Slipping between the cracks? Maximising the effectiveness of prerequisite paths in UTS mathematics degrees

S. Woodcock[1] S. Bush[2]

(Received 20 January 2014; revised 10 July 2014)

Abstract

As with almost all degree programs at major Australian universities, the courses offered by the School of Mathematical Sciences at the University of Technology, Sydney consist of a number of individual subjects which together form several prerequisite paths. That is, certain subjects must be successfully completed by students before they may enrol in more advanced subjects in the same broad academic area. This attempts to ensure that those entering later subjects have the sufficient prior knowledge to succeed in their enrolled subjects. While this model is largely successful, it does have one potential 'loophole'. The minimum mark to pass a subject is 50%, meaning that students can potentially complete a prerequisite subject and advance through

http://journal.austms.org.au/ojs/index.php/ANZIAMJ/article/view/7943 gives this article, Published August 1, 2014, as part of the Proceedings of the 11th Biennial Engineering Mathematics and Applications Conference. ISSN 1446-8735. (Print two pages per sheet of paper.)

their course while demonstrating understanding of only half of the prior material. The implications of applying this pass criterion are generally poorly understood and little quantitative work has been undertaken to assess its efficiency and effectiveness. Here, we present quantitative analyses of recent (2008–2013) subject results and discuss a number of possible concerns and issues that these highlight. Furthermore, we draw conclusions and propose several future initiatives which are being used to inform future practice within the school, both in terms of subject development and assessment and also in providing targeted additional support to students whose prior subject performances suggest they may be at risk of future failure.

Contents

1 Introduction

Currently the School of Mathematical Sciences at the University of Technology, Sydney (UTS) has just over 4500 individual student subject enrolments, equivalent to around 550 full time students (based on a full time load of eight subjects per year). The subjects offered by the school fall into two categories. The majority of the subjects are designed for, and primarily taken by, students studying towards a Mathematics or Statistics major. Additionally, the school offers service subjects to students majoring in other disciplines, including the life sciences, physical sciences and engineering. All of its degree courses are organised in a similar manner, in line with practice at almost all major Australian universities, with numbers of individual subjects organised together to form prerequisite paths. For students majoring in the Mathematical Sciences, there are multiple prerequisite paths which must be completed in order to graduate (calculus/analysis, algebra, statistics/ probability, operations research etc.). However, these subject areas are not quite as separate as the labelling might suggest. For example, to enrol in a final year subject which involves the study of multivariate probability distributions requires successful completion of earlier subjects both in probability and in multivariable calculus. Students majoring in degrees primarily taught by other schools and faculties have just one mathematics prerequisite path to complete.

The benchmark to successfully pass all subjects taught by the school is, irrespective of the exact degree or major being undertaken, 50%. This does leave open the possibility for students to advance while demonstrating understanding of only half of the prior material. The implications of applying this rigid pass criterion are generally poorly understood and little quantitative work has been undertaken to assess its efficiency and effectiveness. There is currently the provision of awarding a student scoring 50% or more marks a Fail grade for a subject where, irrespective of the number of marks accumulated, a student has demonstrated unacceptable gaps in their knowledge or abilities. This provision is included primarily for subjects with practical skills assessments, especially in the health sciences. For example, if a student demonstrates an acceptable level of disciplinary knowledge, but inadequate patient-based skills, then they could be assessed as failing the subject regardless of their examination result. However, in other disciplines, including the mathematical sciences, this failure method is very rarely used, largely because it is seen to be heavy-handed and a blunt tool for addressing what might be an acute problem. In practice, some students occupy a middle ground; their progression, without filling in some of the gaps in their knowledge is arguably setting them up to fail future subjects, but making them resit a subject they largely (but not wholly) understand is too severe, unnecessary and often would create further problems in terms of fitting future subjects into a study plan without prolonging their studies unnecessarily (Like most Australian universities nowadays, UTS has no provision for awarding a 'terminating pass' grade which could be counted towards a credit points total, but not allow progression along a prerequisite path). Working within these university-wide assessment standards we are looking at a number of options to address this issue. We recently trialled a Master Learning approach [2] in some subjects, whereby students must score a higher mark (in our case, 80%) on each of a series of short exams testing the fundamental skills which are deemed core to the subject. These are weighted such that a student who passes each, but obtains no additional marks will still score at least 50% overall and hence pass the subject. Initial data from our implementation of this approach suggest promising results but, at this stage, it is too early to draw any firm conclusions.

The central aim of this project is to identify any places along the prerequisite paths where students appear to be 'slipping between the cracks'—that is, where students are progressing readily into subjects for which they are ill-prepared. This information is being used to inform future curriculum and subject renewal and also to develop targeted online support materials to assist with students identified as being at risk of future failure.

The analyses presented here represent some of the preliminary results arising from a project funded under the UTS Vice-Chancellor's Learning and Teaching Grants Scheme. We examine the performance of all students taking at least one subject taught by the school between Autumn Semester 2008 and Autumn Semester 2013, inclusive. This represents a dataset of around 30 subjects, and 17000 individual student subject enrolments. Additionally, data were sourced regarding the level of students' pre-university mathematics studies. For each subject, student performances are tabulated based on their performances in prior studies.

Currently, subjects are assessed on a subject-by-subject basis. The primary tool for assessing the effectiveness and functionality of a subject is its pass rate. Additional information and feedback is gleaned from responses to a Student Feedback Survey. Subjects which either have consistently high failure rates, or which are not perceived by students as fruitful or reasonable learning experiences are subject to additional scrutiny. Such subjects may be amended or redesigned to address these apparent weaknesses. One of the principal aims of this project is to take a course-level view of subjects and how they function as components along a prerequisite path. Subjects are not 'islands' to students; depth and breadth of knowledge builds gradually throughout the course of the degree and certain subjects, or combinations of subjects, at one level must be satisfactorily completed before more difficult ones in that area can be attempted. Accordingly, we are now taking the view that when assessing and evaluating individual subjects, they should be viewed within the context of the surrounding subjects.

2 Discussion and results

In this article, for reasons of student record confidentiality, subjects are identified only by their position in a standard study plan (i.e. first semester, second year). Subject names and for which majors the subjects are intended are not included. Instead, follow the subjects along three prerequisite paths, labelled simply Path A, Path B and Path C. All subjects, whether Mathematics major subjects or service subjects, are graded the same way: 0–49, Fail; 50–64, Pass; 65–74, Credit; 75–84, Distinction; and 85–100, High Distinction. Since the data for students who have a mark over 50 and fail is very sparse, we combine all students who fail into a single category.

The discussion is centred around three main questions.

1. Coming out of high school, how well prepared are students for undergraduate mathematics?
2. Are there any examples of prerequisite paths failing to function as they should?
3. Do the analyses indicate the need for any changes to prerequisite paths?

We discuss each of these questions in turn with reference to the student results data from the past six years. Where the dataset highlights any issues of concern or potential improvements in curriculum or in teaching practice, some possible refinements are suggested. Some of these changes have already been implemented for future semesters, some require additional analysis before any action, and some may be difficult to implement at all.

2.1 Student preparedness

To examine the preparedness of incoming students, we first classified each student according to his/her level of mathematics studied in high school. For the New South Wales (NSW) Higher School Certificate (HSC), students can study two units of mathematics (Mathematics) or more; either three units (Extension 1) or four units (Extension 2) in total. There is also the provision of a lower-level mathematics subject, General Mathematics, which is intended primarily for students who do not plan to enrol in scientific or quantitative undergraduate degrees. Critically, the General Mathematics subject contains no calculus whatsoever. Only a small number of students study no mathematics at all, but this option is offered. For the purposes of these analyses, students who did not come through the standard NSW system are excluded.

Table 1 shows the distribution of results for one first year, first semester subject which is a compulsory subject for all students majoring in Mathematics or

Table 1: Percentage of students who attained each grade in a first year, first semester subject tabulated against students' levels of mathematics in the NSW HSC (F=Fail, P= Pass, C=Credit, D=Distinction, HD= High Distinction).

	F	P	C	D	HD	Proportion of Cohort
No HSC Maths	100.0	0.0	0.0	0.0	0.0	0.3
General Maths	95.2	4.8	0.0	0.0	0.0	5.6
Maths	55.9	34.3	4.9	2.9	2.0	27.3
Extension 1 Maths	44.0	37.3	14.7	4.0	0.0	40.1
Extension 2 Maths	26.0	26.0	26.0	12.0	10.0	26.7
Overall	45.4	31.6	14.2	5.6	3.2	100.0

Statistics, relative to each of the levels of mathematics in the NSW HSC. These results are striking and reinforce an argument which is frequently made. Students who have done General Mathematics for the HSC are not at all prepared to undertake undergraduate level mathematics subjects.

The level of mathematics students study in high school is not something that the university can readily influence. This is not an issue facing UTS alone; much has been written both in the academic literature [1, 3, 5] and in the national media [4] about declining student numbers into more advanced level high school mathematics subjects. Furthermore, there is evidence that schools are increasingly advising students to do General Mathematics instead of more complex and calculus based subjects in order to maximize their chances of higher university entrance scores [4]. While the universities can urge students not to avoid higher mathematics in high school, the message is unlikely to gain widespread traction, especially if schools are promoting the opposite message. Therefore, the challenge for tertiary education is to find ways to bridge the sizeable gaps between the knowledge levels of General Mathematics students and undergraduate-level mathematics subjects.

One approach which we are taking at UTS is the widespread implementation of a Mathematics Readiness Survey. This is a diagnostic test to assess the ability incoming students have in the areas of high school level algebra and simple univariate calculus. In 2013, we ran a trial for students entering programs to major in the Mathematical and Physical Sciences. The purpose of this survey was to identify an appropriate pathway into studying mathematics at the tertiary level. Based on the level of background that the students demonstrated, they were recommended to either complete a Foundation Mathematics subject or to move straight into the regular first year, first semester subjects for their chosen degree courses. However, this survey was not compulsory in 2013 and, even when taken, some students opted to ignore the advised pathway. From 2014, the survey and its proposed study plan will be compulsory for all incoming students in these courses.

The Readiness Survey has already run for several years with a great degree of success for students taking Engineering degrees at UTS. Incoming students who fail the survey upon enrolment are guided into a Foundation Mathematics subject before commencing the regular stream of mathematics and statistics subjects for Engineering majors. While we are confident that several of the issues related to student preparedness will be resolved by the broadening scope of this survey, we certainly do not believe that it will be a silver bullet for the problem. One semester of additional study is barely an adequate substitute for two years of senior high school mathematics. One other issue which may warrant future analysis and investigation regards the pass criteria for the Readiness Survey. In line with standard undergraduate subjects, a 50% pass criterion is employed. One future avenue would be to examine the progress of students, split by their Readiness Survey score. For example, it may be the case that only students scoring above, say, 75% on the survey are going to pass their first year subjects. In such a scenario, we may wish to refine or raise the pass requirements of the survey.

2.2 Are there any examples of prerequisite paths failing to function as they should?

To examine how effective the current prerequisite paths are in terms of preparing students for future subjects, we tabulate the results for each subject with students categorised according to how they performed in the preceding subject. In an ideal situation, the pass rates for subjects later down a prerequisite path would be close to 100%; all students entering these subjects would be equipped with the necessary skills and knowledge from previous subjects to ensure success. This is not ever likely to be observed in reality. For a number of reasons, there will always be students who fail individual subjects. Some students report that they 'hit the wall' academically and find that study habits which carried them through earlier material may need to be changed to succeed with later topics. This is especially true in first year subjects where, especially for students who have done Extension Mathematics in high school, there is initially some overlap with material previously covered. When encountering entirely new and more advanced topics, there is the risk that students have become a little complacent. Additionally, unforeseen health, family or work pressures lead students to stop attending or to deprioritise their studies. Overall, though, an ideally functioning structure should see slowly increasing pass rates throughout the degree course.

One metric of an 'unhealthy' prerequisite path which we looked out for was where pairs of consecutive subjects had vastly different pass rates. For example, if the earlier subject had a very high failure rate but its following subject had a very low failure rate, then this could be symptomatic of one of two issues. Either the first subject is being needlessly harshly assessed, or perhaps the second subject is too easy or too leniently assessed. In either case, the possible inconsistency in assessment standards along the prerequisite path is not a desirable feature and, arguably, is unfair and confusing to students. Conversely, when the later subject has a much higher failure rate, a case might be made that it is too harshly assessed or, alternatively, that its preceding subject is too easy to pass and is passing students who are ill-equipped to

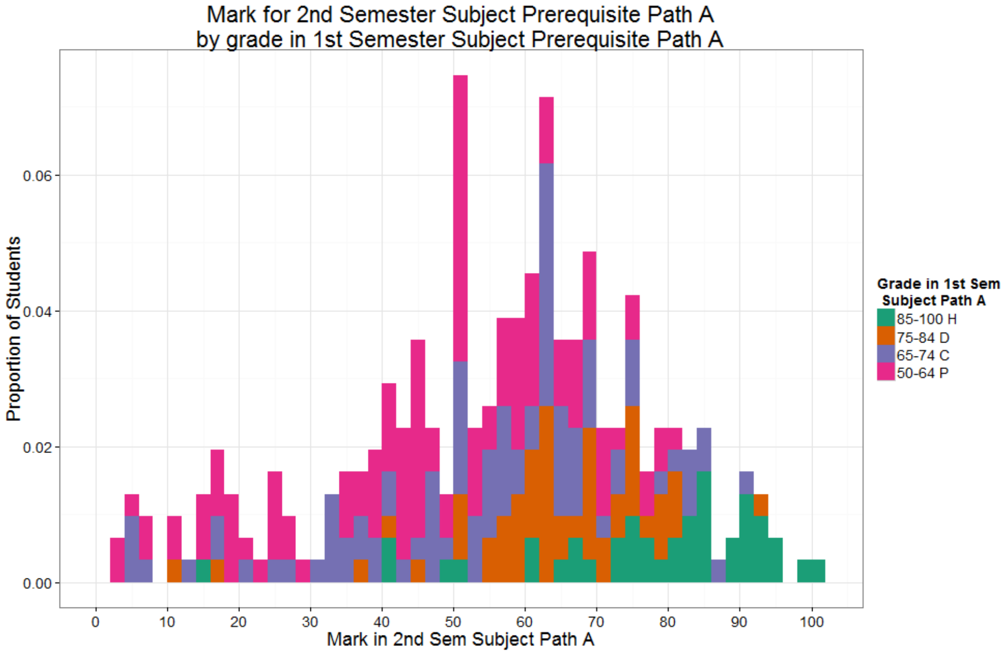

Figure 1: Distribution of results for second semester subjects along prerequisite Path A, tracking student performances in the prerequisite first semester subject.

handle more complex material in later subjects. Alternatively, there could be inherent differences between the two subjects which may account for the discrepancy in student performances. For example, if one subject is very theoretical and the other much more practical and applied, then it may be that students can more readily demonstrate understanding and ability in one of these areas than the other. Even in this case, the differences in subject style and tone might be something which we would wish to examine and potentially address in future subject redevelopment.

Figures 1, 2 and 3 show the progression of students from a first year, first semester subject through to its follow-up subject down three different pre-

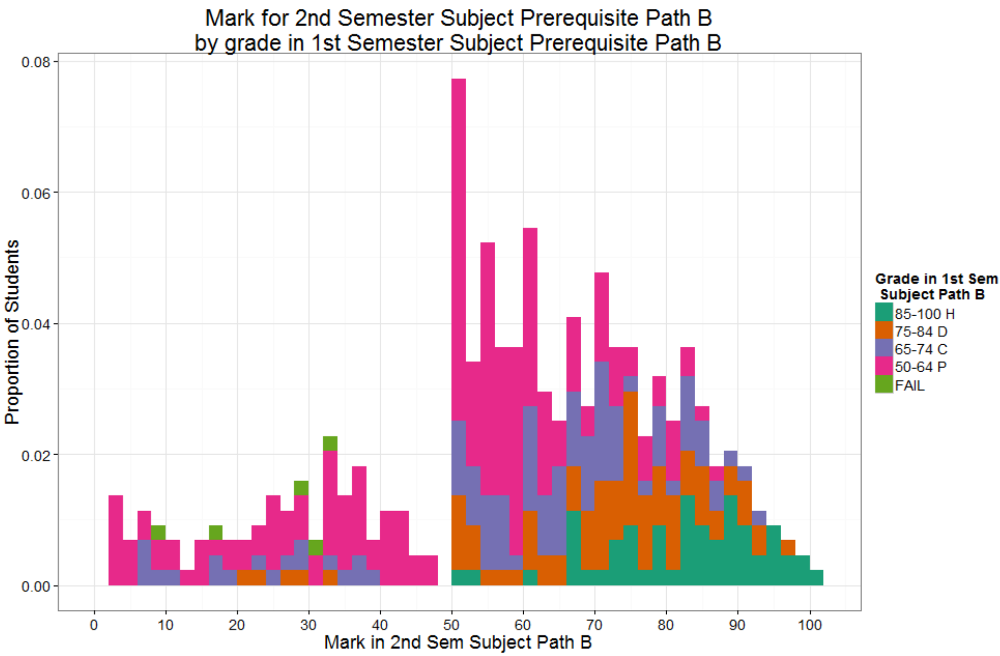

Figure 2: Distribution of results for second semester subjects along prerequisite Path B, tracking student performances in the prerequisite first semester subject.

requisite paths. In all three cases, the majority of the students failing the later subject scored only a Pass or a Credit in the previous subject. This is in itself entirely predictable. However, what is notable, is the difference in the performances of students whose previous result was a Pass (the pink section on the histograms). For the first two prerequisite paths shown (Path A and Path B), while the majority of the later Fails came from students who had previously obtained just a Pass, the majority of the Pass students did indeed successfully pass the subsequent subject. This is not the case for Path C. Extracting the data for just Pass students in the earlier subject, we see a very clear difference between subject Paths A and B and Path C (Table 2).

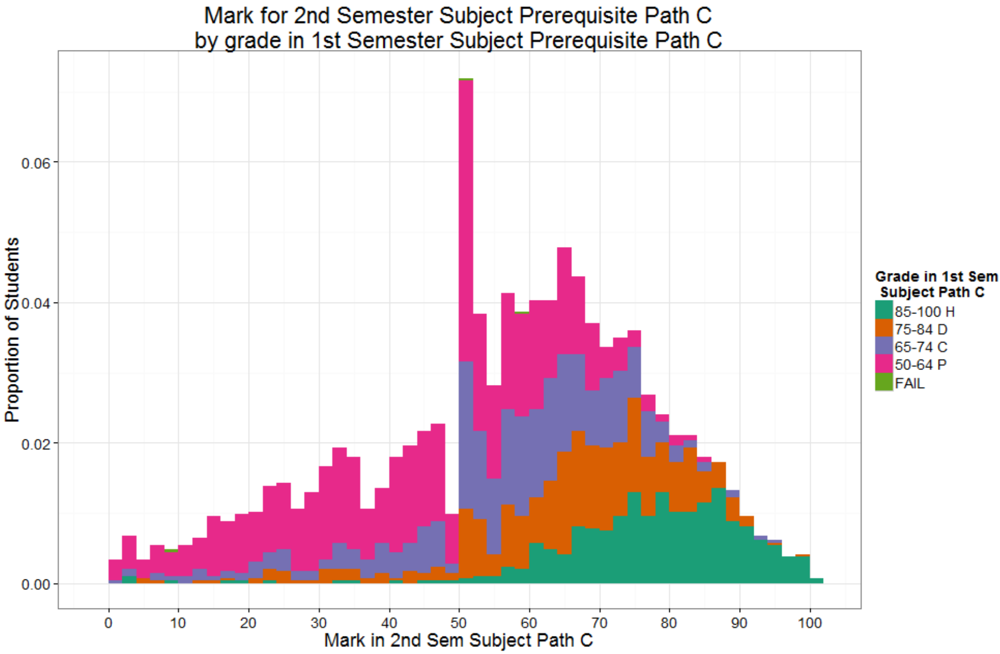

Figure 3: Distribution of results for second semester subjects along prerequisite Path C, tracking student performances in the prerequisite first semester subject.

Table 2: Pass rates (percentages) of students in second semester subjects along three different prerequisite paths. Data are only included for students who obtained a Pass (50–64) in the prerequisite first semester subject.

Mark in first semester subject	Pass rate of second semester subject Path A	Path B	Path C
50–54	79.5	52.0	32.6
55–59	85.0	68.1	46.0
60–64	85.7	74.1	59.5

In the case of Path C, there is clear evidence that obtaining a Pass in the first semester subject is not adequate preparation for the following subject as the majority of those students go on to fail. The exact reasons for this are currently unclear and will require a detailed review of the Pass benchmark for both subjects in question. We are currently in the process of evaluating aspects of academic content, subject delivery, and assessment standards and criteria for both subjects. One simple solution to this issue might be a change in the pass benchmark for one or both subjects. It may be concluded that the requirements to complete the first subject need to be toughened, or it may be decided that the second subject is currently too harshly assessed, or any combination of those two factors. One alternative explanation is that the quantity of material in one of the subjects could be improved. A first semester subject without prerequisites will always have some overlap with high school material, especially for students who have done Extension Mathematics. It might be that the first subject in this prerequisite path overlaps too much with previous material, or conversely that the second subject is too ambitious in terms of scope and quantity. No firm conclusions have been reached at this point, but this project illuminates what is evidently a point of concern with the current progression structure.

One alternative avenue which is also being explored is using these data to inform an 'early warning system' to aid students whose previous results identify them as being at high risk of future failure. For example, in the third of these prerequisite pairs, more than half the students who obtained a Pass in the first subject subsequently failed the second. However, of students who obtained a Credit, fewer than one quarter failed the subsequent course. Especially for such cases, we are developing a suite of targeted online resources for additional support. These are to be specifically aimed at students obtaining a Pass, and they would be encouraged to try to fill in some of the gaps in their knowledge to minimise their risk of future failure. This approach shares the responsibility for ensuring student successes between the individual student and the university. The student has to self-identify, admit academic weaknesses, and engage with the suggested online resources. Equally, the university delivers on its obligation to ensure students enrolling in more advanced subjects are given every opportunity to acquire and strengthen prerequisite knowledge and skills.

2.3 Do the analyses indicate the need for any changes to prerequisite paths?

To examine whether subjects needed to be added to the current prerequisite paths, we analysed the results for each subject, tabulated against other subjects which are not currently prerequisites. Prerequisite paths were traditionally set by a qualitative analysis of subject offerings; teaching staff simply stated which subjects seem necessary. As such, the work of this project provided the first quantitative measure of the effectiveness of this system. The metric used to detect subjects which should be added to a prerequisite path is a non-prerequisite subject that is a very strong predictor of future success or failure in a later subject. Naturally, it is expected that even for subjects in very different discipline areas, there is a reasonably strong correlation in student performance; very capable and motivated students tend to excel in most of their subjects, whereas weaker or less dedicated students struggle in multiple subjects. Unsurprisingly, this trend is indeed observed between all pairs of subjects. We account for this and only look for subjects where the trend is sufficiently strong, implying there is an additional factor in play.

Table 3 shows the data for the pair of subjects which most strikingly indicated a weakness in the existing prerequisite path. The subjects in question are a second year subject in the statistics/probability prerequisite path and a first year subject in the algebra path.

The failure rate of 87.5% for students who had not passed the first year subject is a stark indication that the prerequisite structure is not functioning as intended. This appears to indicate that students are permitted to enrol in a later subject when, based on their prior performances, they are not equipped with the fundamental skills and knowledge to succeed. A more

Table 3: Pass rates (percentages) for a second year subject tabulated against results in a non-prerequisite first year subject.

First year result	Result in second year subject 0–49 F	50–64 P	65–74 C	75–84 D	85–100 HD	Proportion of cohort
F	87.5	12.5	0.0	0.0	0.0	2.5
P	41.8	35.8	15.8	5.4	1.2	51.6
C	34.3	35.8	19.4	6.0	4.5	20.9
D	20.4	30.6	26.5	14.3	8.2	15.3
HD	9.6	6.5	12.9	22.6	48.4	9.7
Overall	35.0	31.6	17.5	8.4	7.5	100.0

detailed look at the data indicates that no student who failed the first year subject scored 55 or higher on the later subject during the six year timespan of the analyses.

Stepping back from the purely data-driven argument, it seems very reasonable to add the first year subject as a prerequisite for the second year subject. Although the later subject requires students to have first completed a first year subject in the most directly related sub-discipline, it also relies on algebraic skills which students who failed the first year algebra subject may lack. While students unquestionably need the statistics and probability background to progress, the evidence is clear that these alone are not sufficiently equipping students for future success. As a direct result of this project, from 2014 onwards, the prerequisite structure is amended to include this additional requirement.

3 Conclusions

This project has, for the first time, delivered a data-driven course-level perspective of the subject offerings of the School of Mathematical Sciences at UTS with analyses spanning six academic years. The points presented in this article represent only a small subsection of the overall set of analyses. However, they do give a clear picture of the sorts of insights delivered by these comprehensive quantitative reviews of subject and course performance. Some of the issues highlighted are perhaps unsurprising, others less so. Even in the case of detecting a previously known phenomenon, the school can proceed with more confidence and authority, with rigorous analyses to support any proposed changes. Other issues were flagged for the first time and hence, in these cases, the scope for curriculum improvement may have been missed with a simple subject-level perspective alone.

The challenges highlighted by this project must not be viewed solely as ones to be faced by academics alone. Identification of which students are, statistically speaking, at higher risk of future subject failure presents an opportunity to develop 'early warning' systems and to encourage students to take charge of their own undergraduate learning experiences. This will be done by developing a bank of selected online materials, such as video lectures, practice questions and full worked solutions to mirror sections of our course offerings. The weaknesses of one student will not be identical to those of any other, and so the suite of resources to which they are directed will be tailored to their specific education needs. The onus then falls to the students to heed the targeted advice and to provide themselves with the basis and skills to progress with their learning. This shared responsibility for achievement more closely mirrors how graduates will handle skills development in the workplace and hence such an ethos will better prepare UTS graduates for their post-university careers.

Now that they have been established, the core tools behind this project's analyses are entirely sustainable and will be repeated each semester to incorporate results as and when they become available. Indeed, the long-term trends will become increasingly well understood as individual subjects run for longer and more data become available. The costs associated with these future analyses will be low, ensuring the sustainability of this approach. Although this project only looks at student learning trajectories within subjects taught by the School of Mathematical Sciences, the underlying processes could, for minimal cost, be offered to all schools across the university to opt into such course-level analyses of student learning trajectories. The general ethos that early intervention and the offering of supplementary online support where needed may be key to minimising student failure rates (and hence, diminishing overall course attrition rates) is certainly one which is of much wider applicability than to just the Mathematical Sciences. The UTS Strategic Plan speaks of "empower[ing] each other and our students to grow, contribute, challenge and make a difference". This project further strengthens this commitment by implementing a novel strategy, informed by recent and relevant data, to ensure all students are equipped with the tools required for success in their future studies. By undertaking rigorous quantitative analyses such as these, we are in a position to ensure that future curriculum redevelopments are meeting the needs of students and delivering coherent and consistent expectations for assessment.

Acknowledgements We thank Bojana Manojlovic for her assistance in preparing this manuscript for publication.

References

[1] Belward, S., Mullamphy, D., Read, W. and Sneddon, G. Preparation of students for tertiary study requiring mathematics. *ANZIAM J.*, 47:C840–C857, 2005. http://journal.austms.org.au/ojs/index.php/ANZIAMJ/article/view/1078 C303

[2] Keller, F. S., and Sherman, J. G. *The Keller Plan handbook: Essays on a Personalized System of Instruction.* Menlo Park, CA.: W. A. Benjamin, 1974. ISBN: 978-0805352399 C300

[3] Rylands, L. J. and Coady, C. Performance of students with weak mathematics in first-year mathematics and science. *Int. J. Math. Edu. Sci. Tech.*, 40(6):741–753, 2009. doi:10.1080/00207390902914130 C303

[4] Tovey, J. Warning as pupils dump courses in 2-unit maths. *Sydney Morning Herald*, 6 November 2013. http://www.smh.com.au/national/education/warning-as-pupils-dump-courses-in-2unit-maths-20131105-2wzdm.html C303

[5] Varsavskya, C. Chances of success in and engagement with mathematics for students who enter university with a weak mathematics background.*Int. J. Math. Edu. Sci. Tech.*, 41(8):1037–1049, 2010. doi:10.1080/0020739X.2010.493238 C303

Author addresses

1. **S. Woodcock**, School of Mathematical Sciences, University of Technology, Sydney, Australia.
mailto:stephen.woodcock@uts.edu.au
2. **S. Bush**, School of Mathematical Sciences, University of Technology, Sydney, Australia.