = p(r)$, $z = 0$, $v_z = 0$, the following equations are obtained:

$$\begin{aligned}
&B_r=\frac{1}{3q}\{\frac{1}{r}\frac{\partial[(p-p_0)_z]}{\partial\theta}+\frac{\partial[(p-p_0)_\theta]}{\partial z}+qB_r(r_0)\}=0\\
&B_\theta=\frac{1}{3q}\{\frac{\partial[(p-p_0)_r]}{\partial z}+\frac{\partial[(p-p_0)_z]}{\partial r}+qB_\theta(r)\}=0\\
&B_z=\frac{1}{3q}\{\frac{1}{r}\frac{\partial[r(p-p_0)_\theta]}{\partial r}+\frac{1}{r}\frac{\partial[(p-p_0)_r]}{\partial\theta}+qB_z(r)\}\\
&=\frac{1}{2qr}\frac{\partial[r(p-p_0)_\theta]}{\partial r}
\end{aligned} \quad . \qquad (19)$$

In the case that the particle momentum is proportionate to the radial position, the popular relation of magnetic rigidity is derived.

$$\begin{aligned}
&(p-p_0)_\theta\equiv\frac{r}{\rho}p_{00}\\
&\rho B_z=\rho\frac{p_{00}}{2qr\rho}\frac{\partial[r^2]}{\partial r}=\frac{p_{00}}{q}
\end{aligned} \qquad (20)$$

The magnetic flux density to satisfy the isochronous condition is derived as follows:

$$\begin{aligned}
&\upsilon_\theta T_0\sim|\upsilon|T_0=2\pi r\\
&\gamma=1/\sqrt{1-(\frac{2\pi}{cT_0})^2r^2}\\
&B_z=-\frac{\pi m_0}{qT_0}\frac{2-(\frac{2\pi}{cT_0})^2r^2}{\{1-(\frac{2\pi}{cT_0})^2r^2\}^{3/2}}
\end{aligned} \quad . \qquad (21)$$

for small value of r, the absolute value of the z component of the magnetic flux density should be increase according to r. The radial variation is used for a horizontal beam focusing, but it works as beam defocusing in vertical direction, which causes serious beam loss as known generally.

$$\begin{aligned}
&|B_z|\sim\frac{2\pi m_0}{|q|T_0}\{1-\frac{1}{2}(\frac{2\pi}{cT_0})^2r^2\}\{1+\frac{3}{2}(\frac{2\pi}{cT_0})^2r^2\}\\
&\sim\frac{2\pi m_0}{|q|T_0}\{1+(\frac{2\pi}{cT_0})^2r^2\}
\end{aligned} \qquad (22)$$

An example to reduce the vertical beam loss can be obtained by supposing drift spaces made by separating the magnet, as shown by eqs. (23) and (24). The first term of eq. (24) indicates that the magnetic flux density is proportionate to the inverse of the radial position, which plays a role to reduce the vertical beam loss. The covariant expression in eq. (16) is useful for understandings of various motions of high-energy particles and required static or quasi-static magnetic field. The concept of the expression was used for a new concept of a rapid cycling circular particle accelerator [6].

$$\upsilon_\theta T_0 \sim |\upsilon| T_0 = 2\pi r + L_D \tag{23}$$

$$B_z = \frac{\pi m_0}{qT_0} \frac{\frac{1}{2\pi}\frac{L_D}{r} + \{1 + \frac{1}{1-(\frac{2\pi r + L_D}{cT_0})^2}\}}{\sqrt{1-(\frac{2\pi r + L_D}{cT_0})^2}} \tag{24}$$

SUMMARY

The possibility to satisfy the isochronous condition in the case of DC and uniform magnetic field is discussed by getting back to the fundamental theoretical approach. The new concepts on particle accelerator designs are expected by the new approach, to open the new frontier of the industrial applications of new particles.

ACKNOWLEDGMENTS

This work was supported by KAKENHI 21540310, KAKENHI 25286089 and KAKENHI 26610074.

REFERENCES

[1] The Large Hadron Collider (LHC).

[2] Ishi Y, *et al.*, Nucl Instr and Meth 2001; A 472: 639.

[3] Table-Top Proton Synchrotron (TTPS).

[4] Superconducting Ring Cyclotron (SRC) at RIKEN.

[5] Rapid Cycling Synchrotron in the Japan Proton Accelerator Comprex (RCS of J-PARC).

[6] Nakamura E, *et al.*, Int J Adv Appl Phys Res 2015; 2(1): 8. http://dx.doi.org/10.15379/2408-977X.2015.02.01.2

[7] Nakamura E, *et al.*, Int J Adv Appl Phys Res 2015; 2(2): 1. http://dx.doi.org/10.15379/2408-977X.2015.02.02.1

[8] Boozer AH, *et al.*, Plasma Physics and Controlled Nuclear Fusion Research 1982, in: Proceedings of the 9th Int. Conf., Baltimore, 1982) (IAEA, Vienna, 1983) Col. 3, p.129.

[9] Wendelstein VII-X in Max-Planck Institute for Plasma Physics.

[10] Nakamura E, *et al.*, Jpn. J Appl Phys 1997; 36: 889-895. http://dx.doi.org/10.1143/JJAP.36.889

[11] Nakamura E, *et al.*, Permanent Magnets for the 500 MeV Accumulator Ring of the Intensity Doubler Project in KEK-PS, in; Proceedings of the 18th International Conference on High Energy Accelerators (HEACC2001), Tsukuba International Congress Center, Tsukuba, Ibaraki, Japan, March 26-30 (2001) P2oth11.

Received on 02-03-2016 Accepted on 20-05-2016 Published on 30-06-2016

http://dx.doi.org/10.15379/2408-977X.2016.02.03