

Dynamic Analysis of the Infinite Plate on Orthotropic Foundation Subjected to Moving Loads

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Abstract: Based on the Kirchhoff thin plate theory and elastodynamics theory, the Kirchhoff small deformation infinite elastic thin plate is adopted to simulate the pavement, and the orthotropic elastic half space is used to simulate subgrade. The mechanical model and dynamic equations in the rectangular coordinate system are established for the infinite elastic plate on orthotropic foundation subjected to moving loads. The integral forms of plane strain dynamic responses are derived by means of Fourier transform and inverse Fourier transform. Numerical examples are conducted on condition that the harmonic vibrating strip load is applied on the plate surface. Studies are conducted to investigate the effect of the soil orthotropic parameters on dynamic response of subgrade and the plate. The results indicate that the anisotropy of the soil has a great influence on the dynamic response of subgrade and pavement interaction, and that dynamic response can be described more accurately by considering the orthogonal anisotropy of foundation.

Keywords: orthotropic foundation; infinite plate; Fourier transform; dynamic responses.

1. Introduction

The research of dynamic response of infinite plate on orthotropic foundation subjected to moving loads in the practical engineering is important and significant, as some of the conclusions may be used in the dynamic behaviors of runways and roadways. The loads on the pavement slab are usually treated as static in some studies about the road surface. In fact, the load on the plate should be the moving loads with uniform speed when vehicles run normally. Under such situation, it is necessary to study the dynamic response of plate. A number of studies have been conducted recently to find the dynamic response of plate on the elastic foundation subjected to moving loads. Cheng[1] studied dynamic response of a thin rectangular plate on Winkler elastic foundation by means of the variational method. Zheng[2] analyzed dynamic response of simply supported rectangular plates on Winkler elastic foundation under moving loads using mode superposition method. Dynamic response of infinite plate on elastic Winkler foundation was obtained using Integral transform method by Sun[3] and Kim[4][5]. Generally, there is a big difference between Winkler foundation model and actual base model. Elastic half-space foundation model can reflect not only the deformation of soil within the range of loads, but also the displacement of soil outside the scope of loads. Experiments on dynamic response of pavement under moving load was investigated by Chen[6]. Jiang[7] analyzed the responses of asphalt pavement on the elastic isotropic foundation by the finite element software. Zhang[8] obtained the dynamic response of the orthogonal anisotropic medium plane strain problem under harmonic loads using the integral transform method. During deposition process, foundation soil shows significant anisotropy[9]. In addition, the reinforced soil displays the obvious anisotropy[10]. However, the dynamic response of the infinite plate on the orthotropic foundation goes largely unexplored. Based on the previous researches, the dynamic responses of an infinite plate on the orthotropic foundation are studied by the integral transformation method. Furthermore, the present study illustrates the influences of different soil parameters on the plate deformation, the soil vertical normal stress and contact stress between the plate and the foundation.

2. The problem description

2.1. The mechanical model

As shown in Fig. 1, a lateral mechanical load $q(x_1, t)$ moves with the constant velocity c parallel to the positive x_1 -axis on the plate surface, and ω is the angular frequency. The subgrade reaction is expressed as

$p(x_1, t)$ that acts on the bottom of plate. According to the law of action and reaction, there is also a force $p(x_1, t)$ on the ground surface and in the opposite direction.

$$q(x_1, t) = \begin{cases} q_0 e^{i\omega t} & |x_1 - ct| \leq b \\ 0 & \text{Other} \end{cases} \quad (1)$$

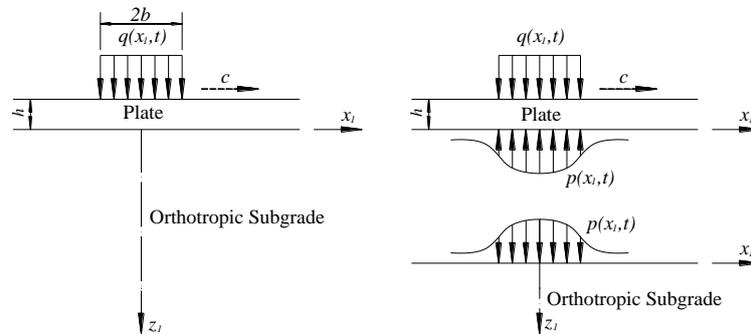


Figure 1 The mechanics model of an elastic plate on an elastic half-space

2.2. The basic equations

Under the dynamic load, the motion differential equation of plate on the elastic foundation is written as

$$D_p \frac{\partial^4 w_1}{\partial x_1^4} + m \frac{\partial^2 w_1}{\partial t^2} = q - p \quad (2)$$

where $D_p = \frac{Eh^3}{12(1-\mu^2)}$ is the flexural rigidity of elastic plate with E being the elastic modulus, μ being the Poisson's ratio, h being the thickness; m is the mass of infinite plate per unit area; and w_1 is the deflection of thin plate on foundation.

The subgrade soil can be viewed as the plane strain state[11]. For the plane strain problem of orthotropic elastic foundation, the equations of motion of the half-space are given as

$$\left. \begin{aligned} c_{11} \frac{\partial^2 u_{x_1}}{\partial x_1^2} + (c_{13} + c_{55}) \frac{\partial^2 u_{z_1}}{\partial x_1 \partial z_1} + c_{55} \frac{\partial^2 u_{x_1}}{\partial z_1^2} &= \rho \frac{\partial^2 u_{x_1}}{\partial t^2} \\ c_{55} \frac{\partial^2 u_{z_1}}{\partial x_1^2} + (c_{13} + c_{55}) \frac{\partial^2 u_{x_1}}{\partial x_1 \partial z_1} + c_{33} \frac{\partial^2 u_{z_1}}{\partial z_1^2} &= \rho \frac{\partial^2 u_{z_1}}{\partial t^2} \end{aligned} \right\} \quad (3)$$

where c_{ij} are the elastic constants of the foundation; u_{x_1} 、 u_{z_1} are the soil displacement along x_1 -axis and z_1 -axis, respectively; ρ is mass densities of the soil.

The stress-strain relationships of soil are

$$\left. \begin{aligned} \sigma_{x_1} &= c_{11} \frac{\partial u_{x_1}}{\partial x_1} + c_{13} \frac{\partial u_{z_1}}{\partial x_3} \\ \sigma_{z_1} &= c_{13} \frac{\partial u_{x_1}}{\partial x_1} + c_{33} \frac{\partial u_{z_1}}{\partial x_3} \\ \tau_{z_1 x_1} &= c_{55} \left(\frac{\partial u_{x_1}}{\partial x_3} + \frac{\partial u_{z_1}}{\partial x_1} \right) \end{aligned} \right\} \quad (4)$$

where σ_{x_1} 、 σ_{z_1} 、 $\tau_{z_1 x_1}$ are the stress components.

Referring to reference[12], we introduce $x = x_1 - ct$ and $z = z_1$. Then a steady pattern is created in the medium with respect to an observer situated in a moving coordinate system.

The variables in the moving coordinate system can be expressed as

$$\Psi(x_1 - ct, z_1, t) = \Psi(x, z) e^{i\omega t} \quad (5)$$

$$\dot{\Psi}(x_1 - ct, z_1, t) = (i\omega\Psi - c\Psi_x) e^{i\omega t} \quad (6)$$

$$\ddot{\Psi}(x_1 - ct, z_1, t) = (c^2\Psi_{xx} - 2i\omega c\Psi_x - \omega^2\Psi) e^{i\omega t} \quad (7)$$

where Ψ represents any variable, $\dot{\Psi}$ is the first derivative of time t , $\ddot{\Psi}$ is the two derivative of the time t .

Thus, Eqs. (2), (3) and (4) can be rewritten as

$$D_p \frac{\partial^4 w}{\partial x^4} + m(c^2 \frac{\partial^2 w}{\partial x^2} - 2i\omega c \frac{\partial w}{\partial x} - \omega^2 w) = q(x) - p(x) \quad (8)$$

$$\left. \begin{aligned} c_{11} \frac{\partial^2 u_x}{\partial x^2} + (c_{13} + c_{55}) \frac{\partial^2 u_z}{\partial x \partial z} + c_{55} \frac{\partial^2 u_x}{\partial z^2} &= \rho(c^2 \frac{\partial^2 u_x}{\partial x^2} - 2i\omega c \frac{\partial u_x}{\partial x} - \omega^2 u_x) \\ c_{55} \frac{\partial^2 u_z}{\partial x^2} + (c_{13} + c_{55}) \frac{\partial^2 u_x}{\partial x \partial z} + c_{33} \frac{\partial^2 u_z}{\partial z^2} &= \rho(c^2 \frac{\partial^2 u_z}{\partial x^2} - 2i\omega c \frac{\partial u_z}{\partial x} - \omega^2 u_z) \end{aligned} \right\} \quad (9)$$

$$\sigma_x = c_{11} \frac{\partial u_x}{\partial x} + c_{13} \frac{\partial u_z}{\partial z}, \quad \sigma_z = c_{13} \frac{\partial u_x}{\partial x} + c_{33} \frac{\partial u_z}{\partial z}, \quad \tau_{zx} = c_{55} \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) \quad (10)$$

2.3. The boundary conditions

The boundary conditions for the half-space surface of elastic foundation are $\lim_{|x| \rightarrow \infty} u_x(x, z, t) = 0$, $\lim_{|x| \rightarrow \infty} u_z(x, z, t) = 0$, $\lim_{|x| \rightarrow \infty} \partial u_x(x, z, t) / \partial x = 0$, $\lim_{|x| \rightarrow \infty} \partial u_z(x, z, t) / \partial x = 0$.

When the plate is cooperated with the ground where it is placed, the bottom surface of plate and foundation surface have the same vertical displacement. Moreover, the plate is assumed in the smooth contact with the foundation[13]. Then, the boundary conditions at the surface ($z = 0$) are given as

$$\sigma_z(x, 0, t) = -p(x)e^{i\omega t} \quad (11)$$

$$\tau_{zx}(x, 0, t) = 0 \quad (12)$$

$$u_z(x, 0, t) = w(x, t) \quad (13)$$

3. The Solution procedure

3.1. Fourier transform

To solve Eqs.(8)and(9), we use Fourier transform with respect to x -coordinate and its inverse transformation that are defined by

$$\bar{f}(\xi, z) = \int_{-\infty}^{+\infty} f(x, z) e^{-i\xi x} dx \quad (14)$$

$$f(x, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \bar{f}(\xi, z) e^{i\xi x} d\xi \quad (15)$$

Fourier transform of the dynamic Eq. (8) gives

$$\bar{w} = (\bar{q} - \bar{p}) / \{D_p \xi^4 + m(-c^2 \xi^2 + 2\omega c \xi - \omega^2)\} \quad (16)$$

Using Eq. (14) to transform the dynamic Eq.(9), and introducing boundary conditions, a new expression in matrix form can be obtained as

$$\begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} \frac{\partial^2}{\partial z^2} \begin{bmatrix} \bar{u}_x \\ \bar{u}_z \end{bmatrix} + \begin{bmatrix} 0 & B_{12} \\ B_{12} & 0 \end{bmatrix} \frac{\partial}{\partial z} \begin{bmatrix} \bar{u}_x \\ \bar{u}_z \end{bmatrix} + \begin{bmatrix} D_{11} & 0 \\ 0 & D_{22} \end{bmatrix} \begin{bmatrix} \bar{u}_x \\ \bar{u}_z \end{bmatrix} = 0 \quad (17)$$

where $A_{11} = -c_{55}$, $B_{12} = -(c_{13} + c_{55})i\xi$, $A_{22} = -c_{33}$, $D_{11} = c_{11}\xi^2 + \rho(-c^2\xi^2 + 2\omega\xi c - \omega^2)$, $D_{22} = c_{55}\xi^2 + \rho(-c^2\xi^2 + 2\omega\xi c - \omega^2)$.

The characteristic equation of Eq. (17) can be formulated as

$$a_1 \lambda^4 + a_2 \lambda^2 + a_3 = 0 \quad (18)$$

where $a_1 = A_{11}A_{22}$, $a_2 = A_{11}D_{22} + A_{22}D_{11} - B_{12}^2$, $a_3 = D_{11}D_{22}$.

Characteristic equation (18) is a quartic equation with the complex coefficients, and it has four roots in forms $\pm\lambda_1, \pm\lambda_2$. λ_j is a complex with the positive real part, namely $\text{Re}[\lambda_j] \geq 0, (j = 1, 2)$, and it can be expressed as follows

$$\lambda_j^2 = (-a_2 \pm \sqrt{a_2^2 - 4a_1 a_3}) / (2a_1) \quad (19)$$

where λ_1^2 and λ_2^2 refer to two different complexes.

The solutions of Eq. (17) can be written as

$$\bar{u}_x = \sum_{j=1}^2 \alpha_j (H_j e^{\lambda_j z} - F_j e^{-\lambda_j z}), \quad \bar{u}_z = \sum_{j=1}^2 (H_j e^{\lambda_j z} + F_j e^{-\lambda_j z}) \quad (20)$$

in which $\alpha_j = -\frac{A_{22}\lambda_j^2 + D_{22}}{B_{12}\lambda_j}$, F_j and H_j are undetermined coefficients.

3.2. General solutions

For the dynamic analysis of the orthotropic half-space ($z \geq 0$), the fluctuation shows a damping trend such as $\lim_{z \rightarrow \infty} u_x(x, z, t) = 0$, $\lim_{z \rightarrow \infty} u_z(x, z, t) = 0$.

Therefore, H_j should be zero in the general solution of integral forms of the displacement component for the orthotropic media under moving load. Then, Eq.(20) can be rewritten as

$$\bar{u}_x = -\sum_{j=1}^2 \alpha_j F_j e^{-\lambda_j z}, \quad \bar{u}_z = \sum_{j=1}^2 F_j e^{-\lambda_j z} \quad (21)$$

Performing Fourier transform to the Eq. (10), and substituting Eq.(21) into its transforming form, we have

$$\left. \begin{aligned} \bar{\sigma}_x &= -c_{11}i\xi \sum_{j=1}^2 \alpha_j F_j e^{-\lambda_j z} - c_{13} \sum_{j=1}^2 \lambda_j F_j e^{-\lambda_j z} \\ \bar{\sigma}_z &= -c_{13}i\xi \sum_{j=1}^2 \alpha_j F_j e^{-\lambda_j z} - c_{33} \sum_{j=1}^2 \lambda_j F_j e^{-\lambda_j z} \\ \bar{\tau}_{xz} &= c_{55} \left(\sum_{j=1}^2 \alpha_j \lambda_j F_j e^{-\lambda_j z} + i\xi \sum_{j=1}^2 F_j e^{-\lambda_j z} \right) \end{aligned} \right\} \quad (22)$$

Performing Fourier transform to the boundary conditions Eqs.(11)and(12) and substituting into Equation (22) gives the coefficient F_j

$$F_1 = -\bar{p}(\alpha_2 \lambda_2 + i\xi) / \Delta, \quad F_2 = \bar{p}(\alpha_1 \lambda_1 + i\xi) / \Delta \quad (23)$$

where $\Delta = -(c_{13}i\xi\alpha_1 + c_{33}\lambda_1)(\alpha_2\lambda_2 + i\xi) + (c_{13}i\xi\alpha_2 + c_{33}\lambda_2)(\alpha_1\lambda_1 + i\xi)$.

Substituting Eqs.(16) and (21)and (23) into displacement boundary condition Eq.(13), the subgrade reaction is obtained as

$$\bar{p} = \bar{q} / \{1 + [D_p \xi^4 + m(-c^2 \xi^2 + 2\omega c \xi - \omega^2)] \cdot [-(\alpha_2 \lambda_2 + i\xi) + (\alpha_1 \lambda_1 + i\xi)] / \Delta\} \quad (24)$$

Thus, the deflection of plate is

$$\bar{w} = \bar{q} / \{D_p \xi^4 + m(-c^2 \xi^2 + 2\omega c \xi - \omega^2) + \Delta / [-(\alpha_2 \lambda_2 + i\xi) + (\alpha_1 \lambda_1 + i\xi)]\} \quad (25)$$

The soil vertical normal stress is

$$\bar{\sigma}_z = \bar{q} \cdot \frac{[c_{13}i\xi\alpha_1(\alpha_2\lambda_2 + i\xi) + c_{33}\lambda_1(\alpha_2\lambda_2 + i\xi)]e^{-\lambda_1 z} - [c_{13}i\xi\alpha_2(\alpha_1\lambda_1 + i\xi) + c_{33}\lambda_2(\alpha_1\lambda_1 + i\xi)]e^{-\lambda_2 z}}{\Delta + [D_p \xi^4 + m(-c^2 \xi^2 + 2\omega c \xi - \omega^2)] \cdot [-(\alpha_2 \lambda_2 + i\xi) + (\alpha_1 \lambda_1 + i\xi)]} \quad (26)$$

Taking Fourier transform of the load on thin plate, we have

$$\bar{q} = 2q_0 e^{i\omega t} \sin(\xi b) / \xi \quad (27)$$

Substituting Eq.(27) into Eqs.(24) and (25) and (26), the integral form solutions can be obtained by the inverse Fourier transform such as

$$w = \frac{q_0 e^{i\omega t}}{\pi} \int_{-\infty}^{+\infty} \frac{1}{\xi} \frac{\sin(\xi b)}{D_p \xi^4 + m(-c^2 \xi^2 + 2\omega c \xi - \omega^2) + \Delta / [-(\alpha_2 \lambda_2 + i\xi) + (\alpha_1 \lambda_1 + i\xi)]} e^{i\xi x} d\xi \quad (28)$$

$$p = \frac{q_0 e^{i\omega t}}{\pi} \int_{-\infty}^{+\infty} \frac{1}{\xi} \frac{\sin(\xi b)}{1 + [D_p \xi^4 + m(-c^2 \xi^2 + 2\omega c \xi - \omega^2)] \cdot [-(\alpha_2 \lambda_2 + i\xi) + (\alpha_1 \lambda_1 + i\xi)] / \Delta} e^{i\xi x} d\xi \quad (29)$$

$$\sigma_z = \frac{q_0 e^{i\omega t}}{\pi} \int_{-\infty}^{+\infty} \frac{\sin(\xi b)}{\xi} \frac{[c_{13}i\xi\alpha_1 + c_{33}\lambda_1](\alpha_2\lambda_2 + i\xi)e^{-\lambda_1 z} - [c_{13}i\xi\alpha_2 + c_{33}\lambda_2](\alpha_1\lambda_1 + i\xi)e^{-\lambda_2 z}}{\Delta + [D_p \xi^4 + m(-c^2 \xi^2 + 2\omega c \xi - \omega^2)] \cdot [-(\alpha_2 \lambda_2 + i\xi) + (\alpha_1 \lambda_1 + i\xi)]} e^{i\xi x} d\xi \quad (30)$$

4. The computational examples

To check the numerical results, a particular highway is chosen at which these values of consents have been measured. The parameters of load are $b=0.075\text{m}$; $q_0=100\text{kPa}$; $f=8\text{Hz}$; $c=35\text{m/s}$. The elastic parameters of plate are $E=30\text{GPa}$; $\mu=0.15$; $h=0.25\text{m}$; $m=2400\text{kg/m}^3$. The parameters of soil are $E_x=50\text{MPa}$; $\mu_{xy}=0.25$; $G_{zx}=24\text{MPa}$; $\rho=1815\text{ kg/m}^3$; $c_R=106.4\text{m/s}$; $c_s=115\text{m/s}$; damping ratio $\eta=0.05$. This paper introduces proportionality coefficient k_m ($m=1,2,3,4$) to describe orthogonal anisotropy of the soil, namely, $E_y = k_1 E_x$, $E_z = k_2 E_x$,

$\mu_{xz} = k_3\mu_{xy}$, $\mu_{yz} = k_4\mu_{xy}$. The soil is isotropic when k_m is 1. In practical engineering, the anisotropy of soil is usually expressed by engineering constants, and the relationship between the stiffness coefficient and the engineering constant is show in literature[14]. Based on the above method and fast Fourier transform theory[15], the numerical computation is implemented by MATLAB software. If the plate is infinitely large and the velocity is constant, the deflected shapes subjected to the moving load are the same at any instant along the moving axis, which means that the deflected shape is moving with the load[5]. The zero point represents the location of the load center. μ_{ij} has little change and little impact on the calculation results so that this paper only considers the influence of E_i on dynamic response. In this paper k_3 is 1.2 and k_4 is 1.6.

Figure 2(a) shows plate deformation curves due to different k_1 on condition that k_2 is 0.8. Obviously, the anisotropy of the soil has a certain influence on the plate deformation. The vertical displacement of plate on the isotropic soil is small and flattens out, and its maximum is only greater than that of $k_1=0.5$. It is clear that plate displacement increases with the increase of the k_1 value. But the increase is very small, and the value can be ignored. Especially plate deformation curve almost overlaps when k_1 is 1.2 and k_1 is 2.0. It shows that the elastic modulus E_y of soil has very little influence on displacement of plate. The effect of k_2 on the plate deformation shapes can be observed in Figure 2(b) on condition that k_1 is 1.2. As the value of k_2 increases, plate maximal displacement decrease obviously. The maximum vertical displacement of plate on the isotropic soil is only less than that of $k_2=0.8$. If the plate deformation is too large, we can increase the elastic modulus E_z of soil appropriately, the effect is not obvious to change E_y .

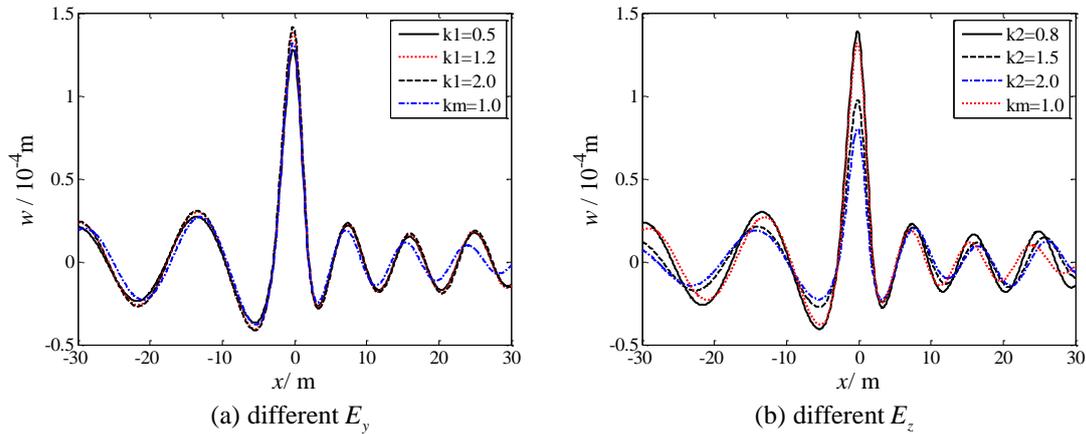


Figure 2 Plate deformation curves

Figure 3(a) reflects contact stress curves between the plate and the foundation due to different k_1 when k_2 is 0.8. As the value of k_1 increases, contact stress decreases. But the decrease is very small, and the decrement can be ignored. Especially contact stress curve almost overlaps when $k_1=1.2$ and $k_1=2.0$. The contact stress maximum of the isotropic soil is only less than that of $k_1=0.5$. It shows that the elastic modulus E_y of soil has very little influence on contact stress. As can be seen in Figure 3(b), the effect of k_2 on contact stress is obvious on condition that k_1 is 1.2. As the value of k_2 increases, maximal stress decrease obviously. The maximum stress on the isotropic soil is only greater than that of $k_2=0.8$. On the right side of the loading area, oscillation amplitude of the contact stress decreases as the value of k_2 increases. About 4 meters away from the left side of the loading area, the anisotropy of the soil have little impact on contact stress, and contact stress is almost close to zero. If the contact stress is too large, we can decrease the elastic modulus E_z of soil appropriately, the effect is not obvious to change E_y .

The effect of k_1 on the soil vertical normal stress shapes is shown in Figure 4(a) when k_2 is 0.7. As the value of k_1 increases, vertical normal stress decreases. Stress curve of $k_1=0.5$ is close to that of $k_m=1$. The maximum value of vertical normal stress appears at about depth of 0.1m. The depth of tensile stress is smaller with the value of k_1 increasing. Figure 4(b) shows soil vertical normal stress curves due to different k_2 on condition that k_1 is 1.25. As the value of k_2 increases, vertical normal stress increases obviously. The stress on the

isotropic soil is only less than that of $k_2=0.8$. The maximum value of vertical normal stress appears at about depth of 0.1m.

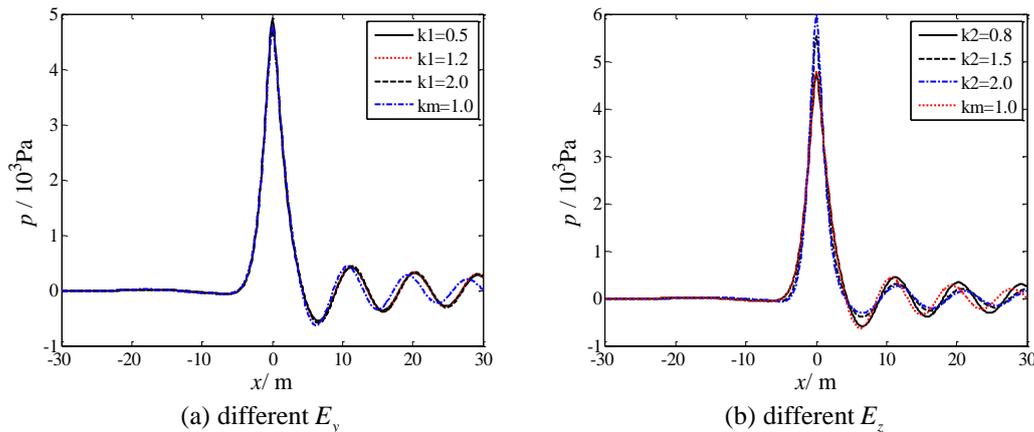


Figure 3 Contact stress curves between the plate and the foundation

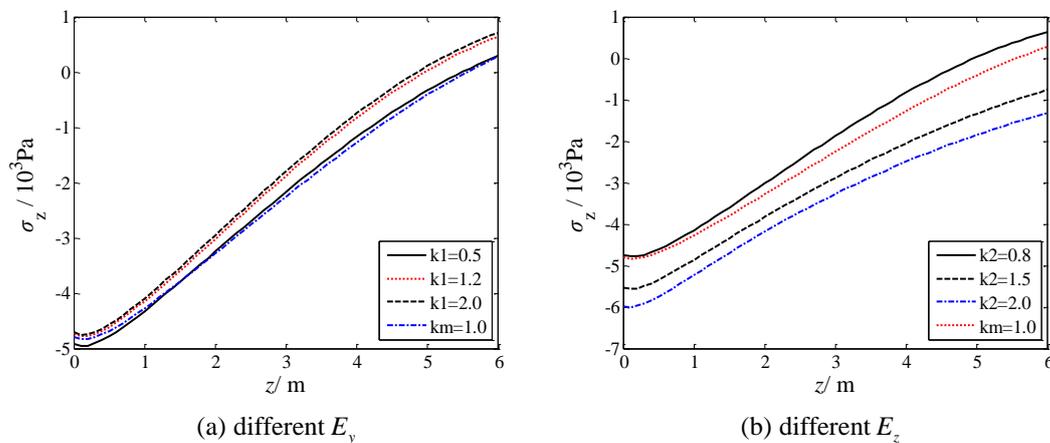


Figure 4 The soil vertical normal stress curves at the center of load

5. Conclusions

The work studied the dynamic response of infinite plate on the orthotropic foundation subjected to the harmonic moving loads in rectangular coordinate system. Through the numerical analysis, several significant conclusions are drawn as: (1)The anisotropy of the soil has a great influence on the plate deformation, the contact stress and the soil vertical normal stress. It is suggested that the effect of the orthotropic soil on the dynamic response of the foundation is considered in the actual projects. (2)If the plate deformation is too large, the elastic modulus E_x of soil should be increased appropriately. If the contact stress is too large, the elastic modulus E_z of soil should be decreased appropriately. The effect is not obvious to change E_y . (3)The maximum value of vertical normal stress appears at about depth of 0.1m. As the value of E_y increases, vertical normal stress decreases. As the value of E_z increases, vertical normal stress increases obviously. Various factors should be considered to choose the appropriate soil parameters. The above conclusions provide certain theoretical foundation for further dynamic analysis and research of road surface.

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