

# Method for Investigation of Structures for Temperature Effects

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**Abstract:** In this paper we consider the developed method which, in the presence of the appropriate indicated types of collateral, is an effective technology for performance of investigations of structures exposed to virtual temperature fields. Its distinctive features are the simplicity of experimental part, the low cost of actual experiment, the physical clarity and logical foundation of results being obtained. In the theoretical bases of the method there is the energy principle of Betty-Maxwell reciprocity of works. It was shown that thermal displacement of the element (structure) in a predetermined direction and at a given time depends on the modulus of deformation of the 1st kind  $E$ , coefficient of transverse deformations  $\nu$ , the thermal expansion coefficient  $\alpha$  and the 1st invariant of deformation tensor defined in a predetermined number of points of the heated (cooled) element loaded by a single force. A program of experimental investigations is suggested. Results of a full-scale experiment are given by the example of a joint of monolithic reinforced concrete slab and column. The comparison of the values obtained experimentally and by using the proposed technique made the difference of 5-15%. By means of expanding the technical base, the possibilities of the method can be significantly expanded. This method allows testing the types of structures varying in shape and application: bodies, shells, plates, rods, combinations thereof, etc.

**Keywords:** temperature effects; reinforced concrete structures; numerical methods; photogrammetry; frame systems.

## 1. Introduction

As is known, laboratory, and in particular, full-scale tests of structures exposed to high temperatures, are rather complex and high budget procedures [1, 2]. In this connection, the search for effective and low-cost methods for investigations of structural systems and components being under the conditions of mentioned effects is a promising and timely direction of solving this problem. Particular interest is aroused by the idea, based on an equivalent replacement of a body being heated or cooled by an unheated one [3]. At the heart of such replacement there is an ideology, grounded on the energy principle of Betty-Maxwell reciprocity of works. The manner of creating such a method is specified in the above cited work; however, its introduction into practice has become impossible due to a variety of reasons. First of all, these should include the following:

- 1) complexity of recording of displacements in the interior points of the region occupied by a test body;
- 2) complexity of taking measurements of displacements in a significant number of points on the body surface;
- 3) need to perform a large number of computing operations associated with the processing of experimentally obtained information and its presentation in a form predetermined by a special algorithm.

Professor V. Meisel, the author of this approach, chooses from a variety of structures and situations only those for which, by means of a prior decision of relevant tasks of the theory of elasticity, it is permissible to express the first deformation tensor invariant via the vector of displacements of points lying on the surface. Such constructs include mainly thin plates.

Integrating the listed items, together with the regard of the current state of existing measurement technology and application software, it is considered reasonable to create a new experimental and theoretical method of testing structures for action of temperatures.

## 2. Theoretical bases of the method

The following hypotheses and assumptions were adopted in the development of the method:

- 1) full equivalence of heated and unheated bodies in terms of geometry (shape, dimensions) and materials;
- 2) full equivalence of support conditions (edge conditions) of heated and unheated bodies;
- 3) presentation of a structure in the form of not simply connected heterogeneous isotropic body, the characteristics of which, as a consequence, are functions of coordinates and time  $E=E(x,y,z,t)$ ;  $\nu=\nu(x,y,z,t)$ , where  $E$  is modulus of deformation of the 1st kind;  $\nu$  is coefficient of transverse deformations;  $x, y, z$  are Cartesian system coordinates;  $t$  is time;
- 4) possibility to set the temperature distribution  $T$  in the form of stationary or nonstationary fields -  $T(x, y, z, t)$ ;

5) possibility of approximate identifying of the relationship between deformation intensity and stress intensity with the relationship between stress and deformation, obtained experimentally for a uniaxially loaded standard sample (analogy of the hypothesis of the theory of small elasto-plastic deformations).

The formula obtained in work [3] is used as a theoretical base of the method. The said formula is based on the above hypotheses and assumptions as well as Betty-Maxwell concept. The formula allows determining the displacement in any point of the heated (cooled) body being studied. For that, instead of the body (structure) experiencing the preset temperatures, the same, but unheated constructive is introduced into the consideration, which is loaded with unit force applied at an arbitrary point C and in the direction of the temperature displacement of interest. The formula in Cartesian coordinates is as follows:

$$Q_c = \iiint_{V_B} \frac{\alpha E}{1-2\nu} T \cdot \theta \, dV_B, \quad (1)$$

where  $Q_c$  is the displacement of the structure at the point C;

$\alpha = \alpha(x, y, z, t)$  is the thermal expansion coefficient;

$T - T(x, y, z, t)$  is the preset temperature field;

$\theta^*$  is the unit volumetric deformation due to the action of the unit force applied at the point C;

$V_B$  is the volume occupied by the structure.

This formula is the reason for the creation of the proposed method.

### 3. Mathematical model

Let us introduce the following designation:

$$f(x, y, z, t) = \frac{\alpha(x, y, z, t) \cdot T(x, y, z, t)}{1 - \nu(x, y, z, t)} \quad (2)$$

Then formula (1) will take the following form:

$$Q_c = \iiint_{V_B} f(x, y, z, t) \operatorname{div} R \, dV_B, \quad (3)$$

where  $\{R\}^T = \{u, v, w\}$

$u = u(x, y, z, t)$ ;  $v = v(x, y, z, t)$ ;  $w = w(x, y, z, t)$ ;

$\operatorname{div} R = I_1$ ;

$I_1$  is the 1st invariant of deformation tensor;

$u, v, w$  are projections of vectors of displacements along the axes  $X, Y, Z$  respectively  $f; u, v, w \in C^1$

In case of a preset stationary temperature field  $T = \text{const}$ . If, herein, the body is isotropic and uniform, then  $E = \text{const}$ ,  $\nu = \text{const}$ ,  $\alpha = \text{const}$ , and as a result,  $f = \text{const}$ , then formula (3) becomes:

$$Q_c = f \cdot \iiint_{V_B} \operatorname{div} R \, dV_B, \quad (4)$$

Further, Ostrogradskii-Gauss formula is used:

$$Q_c = f \cdot \iiint_{V_B} \operatorname{div} R \, dV_B = \iint_S (Ul + Vm + Wn) dS \quad (5)$$

where  $S$  is the surface area of the body under consideration;

$l, m, n$  are direction cosines of the outward normal, and integral on the right side is the surface integral of the 1st kind.

Thus, formula (5) enables only by displacements occurring on a surface of a heated body under consideration, which is loaded with unit force, to determine the displacement of the preset point, in the preset direction and at the preset timepoint, of the heated (cooled) body.

For general case, when a non-stationary field is preset, formula (3), after integration by parts, will take on the following form:

$$Q_c = \iiint_{V_B} f(x, y, z, t) \operatorname{div} R \, dV_B = \iint_S (Ul + Vm + Wn) dS + \iiint_V \left[ U \frac{\partial f}{\partial x} + V \frac{\partial f}{\partial y} + W \frac{\partial f}{\partial z} \right] dV_B; \quad (6)$$

Presence of the second summand (volume integral) on the right side of (6), complicates the problem as compared to the procedure defined by (5). That is, in case of constant temperatures and homogeneity of material, it is sufficient to measure the displacements of the body points located on the surface, and then to determine the thermal displacement according to (5).

As for general case (unsteady temperature field), in order to implement the sequence of actions according to (6), the following algorithm is proposed:

- 1) discretization of the computational model of a body by finite element method (FEM);
- 2) determination of function  $f(x, y, z, t)$  by the preset temperature and physical-mechanical parameters;
- 3) analytical (in case of approximation) or numerical calculation of the derivatives of  $f$  according to coordinates  $x, y, z$ ;
- 4) determination of FEM (if necessary, taking into account physical and geometrical nonlinearities) of the displacements at all nodes of the finite elements (FE) mesh due to the action of the unit force applied at the preset point;
- 5) experimental measurement of displacements at the nodes of the FE mesh located only on the surface of the body;
- 6) calculation of the difference between the movements in FE mesh nodes located on the surface of the body, measured experimentally and calculated theoretically;
- 7) calculation of the movement at all FE mesh nodes from the found difference of displacements applied to the surface of the body;
- 8) calculation of the total displacements at all nodes of the FE mesh due to the action of the unit force and the surface displacements difference;
- 9) definition of the temperature displacement in question according to (6).

The adopted experimental-theoretical approach allows you to adjust the displacement field obtained by calculation in such a way, that on the body (structure) surface the displacements would be equal to those experimentally measured.

The given algorithm substantiates the suggested experimental-theoretical method in general case.

In some special but practically important cases, the method procedure can be simplified. They primarily include constructives in form of thin plates and shells. A special feature here is that the first invariant of the deformation tensor can be expressed via vertical displacements (deflections) of the middle surface of the unheated plate under the action of the unit vertical force [4].

Formula (3) then takes the following form (Fig. 1) [3]:

$$Q_c = - \iint_S \frac{\alpha \cdot E}{1 - \nu} \cdot \nabla W(\xi, \eta, x, y) \left[ \int_{-h/2}^{h/2} T(x, y, z, t) z dz \right] dx dy, \quad (7)$$

where  $h$  is the plate thickness;

$\nabla$  is the harmonic Laplace operator;

$W(\xi, \eta, x, y)$  is the deflection of the point  $(x, y)$  of the middle surface of the unheated plate under the action of the unit vertical force applied at the point  $(\xi, \eta)$ . As before, the temperature field in (7) can be of any kind.

By positioning the mesh of points on the lower surface of the plate (Bernoulli-Kirchhoff-Love hypotheses are considered fair) and replacing the harmonic operator with its finite-difference representation, instead of (7) we get the following:

$$Q_c = - \sum_{i=1}^N \sum_{j=1}^M \frac{\alpha_{ij} \cdot E_{ij}}{1 - \nu_{ij}} \cdot \nabla_{ij} W \left[ \int_{-h/2}^{h/2} T(x, y, z, t) z dz \right], \quad (8)$$

where  $N, M$  is the number of points on the surface of the plate along the axes  $X$  and  $Y$ , respectively.

$\nabla_{ij}$  is the discrete (finite-difference) analogue of harmonic Laplace operator.

$i \in [1, N]$ ;  $j \in [1, M]$  are running indexes.

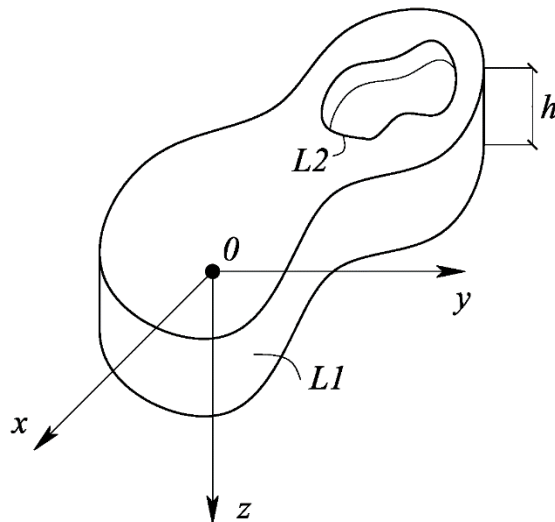


Figure 1 Thin plate. L1, L2 are contours.

At the same time, Laplace operator takes the following form:

$$\nabla_{ij} w = \frac{1}{\lambda_x^2} (W_{i+1,j} - 2W_{i,j} + W_{i-1,j}) + \frac{1}{\lambda_y^2} (W_{i,j+1} - 2W_{i,j} + W_{i,j-1}), \quad (9)$$

where  $\lambda_x$ ;  $\lambda_y$  are steps of the mesh along the coordinate axes;

$W_{i,j}$  is deflection of the plate at the point  $(i, j)$ .

From (9) it follows that by measuring the displacements of the plate due to the action of the unit force only on its lower surface, it is possible to determine the temperature displacement at the point of its application.

A similar manner can be used for shells, and taking into account certain additional prerequisites for thick plates as well [3].

#### 4. The experimental component of the method

Let us denote the attributes of the method.

*The object of the investigation* is a structure (body), undergoing the preset temperature effect (field).

*The subject of the investigation* is temperature displacements of the body being studied.

*The purpose of the method* is to determine thermal displacements of the structure by means of experimental-theoretical investigation of its unheated analogue.

The method involves special hardware and software.

Hardware includes loading system, measuring system and computing complex (PC).

*Loading system* consists of a lifting jack, an oil-pumping station and a support frame (Fig. 3). This system is easy to use, it is mobile, and does not require big starting and operating costs. It is designed for use in tests both in laboratory conditions (small models), and directly on finished structures (full-scale tests, tests of large-scale models).

*Measuring system* is a photogrammetric or laser installation which allows experimental measuring of all three projections of displacement on the surface of the sample (body) [5], and, practically, at any number of points, for example of Vic-3D type [6].

Such measuring system has the following advantages as compared to other measuring systems:

- 1) time and cost savings as compared to the use of resistive-strain sensors;
- 2) absence of direct contact with measured object (sample), allowing the use of the system to measure deformation field on a real operating object or on samples which are sensitive to mechanical effect;
- 3) use of common white light to illuminate the measured object, and absence of potentially dangerous laser emissions;
- 4) visualization of display of obtained results in form of full-color 3D diagrams superimposed on an image of a real object; obtaining of a complete picture of deformations over the entire surface, in addition (background

effect), use of the system for rapid analysis of the structure during loading and search for mechanical concentrators;

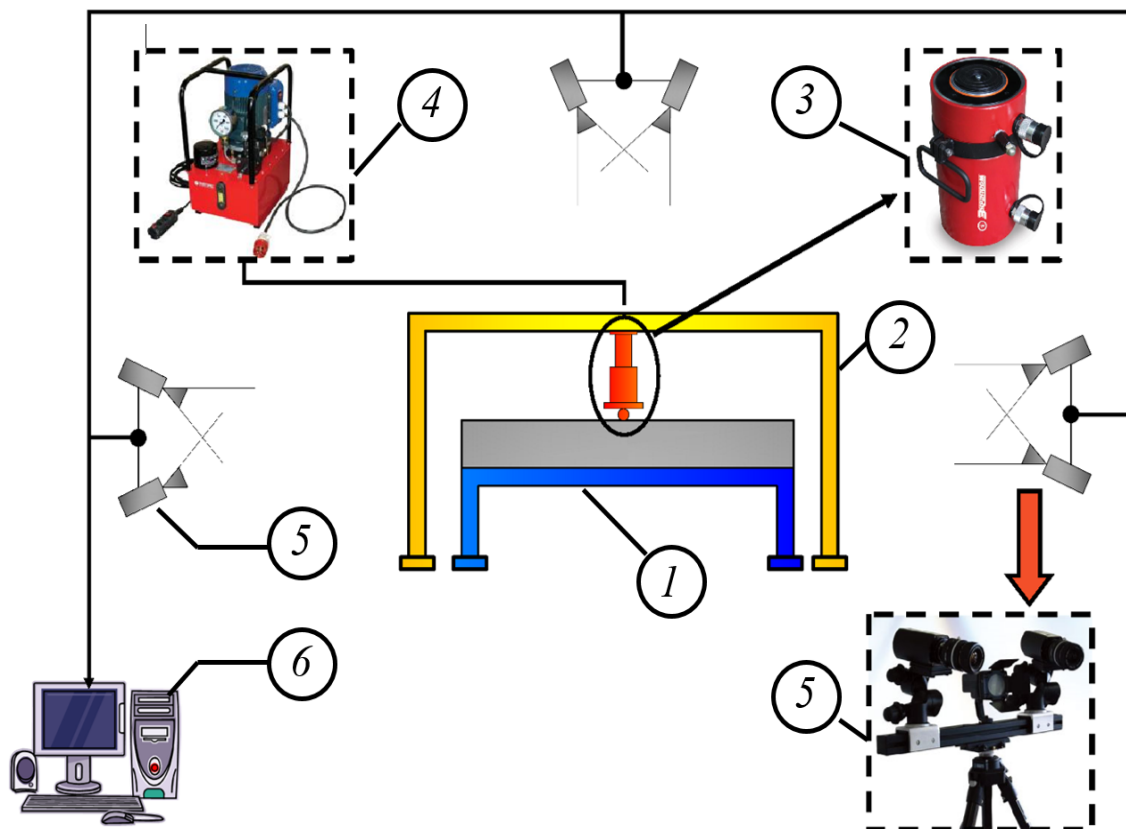
5) availability of standard libraries for calculations of deformations  $\varepsilon_1$ ,  $\varepsilon_2$ ,  $\varepsilon_{xx}$ ,  $\varepsilon_{yy}$ ,  $\varepsilon_{xy}$  of Young's modulus and Poisson's ratio;

6) synchronization with the test machine, and use of integrated functions of “virtual extensometer” and “virtual resistive-strain sensor” for obtaining of loading diagram in true coordinates;

7) simple export of diagrams, for example, into standard applications of Microsoft Office, ensures the formation of a full report of conducted tests;

8) high resolution of the measuring system, which allows capturing the changes in deformation associated even with internal damage of lamellas of the composite material.

Software includes a proprietary system for processing of taken surface displacements, any software package that implements the finite element method and its own set of exclusive utilities, which ensure co-function of the systems of loading, measuring and calculation. Structural diagram is shown in Figure 2.



**Figure 2 Structural diagram of the method:**  
 1 – investigation object; 2 - support frame; 3 - hydraulic lifting jack; 4 – oil-pumping station;  
 5 - photogrammetric or laser installation; 6 - personal computer equipped with PC FEM.

## 5. The experimental verification of the method

In order to verify the proposed method, the results of experimental investigation of reinforced concrete slabs for pushing upon heating [7] were taken as the initial data. The said investigation was performed using a specially designed unique installation for testing of reinforced concrete junctions of a column and a plate (Fig. 3 and Fig. 4).

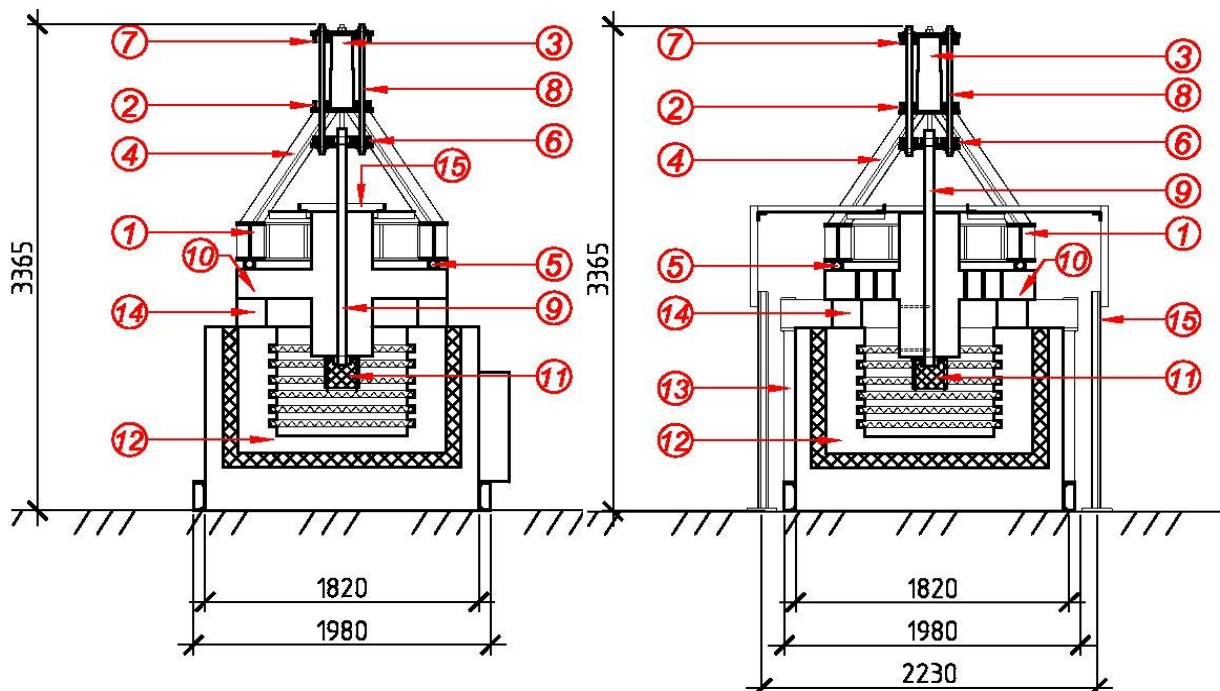


Figure 3 Sectional view of the installation:

1 - lower base of the upper frame of an I-beam; 2 - plate of the upper base of the frame; 3 - lifting jack; 4 - inclining braces; 5 - fluoroplastic gaskets; 6 - lower plate of the reverser; 7 - upper plate of the reverser; 8 - tension bars of the reverser; 9 - main tension bar; 10 - sample; 11 - heat insulating case; 12 - electric furnace; 13 - racks of the lower frame; 14 - girthing-rail structure of the lower frame; 15 - frame for measuring instruments.

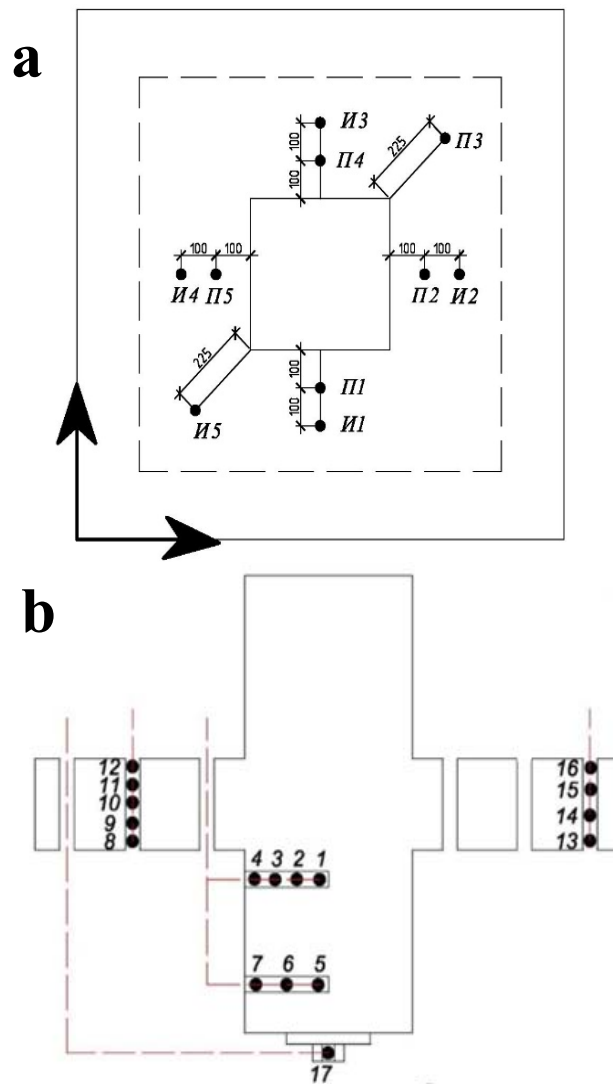


Figure 4 Life-size view of the installation

In work [7] the results of tests of 8 samples are given, which were the junctions of a column with a slab and were different in plate thickness, presence of transverse reinforcement, as well as nature of temperature load application. As the initial data, we selected the test results of 7-PK2000 sample (without transverse reinforcement) at the maximum heating temperature of the lower surface of the plate to  $\theta=479^{\circ}\text{C}$  without force loading (for more “purity” and ease of calculation). Location of thermocouples, dial gauges and Maximov’s deflectometers is shown in Figure 5. Temperature distribution in time for Cr/Al thermocouples is given in Table 1.

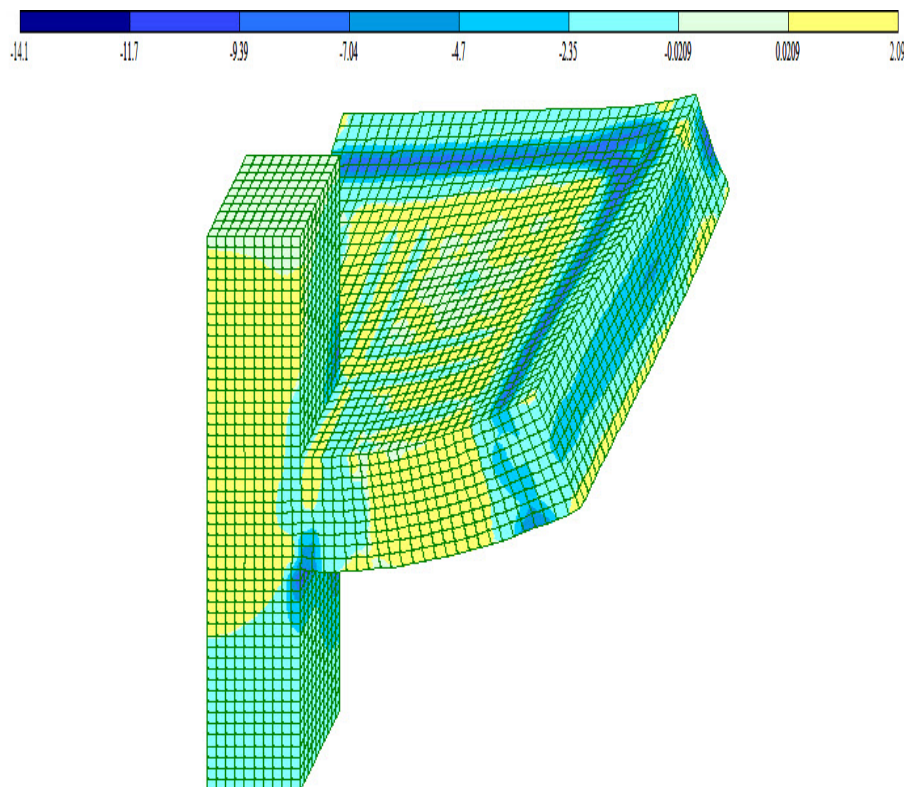
**Table 1 Temperature distribution**

Sampling time	Temperature values for thermocouples, °C								
	T1	T2	T3	T4	T5	T6	T7	T8	T9
9:40	27.4	29.0	27.3	27.5	29.0	28.7	27.1	25.6	24.8
10:00	26.6	27.9	32.8	95.3	30.4	131.3	26.9	66.7	27.8
10:30	36.3	42.4	62.3	180.6	59.9	247.0	41.1	138.4	51.3
11:00	52.5	66.3	98.9	254.0	102.5	333.0	63.2	204.0	79.3
11:30	71.9	94.1	139.2	318.0	150.7	397.0	88.4	261.0	108.1
12:00	93.9	124.9	180.3	370.0	200.0	458.0	116.9	317.0	137.2
12:30	119.5	158.3	220.0	422.0	247.0	501.0	147.9	348.0	164.9
13:00	145.6	189.2	257.0	462.0	288.0	536.0	179.3	384.0	191.1
13:30	170.7	214.0	291.0	496.0	324.0	565.0	208.0	417.0	214.0
14:00	200.0	243.0	328.0	537.0	361.0	603.0	242.0	452.0	239.0
14:30	219.0	257.0	354.0	562.0	385.0	627.0	265.0	479.0	256.0



**Figure 5 Measuring system on 7-PK2000 sample**  
 a) dial gauges and Maximov's deflectometers;  
 b) Cr/Al thermocouples.

For calculation by the proposed technique, we selected the values of displacements due to temperature at different time points for I2 and I3 dial gauges, and P2, P3 and P4 Maximov's deflectometers. In order to determine the first invariant of the deformation tensor due to the unit load, we created a FE model in PC Lira-Sapr, which was (because of the symmetry of geometry and load), a quarter of the sample being investigated (Fig. 6). The results are presented in Table 2. The comparison of the values obtained experimentally and by using the proposed technique made the difference of 5-15%.



**Figure 6 Stress isofields for  $N_x$  (MPa) under the joint action of temperature and static loading (1/4 specimen)**

**Table 2 Comparison of experimental values of displacements with those obtained according to the proposed technique.**

Sampling time	Dial gauge I2		Dial gauge I3		Deflectometer P2		Deflectometer P3		Deflectometer P4	
	I*	II**	I*	II**	I*	II**	I*	II**	I*	II**
9:40	0	0	0	0	0	0	0	0	0	0
10:00	0.17	0.1683	0.05	0.0495	0.04	0.0396	0	0	0.04	0.0396
10:30	0.51	0.561	0.39	0.4056	0.34	0.2958	0	0.001	0.32	0.2958
11:00	0.89	1.0413	0.69	0.7797	0.73	0.7665	0.06	0.063	0.6	0.7665
11:30	1.26	1.134	0.99	0.9405	1.08	0.9396	0.1	0.103	0.88	0.9396
12:00	1.53	1.3464	1.2	1.188	1.31	1.3362	0.12	0.1176	1.07	1.225
12:30	1.79	1.8437	1.42	1.4342	2.55	2.8815	0.13	0.1417	1.23	1.145
13:00	2.01	1.7889	1.59	1.5264	1.78	1.7088	0.27	0.2646	1.37	1.557
13:30	2.14	2.2256	1.7	1.768	1.91	2.0437	0.27	0.2889	1.46	1.38
14:00	2.22	2.5086	1.77	1.9293	2.04	2.1012	0.27	0.2781	1.53	1.6
14:30	2.21	2.34	1.79	1.8616	2.11	2.1944	0.27	0.2889	1.56	1.43

\* Experimental values.

\*\* Values obtained according to the proposed technique.



## 6. Conclusions

The developed method, with the availability of appropriate indicated types of collateral, is an effective technology for investigations of structures exposed to virtual temperature fields. Its distinctive features are the simplicity of the experimental part, the low cost of the actual experiment, the physical clarity and logical foundation of the obtained results. The listed items, in the first place, are predetermined by performance of the necessary procedures on an unheated body being loaded in the simplest way – with a unit force. By means of expanding the technical base, the possibilities of the method can be significantly expanded (we mean devices which create simultaneous effect of forces applied in different directions, for example, experimental simulation of a “center of compression”). The important point here is also the possibility of testing structures of different types: bodies, shells, plates, rods, and combinations thereof, etc. Also, there is practically no limit for qualitative and quantitative parameters of temperature field.

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