Summer 2017

Differentiating Instruction To Develop Conceptual Understanding In Multiplication

Christopher A. Kamrath
Hamline University

DIFFERENTIATING INSTRUCTION TO DEVELOP CONCEPTUAL UNDERSTANDING IN MULTIPLICATION

By Christopher A. Kamrath

A capstone project submitted in partial fulfillment of the requirements for the degree of

Master of Arts in Teaching

Hamline University

Saint Paul, Minnesota

August 2017

Capstone Project Facilitator: Laura Halldin and Susan Manikowski

Content Expert: Tom Mulroy

TABLE OF CONTENTS

CHAPTER ONE

Introduction

Overview

“I was never good at math.” “I never liked math.” These are two common statements I hear often from parents or students describing their experiences with mathematics. My experience with mathematics was different as my father has a mechanical engineering degree who tried to teach my brother and I, as second graders, how to replace numbers with variables. While I unfortunately resisted my father’s help, I still remember feeling successful in my math classes. In elementary school, I liked using manipulatives or tools to represent the problems I was solving and I liked the timed tests. While taking timed tests, I remember trying to prove I was quicker than my second grade teacher with addition which made math fun for me. I enjoyed mathematics in middle school and high school and was successful in my Advanced Placement math classes with minimal effort applied outside of school. However, once I was in a calculus class with five hundred classmates at the University of Wisconsin-Madison, I began to struggle. The weekly assignment of a hundred problems did not help me learn the required concepts. Looking back on my experiences in college, I was focused on completing my assignments rather than developing an understanding of concepts taught during my undergraduate experiences. This led me to struggle and eventually end my journey with math in college.

My journey with math commenced again when I became a classroom teacher and math interventionist. I noticed many students struggled to correctly complete multiplication problems in the classroom after receiving extended instruction on the topic. Their struggles with understanding multiplication were further compounded when the curriculum moved to division. Seeing this struggle led me to my research question: *How can curriculum be created to increase student conceptual understanding of multiplication?*

When I went back to school to get my teaching license, I was reintroduced to Cognitively Guided Instruction (CGI) during my elementary math instructional courses. CGI is an instructional mindset of helping students make sense of math (Carpenter, Fennema, Franke, Levi & Empson, 2015). In CGI led classrooms, instruction is focused on making sense of student knowledge and developing student thinking (Carpenter et al., 2015). During my mathematics class, a light bulb went off and math became fun again. Not only was my professor the one who taught CGI to me as an adult, he also had been my first grade math teacher. The reintroduction to the thought process of problem solving and mental math seemed natural. Using what I know to make the problems easier were concepts I lost as I grew older. The focus turned towards formulas and finishing my homework rather than learning the concepts. During my math class as an aspiring teacher, I took a copious amount of notes and promised myself I would utilize these tools as soon as possible.

In this chapter, I provide context and background for why I picked my research question: *How can curriculum be created to increase student conceptual understanding of multiplication?* I explain my personal interest and motivation in my topic, I examine problems I had teaching math in the classroom and I state a rationale for the focus on developing understanding. By learning about teaching conceptual understanding with multiplication, I hope to become a more efficient and effective math teacher. I can use my experiences to help students of all grades have a better conceptual understanding with numbers. Additionally with my project, I will create resources my colleagues and I can use to provide instruction.

First Teaching Experiences

My first years teaching math were hectic. As a long-term substitute and as a classroom third grade teacher, I did not have much overarching guidance or training and I relied on the curriculum. Consequently, I did not feel I was effectively teaching math in a manner which allowed students to develop a conceptual understanding. With an expectation to follow the curriculum and without CGI materials ready for use, I found it difficult to squeeze in time to create materials I knew I needed to use for effective student learning that followed the curriculum. After being uncertain of my effectiveness during my first full year in the classroom, I was hired as a math interventionist and was given a binder of curriculum created for my school district labeled Conceptual Place Value (Anoka-Hennepin Independent School District #11, 2013). The focus of conceptual place value instruction is to teach the conceptual understanding of addition and subtraction using number problems (Wright, Stanger, Stafford, & Martland, 2006). According to Conceptual Place Value there are eight stages of conceptual development within addition and subtraction (Anoka-Hennepin Independent School District #11, 2013). My school district has established a goal that by third grade, students have moved from Conceptual Place Value level one of direct modeling of counting numbers by ones to level eight in which students are able to add and subtract three digit numbers mentally. The students who do not master these concepts are monitored and receive additional intervention instruction as third, fourth and fifth graders.

Success in the Classroom

For two years I utilized the Conceptual Place Value curriculum and worked with a new group of students every ten days in grades two through five on developing mental addition and subtraction strategies. I became effective at noticing the stages of addition and subtraction development in students. My experiences showed if students were unable to add numbers together mentally such as make a ten (6+4=10) or go to the nearest decade (48+5=48+2+3=50+3=53), then they would not be able to solve a problem mentally with subtraction. I knew when they had mastered each respective concept which allowed me to provide instructional material within their zone of learning.

Throughout this process I was able to fine-tune my materials into a set of steps and activities which allowed me to help students reach mastery at all eight levels of addition and subtraction. As these students were assessed throughout the year, I noticed the younger grades would consistently increase their stage of development with addition and subtraction or remain at the level they last received an intervention throughout the school year. This data showed my successes was lasting and gave me confidence in my abilities. Additionally, teachers would tell me how the participation of students I worked with had greatly improved during whole group instruction and how the intervention cycle was increasing students' confidence and abilities in the classroom.

The improvement during the interventions with second and third grade students translated to the classroom and resulted in those students becoming more successful with their classroom and independent work. However, I noticed many of the fourth and fifth grade students would cycle through my groups again and again and they would struggle using the mental strategies. They did not understand the value of the numbers. Often these students preferred traditional algorithms even though they often did not use them accurately. They would make common conceptual mistakes even when using pencil and paper. Many students who were successful in their interventions did not have their success with addition and subtraction translate to noticeable improvement in their math classroom schoolwork. This led me to start thinking about what materials I could create to further support learning in fourth and fifth grade mathematics.

For my third year at the same school, I was moved from a math interventionist position to a fifth grade math classroom teacher. Many of the same students I taught as a small group math interventionist in third and fourth grade were now in my class as fifth graders. My students had conceptual understanding of addition and subtraction, but I noticed it did not translate to multiplication and division. They would struggle using traditional algorithms and partial products methods. The curriculum did not promote math talks and development of the concepts my students needed. This led me to realize I required something more than what was provided by the district. This experience led me to remember my previous training with Conceptual Place Value and how effective teaching conceptual understanding was. For whole group instruction and intervention periods, I would create my own material with the goal of developing a deeper understanding of the multiplication problems presented. While I felt these materials were effective, often I was scrambling to put together materials without much previous evidence of success or a way to accurately record progress. My assessments would frequently tell me if students knew how to solve two-digit multiplication problems or not, but would not provide me with sufficient data on what concepts my students understood. Due to the relationship between multiplication and division, it is crucial for students to learn multiplication. A development of conceptual understanding in multiplication would result in a greater chance of proficiency with division as well (Pope, 2012).

While I felt moderately successful with my instruction in the classroom, I did not feel I had the same impact as a math intervention teacher or had the assessments to measure progress of understanding with multiplication. This experience as a fifth grade teacher led me to my research question: *How can curriculum be created to increase student conceptual understanding of multiplication?*

Conceptual Understanding

My experiences teaching and learning CGI demonstrated the importance of teaching students how to think about math problems. As a math interventionist, I observed when students learned a procedure to solve a problem which would result in limited transfer of understanding to different concepts. For instance, being able to solve a double-digit multiplication problem and transferring that knowledge to solve a triple-digit multiplication. Boaler's (2009) research demonstrated that frequently math classes in the United States have students who are passive learners, meaning they are not required to think. Students are shown a method from the teacher and asked to practice it (Boaler, 2009). This type of passive learning is used pervasively in mathematics classes throughout the United States and has proven to be an ineffective method of teaching (Boaler, 2009). Developing conceptual understanding and number sense has proven to be an effective way to teach mathematics (Boaler, 2016; Carpenter et al., 2015; Empson & Levi, 2011; Wright, Ellemor-Collins, & Tabor, 2012). In order to create effective mathematics instruction, teachers need to ensure learning is based on concepts and students are given time to practice the skills in a purposeful way (Tomlinson, 1999).

Going Forward

In this first chapter I have discussed my personal and professional reasons for picking my question of interest: *How can curriculum be created to increase student conceptual understanding of multiplication?* My capstone project has a focus on studying the steps of conceptual understanding with multiplication and developing the materials to effectively differentiate multiplication instruction in my classroom. In Chapter 2, I explore research on the stages of conceptual understanding with multiplication. I focus on learning effective multiplication teaching strategies with conceptual understanding which moves students from a direct modeling level to a mastery level. In Chapter 3, I provide a project description and provide a rationale for using stations as an effective way to teach conceptual understanding of multiplication. In Chapter 4, I reflect on my research project and my findings. My project follows Chapter 4 and provides materials I can utilize to teach multiplication with stations.

CHAPTER TWO

Review of Literature

Introduction

The common core state standards for mathematics dictate that conceptual understanding is the understanding of a relationship between numbers in a mathematical process (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Research in conceptual understanding with multiplication provides information on how to effectively develop mathematics instruction. Carpenter et al. (2015) stated that students begin school with a plethora of mathematical knowledge and students need to be taught mathematical concepts first rather than traditional algorithms or formulas in order to increase success with mathematics. This chapter will provide an overview of research discussion on the importance of developing conceptual understanding and help answer: *How can curriculum be created to increase student conceptual understanding of multiplication?* Section one provides an overview of research on how developing conceptual understanding impacts student learning. Section two gives an overview on the stages of development of multiplication. Section three discusses developing conceptual understanding in the classroom using a growth mindset.

Conceptual Understanding

This section discusses how students learn, reviews research on student learning, and demonstrates the impact discovery has on student learning. It further discusses the importance peers make on student learning and potential stages of learning for students. This section investigates the research conducted specifically on the effectiveness of conceptual and fact based instruction. The first portion of this section examines theories on student learning and conceptual thinking. The second portion of this section reviews research on conceptual versus fact based instruction.

Conceptual thinking. Piaget and Inhelder (1972) and Carpenter et al. (2015) concluded that students learn differently than adults. Teachers become effective multiplication instructors when they learn how children develop their own thinking (Carpenter et al., 2015). Piaget's Cognitive Theory stated in the concrete phase of learning, as students mature, they need to learn through creating their own hypothesis and testing it out (Piaget & Inhelder, 1972). According to Piaget's Cognitive Theory, "Maturation, activity, social transmission and need for equilibrium all influence the way thinking processes and knowledge develop" (as cited in Woolfolk, 2011, p. 57). Student development occurs when students are trying to understand the world and create knowledge through problem solving with their personal experiences with objects, people and ideas (Piaget & Inhelder, 1972). Piaget's belief that students should learn with an inquiry perspective to be developed from their own experiences has proven to be effective in mathematics (Cramer, 2003). Carpenter et al. (2015) found students will learn complex multiplication strategies naturally if students are given time, manipulatives, and are exposed to different strategies. Students' learning benefits when their thinking is challenged to develop their own strategies to solve problems (Carpenter et al., 2015; Cramer, 2003; Piaget & Inhelder, 1972).

One of the main differences between the research of Piaget and Inhelder (1972) and Carpenter et al. (2015) is the role peers play in academic development. Piaget stated students will develop their conceptual thinking through individual explorations, equal minded peer interaction and an environment of learning (as cited in Woolfolk, 2011). Carpenter et al. (2015) has shown students adapt their strategies when they are exposed to more effective or complex methods during social interaction times, such as math talks or group work time. Piaget and Inhelder (1972) stated students learn best from equal minded peer interactions, when peers share a similar understanding of concepts, and when the peer challenges their thinking.

Vygotsky's (1934) Sociocultural Theory of Development disputes Piaget's requirement of age to master a concept. Vygotsky's (1934) theory stated scaffolding, providing hints or tips to the learner, will help the learner master new concepts. Scaffolding is a common teaching tool used in teaching today. Taking the time to display or discuss new strategies from students has been proven to improve students' strategies and encourage students to try different types of strategies and new ways to solve problems (Boaler, 2009; Carpenter et al., 2015; Humphreys & Parker, 2015; Parrish, 2010). Once students have been exposed to different and more efficient strategies, students can develop further understanding of a concept. As an example, Carpenter et al. (2015) demonstrated some kindergarteners and first graders are able to do multiplication and division problems using direct modeling exclusively. As students get older, students may develop different and more effective strategies. Students begin solving multiplication problems using counting strategies or using number facts, which increases accuracy and efficiency. Engaging students in their peers' mathematical ideas is an effective method to scaffold student learning (Boaler, 2009; Carpenter et al., 2015; Humphreys & Parker, 2015; Parrish, 2010).

Carpenter et al. (2015), and Piaget and Inhelder (1972) stated there are stages of development in student learning. Carpenter et al. (2015) demonstrated the stages of development with mathematics is a fluid process. Piaget and Inhelder (1972) stated depending on the stage of development and age which a student is, students are either ready to learn and master a concept or they are not. Piaget may have underestimated the abilities of younger students (as cited in Woolfolk, 2011). Cramer (2003) has shown direct interactions with patterns and procedures can lead to conceptual understanding for students at different ages. Skills which students struggle to learn independently yet are able to learn with assistance is the basis for Vygotsky's (1934) Cognitive Development Theory asserting that students learn from others in their zone of proximal development. The need to create an environment in which students can develop conceptual understanding at their own pace is crucial to increasing student learning. To increase success in mathematics, involving peers and developing understanding should take precedent over memorization. Memorizing facts and procedures leads to non-engagement with students as the brain is more effective when learning concepts compared to rules and methods (Boaler, 2016). Carpenter et al. (2015), Piaget and Inhelder (1972) and Vygotsky (1934) stated that students need opportunities to practice their problem solving skills in order to learn or master a subject. Learning ideas at the student's pace is crucial to a child's development. The next section focuses on research which demonstrates the impacts of conceptual-based instruction and fact-based instruction.

Conceptual based and fact based instruction. Poncy, Skinner and Axtell (2010) demonstrated mastering basic fact numbers can make multiplying numbers with more digits (such as 5x20 and 5x200) easier to solve if basic facts are known. Developing fluency with single digits is important to furthering development with multiplication. Frequently in curriculum there is a high priority on giving students practice with times tables or math facts in order to teach multiplication. Woodward (2006), Bansilal (2013) and Poncy, Skinner and Axtell (2010) all have concluded learning math facts in an intervention group does increase math fact fluency.

Bansilal (2013) followed one student named Lizzy and tracked her progress with multiplication facts of 7 and 8. Despite receiving instruction focused on memorizing addition, subtraction and multiplication facts throughout primary school, Lizzy was unable to recall the multiplication facts after repeated practice with a times table of 7 and 8 and when the numbers became larger, she continued to struggled (Bansilal, 2013). Similarly, Woodward (2006) assessed fifty-eight fourth grade students, fifteen of whom had a documented learning disability, who were a year behind grade level in math despite receiving extensive practice with multiplication facts. Woodward (2006) found students who received integrated conceptual instruction and timed test practice were more effective when compared to students who received solely math fact practice as instruction. This was true for students with and without a learning disability (Woodward, 2006). Similarly, Lizzy became successful multiplying once she learned how to add the numbers 7 and 8 with automaticity (Bansilal, 2013).

Both studies found providing scaffolding and direct instruction on specific conceptual building blocks of multiplication helped students gain mastery (Bansilal, 2013; Woodward, 2006). Woodward (2006) showed that students taught in an integrated approach with conceptual understanding instruction and timed test practice scored significantly higher with extended facts (5x6, 5x60) and approximation (multiplying numbers after rounding to the nearest double or triple digit number) compared to the group taught algorithm and math facts. Through an increase in multiplication fluency Lizzy was successful with fraction concepts which involved multiplication (Bansilal, 2013). Both studies affirmed that a conceptual focus resulted in students being more successful at level of instruction and in applying those skills to problems with greater values or to another mathematical setting (Bansilal, 2013; Woodward, 2006). These studies demonstrated the process of learning multiplication should not focus solely on memorizing facts but rather focus on developing an understanding (Bansilal, 2013; Woodward, 2006). Juggling the process is part of the challenge. Woodward (2006) stated for the students in his study their next step of instruction was to learn new numbers despite having not yet mastered the first easier set of numbers. Additionally, the students who were behind were not given extra time to learn the facts they had not mastered (Woodward, 2006). Moving forward prior to mastery creates holes in students' learning which can take time and interventions to teach as Lizzy demonstrated.

Learning basic facts is important in developing fluency with multiplication (Poncy, Skinner, & Axtell, 2010). A concern in the research of the impact of timed multiplication tests as stated by Poncy, Skinner, and Axtell (2010) is that more research is needed to prove focusing on learning math facts in intervention increases development in mathematical fluency. Boaler (2014) demonstrated timed tests create anxiety in students which lead to a negative association with math and students feel they have to "perform - to show that they know math" (p. 470). Timed tests inhibits students' perceptions of math as a topic they can master through solving problems if they put forth the effort (Boaler, 2014). Hurst and Hurrell (2016) demonstrated over half of the students in their study who knew their basic single digit multiplication facts did not have multiplicative understanding. While students may improve their multiplication facts using timed tests, effective instruction develops an understanding of multiplicative relationships between numbers (Bansilal, 2013; Boaler, 2014; Hurst & Hurrell, 2016; Woodward, 2006). When given conceptual understanding instruction, students will master the basic single digit facts and it will result in an increased proficiency in applying the concept taught to larger numbers and translating those skills to other mathematical topics (Bansilal, 2013; Woodward, 2006). Taking the time to ensure students understand concepts versus memorizing facts will result in increased mastery of current and future topics.

Summary. The push in current mathematics instruction is to have an emphasis in conceptual understanding of mathematical concepts in order to increase proficiency in other related mathematical topics. Developing concepts early in elementary school such as an understanding with addition does help to develop understanding with multiplication. The official position for the National Council of Teachers of Mathematics (2014) is that in order to reach procedural fluency, students need to become comfortable using a variety of strategies to develop student comfort in picking the correct procedure to solve a problem. The National Council of Teachers of Mathematics (2014) has shown that when students learn with procedures connected with an understanding of concepts, students are more likely to be able to remember the procedure and to use it in new situations. The challenge for teachers is taking the time to develop conceptual understanding in students particularly when a small group of students has not yet mastered the topic. In chapter three and four, materials are created to develop conceptual understanding. Included are materials for students who fall behind to ensure every student receives instruction and activities within their proximal zone of learning to reach proficiency in multiplication. The next section of this paper will focus on the conceptual developmental stages in multiplication.

Stages of Development

This section provides a summary on how to develop a conceptual understanding of multiplication. A review of research on the different ways children solve multiplication problems will be provided. The first part of this section provides an overview on the thought process of elementary students with multiplication problems. The second part of this section provides an overview on different multiplication strategies. In order to effectively teach multiplication, it is crucial to understand how children develop their thinking in math and with multiplication problems.

Thought process of children with multiplication. Brickwedde (2012b) and Carpenter et al. (2015) demonstrated there are three methods in which children generally use to solve multiplication problems: direct modeling, counting strategies, and derived facts. Throughout these three methods, flexible thinking and relational thinking are seen.

Elementary children typically approach solving math problems with direct modeling (Brickwedde, 2012b; Carpenter et al., 2015). Direct modeling is when a student uses manipulatives such as snap cubes to represent their thinking. A direct modeler will have to represent their thinking using manipulatives and will count everything. If given an example of adding 5+5, a direct modeler will count one, two, three, four, five for the first set and one, two, three, four, five for the second set. Followed by counting all of the cubes together such as one, two, three, … ten (Brickwedde, 2012b). Brickwedde (2012b) and Carpenter et al. (2015) showed that a direct modeler in multiplication would solve the problem 4x5 by counting out one group of five unifix cubes (one, two, three, four, five), then count unifix cubes for the second group, the third group and the fourth group separately by ones. After the student completed representing all of the groups, the student would count all the unifix cubes starting with group one and continuing to count to get a total sum.

After beginning with direct modeling, students start to utilize counting strategies. A common counting strategy is skip counting, in which students count by the number of groups they have (Brickwedde, 2012a; Carpenter et al. 2015; Wright et al., 2006). For instance in the problem 4x5 a student will count and say five, ten, fifteen, twenty. With numbers students are not familiar with, they may incorporate direct modeling within a counting strategy. For instance in a problem like 3x7, students may skip count seven, fourteen, followed by counting fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty-one (Carpenter et al. 2015). In conjunction with skip counting, students also may use an additive counting strategy in which they repeatedly add the numbers multiplied.

For an additive solution in a problem like 3x6, students would repeatedly add the number six (6+6+6) to get the answer (Carpenter et al. 2015).

After students have learned counting strategies, students begin to utilize derived facts they know (Brickwedde, 2012b; Carpenter et al. 2015). When students begin to learn derived facts, they begin to develop their thinking and utilize derived fact strategies involving doubling numbers (2x3) and using memorized answers of squares (3x3, 4x4, etc.) (Wright et al. 2006). For a problem like 3x7, they might know 2x7 is fourteen and then they need to add one more seven to get to twenty-one. This is also an example of relational thinking, using facts students know in order to help solve the problem (Brickwedde, 2012b). An example of a flexible thinker in the derived fact stage would be a student who uses facts they know such as 2x7+2x7 in order to solve 4x7. Throughout the stages of multiplication, the student could be thinking of the problem flexibly meaning the student could change the problem to help them solve it (Brickwedde, 2012b). A student who does not know how to solve a problem with two groups of seven (2x7) would demonstrate flexible thinking if the student solved the problem with seven groups of two (7x2) (Brickwedde, 2012b; Carpenter et al. 2015).

When developing curriculum for conceptual understanding in children, the Lesh translation model demonstrated “manipulatives, pictures, real-life contexts, verbal symbols, and written symbols” are crucial to developing student conceptual understanding in elementary mathematics (Cramer, 2003, p. 450). These five themes are not taught or learned independently but rather integrated and the translation between these five topics are crucial to developing student conceptual understanding. In order to build conceptual understanding with multiplication, it is necessary to understand how students process multiplication problems. Learning multiplication needs to be a fluid process as “there is a progression of direct modeling, counting, flexible thinking, and derived & number fact levels, it is not necessarily a linear one” (Brickwedde, 2012a, p. 3). Students take a series of steps when developing their multiplication skills but depending on a variety of factors such as the value of the numbers and the vocabulary of a problem, students may fluctuate between the learning methods of multiplication. Creating a relational understanding of these steps in mathematics is crucial to developing a conceptual understanding (Miracle-Meiman & Thomas, 2014). Knowing the initial stages students learn in multiplication allows teachers to identify where students are in the spectrum of conceptual development and their areas for growth. The next section will focus on the derived fact stage and will discuss different strategies students can use when solving multiplication problems.

Multiplication strategies. While there are a wide variety of strategies to solve a multiplication problem, it is essential to allow students to develop strategies they know or feel comfortable using. The teacher needs to introduce a spectrum of strategies in order to expose students to different ways of solving problems and to develop comfort in a multitude of strategies (Heibert & Wearne, 1996). When the classroom expectation is to achieve the most efficient strategy to solve a problem, students use multiplicative relationships and properties (Empson & Levi, 2011).

When solving multiplication problems, students will often start as a direct modeler (Brickwedde, 2012 & Carpenter et al., 2015). Then they will frequently progress to utilizing repeated addition and skip counting (Parrish, 2014; Wright et al., 2006). When students begin to learn their derived facts they are able to apply what they have learned previously to new problems. Students utilize known facts they have used previously in order to simplify problems (Parrish, 2014). When solving the problem 6x25, they could take the known fact 2x25 is fifty and break up the problem into (2x25)+(2x25)+(2x25)= 50+50+50=150. Students may utilize doubling and halving to change problems to have numbers they understand such as 2x32 is changed into 4x16 which could be changed into 8x8 (Boaler, 2014; Parrish, 2014).

Another strategy which gives students a visual representation of a problem utilizes an array model (Parrish 2014). Array models with boxes or dots can be used to show every square represented (Figure 1). Array models can be used to help young learners move from counting by one to skip counting (Wright et al., 2006). Research has demonstrated arrays encourage students to see patterns with repeated addition, commutative property and to give students a visual representation to support or disapprove solutions they are unsure of (Day & Hurrell, 2015; Wright et al., 2006).

Array models can be represented with an open array in which numbers are represented and students find the area of the open array (Figure 1) (Parrish 2014). Array models can be used to introduce the partial products multiplication strategy which utilizes the distributive property of multiplication. The problem 6x25 using the partial products method would solve 6x5 and 6x20 independently and then add the products together. The partial products method develops conceptual understanding of what the numbers represent when using the traditional algorithm (Parrish, 2014). The partial products method utilizes the distributive property which is used extensively in algebra and throughout middle school and high school (Empson & Levi, 2011). Mastery of the partial products sets students up for success in future mathematics classes.

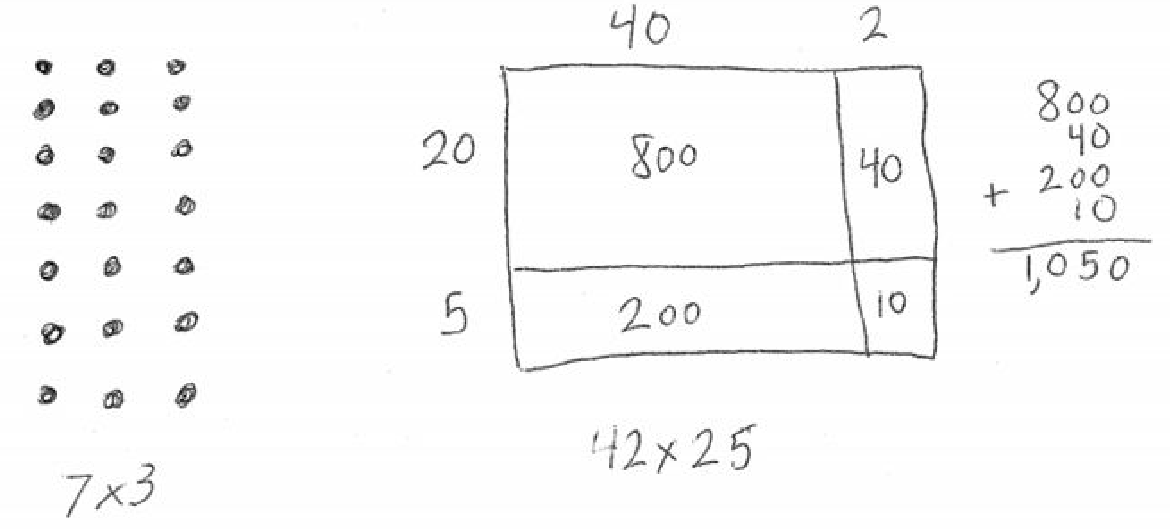


Figure 1. Examples of two different array models, dot array and open array.

The traditional algorithm utilizes the partial products method but it is compacted so the numbers can be solved by each place value starting from the ones and then moving on to the tens, etc. (see Figure 2). Using the traditional algorithm in a two digit problem, the ones and tens are solved separately and then the products are added together. While the traditional algorithm has many strengths, "their conciseness, their dependence on symbol manipulation and generalisability also constitute their major weakness" (Thompson, 1996, p. 43). Students who do not have a conceptual understanding do not grasp how the numbers are related and will not remember steps of the procedure (Thompson, 1996). The procedural steps of carrying and adding those numbers are not obvious and are not easy for students to remember (Thompson, 1996). Students who create their own strategies from their own understanding and are challenged to understand the strategies presented from others develop a better understanding of the concept compared to students who are taught with a procedural focus (Bansilal, 2013; Heibert & Wearne, 1996; Hurst & Hurrell, 2016). Hurst and Hurrell (2016) demonstrated that students knowing math facts or the algorithm does not mean they understand the multiplication process. Teaching students the traditional algorithm before students have an understanding of the value of the numbers will hamper the ability of students when the numbers become larger in value or when they are asked to use an unfamiliar strategy they have not mastered with a new topic such as fractions and decimals (Heibert & Wearne, 1996; Hurst & Hurrell, 2016; Thompson, 1996).

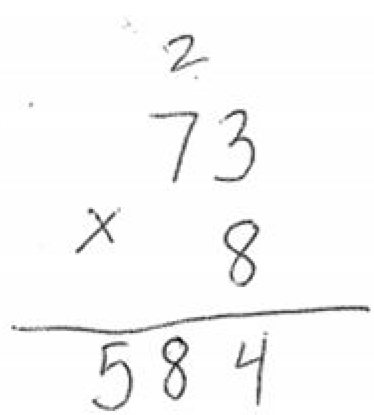


Figure 2. Example of traditional algorithm.

Summary. There are many different strategies available to use in solving multiplication problems. Each strategy is useful in developing conceptual understanding. These strategies will be at the focus of the curriculum when creating materials. Knowing the different strategies enables teachers to help students develop their thinking. Brickwedde (2012) and Carpenter et al. (2015) have demonstrated there are three categories for growth within multiplication: direct modeling, counting and derived facts. These stages are intertwined and a student can change stages depending upon the student's conceptual understanding of the problem. The use of arrays and partial products strategies were discussed as two strategies being effective in developing conceptual understanding. Chapters three and four utilize the development of multiplicative strategies in order to differentiate student work, guiding the creation of assessments and future student work. Knowing the different stages of development in multiplication allows the teacher to differentiate to the meet students' needs. The next section discusses the impact of guided multiplication instruction and how to format the classroom into stations to maximize time with guided instruction with a growth mindset.

How to Develop Conceptual Understanding in the Classroom

This section discusses developing a growth mindset and its impacts on problem solving in the classroom. It reviews research on the impact that mistakes and growth mindset have on students. It includes strategies on how to promote a growth mindset through number talks and grouping in the classroom. The first section examines fixed and growth mindsets. The second section has an overview on number talks. The third section discusses research on the impact grouping has on growth mindset in the classroom. The fourth section provides an example of an effective grouping strategy.

Growth mindset with problem solving. Dweck (2008) demonstrated there are two types of mindsets, a fixed mindset and a growth mindset. People with a fixed mindset believe they have a fixed ability and their work is proving their talent and ability (Dweck, 2008). People with a growth mindset believe their talent and brain can grow with hard work and effort (Dweck, 2008). Students who demonstrate a growth mindset adapt to problems (Dweck, 2008). Moser, Schroder, Heeter, Moran and Lee (2011) stated when students make a mistake in math, their brain grows and when they complete the problems correctly, there is no brain growth. To develop brain growth, it is crucial for teachers to teach students to adapt and learn from mistakes. Students receive a mixed message from mathematics teachers as timed tests promote memorizing math facts over understanding the math facts (Boaler, 2014). Through testing, students learn that to be effective in mathematics, one must be quick with the answer (Boaler, 2014). Quickness in answers and on tests promotes that mathematics is based on ability, a fixed mindset. Moser et al. (2011) demonstrated when students with a growth mindset made mistakes, the brain has "enhanced Pe amplitude - a brain signal reflecting conscious attention allocation to mistakes" and resulted in an improved rate of success on the next attempt (p. 1487). Therefore, every time a mistake is made, the brain grows (Moser et al., 2011). Additionally, adults who make more mistakes have proven to be more successful (Boaler, 2016). A classroom environment where mistakes are encouraged promotes a focus on developing understanding rather than solving for the solution. Number talks are a successful method to discuss problem solving in the classroom, resulting in discussion of mistakes and encouraging flexible thinking (Boaler, 2009; Humphreys & Parker, 2015).

Number talks. Number talks are a daily practice in which students mentally solve problems and discuss their strategies (Boaler, 2009; Humphreys & Parker, 2015).

Low-achieving students require procedures or algorithms when solving a mathematics problem while high achievers use the numbers flexibly (Boaler, 2009). Students are not pursuing upper level courses and careers in mathematics which involve mathematics because they are taught to memorize procedures and have difficulty applying the conceptual skills they have learned to algebra (Parrish, 2010). Number talks are an effective method to help students make sense of the numbers (Boaler, 2009; Humphreys & Parker, 2015; Parrish, 2010). This is an effective method to encourage a growth mindset with mathematics as the discussion relies on students' thinking and allows students to build connections to conceptual mathematical ideas (Boaler, 2009; Humphreys & Parker, 2015). Number talks are important to develop conceptual understanding and to prevent students dependence on traditional algorithms and procedures (Humphreys & Parker, 2015). During number talks, the teacher's role is to act as a guide on the side, a facilitator of the discussion (Parrish, 2010).

While number talks can be of great benefit to many students, a concern is when students do not pay attention during number talks or participate in the discussion. Ways to develop accountability in what is being taught is to hold small-group number talks to target specific topics to groups of students and to use exit slips with problems which involve the discussed strategies (Parrish, 2010). Another possible way to hold students accountable is to give a weekly assessment and require students to solve the problem at least two different ways to encourage students to participate in number talks (Parrish, 2010).

Grouping. Number talks expose different strategies to high and low achievers. Ability grouping negatively affects the exposure of strategies and achievement for all students (Boaler, 2009). Starting in middle school, many schools begin tracking their mathematics program, a system designed to teach lower and high level mathematics to different groups of students (Boaler, 2016). In tracked classes with struggling learners placed in one classroom group, research has shown their teachers' expectations of students decrease and the material they receive is simplified (Tomlinson, 1999). Asking over 800 mathematics teacher leaders, Boaler (2016) found ability grouping is the number one cause of fixed mindset in mathematics. The most effective countries in mathematics utilize grouping the least in their schools (Boaler, 2016). In order to utilize grouping with a growth-mindset, instruction needs to be taught at a high level for all and groups need to be mixed abilities (Boaler, 2016). Commonly, in a mixed ability classroom, advanced students will do more of the work and struggling students' needs will not be met at their current conceptual understanding (Tomlinson, 1999). Developing curriculum to differentiate for students' individual knowledge and yet hold all students to high expectations should be the teacher's classroom goal.

Stations. Differentiating materials in stations is an effective method to meet students' needs while maintaining a mixed ability classroom (Andreasen & Hunt, 2012; Tomlinson, 1999). Stations allow students to do differentiated tasks with different time lengths (Tomlinson, 1999). A general setup for stations is to use four different stations including a teacher's station, a shop, practice plaza and a proof place (Andreasen & Hunt, 2012). The teacher's station is where students receive reteaching or enrichment in the materials. Students could participate in number talks while the teacher moderates. The students in the teacher's station may work on problems or topics with whiteboards or notebooks while the teacher circulates amongst the groups at the stations (Tomlinson, 1999). The shop utilizes unit projects, error analysis and written analysis (Andreasen & Hunt, 2012). The practice plaza is a place where students receive practice with differentiated materials or with new representations of the concepts they are learning (Andreasen & Hunt, 2012). The proof place is a station where students provide evidence using tools or models to solve mathematical problems (Andreasen & Hunt, 2012). Students could work on proving the mathematical problems independently and then after a certain period of time, share their strategy with a partner (Tomlinson, 1999). A consideration to be made is if the rotation does not progress in a certain order or time length, then students do not feel as if they are split up into ability math groups (Tomlinson, 1999). Utilizing a list of tasks students have to complete by a certain time period, would be a way to monitor student completion of tasks (Tomlinson, 1999). This set up of stations promotes a growth mindset as it allows students to complete open-ended tasks, offers them a choice and differentiates individual tasks for students to complete (Boaler, 2016).

Summary. Dweck (2008) and Boaler (2009, 2014, 2016) demonstrated how students' responses to mistakes are crucial in the development of mathematical understanding. Students who learn from their mistakes become far more effective in mathematics than students with a fixed mindset. An effective method to encourage a growth mindset is to have daily number talks with a focus on sharing mental strategies

(Humphreys & Parker, 2015; Parrish, 2010). This gives students the opportunities to share and learn from their peers. Utilizing effective grouping with stations to differentiate learning has proven to be effective with student learning and promotes a growth mindset (Andreasen & Hunt, 2012; Tomlinson, 1999). Due to the potential for brain growth when mistakes are made, the curriculum materials in chapter four will be developed to foster a growth mindset climate in the classroom where students can share their strategies and not be afraid to make mistakes (Moser et al., 2011). The materials utilize number talks and effective grouping in stations to differentiate materials for students and to promote developing understanding.

Conclusion

Mathematics is a topic dependent on students developing number sense, relational thinking and a growth mindset. Elementary students who do not have a solid foundation of understanding and learn mathematics primarily through memorizing facts, procedures and algorithms will eventually struggle when they have to apply their skills to more advanced and unfamiliar topics. Teachers who develop an understanding of how students think mathematically can be intentional with their instruction to promote a growth mindset and conceptual understanding of multiplication. The research in this chapter will be used as a framework to create materials in chapter four to move students from a direct modeler stage to using several multiplication strategies independently. The next chapter introduces the methods for the research project. It provides information on the project design, the curriculum method, assessments and setting for the research.

CHAPTER THREE

Project Description

Introduction

When teachers understand how students think and develop number sense with mathematics strategies, instruction can have a significant positive impact on student learning (Boaler, 2009). Developing conceptual understanding is crucial to students' success in mathematics (Boaler, 2014; Briars, 2016; Brickwedde, 2012b; Carpenter et al., 2015; Empson & Levi, 2011). The purpose of this chapter is to describe the methods and plan for this research curriculum project. The curriculum provided is a resource for teachers to develop student conceptual understanding of multiplication. The rationale is due to the lack of materials focused on independent student learning to provide time for guided instruction. This led to my research question: *How can curriculum be created to increase student conceptual understanding of multiplication?*

In Chapter 2, I described the importance of developing a conceptual understanding in mathematics, explored the different stages of development with multiplication and the impact of developing a growth mindset. This chapter starts with an explanation of the project description. The rationale used for this research project to create an Understanding by Design (UbD) curriculum to develop mathematical conceptual understanding with multiplication. The steps for the UbD section provide an overview of the UbD curriculum process and provides details on how the UbD curriculum will be used to create stations. The audience and setting section describes the grade of the students, demographics of the school, and classroom details.

Project Description

The purpose of the project is to design a curriculum to support conceptual understanding of multiplication which students can complete independently, or in small groups without adult assistance while the instructor takes a small group of students for an intervention period. Materials will be differentiated to meet student needs. Differentiated instruction is equitable instruction which allows for “reasonable and appropriate accommodations be made and appropriately challenging content be included to promote access and attainment for all students” (National Council of Teachers of Mathematics, 2000, p. 2). The UbD framework will be used as a guide in the creation of materials for stations. Findings from the research on conceptual understanding of multiplication in chapter two will be utilized to formulate stage one in the UbD plan. Formative and summative assessments modified from the research of Carpenter et al. (2015) and Wright et al. (2006) utilize a guided staging assessment to assess student understanding. After assessments have been created, curriculum materials will then be created for stations. Materials will be created for stations so students can work independently in small groups while the instructor works with students at the teacher's station. Having stations allows the instructor to provide direct intervention instruction to students individually or in small groups and provide differentiated practice to students to reach proficiency in multiplication (Tomlinson, 1999). Both Tomlinson (1999), and Andreasen and Hunt (2012) demonstrated stations can be utilized to effectively differentiate instruction to meet the learning needs of students.

The materials will be implemented over a time period of six weeks for the first time in the fall of 2017 for forty-five minutes everyday. For the duration of the forty-five minute intervention period, every fifteen minutes there will be a rotation to a different station or task. The instructor will be able to provide interventions to three different groups. Each student will be in three different stations daily. The curriculum materials will be utilized after a whole group instructional time of fifteen minutes. The next section will discuss the curriculum method used to create the station materials.

Rationale for Understanding by Design

The curriculum method for this project is Wiggins and McTighe's UbD (2005). UbD is a curriculum framework that utilizes backward design, a process which plans the desired student outcomes first. The curriculum is then designed around a desired student outcome. Senk and Thompson showed how students in elementary schools who were given math curriculum which focused on developing understanding were more successful with difficult calculations embedded in word problems and in explaining how different operations worked (as cited in McTighe & Sief, 2003). Developing understanding is a focus of the UbD curriculum as all materials and lesson plans are based on a desired student understanding of a specific goal (Wiggins & McTighe, 2005). After the student centered goal is made, performance assessments, teacher instruction and activities are created based on the desired student understanding. The research of Newman and Associates of twenty-four different schools demonstrated that inequalities between high and low achieving students were greatly decreased when schools utilized effective teaching and strong assessments (as cited in McTighe & Sief, 2003). The UbD curriculum plan emphasizes effective assessments for students to demonstrate understanding of the concept (Wiggins & McTighe, 2005). The utilization of the UbD curriculum allows for student centered instruction with a strong understanding of their knowledge through assessments. The UbD curriculum project materials will be assessed by measuring the level of student understanding by comparing the curriculum's assessment at the beginning of the school year and at the end of the school year. The assessment consists of a comprehensive one-on-one verbal assessment.

Steps of Understanding by Design

There are three stages in UbD curriculum (Wiggins & McTighe, 2005). Stage one is the desired results, a goal the unit or lesson focuses on. In stage one, the goals for student learning after instruction are given: what students will understand, will know and will be able to accomplish. Questions are created to provide direction in creating materials. Additionally, these questions are utilized to deepen understanding and to promote inquiry. For this project, stage one includes: a) Students understand that numbers can be flexibly changed in order to solve problems (Anoka-Hennepin Independent School District #11, 2014); b) Students will know how to represent a single digit multiplication problem; c) A question asked: Which strategy is most efficient? Developing desired student understanding focuses the curriculum writing on achieving the stage one goals throughout the unit (Wiggins & McTighe, 2005).

The second stage of UbD is the assessment stage. Formative and summative tasks or assessments are created to demonstrate understanding of the goal. For this project, a guided staging assessment titled a Conceptual Multiplication Assessment was modified from the research of Carpenter et al. (2015) and Wright et al. (2006). It will be used to gauge student understanding throughout the curriculum. The assessment categorizes five separate stages of learning with multiplication and it utilizes a one-on-one verbal assessment that tests each student's understanding of the stages of multiplication. It will be given at the beginning of the school year to create student groups. Each cycle of interventions lasts ten days. Students will be reassessed with the Conceptual Multiplication Assessment after a ten day intervention period if students have demonstrated progress. If students pass the assessment, then they move up to the next stage; if they do not pass they need additional practice at their current stage. The assessment will be given at the end of each trimester to assess the effectiveness of the curriculum for the year.

Stage three is the learning plan, where the activities and instruction are created. Stage three provides the activities to empower students to be able to achieve the goals previously established. Specific descriptions of how the teacher plans to differentiate instruction, test student thinking and allow for student reflection are included in stage three. For this project, stage three focuses on creating materials to use in stations. Materials were created for the four stations: teacher's station, proof station, game station and practice station (Andreasen & Hunt, 2012). For the teacher station, content and resources were provided from Brickwedde (2012a, 2012b, 2016), Carpenter et al. (2015), and Wright et al. (2006). For the proof station, real world questions were created and require students to show their work. Questions are organized for the proof station by each multiplication grouping stage and additional resources are provided through the third and fourth grade curriculum of Brickwedde (2016). Both the game station and practice station primarily use content and materials from the math curriculum Everyday Math. Students in the game station will practice number sense and multiplication skills in a game format. In practice station, students will complete problems that correspond to the curriculum Everyday Math and use the word problems from the fourth grade curriculum of Brickwedde (2016).

Audience and Setting

The target audience for this project is a fourth grade classroom with a range of twenty-five to thirty-five students. The site is a Title I school, meaning a minimum 40% of the students that live in the school boundary qualify as having a low income family (Minnesota Department of Education, 2016). The setting for the project is in a classroom with desk spots for each student and with a promethean board. The students in the school have a variety of ethnic and cultural backgrounds, including a growing population of Arabic speakers. The following statistics were pulled from the Minnesota Department of Education Report Card in the spring of 2017 regarding the population of the elementary school. The demographics of the students at the elementary school site are 66.1% White, 12.4% Black/African American, 8.6% two or more races, 6.9% Hispanic, 5.3% Asian, .5% American Indian/Alaskan native. The school's population includes 45.1% students who qualify for free and reduced lunch, 19.1% classify as special education students, 8.1% are English language learners and 3.8% are homeless students.

Conclusion

This chapter introduces and discusses the rationale for the UbD curriculum design model for this project followed by a description of the setting and its participants. It gives a description of the UbD curriculum and provides examples of how the project resources will be implemented.

Through examination of literature and research on the stages of conceptual understanding with multiplication, this project examines the research question: *How can curriculum be created to increase student conceptual understanding of multiplication?* Research on the stages of development and a growth mindset has been been analyzed and utilized to create materials to scaffold students' understanding in multiplication. This curriculum provides resources for elementary teachers to support their students in developing conceptual understanding of multiplication. In chapter four, I discuss the results of my curriculum development project. The created curriculum materials follow in the appendices.

CHAPTER FOUR

Reflection

Introduction

This final chapter reflects on the project as a whole through the research and the materials created. It includes a personal reflection on the impacts of completing the project. This chapter reviews research which was most useful when creating the project from the literature review and from research which was not in the literature review. It identifies the limitations of this curriculum project and options for further research. The conclusion reflects on how this project is impactful to other educators and how the curriculum project can be used to help teachers solve the question: *How can curriculum be created to increase student conceptual understanding of multiplication?*

Personal Self-Reflection

Prior to starting my masters I did not have intrinsic motivation to complete my masters. Getting my masters was the next step on my teacher journey, the benefit to me was the degree. However, I did not expect to grow professionally. Making time to thoroughly research a topic to develop a deep understanding was enjoyable because I saw the direct application to my everyday teaching. While I was completing my research, I was able to utilize different techniques I learned such as using subitizing cards. I practiced using subitizing cards in summer school while I was in the process of completing this project. Having a deadline helped me stay motivated and organized to create a project which I am excited to use next year in my classroom.

A big takeaway from my research is to never stop learning from others. There are professionals who have dedicated their lives to learning and mastering even the smallest of topics. I have always read books to learn but I found that by taking the time to talk to a colleague or a professor for even fifteen minutes, I could learn something new. I was surprised to notice that my desire to research proven methods has grown beyond studying multiplication and influenced my life in other ways. I have sought out other professionals to help me master my craft and continue to grow. While I have been researching my topic, I have been reading about growth mindset in math through Dr. Jo Boaler's online tutorial, discussed homeless student population needs with my school social worker and the school district's superintendent, learned how to stay motivated and how to motivate others through podcasts by Dr. Eric Thomas and attended basketball coaching clinics given by collegiate coaches. The next section will discuss the impacts of the research on the project.

Review of Literature Review

The most effective takeaways from the literature review were the student problem examples, discussions on the mindset students need and available curriculum materials. Being intentional in the types of problems given was inspired by the work of Carpenter et al (2015). Since students think differently than adults, Carpenter et al. (2015) demonstrated how to break down word problems using Cognitively Guided Instruction to develop mathematical number sense in children. Brickwedde (2012b) built upon the research of Carpenter et al. (2015) and provided research on how children would respond to different problems. Brickwedde provided examples of attainable goals students needed to display to demonstrate mastery of the concept. I created the curriculum based largely upon how students reacted to the word problems in the research of Carpenter et al. (2015) and Brickwedde (2012a).

The research of Boaler (2009, 2014, 2016) and Dweck (2008) was impactful as it showed how developing differentiated materials led to increased mathematical number sense with students. To differentiate materials, I designed the curriculum in stations as teachers would be able to meet student needs in smaller groups. Boaler's (2014) research on the ineffectiveness and detrimental effects timed tests had on students' learning mathematics were impactful in the project as the curriculum was formatted to provide students opportunities to work independently, in small groups and to have sufficient time to complete assessments.

The assessment stages used in the Conceptual Multiplication Assessment were modified from the different activities provided through the research of Wright et al. (2006). Having a researched based framework upon which to build allowed me to create an effective assessment with similarities to the Conceptual Place Value (Anoka-Hennepin Independent School District #11, 2013) assessment which I had previous success with as a math interventionist teacher.

The literature review emphasized the importance of breaking down student thinking and developing differentiated materials, which led to a search of other types of curriculum with a conceptual development in mathematics. The following resources discussed were not included in the literature review but were impactful in creating the curriculum materials. The Anoka-Hennepin Independent School District #1 (2013)

Conceptual Place Value Curriculum which is focused on addition and subtraction was used as a framework to create an assessment utilizing multiplication. The Anoka-Hennepin Independent School District #11 (2014) fourth and fifth grade UbD lessons were utilized as a model for the format and the types of questions and statements included in stage one of the UbD lesson plan. Additionally, the third, fourth and fifth grade math curriculum on Brickwedde's website www.projectmath.net provided activities which promote the development of conceptual understanding of numbers. Additionally, Brickwedde's website supplied effective word problem worksheets for the teacher station, proof station and practice station. My school district's math curriculum Everyday Math had a multitude of games which aligned with developing conceptual understanding. The existing curriculum materials were beneficial as I could format and organize the materials in stages corresponding to the Conceptual Multiplication Assessment to differentiate materials for students. The next section will discuss the process of developing the curriculum.

Curriculum Development

After finishing the literature review, the research findings were utilized to create the curriculum using the UbD curriculum model from McTighe and Seif (2003). The stages were an effective method to categorize the project. By thinking about the end goal first I was able to dissect what students need to know and align the curriculum with the research on how math concepts need to be broken down to stages students can understand. Being goal driven allowed me to dedicate time to find resources and create materials with a focus on developing number sense with multiplication.

After the goals for students were completed, the stage two assessment period was designed. I created a method titled Conceptual Multiplication Assessment which identified concepts students need to understand to accurately solve double digit multiplication problems from the research of Wright et al. (2006), Carpenter et al. (2015) and Brickwedde (2012b). I will assess and compare the progress of other fourth grade classes which do not utilize the project materials during their intervention cycles to measure the validity of the curriculum materials.

When the goals and assessment in stage one and two were finished, the activities in stage three were then created. Materials were organized and developed with a real life focus. Research demonstrated context in math increases student engagement (Brickwedde, 2012a) The context of the problem allows students to draw a picture or imagine the problem to solve it. This is demonstrated in Word Problem 1.3:

Natalie went to Cub foods and bought two bags of apples. Each bag had three apples. How many apples did Natalie buy?

The word problem is a real life situation students can relate to. The names can be changed to the names of students in the classroom to increase engagement.

The intervention and activities will be effective as the curriculum materials allow the teacher to be intentional in teaching multiplication concepts and provide a method to monitor student learning. Students will be engaged as they have the opportunity to practice through games and working in teams with real world problems. Engaged practice in a topic develops a deeper understanding (Gladwell, 2008). The next section will discuss the limitations of the curriculum project.

Limitations

In order to accurately evaluate the impacts of the designed curriculum, reflection is an important method in teaching to develop understanding and to assess the effectiveness of material taught (Miracle-Meiman & Thomas, 2014). The following discussion is a reflection to identify possible areas for improvement.

As a classroom math teacher and a former math intervention specialist, the materials I created or selected were based on curriculum or experiences I had previous success with. While I used my personal experiences relating to materials I could see myself effectively implementing in an intervention session, I did not teach using the curriculum project. Therefore, a shortcoming of the intervention materials created is the lack of an opportunity to implement or test the curriculum. In particular, the teacher intervention station has materials for each stage of development in which there is not a daily instructional schedule. This allows for modification of the intervention based on student need. After implementing the curriculum, a set schedule or a series of paths could be created in order to ensure fidelity of the curriculum when other teachers implement it. The next section follows with topics for possible further research.

Next Steps for Research

This section discusses the next steps for research to further develop an understanding of multiplication strategies in students. Future research should investigate the connection between multiplication and division. Students' initial discussions with multiplication and division involve dividing or grouping a number equally, linking the development of the two topics (Carpenter et al., 2015). Integrating division strategies into the multiplication curriculum would be beneficial to further develop number sense and strategies for multiplication and division. After implementing the curriculum and researching division strategies, the stages in the Conceptual Multiplication Assessment for the project curriculum should be modified to assess both multiplication and division concepts.

Additionally, further research is needed on how to effectively implement Cognitively Guided Instruction and problem solving for English language learner students. The word problems created for students to complete in this curriculum require an understanding of English vocabulary which some students may not have. Further research is needed on how to effectively teach math to English language learners and how to incorporate it with Cognitively Guided Instruction.

Implementation Plan

The plan to enact this project initially will be September 2017 in my fourth grade classroom in the Anoka-Hennepin School District. I plan to implement this curriculum project during our intervention math blocks starting at the beginning of the school year. I hope to use the results of the Conceptual Multiplication Assessment to monitor progress the students are making and to differentiate materials for students based on their understanding.

I will share the project curriculum resources with my teacher teammates to use during our daily intervention periods. In my school, we have a weekly Professional Learning Community meeting with the other content specific teachers in our school. I will use this collaboration time with the other fourth grade math teachers to receive feedback on how students are performing and to discuss effective and noneffective instruction during the intervention periods. This feedback will be used to modify and adapt the curriculum project materials. I plan to use and communicate the end results of Conceptual Multiplication Assessment with the math specialist at my school at the end of the project unit and at the end of each trimester.

In the future, if I am moved back to a small group math interventionist position, the teacher station portion of the project will be a resource for me to use and to implement. Additionally, I will be able to share this resource with classroom teachers when I go into the classroom to do interventions in order to coordinate instruction during daily intervention periods.

Summary

This chapter reflected on my experiences designing and creating this curriculum project. The knowledge professionals had to share was beneficial and led me to seek out additional input for my success in and outside of my school environment. Having reflected on research from the literature review, what was most helpful included the student thinking process and how to improve student mindset towards math. This chapter discussed how existing curriculum was beneficial in creating station materials. Also reviewed are the limitations due to not having an opportunity to teach the curriculum in a classroom setting. Included are possible future research topics on integrating division strategies into the assessment and curriculum materials. Lastly, an implementation plan for the next school year is included.

Conclusion

To teach students how to solve mathematical problems using a conceptual understanding there needs to be intentional instruction in elementary school (Boaler, 2014). This curriculum project provides teachers with a resource to develop a conceptual understanding of multiplication. In doing so, students will approach mathematics with a more positive mindset and be able to understand why the multiplication process works. Students can take what they have learned and apply it to more complex mathematical tasks. Developing an attitude in children to understand why mathematical procedures work will enable students to become lifelong learners of mathematics.

REFERENCE LIST

Andreasen, J. B., & Hunt, J. H. (2012). Using math stations for commonsense inclusiveness. *Teaching Children Mathematics, 19*(4), 238-246.

Anoka-Hennepin Independent School District #11. (2013). *Conceptual Place Value Intervention Guide*: *Assessments and Resources*. Anoka, MN: Author.

Anoka-Hennepin Independent School District #11. (2014). *Anoka Hennepin K-12 Curriculum Unit Plan*. Anoka, MN: Author.

Bansilal, S. (2013). Lizzy's struggles with attaining fluency in multiplication tables. *Perspectives in Education, 31*(3), 94-105.

Boaler, J. (2009). *What's math got to do with it? How parents and teachers can help children learn to love their least favorite subject*. New York City, NY: Penguin.

Boaler, J. (2014). Research suggests that timed tests cause math anxiety. *Teaching Children Mathematics, 8*, 469-474.

Boaler, J. (2016). *Mathematical mindsets*. San Francisco, CA: Wiley.

Brickwedde, J. (2012a). *Windows into children's mathematical thinking: Instructional tools focusing on multiplication and division in the intermediate grades.* Retrieved from http://www.projectmath.net/professional-development-resources.html.

Brickwedde, J. (2012b). *Windows into children's mathematical thinking: Problem types, levels of development & children's solution strategies: Addition, subtraction, multiplication, & division*. Retrieved from http://www.projectmath.net/professional-development-resources.html.

Carpenter, T. P., Fennema, E., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic & algebra in elementary school.* Portsmouth, NH: Heinemann.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's mathematics cognitively guided instruction* (2nd ed.). Portsmouth, NH: Heinemann.

Cramer, K. A. (2003). Using a translation model for curriculum development and classroom instruction: Models and modeling perspectives on mathematics pr. In R. Lesh, & H.Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics pr* (pp. 449-464). Mahwah, NJ: Lawrence Erlbaum Associates.

Day, L., & Hurrell, D. (2015). An explanation for the use of arrays to promote the understanding of mental strategies for multiplication. *Australian Primary Mathematics Classroom, 20*(1), 20-23.

Dweck, C., S. (2008). *Mindset: The new psychology of success*. New York City, NY: Ballantine Books.

Empson, S. B., & Levi, L. (2011). *Extending children's mathematics: Fractions and decimals*. Portsmouth, NH: Heinemann.

Gladwell, M. (2008). *Outliers: The story of succes*s. NY: Little, Brown and Company

Glaserfeld, E. v. (2013). Review of “Beyond constructivism”. *Zentralblatt Für Didaktik Der Mathematik, 35*(6).

Hiebert, J., & Wearne, D. (1996). Instruction, understanding, and skill in multidigit addition and subtraction. *Cognition and Instruction, 14*(3), 251-283.

Humphreys, C., & Parker, R. (2015). *Making number talks matter: Developing mathematical practices and deepening understanding, grades 4-10*. York, Maine: Stenhouse Publishers.

Hurst, C., & Hurrell, D. (2016). Multiplicative thinking: Much more than knowing multiplication facts and procedures. *Australian Primary Mathematics Classroom, 21*(1), 34-38.

Lesh, R., Landau, M., & Hamilton, E. (1983). Conceptual models in applied mathematical problem solving research. In R. Lesh & M. Landau (Eds.), *Acquisition of Mathematics Concepts & Processes* (pp. 263-343). NY: Academic Press.

Miracle-Meiman, B., & Thomas, J. (2014). Making a mathematical symphony: Emphasis on relational thinking and connections. *Ohio Journal of School Mathematics, 70*, 11-15.

McTighe, J., & Seif, E. (2003). *A summary of underlying theory and research base for understanding by design.* Retrieved from https://jaymctighe.com/wordpress/wp-content/uploads/2011/04/UbD-Research-Base.pdf

Minnesota Department of Education. (2016) *Title I, Part A.* Retrieved from http://education.state.mn.us/MDE/dse/ESEA/parta/index.htm

Moser, J. S., Schroder, H. S., Heeter, C., Moran, T. P., & Lee., Y. (2011). Mind your errors: Evidence for a neural mechanism linking growth mind-set to adaptive posterror adjustments. *Psychological Science, 22*(12), 1484-1489.

National Council of Teachers of Mathematics. (2000). *Executive Summary: Principals and Standards for School Mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2014). *Procedural Fluency in Mathematics*. Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Retrieved from http://www.corestandards.org.

Parrish, S. (2010). *Number talks: Helping children build mental math and computation strategies*. Sausalito, CA: Math Solutions.

Piaget, J., & Inhelder, B. (1972). *The psychology of the child.* (2nd ed.). New York: Basic Books.

Poncy, B. C., Skinner, C. H., & Axtell, P. K. (2010). An investigation of detect, practice, and repair to remedy math-fact deficits in a group of third-grade students. *Psychology in the Schools, 47*(4), 342-353. doi:10.1002/pits.20474.

Pope, S. (2012). The problem with division. *Mathematics Teaching,* (231), 42-45.

Thompson, I. (1996). "User friendly" calculation algorithms. *Mathematics in School, 25*(5), 42-45. Retrieved from http://www.jstor.org/stable/30215271

Tomlinson, C. A. (1999). *The differentiated classroom: Responding to the needs of all learners.* Alexandria, VA: Association for Supervision and Curriculum Development.

Vygotsky, L.S. (1934/1986). *Thought and language*. Cambridge, MA: MIT Press.

Wiggins, G. P., & McTighe, J. (2005). *Understanding by design* (Expanded 2nd edition). Alexandria, VA: Association for Supervision and Curriculum Development.

Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disability Quarterly, 29*(4), 269-289.

Woolfolk, A. (2011). *Educational psychology: Active learning edition* (11th edition). Boston, MA: Allyn & Bacon.

Wright, R. J., Ellemor-Collins, D., & Tabor, P. D. (2012). *Developing number knowledge* (1st ed.). London: Sage Publications Ltd.

Wright, R. J., Stanger, G., Stafford, A. K., & Martland, J. (2006). *Teaching number in the classroom with 4-8 year-olds*. Los Angeles, CA: Sage Publications Ltd.

Appendix A
Understanding by Design Stage One - Desired Results

Established Goals:
Minnesota Standards-
3.1.2 Add and subtract multi-digit whole numbers; represent multiplication and division in various ways; solve real-world and mathematical problems using arithmetic.

4.1.1 Demonstrate mastery of multiplication and division basic facts; multiply multi-digit numbers; solve real-world and mathematical problems using arithmetic.

4.2.2 Use number sentences involving multiplication, division and unknowns to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.

Understandings: Students understand that numbers can be flexibly changed in order to solve problems (Anoka-Hennepin Independent School District #11, 2014).
Students understand there are many patterns and strategies to solve multiplication problems.

Essential Questions:
How does knowing a variety of strategies help me solve multiplication and division problems?
How can I solve real-world problems using multiplication and division?
How can I represent my thinking in order to help me solve multiplication and division problems?
Which strategy is most efficient?

Students will know…
Students will know how to represent a single digit multiplication problem.
Students will know how to solve multiplication basic facts (0-10).
Students will know how to explain an equal grouping problem describing the number of objects and the number of groups.

Students will be able to….
Students will be able to use known facts to solve unknown facts.
Students will be able to represent their thinking verbally and visually.
Students will be able to apply multiplication and division processes to real world problems.

Students will be able to use multiple strategies to solve problems.

(Anoka-Hennepin Independent School District #11, 2014; Wiggins and Mctighe, 2005)

Appendix B
Understanding by Design Stage Two - Assessment Evidence

Summative assessments:

Conceptual Multiplication Assessment - Completed every ten days (Appendix D)
District multiplication assessment

Formative assessments:

Guided whole group discussions
Materials completed during Proof station and Practice station
Work during Game station

Students will self assess their progress made through their Conceptual Multiplication Assessment and through comparison of scores in the Everyday Math unit diagnostic and unit post-test.

Assessment Timeline:
The Conceptual Multiplication Assessment (Appendix D) will be administered by the instructor after every ten days of instruction. Instruction will be differentiated depending on how students score on the assessment.

Appendix C
Understanding by Design Stage Three - Activities

The curriculum materials are designed for a forty five minute time period every day for a duration of six weeks. There should be at least three stations daily with each respective station lasting for fifteen minutes. After each fifteen minutes, students rotate to a new station. Students should be grouped by stages as determined by how they perform on the Conceptual Multiplication Assessment (Appendix D). Every tenth day students will be reassessed and new student groups should be created. The new groups should reflect the new assessment scores.

During the station period there will be up to four stations possibilities:

Teacher Station - This station will be completed in a small group with teacher instruction. Teachers will use the assessment to decide what instruction to provide in this intervention period. Effective instruction includes math talks, games using practiced skills and discussion of word problems that students have completed during proof station. Materials are provided and directly correlate to the level that students placed in their assessment. See Appendix F for further directions. See Appendix G for materials.

Game Station - Students will play games that directly correlate with the skills they are learning. Some game materials are provided. Some game materials are not provided as they are from the Everyday Math Curriculum. See Appendix H for further directions.

Proof Station - Students will be given word problems in which they have to use manipulatives and/or show their work in order to prove their solution. Students will be held accountable for this work through discussion in the teacher station. For extension, students could be given word problems a level above their current level to spark discussion. Materials are provided in Appendix I and Appendix J.

Practice Station - Students will work in their groups to complete district developed curriculum that directly correlates with developing multiplicative number sense. Materials are provided from Everyday Math Curriculum. Some materials are provided in Appendix K for use.

(Andreasen & Hunt, 2012; Tomlinson, 1999)

Appendix D

Conceptual Multiplication Assessment

Name: __________________________ **Assessor: _____________________**

Date			
CMAScore			

Stage 1 - Skip Counting

Can you count by 2s? By 5s, 10s?

Answer:

Can count by 2, 5, 10 - **Go to 2.1**

Incorrect/Count by 1s - **Stage 1**

Stage 2 - Repeated Equal Groups Visible

Place five cards with three dots face down. Show the student one card and tell student that each card has three dots. How many dots are there altogether? Flip over all cards.

2.1 Show five groups of three.

Answer:

Skips counts - **Go to 2.2**

Automatic/known facts - **Go to 2.2**

Incorrect/Count by 1s - **Stage 2**

2.2 Show six groups of 4.

Answer:

Skips counts - **Go to 3.1**

Automatic/known facts - **Go to 3.1**

Incorrect/Count by 1s - **Stage 2**

Stage 3 - Repeated Equal Groups with Items Blocked and Groups Visible

Place all the cards flipped over with dots facing down. Turn over one card and tell them that each card has the same number of dots. Ask them how many total dots there are.

3.1 Five cards with two dots.

Answer:

Skips counts - **Go to 3.2**

Automatic/known facts - **Go to 3.2**

Incorrect/Count by 1s - **Stage 3**

3.2 Six cards with 4 dots

Answer:

Skips counts - **Go to 4**
Automatic/known facts - **Go to 4**
Incorrect/Count by 1s - **Stage 3**

Stage 4 - Repeated Equal Groups with Items and Groups Blocked and Arrays
Students are told that you have cards with a certain number of dots on each card. Students are asked how many total dots there are.

4.1 I have two groups of four. How many dots do I have?

Answer:

Skips counts - **Go to 4.2**
Automatic/known facts - **Go to 4.2**
Incorrect/Count by 1s - **Stage 4**

4.2 I have five groups of six. How many dots do I have?

Answer:

Skips counts - **Go to 4.3**
Automatic/known facts - **Go to 4.3**
Incorrect/Count by 1s - **Stage 4**

4.3 Show student one row from array (five dots). Ask student what they see. The array has seven rows of five. How many dots are there altogether?

Answer:

Skips counts - **Go to 5**
Automatic/known facts - **Go to 5**
Incorrect/Count by 1s - **Stage 4**

Stage 5 - Problem Solving and Relational Thinking
5.1 Show student the 7x4 card and ask them to solve it. If student is correct ask if student can use this to solve 4x7.

Answer:

Explains Commutative property - **Go to 5.2**
Skips counts - **Go to 5.2**
Automatic/known facts - **Go to 5.5**
Incorrect/Count by 1 - **Stage 5**

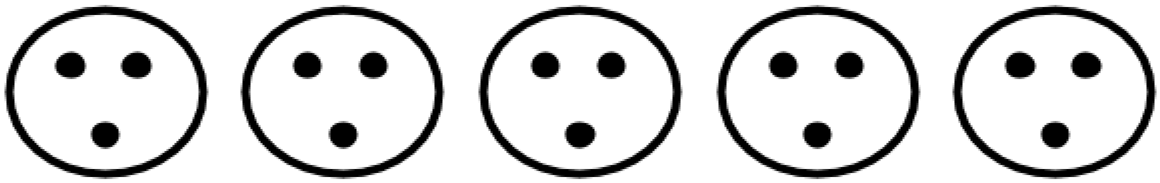
5.2 Show student the 14x4=56 card. If fourteen times four equals fifty-six. What is the answer to 15x4?

Correct and states needs one more group of four - **Exit**
Correct but makes no connection - **Stage 5**
Incorrect - **Stage 5**

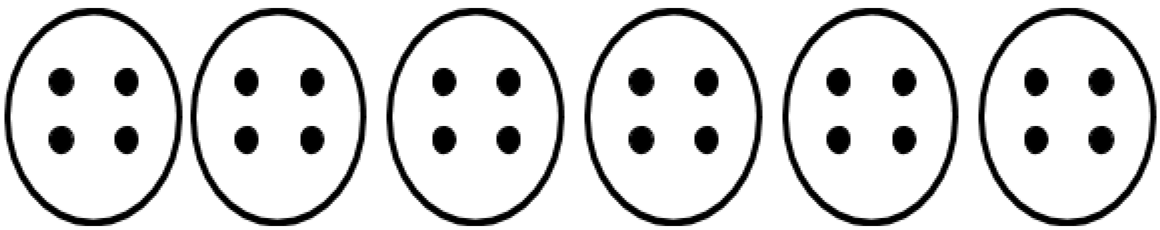
(Anoka-Hennepin Independent School District #11, 2013; Wright et al., 2006)

Appendix E
Conceptual Multiplication Assessment Materials

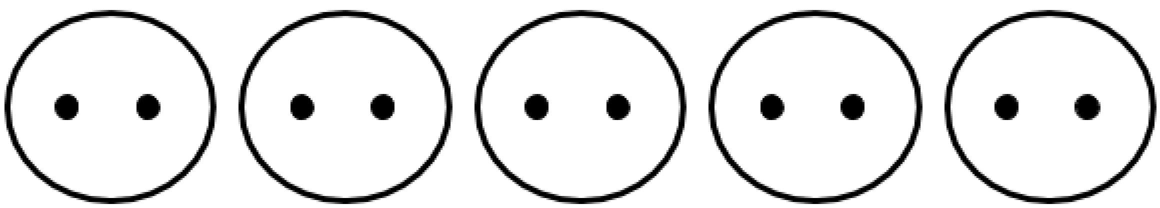
Stage 2.1



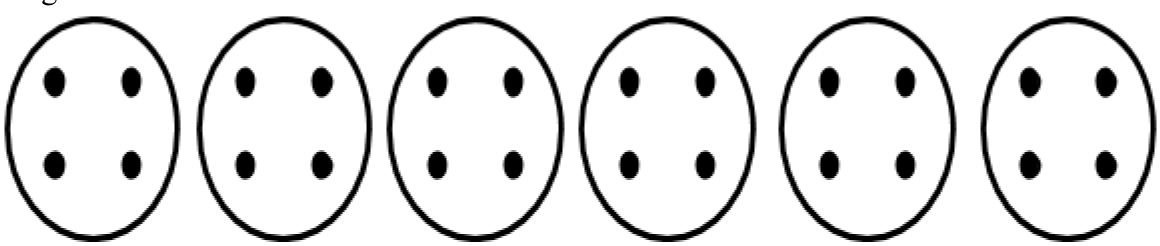
Stage 2.2



Stage 3.1



Stage 3.2



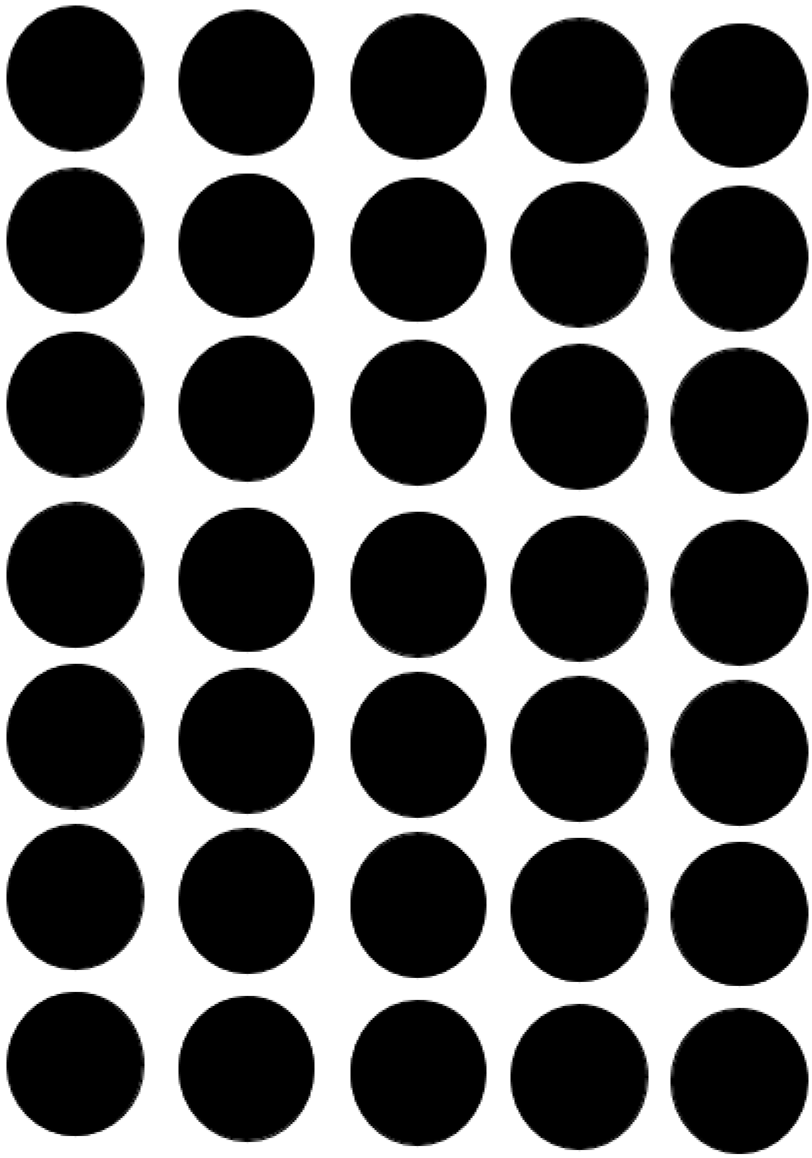
Stage 4.1

I have two groups of four dots.
How many total dots are there?

Stage 4.2

I have five groups of six dots.
How many dots do I have?

Stage 4.3



5.1	5.1	5.2	5.2
4x7=	7x4=	14x4=	15x4=

(Anoka-Hennepin Independent School District #11, 2013; Wright et al., 2006)

Appendix F
Teacher Station Instruction Guide

Stage 1: Mastery students are able to skip count by 2, 5, 10.

- Students practice multiples (Wright et al, 2006).
  Have students count and raise hand/say buzz/give thumbs up/clap with number sequence as students are learning. For instance when learning multiples of five students would count and then clap at bolded numbers: 1, 2, 3, 4, **5**, 6, 7, 8, 9, **10…** When students demonstrate mastery count only multiple factors such as 5, 10, 15…

  Suggestions:
  -Practice counting forwards (10, 20, 30…) and backwards (60, 55, 50...)
  -Start at different places (54, 56, 58…)

- True/False. Have students directly model repeating single digit addition problems such as 2+2+2, 3+3+3, etc. (Carpenter et al., 2003).

  Suggestions:
  -Use True and False number sentences for discussion such as:
  1) __=5+5+5
  2) 15=5+5+5+0
  3) 15=0+5+5+5
  4) 5+5+5+_+_=25

  -Utilize other multiples (4, 7, 8, etc) for additional development of relational thinking
  -Use open number sentences such as in number 4 (6+__+__=18)
  -Use a variety of addends 4+4+4+4, 2+2+2+2+2, etc

- Have students double a single digit number. Have students flip over a card and use a deck of playing cards with single digit numbers (Brickwedde, 2012a).
- Have students jump from a number such as 58->100, 216->300, 62->102. Have students go backwards from a decade number such as 43 go back 6, 34 go back 7, 94 go back 5. Practice using decade number and jumping by tens. (taken from: Brickwedde, 2016).
- Develop place value by a rate of tens with small group discussion (taken from: Brickwedde, 2016).

  4 10 40 Do you agree that there are 4 tens in 40?
  __ 10 80 If you know there are 4 tens in 40, how many tens are in 80?

__ 10 160 How many tens are in 160?

5 10 50 Do you agree that there are 5 tens in 50?
__ 10 70 If you know there are 5 tens in 50, how many tens are in 70?
__ 10 140 How many tens are in 140?

- Subitizing Cards - Flash a card and have students discuss answers. Do not reveal answer. Then show the card again and have students explain how they knew what the answer was. Sets to complete- Subitisation Dice and Card Set, and Subitisation Two Collections 1-5 (State Government of Victoria, 2012). See Appendix L for subitizing cards resource.
- Discuss how to solve problems. Give example of 2x2 and 2x364. Ask students which one they know and what they need to figure out. After completing individual sheets P1-P9 in Appendix J, discuss how we can use what we know to solve problems we do not know. Give students examples such as 3 groups of 3 is 9, how can that be used to solve 6 groups of 3. Ask students if 3x3=9 and 4x3=12, what can we use to solve 11x3? (Brickwedde, 2016).
- Have a guided group discussion on word problems 1.1 - 1.6 in Appendix I or word problems in Appendix J. Problems from Appendix I and Appendix J can be completed or worked on during proof station. Have students use manipulatives to model problems and discuss solutions (Brickwedde, 2012b; Brickwedde 2016).

    Suggestions:
    Discuss student strategies that demonstrate methods that involve array models, repeated addition and equal groups.
- Games
  -Trio for Multiples.
  Materials: At least 24 cards. For a deck of 5s you would have 4 cards of each multiple of 5 such as 5, 10, 15, 20, 25, 30, etc.
  Rules: Students are dealt five cards. They try to get three multiples in a row. After every turn pick a new card and discard a card. For instance 15, 20, 25 would be a winner. Can increase difficulty can increase number of multiples such as have multiples of three and five (Wright et al, 2006).

Stage 2 - Students are able to solve an equal groups problem with the problem modeled

- True/False. Have students directly model single digit multiplication problems such as 4x5, 6x4, 3x8, etc. (Carpenter et al., 2003).

    Suggestions:
    -Use True and False number sentences for discussion such as:
    1. 4x6=6+6+6+6

2. 3x9=9+9+9
3. 3x7=7+7+7
4. 7x3=3+3+3+3+3+3 (False)

For extension of discussion integrate multiplication facts that students have already learned in the problem

5. 4x6=12+6+6
6. 4x6=12+12
7. 3x7=14+7
8. 3x6=12+3 (false)

- Have students double a single digit number and eventually a smaller two-digit number. Have students flip over a card and use a deck of playing cards or cards made from 0-100 number chart. Start with small numbers and expand to larger numbers when doubling, tripling and quadrupling. When beginning tripling or quadrupling numbers start again with smaller numbers and expand to larger numbers when students demonstrate mastery. (Brickwedde, 2012a)
- Have students jump from a number such as 43->71, 88->110, 93->206. Have students go backwards from a decade number such as 35 go back 9, 71 go back 26, and 100 go back 64. Practice using decade number and jumping by tens. (taken from: Brickwedde, 2016).
- Subitizing Cards - Flash a card and have students discuss answers. Don't reveal answer. Then show the card again and have students explain how they knew what the answer was. Sets to complete- Subitisation Two Collections 1-5, Ten Frame Doubles Set, and Ten Frame Build on Five Set (State Government of Victoria, 2012). See Appendix L for subitizing cards resource.
- Develop scaling up and using known numbers to solve problems (taken from: Brickwedde, 2016).

  3 x 7 What are three sevens?
  6 x 7 If you know what 3 sevens are, can you use that to figure out six sevens?
  12 x 7 If you know three sevens and six sevens, how can you use either of those to figure out what 12 sevens are?

  4 x 7 What are four sevens?
  8 x 7 If you know what 4 sevens are, can you use that to figure out eight sevens?
  16 x 7 If you know four sevens and eight sevens, how can you use either of those to figure out what 16 sevens are?

  4 x 8 What are four eights?

2 x 8 If you know what 4 eights are, can you use that to figure out two eights?
8 x 8 If you know four eights and two eights, how can you use either of those to figure out what eight eights are?

- Have a guided group discussion on word problems 2.1 - 2.6 in Appendix I or word problems in Appendix J. Problems from Appendix I and Appendix J can be completed or worked on during proof station. Have students use manipulatives to model problems and discuss solutions (Brickwedde, 2012b; Brickwedde, 2016).

  Suggestions:
  Discuss student strategies that demonstrate methods that involve array models, repeated addition and equal groups.
- Games

  -Draw Multiples (Wright et al, 2006)
  Need: 4 sets of cards for the first ten set of multiples (2, 4, 6, 8, 10, 12, 14, 16, 18, 20)
  Rules: Players are dealt 15 cards and are placed in a pile face down in front of each player. Remaining 10 cards are split 5 for each player and put in front of each player. Each player draws three cards from their pile of 15 cards. At the same time players flip over one card from pile in front of them. They may play a card from their hand that goes either forward or backward. They continue to play one card at a time and may flip over a card or add a card. They may have only 3 cards in their hand but draw from their draw pile after each card is played. If a card of 12 is drawn the player may play a 10 or 14 next to it. Winner has used their entire draw pile and cards are in order either forward or backward. An example might be 8, 10, 12, 10, 8, 6, 4.
  Cards can be made from any multiple such as 3, 5, 10
  -Rolling Groups (Wright et al, 2006)
  Materials: Two dice and grid paper
  Rules: Have students roll two dice and one dice represents the groups and the other dice represents the amount in the groups. Have students shade in amount on grid paper

  Suggestions
  -Use manipulatives or drawings to represent the problem
  -Students use numbers they are unfamiliar with
  -Students count in multiples

Stage 3 -Students are able to solve an equal groups problem with groups visible but items in the groups not visible.

- True/False. Have students model single digit multiplication problems. Build up basic facts background (Carpenter et al., 2003).
    1. 3x8=16+8
    2. 9x3=2x9+9
    3. 3x7=14+7
    4. 3x6=12+3 (false)
    5. 7x6=7x5+6 (false)

  Develop relational thinking, using bigger numbers encourages students not to solve the problem but rather think about the relationship.

    6. 7x4=4x7
    7. 32x5=4x33 (false)
    8. 250x10=10x250
    9. 6x8=8x6
    10. 60x8=8x60
- Have students double a two-digit number and have students triple a single digit or small two-digit number. Use a deck of playing cards or cards made from 0-100 number chart. Start with small numbers and expand to larger numbers when doubling, tripling and quadrupling. When beginning tripling or quadrupling numbers start again with smaller numbers and expand to larger numbers when students demonstrate mastery. (Brickwedde, 2012a)
- Develop place value by a rate of tens with small group discussion (taken from: Brickwedde, 2016).

  10 10s in 100
  ___ 10s in 1000
  ___ 10s in 1230

  10 10s in 100
  ___ 10s in 1000
  ___ 10s in 2000
  ___ 10s in 1350
- Subitizing Cards - Flash a card and have students discuss answers. Do not reveal answer. Then show the card again and have students explain how they knew what the answer was. Sets to complete- Subitisation Two Collections 1-7 and Ten Frame Random Set (State Government of Victoria, 2012). See Appendix L for subitizing cards resource.

- Subitizing Cards - Flash students a card with a grid of dots such as a 5 by 8 grid. Ask students to find as many different strategies as possible besides counting by one (Brickwedde, 2016). See attached Grid Cards.
- Have a guided group discussion on word problems 3.1 - 3.7 in Appendix I or word problems in Appendix J. Problems from Appendix I and Appendix J can be completed or worked on during proof station. Have students model problems and discuss solutions (Brickwedde, 2012b; Brickwedde, 2016).

    Suggestions:
    -Discuss student strategies that demonstrate methods that involve array models, repeated addition and equal group
- Math Games

    -Multiplication top it
    Materials: Dice, recording method (paper, whiteboards, etc)
    Rules: Students take turns rolling two dice and multiply the two numbers that are rolled by the dice. They write down the product. The other player than rolls and finds the product. Keep tallies for the higher score.

Stage 4 - Students are able to solve an equal groups problem with the groups and items not being visible.

- True/False. Encourage students to use basic facts they know when working on single digit multiplication relationships. (Carpenter et al., 2003)
    1. 9x9=9x8+9+9 (false)
    2. 7x6=4x6+__x6
    3. 8x_=6x9+3x9
    4. 6x10=6x5+6x5
    5. 8x4=8x3+4 (false)

    Begin to introduce students proving double or triple digit numbers multiplied by a single digit digit 12x3.

    6. 16x4=_____
    7. 6x5+6x6=6x11
    8. 60x5=2x60+2x60+60
    9. 2x24=2x10+2x10+2x4
    10. 9x25=4x25+4x25+25
    11. 24x6=7x23 (false)
    12. 32x5=5x32
- Have students double a three digit number and triple a small two-digit number. Possibly have students quadruple a single digit number or a small two-digit number. Use a deck of playing cards or cards made from 0-100 number chart. Start with small numbers and expand to larger numbers when double, tripling and quadrupling. When beginning triple or quadruple start again with smaller numbers

and expand to larger numbers when students demonstrate mastery (Brickwedde, 2012a).

- Subitizing Cards - Flash a card and have students discuss answers. Do not reveal answer. Then show the card again and have students explain how they knew what the answer was. Sets to complete- Two Ten Frame Set and Subitisation Three Collections 1-5 (State Government of Victoria, 2012). See Appendix L for subitizing cards resource.
- Subitizing Cards - Flash students a card with a grid of dots such as a 5 by 8 grid. Ask students to find as many different strategies as possible besides counting by one (Brickwedde, 2016). See attached Grid Cards.
- Have a guided group discussion on word problems 4.1 - 4.7 in Appendix I or word problems in Appendix J. Problems from Appendix I and Appendix J can be completed or worked on during proof station. Have students model problems and discuss solutions (Brickwedde, 2012b; Brickwedde, 2016).

    Suggestions:
    -Discuss student strategies that demonstrate methods that involve array models, repeated addition and equal group
- Math Games

    -Lemonade Stand Game (Wright et al, 2006)
    Materials: cups and 60 snap cubes, record sheet
    Rules: Students in pairs take turn preparing the lemonade order of their peer. One student rolls two dice. The first one is the number of cups the player orders. The second dice roll is the number of ice cubes inside the cup. While the first player figures out how many ice cube were ordered, the second player prepares it. Then they reverse roles and whoever has more ice cubes is the winner. Student sheet is in Appendix G.
    -Array Go Fish (Wright et al, 2006)
    Materials: Cards that are split with numbers and a visual representation
    Rules: Each student draws five cards. When they have a pair they put the cards down for everybody to see. For every players turn, the player asks an opposing player if they have a card that matches with one in their hand. If the opposing player has it, he/she must give the card to the player asking and the original player can ask for another card. If the opposing player doesn't have the card then that player will say “no go fish”. If the player says “no go fish,” the original player must draw one card. Then it is the next player's turn. When one player has no cards left in their hand the game is over. Cards are in Appendix G.

Stage 5 - Students are able to solve problem solving and/or relational thinking to solve a single by double digit multiplication problem.

- True/False. Have students solve and prove double or triple digit numbers multiplied by a single digit digit 12x3, 62x4, etc. (Carpenter et al., 2003)
    1. ______=18x3
    2. 7x13=___
    3. 15x5=15x2+15x2+15
    4. 24x5=24x3+24 (false)
    5. 50x7= 300+50
    6. 240=24x__
    7. 2x24=2x10+2x10+2x4
    8. 60x5=2x60+2x60+60
    9. 16x6=15X5+____
- Have students double a three or four digit number and triple two-digit number. Students quadruple a single digit number or a small two-digit number. Use a deck of playing cards or cards made from 0-100 number chart. Start with small numbers and expand to larger numbers when doubling, tripling and quadrupling. When beginning tripling or quadrupling numbers start again with smaller numbers and expand to larger numbers when students demonstrate mastery (Brickwedde, 2012a).
- Subitizing Cards - Flash a card and have students discuss answers. Do not reveal answer. Then show the card again and have students explain how they knew what the answer was. Sets to complete- Subitisation Three Collections 1-7, Subitisation Four Collections 1-5, and Subitisation Four Collections 1-7 (State Government of Victoria, 2012). See Appendix L for subitizing cards resource.
- Have a guided group discussion on word problems 5.1 - 5.7 in Appendix I or word problems in Appendix J. Problems from Appendix I and Appendix J can be completed or worked on during proof station. Have students model problems and discuss solutions (Brickwedde, 2012b; Brickwedde 2016).

    Suggestions:

    -Discuss student strategies that demonstrate methods that involve array models, repeated addition and equal group
- Math Games

    -Dueling Arrays Game (Wright et al, 2006)

    Materials: Dueling Array Cards

    Rules: Students line up in two lines. Show students the card and the quickest student to get it correct goes to end of line and the other student sits down.

-Multiplication top it

Materials: Dice, recording method (paper, whiteboards, etc)

Students take turns rolling two dice and recording product. Keep tallies for the higher score.

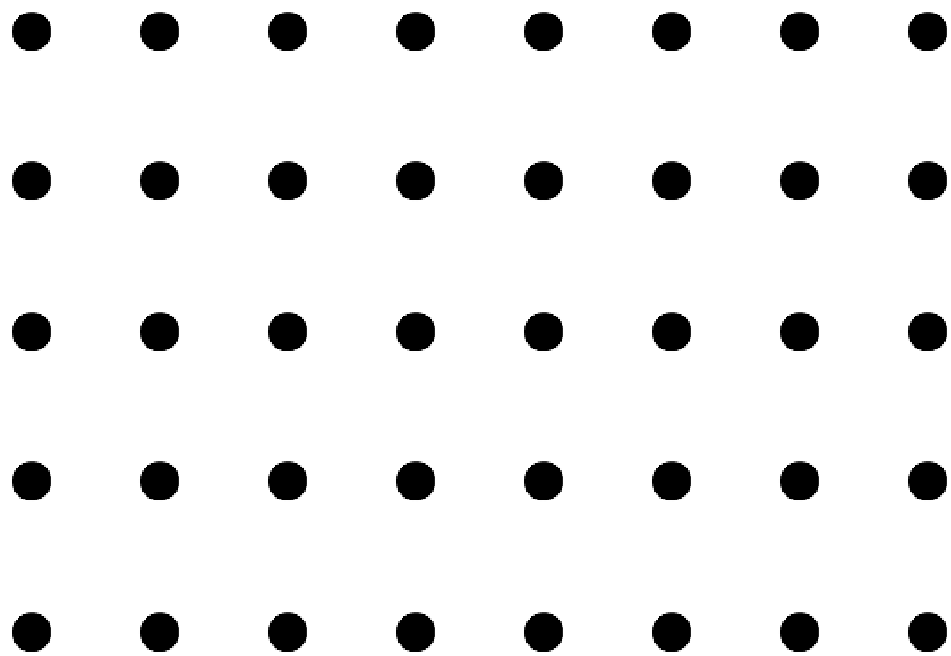
Suggestions

-Have students check answer

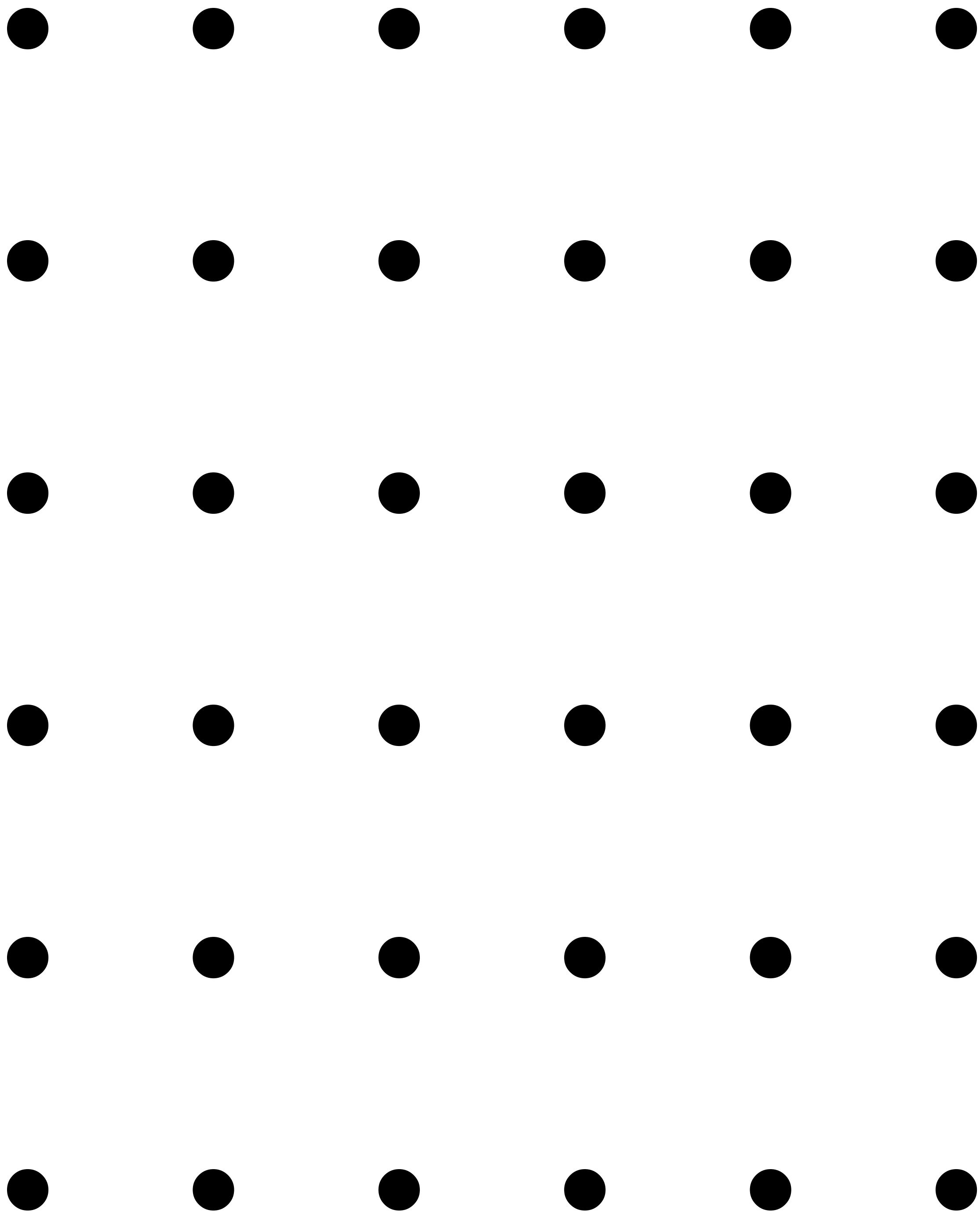
-Use three dice for their dice roll and have students do a double digit number multiplied by a single digit number(such as 24x3=)

Appendix G
Teacher Station Materials

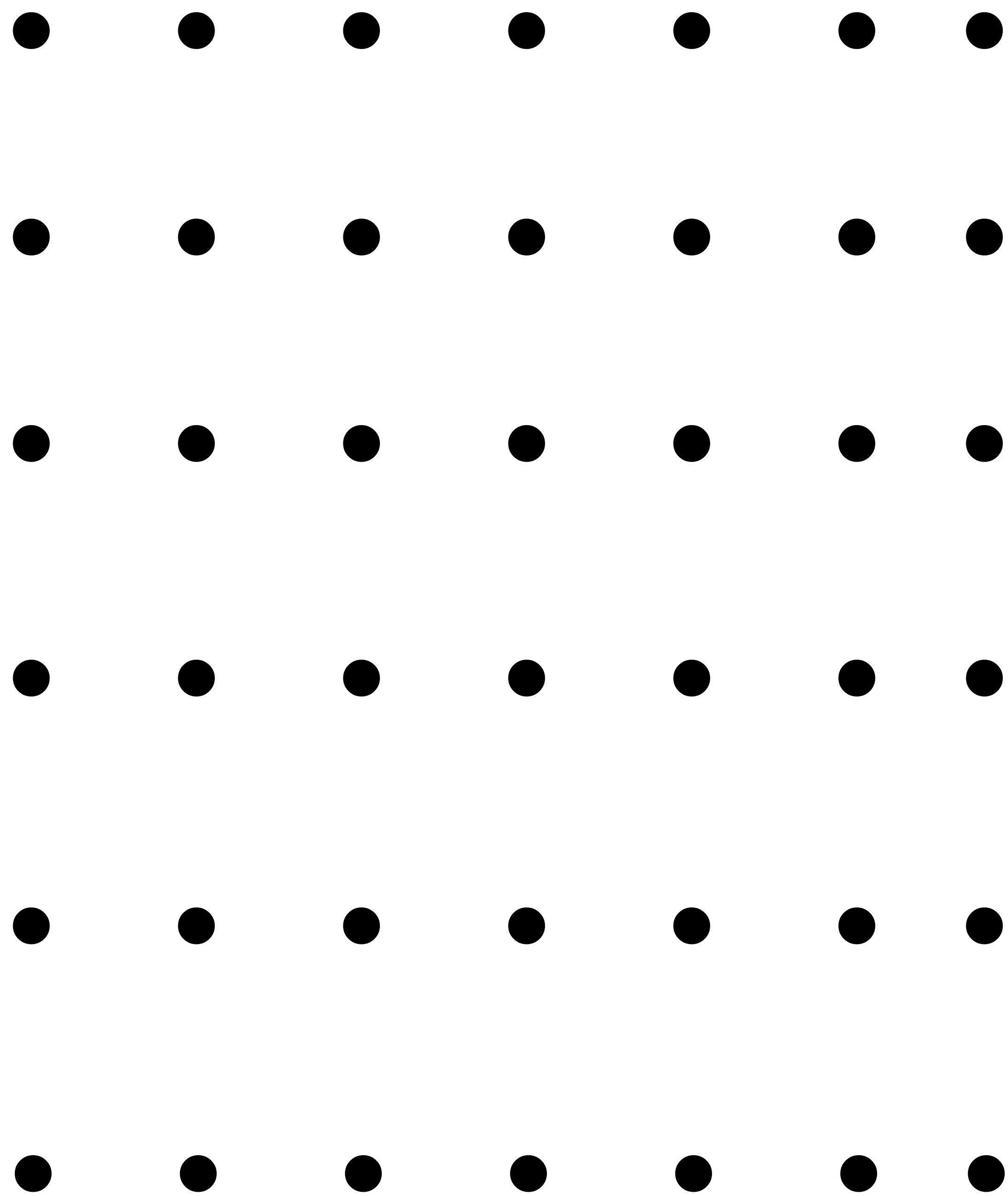
Stage 3 or 4 Subitizing grid cards for discussion (Brickwedde, 2016)



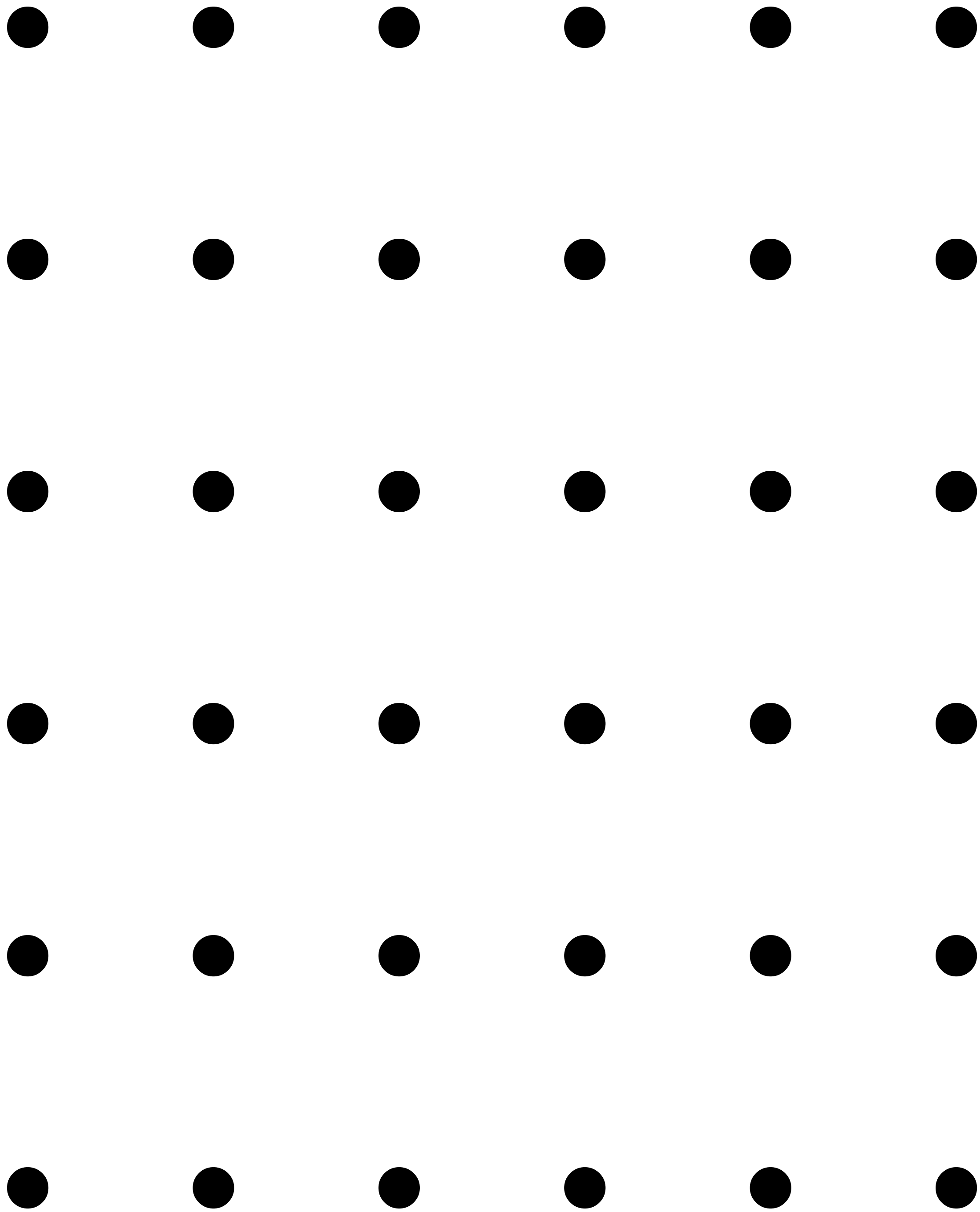
Stage 3 or 4 Subitizing grid cards for discussion (Brickwedde, 2016)



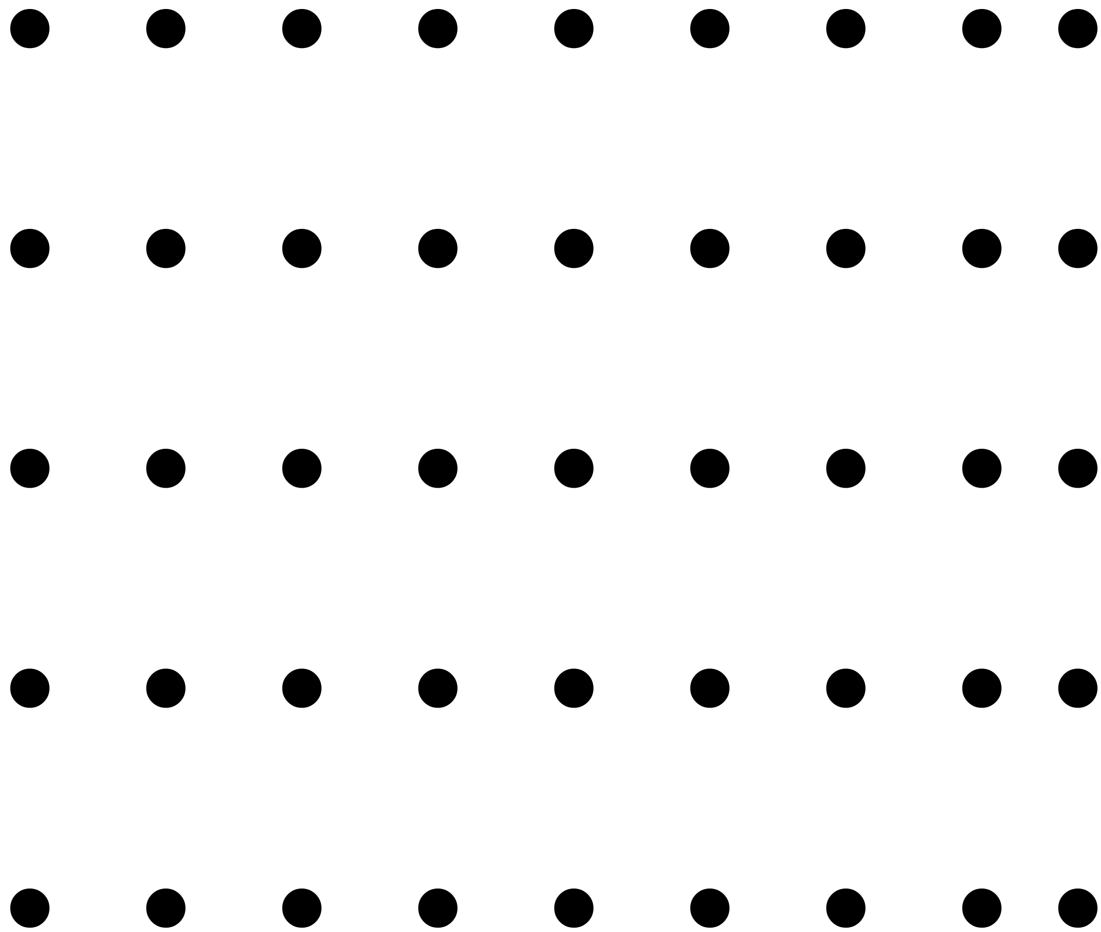
Stage 3 or 4 Subitizing grid cards for discussion (Brickwedde, 2016)



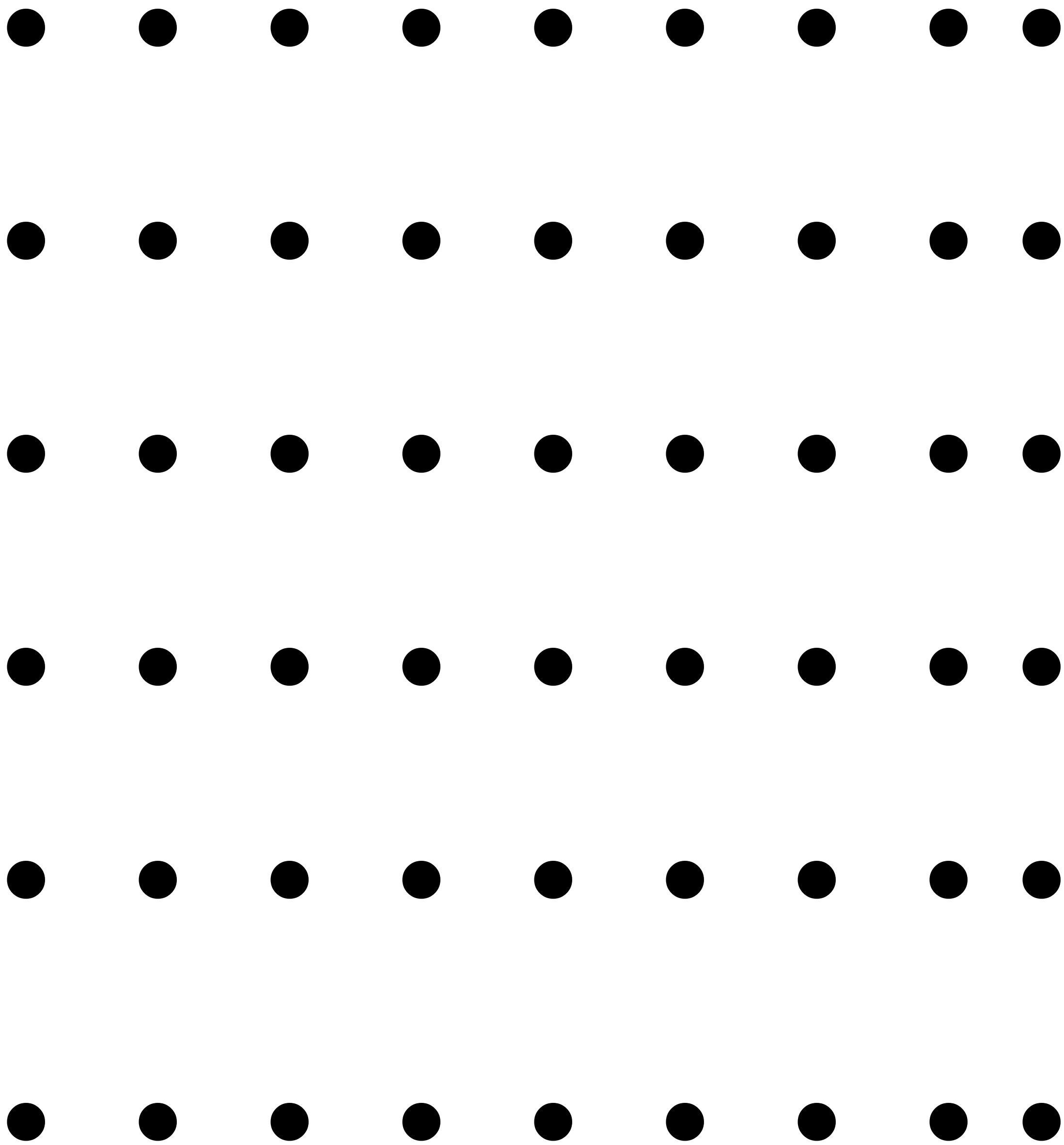
Stage 3 or 4 Subitizing grid cards for discussion (Brickwedde, 2016)



Stage 3 or 4 Subitizing grid cards for discussion (Brickwedde, 2016)



Stage 3 or 4 Subitizing grid cards for discussion (Brickwedde, 2016)



4.1 Lemonade Stand (Wright et al, 2006)

Player 1			Player 2		

Round 1

Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes	Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes

Player 1			Player 2		

Round 2

Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes	Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes

Player 1			Player 2		

Round 3

Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes	Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes

Player 1			Player 2		

Round 4

Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes	Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes

Player 1			Player 2		

Round 5

Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes	Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes

Player 1			Player 2		

Round 6

Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes	Number of Cups	Ice-cubes in one cup	Total number of Ice-cubes

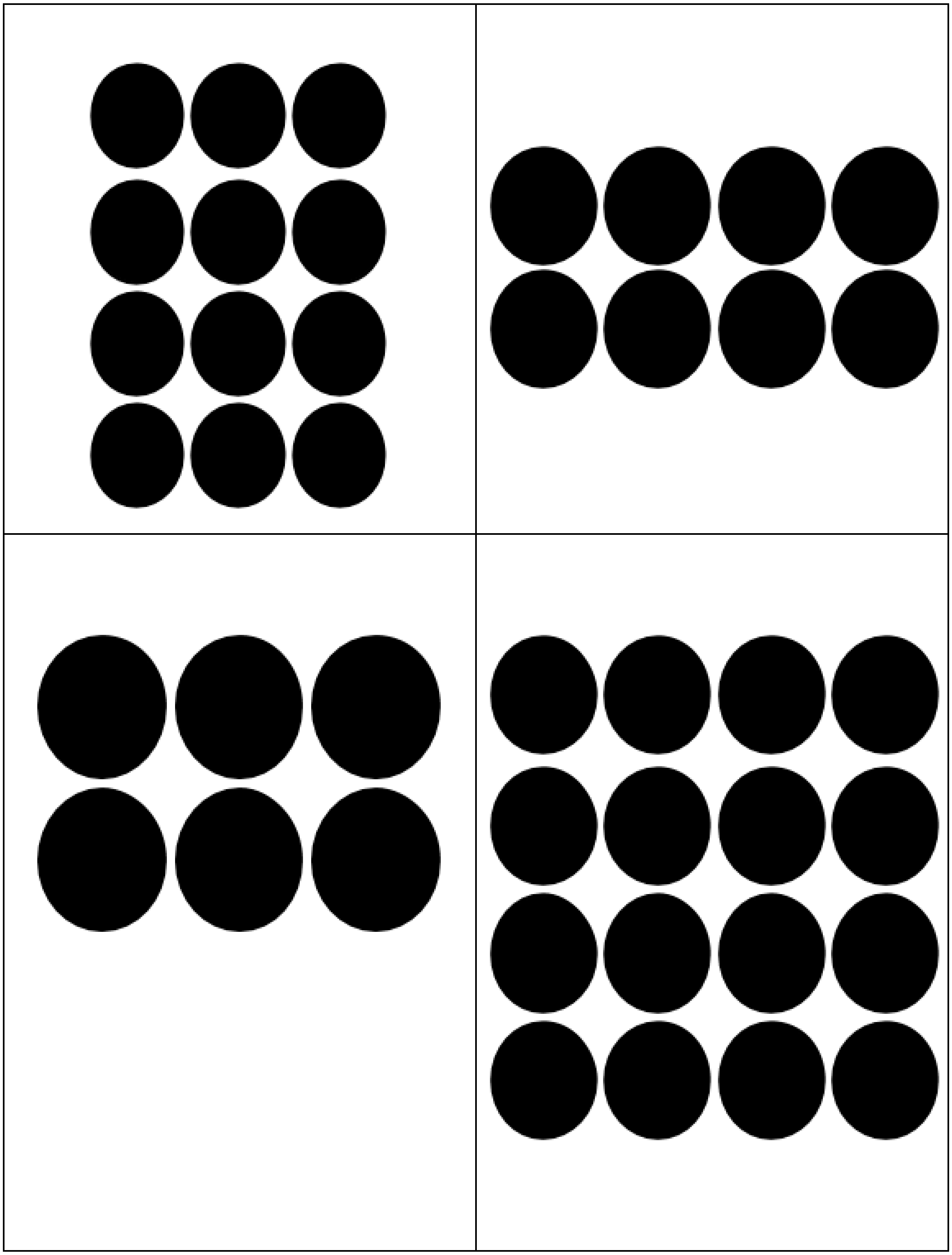
4 Stage- Array Go Fish

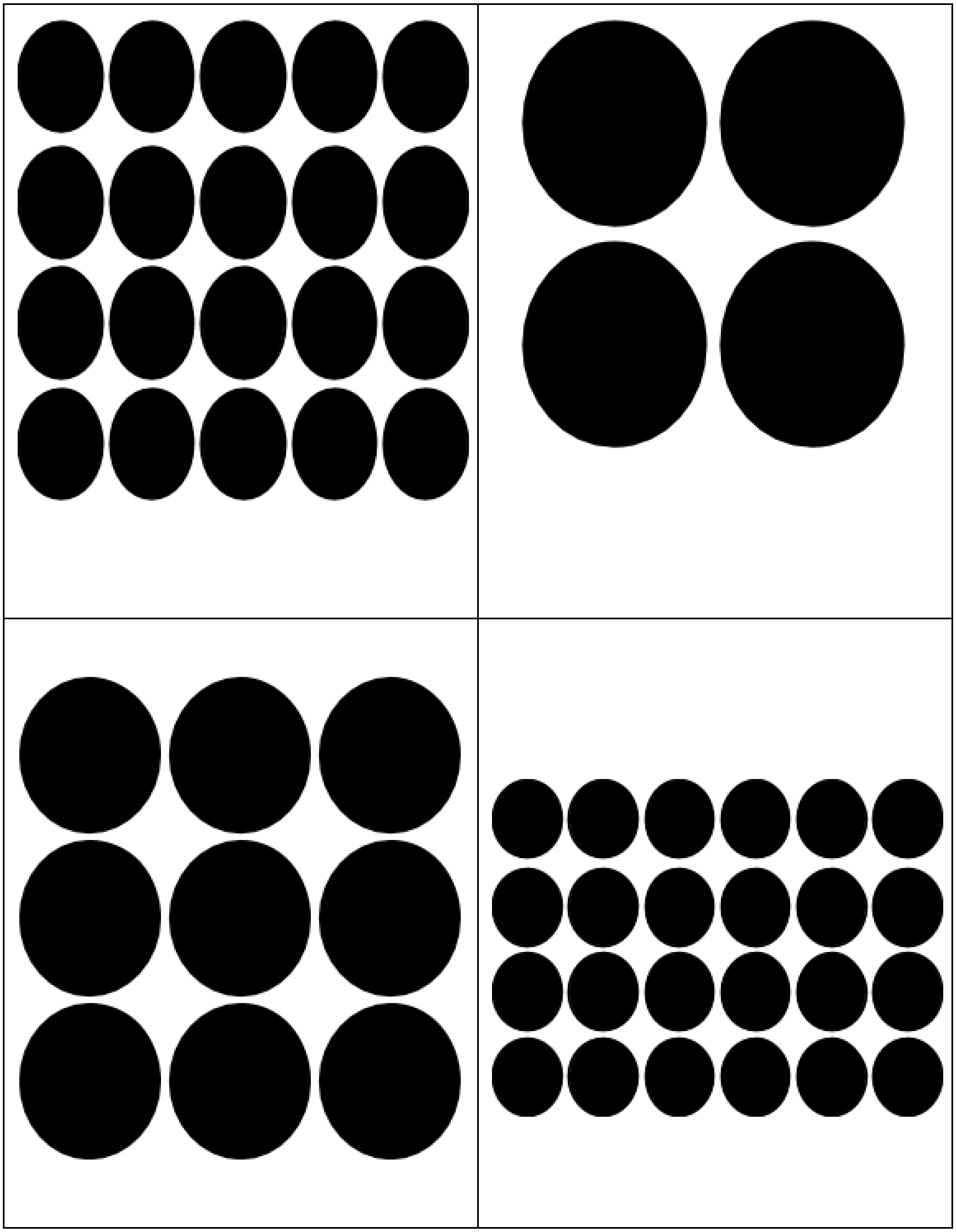
12	8
6	16

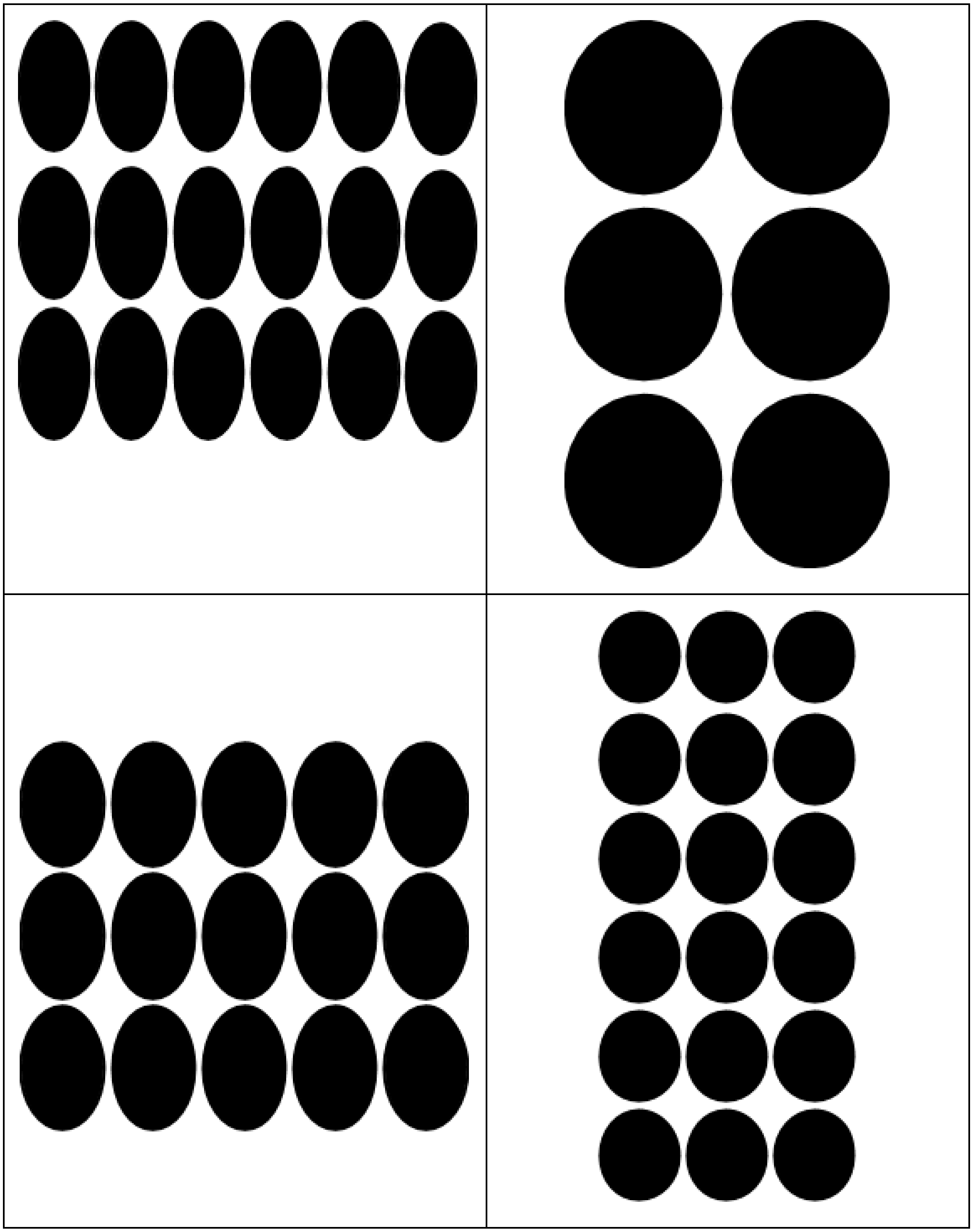
4	20
24	9

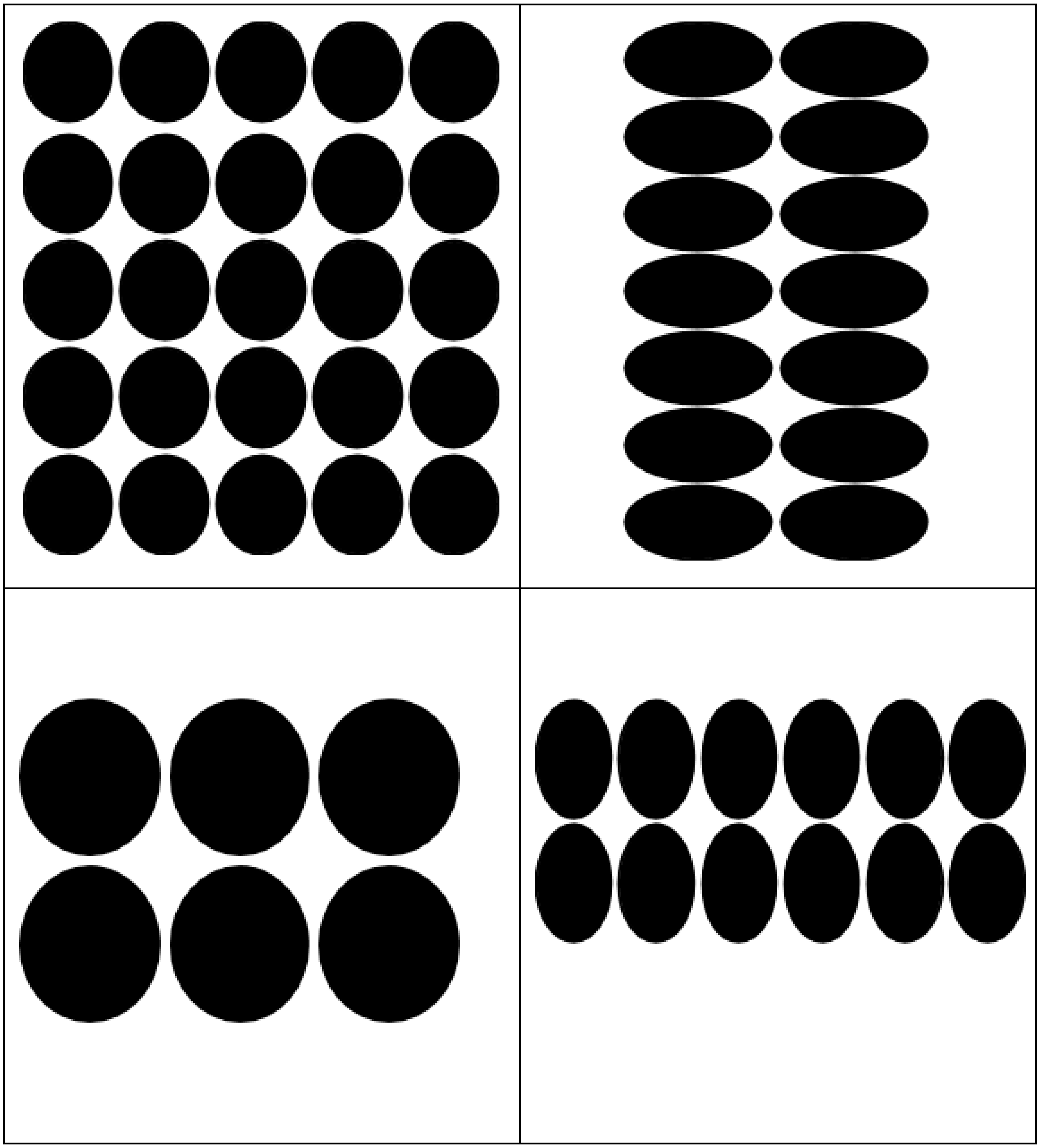
6	18
15	21

20	14
6	12

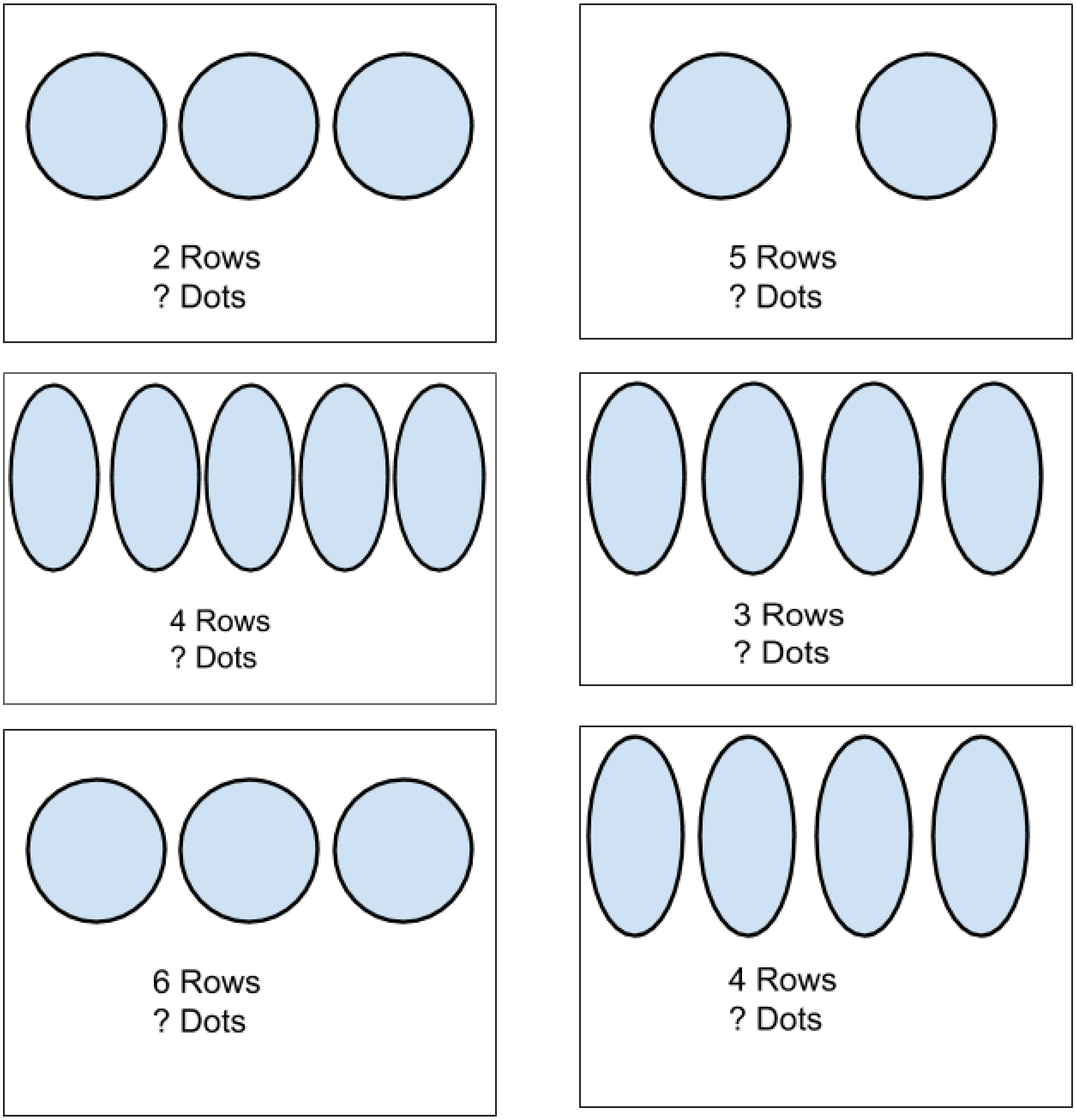


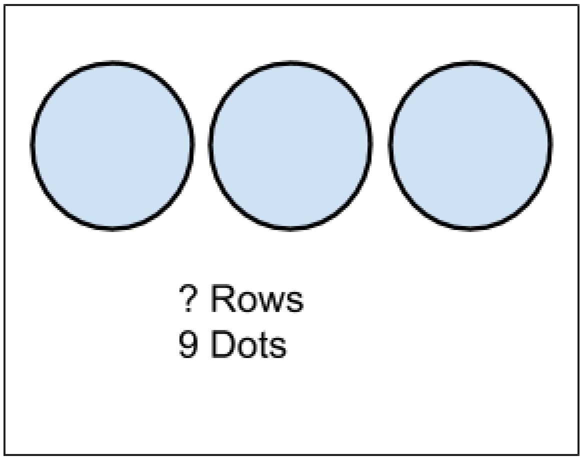


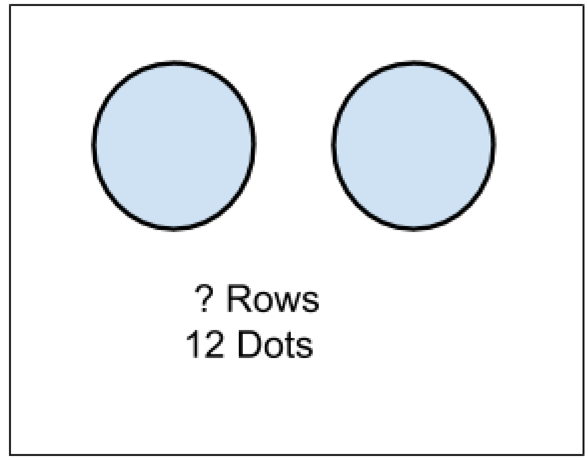


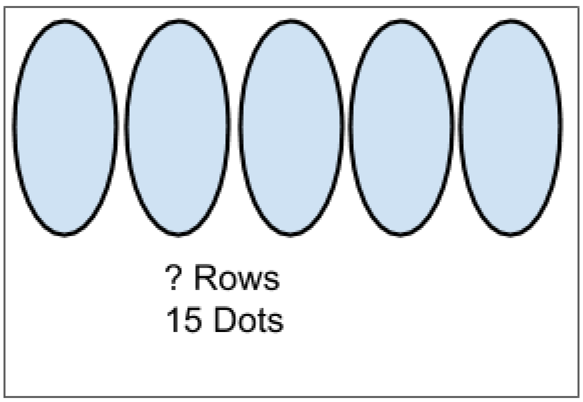


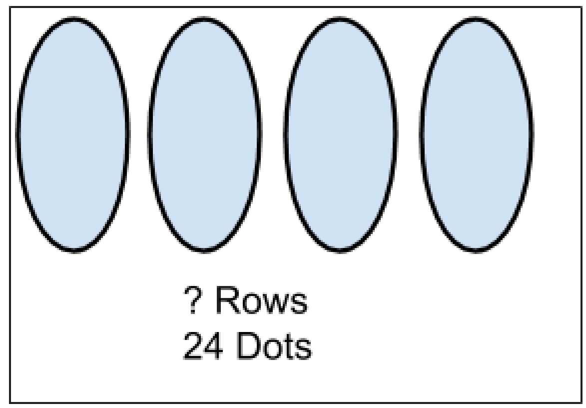
5 Stage-Dueling Array (Wright et al, 2006)

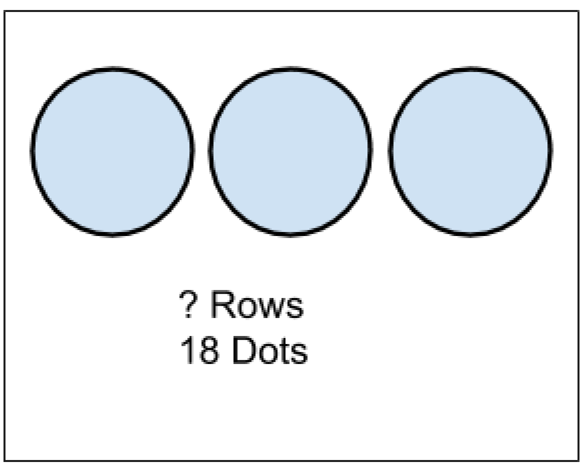


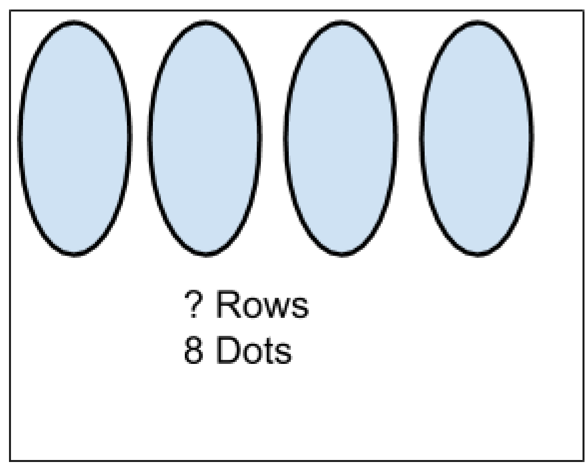

? Rows
9 Dots

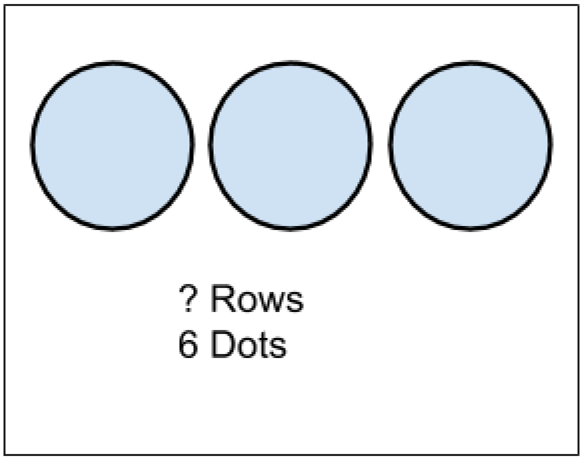

? Rows
12 Dots

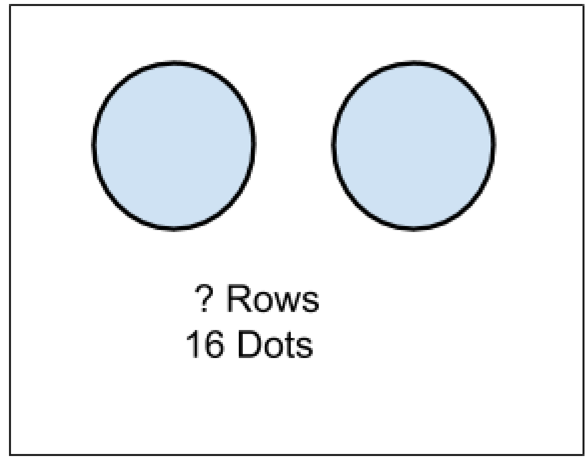

? Rows
15 Dots

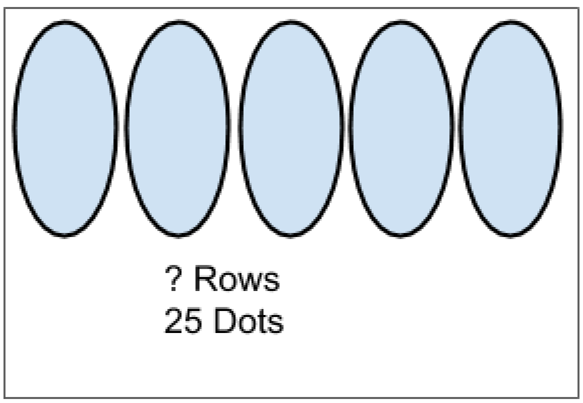

? Rows
24 Dots

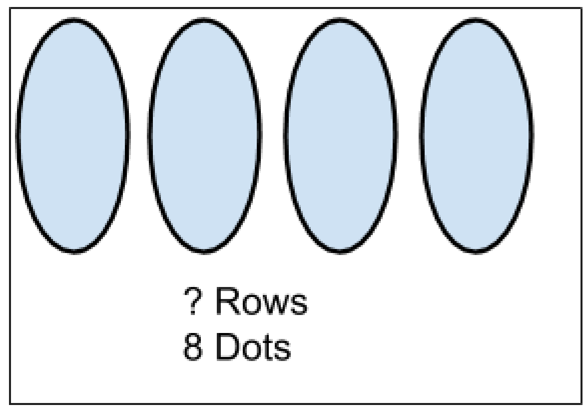

? Rows
18 Dots

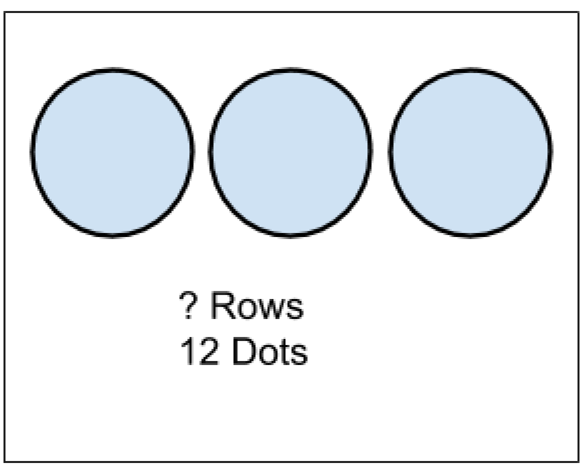

? Rows
8 Dots

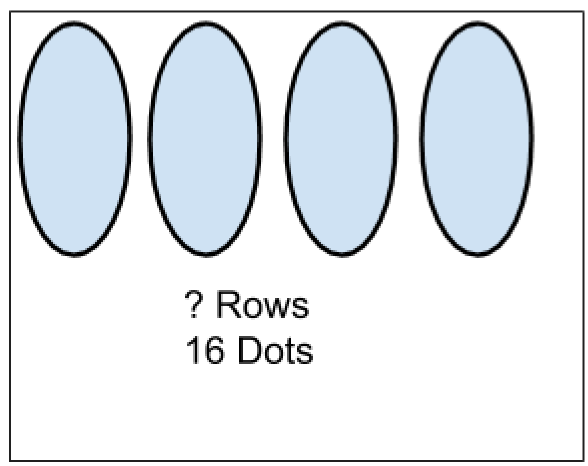

? Rows
6 Dots


? Rows
16 Dots


? Rows
25 Dots


? Rows
8 Dots


? Rows
12 Dots


? Rows
16 Dots

Appendix H
Game Station

Game Station: To be completed in small groups during stations. The games are grouped by stages to best meet the needs of students. If the game is designated with an asterisk, rules and materials are found in the Everyday Math Curriculum. The descriptions of the rules have been modified from the Everyday Math Curriculum.

Stage 1
24
Addition Top It*
Multiplication Top it*
Name that Number*
Number Toss
Snake - Adding
Subtraction Top It*
Who am I (with addition)

Stage 2
24
Addition Top It*
Multiplication Top it*
Name that Number*
Number Toss
Snake - Adding
Subtraction Top It*
Who am I (with addition)

Stage 3
24
Multiplication Compare
Multiplication Top it*
Multiplication Toss
Name that Number*
Number Toss
Who am I

Stage 4
24

Beat the Calculator*
Factor Captor*
Multiplication Compare
Multiplication Top it*
Multiplication Toss
Number Toss
Product Pileup*
Snake
Who am I

Stage 5
24
Beat the Calculator*
Division Arrays*
Fact Triangles flip*
Factor Captor*
Multiplication Compare
Multiplication Top it*
Multiplication Toss
Number Toss
Product Pileup*
Snake
Who am I

Game Descriptions:
24 - Students take a card which has four numbers on it. They must use all four numbers and may add, subtract, multiply or divide to reach the number 24. The game is differentiated as the one dot cards are the easiest and the three dot cards are the hardest to solve. This is a game that can be purchased for classroom use.

Addition Top it* - Every player draws two cards from a pile in the middle and states the sum for their pair of cards. The player with the highest sum gets all of the cards. In case of a tie each player draws two more cards and calls out their sum. The winner takes all of the cards. The player who collects the most cards wins.
Extended version ideas - Have players draw three cards and find the sum. Have players draw four cards and make two double digit numbers to add together.

Beat the Calculator* - Use Number cards 1-10. Have one player be the Caller, Calculator and Brain. The caller draws two cards and asks for the product of the two numbers. The

Calculator solves the problem using the calculator. The Brain solves it without the calculator. The Caller states who got the correct answer first. Switch roles every 7 turns or so.
Extended version - Have the caller attach a zero to one or both factors before asking for the product.

Division Arrays* - Players shuffle cards (6-18) and place facedown. For each turn, each player takes one card. On the graph paper students mark that number with counters. Then the player rolls a dice once and that is the number of equal rows a player must have. Each player scores the number of counters in the row. If there are no leftovers, the score is double the number of counters in 1 row. So if a player gets a 15 and rolls a 2, the player puts down two rows of seven counters with one left over. The player's score is a seven. Highest score wins at the end.

Factor Captor* - Player one covers a number on the grid with a counter and records the number for player one's score. Numbers can only be used once. Player two puts counters on all the factors for player one and adds them up for Player two's score. If Player two missed any factors, then player one can cover them and add the points to his/her score. Now players switch roles and repeat, Player two picks a number and Player one finds the factors. The player with the highest score wins. Grid 1(Beginning), Grid 2(More advanced). May use a calculator if needed.

Fact Triangles flip* - Use fact triangle cards and have students flip over the cards and write all the fact families. The winner is the first to write the fact family correctly. The cards can be fact triangle cards or arrays.

Multiplication Compare - Each player draws two cards and finds the product for their pair of cards. Players must record the equation on their board and the player with the highest product gets a point for the round (Brickwedde, 2016).

Multiplication Top it* - Every player draws two cards from a pile in the middle and states the product for their pair of cards. The player with the largest product gets all of the cards. In case of a tie each player draws two more cards and states their product. The winner takes all of the cards. The player who collects the most cards wins.
Extended version ideas - Have students draw three cards and find the product. Another possibility is to have each player make a double digit and a single digit number and find the product

Multiplication Toss - Two or more players roll 2 six-sided dice or 2 ten-sided dice. The players attempt to fill the grid as much as possible without overlapping. If a roll of 3 and 8 is rolled. Then the player may draw a border around 3 rows of 8 or 8 rows of 3. A player may partition the region as well. So if a player rolled 5 sixes, the player could draw a border around two separate areas of 2 rows of 6 and 3 rows of 6 or 4 rows of six and 1 row of six (State Government of Victoria; 2012).

Name that Number* - Place five cards face up and place the rest of the cards face down in a pile. Flip over the top card in the pile, that is your target number. You may use any of the five cards only once by adding, subtracting, multiplying or dividing to get to the target number.

Number Toss - Two students are playing with seven dice with one dice represents each place value, ones, tens, hundreds to the milliones. The students roll the dice and read and write the number. Students then can use the greater than and less than symbol with their partner.

May use all 7 dice or less dice depending on student background/comfort

Dice may be purchased from: www.eaieducation.com, EAI® Education Place Value Dice - Ones to Millions: Set of 7.

Product Pileup* - Deal 12 cards (Need eight cards each for numbers 1-10) to every player and put the rest of the cards face down. First player picks two cards and states the product. Each subsequent player tries to play two cards that have a higher product than the last product played. If the player cannot play then must draw two cards and may play. If the player still cannot play then the player must pass. The winner is the player that gets rid of all of their cards first or the player with the fewest number of cards when their are no cards left to draw.

Snake - Students roll two dice and write the product down. After each roll the player may exit and keep their score or continue to roll and add the new product to their current total. The round ends if they roll a 1 and their score on this turn is a zero. If they roll a 1 they keep their score from the previous round. If the player rolls a Snake Eyes of two 1s then the player loses all points they have accrued in the game. If a player rolls a 1 or stays at its score, it is the next player's turn. First player to 100 points wins.
This game can be adapted for addition practice.

Subtraction Top it* - Every player draws two cards from a pile in the middle and subtracts their smaller number from their larger number. The player with the largest difference gets all of the cards. In case of a tie each player draws two more cards and

states the difference. The winner with the largest number takes all of the cards. The player who collects the most cards wins.
Extended version ideas: Have each player draw three cards and find the sum of two cards in their hand. Then players subtract the smallest number from the largest. Have players draw four cards and make two double digit numbers to subtract.

Who am I - In groups of three with cards (1-10), have two players pick a card face down and put it on their forehead. The third player states the product. The winner is the student that says their own number first. Can be adapted to complete with addition as well.

Appendix I
Proof Station

Proof Station:
The word problems in Appendix I and Appendix J are to be utilized during the proof station working in their small groups and/or independently. These problems can be turned in and/or reflected upon during small group teacher guided instruction. The problems 1.1-5.7 were developed to match each stage of multiplication development (Appendix I). The worksheets in Appendix J were taken from the Project for Elementary Mathematics website which provide alternative activities that promote conceptual development of multiplication and number sense. Appendix J is a tool to use during discussion in the teacher guided group station. The questions from Project of Elementary Mathematics curriculum are to be completed based on student readiness and used for math talks during the teacher station.

Name______________________

1.1 WORD PROBLEM

Michael bought four bags of stickers with two stickers in each bag. How many stickers does Michael have?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

1.2 WORD PROBLEM

Joe returned to the library two bags with three books in each bag. How many books did he return to the library?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name____________________

1.3 WORD PROBLEM

Natalie went to Cub foods and bought two bags of apples. Each bag had three apples. How many apples did Natalie buy?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

1.4 WORD PROBLEM

Josh's dog eats four pounds of food every day. How much food does Josh's dog eat in three days?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

1.5 WORD PROBLEM

How much would three pizzas cost if they each cost two dollars?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

1.6 WORD PROBLEM

Three pens will cost Ma'laika five dollars. How much will nine pens cost Ma'laika?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

2.1 WORD PROBLEM

There are four basketball teams playing and each team has five players. How many players are there?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name_____________________

2.2 WORD PROBLEM

Jose read fifteen books in five days. How many books did Jose read every day?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

2.3 WORD PROBLEM

Rosita bought six bags of candy that had seven pieces of chocolate in each bag. How many pieces of chocolate did Rosita have?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

2.4 WORD PROBLEM

Jordan ate eight cookies and each cookie had four raisins inside of them. How many raisins did Jordan eat?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

2.5 WORD PROBLEM

Bella watched six movies each month for eight months. How many movies did Bella watch?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name____________________

2.6 WORD PROBLEM

How many days are there in eleven weeks?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

3.1 WORD PROBLEM

Washington thirty crayons in his desk. He wants to give his crayons equally to six friends. How many crayons did each friend get?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

3.2 WORD PROBLEM

Georgine had seven boxes of cupcakes for her birthday party. Each box had six cupcakes in it. How many cupcakes did Georgine have?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

3.3 WORD PROBLEM

Hannah had eight gift bags with M&Ms. Each bag had 12 M&Ms.
How many M&Ms did Hannah have?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name_______________________

3.4 WORD PROBLEM

Jordy had twenty-four marbles and wanted to give all of his marbles to his three friends equally. How many marbles did each friend get?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

3.5 WORD PROBLEM

Madison walked her dog twenty-eight times in a week. She walked the dog the same number of times each day. How many times did she walk her dog in one day?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name____________________

3.6 WORD PROBLEM

Karissa had twenty-seven stuffed animals in her room. Her mom wants her to put them in three boxes. How many will go in each box?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name____________________

3.7 WORD PROBLEM

Samantha has vacuumed her room three times every month for a year. How many times has she vacuumed her room in one year?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

4.1 WORD PROBLEM

Alan bought six boxes of Pokemon cards. Each box has nine cards. How many Pokemon cards did Alan buy?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

4.2 WORD PROBLEM

On a road trip the Johnson family traveled sixty miles per hour for six hours. How many miles did the Johnson family travel?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

4.3 WORD PROBLEM

Jesse bought ten new pencils and six new notebooks for school. At the store, Jeese spent \$0.74 for each pencil and \$1.25 for each notebook. How much money did Jesse spend?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

4.4 WORD PROBLEM

While walking in the park, Hernandez saw four birds in eight different trees. How many birds did Hernandez see on his walk?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

4.5 WORD PROBLEM

Jafaar had six quarters in his jar and five quarters in his wallet. How many quarters does he have? How much money does he have?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name____________________

4.6 WORD PROBLEM

Carmelo and Rufio were playing a basketball game. Each basket was three points. Carmelo scored twenty four points and Rufio scored thirty-six points. How many baskets did each boy make?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

4.7 WORD PROBLEM

Brandon bought seven boxes of oranges. In each box was six oranges. How many oranges does Brandon have?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name_____________________

5.1 WORD PROBLEM

Josey had six quarters in his jar and five quarters in his wallet. How many quarters did he have? How much money did he have?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

5.2 WORD PROBLEM

Isabella had some money. Her dad gave her eight dimes and 5 quarters. Now she has $42.25. How much money did she have to start with?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

5.3 WORD PROBLEM

A football team has eleven players. At the tournament this weekend there were fifteen teams. How many players were at the tournament?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

5.4 WORD PROBLEM

Chris ate five pieces of pizza. He bought nine pizzas with eight slices each. How many pizza slices does he have left?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

5.5 WORD PROBLEM

I have six puzzles. Each puzzle has 75 pieces. How many total pieces do I have?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name______________________

5.6 WORD PROBLEM

A carton of eggs holds twelve eggs. I have 22 cartons of eggs. How many eggs do I have?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

Solve the problem (show your work!):

Name____________________

5.7 WORD PROBLEM

Every student in our school has six notebooks. There are five hundred students in our school. How many notebooks are in our school?

Tell me what you need to do with the numbers to answer the question.

Create a Number Sentence:

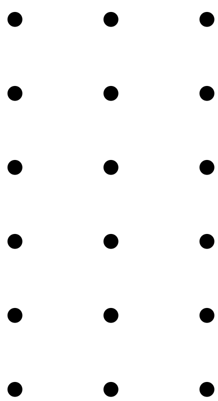
Solve the problem (show your work!):

Appendix J

For stages 1-2 - Taken from: Grade 3 - Brickwedde, 2016 #P1

Name: ______________________ Date: ____________________

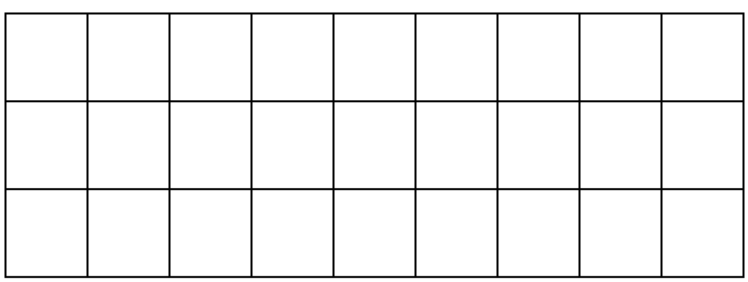
Directions: **How can you find the number of dots or boxes in the figure using chunks of numbers?**
Do not count by ones!



How many rows? ________ How many columns? _________

How many total dots? __________________

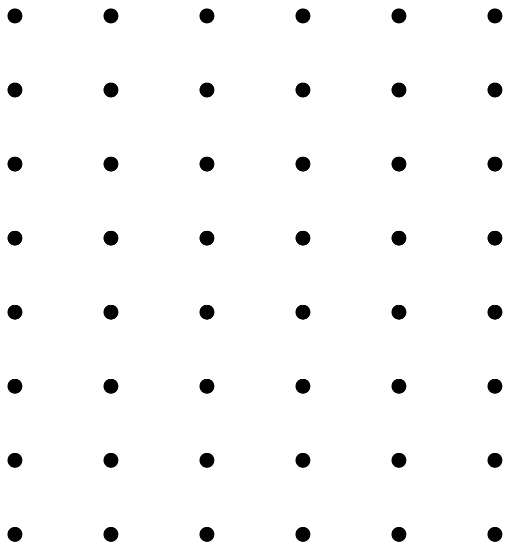
Show your work here:



How many rows? ________ How many columns? _________

How many total squares? __________________

Show your work here:

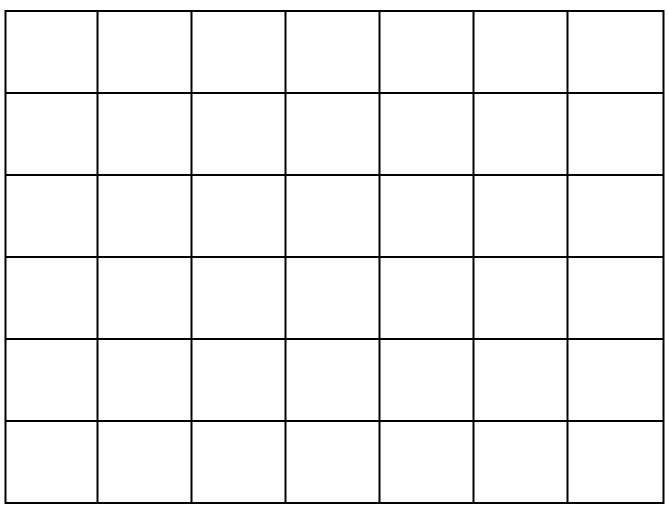


How many rows? _______

How many columns? ________

How many total dots? ________

Show your work here:



How many rows? ________ **How many columns? _________**

How many total squares? __________________

Show your work here:

For stages 1-2 - Taken from: Grade 3 - Brickwedde, 2016 #P2

Name: ________________________ Date: ____________________

Look at the following basic facts combinations. If you know it without counting, write the answer then circle the phrase "know it." If you have to count **leave the answer blank** ***and circle "have to count." Be honest with yourself!***

1 x 3	______	know it	have to count
2 x 3	______	know it	have to count
5 x 3	______	know it	have to count
10 x 3	______	know it	have to count

1 x 5	______	know it	have to count
2 x 5	______	know it	have to count
5 x 5	______	know it	have to count
10 x 5	______	know it	have to count

1 x 4	______	know it	have to count
2 x 4	______	know it	have to count
5 x 4	______	know it	have to count
10 x 4	______	know it	have to count

1 x 6	______	know it	have to count
2 x 6	______	know it	have to count
5 x 6	______	know it	have to count
10 x 6	______	know it	have to count

For stages 1-2 - Taken from: Grade 3 - Brickwedde, 2016 #P3

Name: ________________________ Date: ____________________

Practice developing a plan.

- **Step one: Go through the list and do the ones you know *without counting.***
- **Step two: Write out a plan for how you can *use the ones you know to figure out the ones you have to count.***

1 x 3 = **Plan: ____**

2 x 3 = **Plan: ____**

3 x 3 = **Plan: ____**

4 x 3 = **Plan: ____**

5 x 3 = **Plan: ____**

6 x 3 = **Plan: ____**

7 x 3 = **Plan: ____**

8 x 3 = **Plan: ____**

9 x 3 = **Plan: ____**

10 x 3 = **Plan: ____**

11 x 3 = **Plan: ____**

12 x 3 = **Plan: ____**

For stages 1-2 - Taken from: Grade 3 - Brickwedde, 2016 #P4

Name: ________________________ Date: ____________________

Practice developing a plan.

- **Step one: Go through the list and do the ones you know *without counting.***
- **Step two: Write out a plan for how you can *use the ones you know to figure out the ones you have to count.***

1 x 4 = __________ Plan: ____

2 x 4 = __________ Plan: ____

3x 4 = __________ Plan: ____

4 x 4 = __________ Plan: ____

5 x 4 = __________ Plan: ____

6 x 4 = __________ Plan: ____

7 x 4 = __________ Plan: ____

8 x 4 = __________ Plan: ____

9 x 4 = __________ Plan: ____

10 x 4 = __________ Plan: ____

11 x 4 = __________ Plan: ____

12 x 4 = __________ Plan: ____

For stages 1-3 - Taken from: Grade 3 - Brickwedde, 2016 #P5

Name: ________________________ Date: ____________________

Solve the following questions.

Who gets more strawberries or do they get the same, a person who buys 3 boxes of 16 strawberries in each box or a person who buy 8 boxes with 6 strawberries inside each box?

____

Answer with a label

Who gets more cherries or do they get the same, a person who buys 12 boxes with 3 cherries inside each box, or a person who buy 4 boxes with 10 inside each box?

____

Answer with a label

For stages 1-3 - Taken from: Grade 3 - Brickwedde, 2016 #P6

Name: ______________________ Date: ___________________

Practice developing a plan.

- **Step one: Go through the list and do the ones you know *without counting.***
- **Step two: Write out a plan for how you can *use the ones you know to figure out the ones you have to count.***

1 x 5 = __________ Plan: _________________________________

2 x 5 = __________ Plan: _________________________________

3 x 5 = __________ Plan: _________________________________

4 x 5 = __________ Plan: _________________________________

5 x 5 = __________ Plan: _________________________________

6 x 5 = __________ Plan: _________________________________

7 x 5 = __________ Plan: _________________________________

8 x 5 = __________ Plan: _________________________________

9 x 5 = __________ Plan: _________________________________

10 x 5 = __________ Plan: _________________________________

11 x 5 = __________ Plan: _________________________________

12 x 5 = __________ Plan: _________________________________

For stages 1-3 - Taken from: Grade 3 - Brickwedde, 2016 #P7

Name: ________________________ Date: ____________________

Practice developing a plan.

- **Step one: Go through the list and do the ones you know *without counting.***
- **Step two: Write out a plan for how you can *use the ones you know to figure out the ones you have to count.***

1 x 6 = __________ Plan: ___________________________________

2 x 6 = __________ Plan: ___________________________________

3 x 6 = __________ Plan: ___________________________________

4 x 6 = __________ Plan: ___________________________________

5 x 6 = __________ Plan: ___________________________________

6 x 6 = __________ Plan: ___________________________________

7 x 6 = __________ Plan: ___________________________________

8 x 6 = __________ Plan: ___________________________________

9 x 6 = __________ Plan: ___________________________________

10 x 6 = __________ Plan: ___________________________________

11 x 6 = __________ Plan: ___________________________________

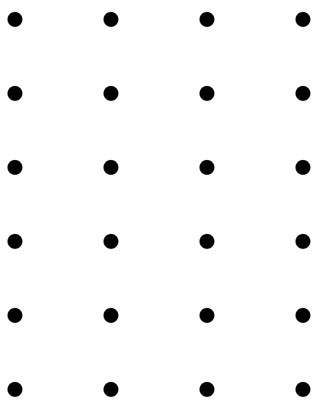
12 x 6 = __________ Plan: ___________________________________

For stages 1-3 - Taken from: Grade 3 - Brickwedde, 2016 #P8

Name: ________________________ Date: ____________________

Directions: **How can you find the number of dots or boxes in the figure using chunks of numbers?**

Do not count by ones!

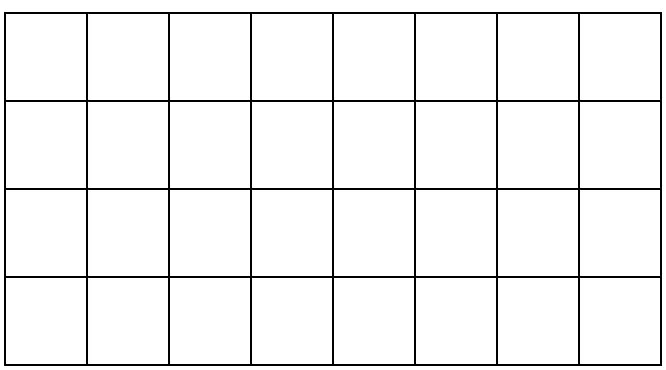


How many rows? ________

How many columns? _________

How many total dots? _________

Show your work here:

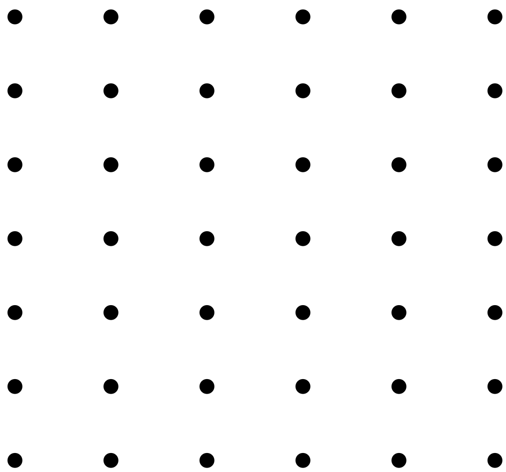


How many rows? ________ How many columns? _________

How many total squares? __________________

Show your work here:

For stages 1-3 - Taken from: Grade 3 - Brickwedde, 2016 #P8

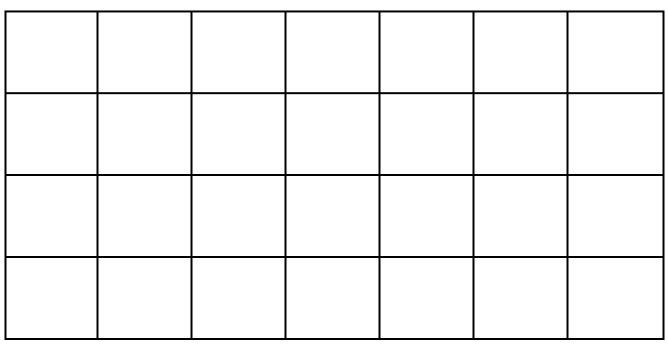


How many rows? _______

How many columns? ______

How many total dots? _______

Show your work here:



How many rows? _________ How many columns? __________

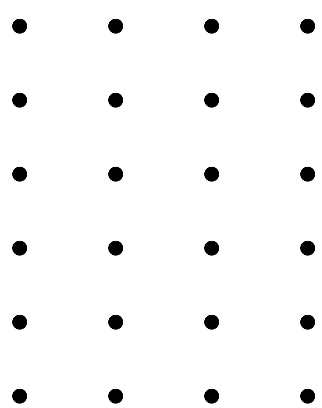
How many total squares? ____________________

Show your work here:

For stages 1-3 - Taken from: Grade 3 - Brickwedde, 2016 #P9

Name: ________________________ Date: ____________________

Directions: **How can you find the number of dots or boxes in the figure using chunks of numbers?**
Do not count by ones!

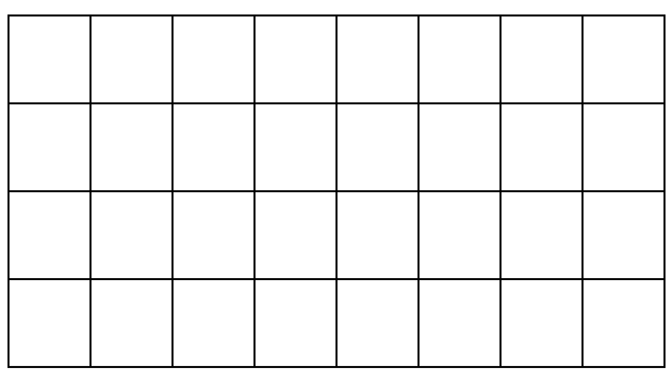


How many rows? ________

How many columns? _________

How many total dots? __________________

Show your work here:

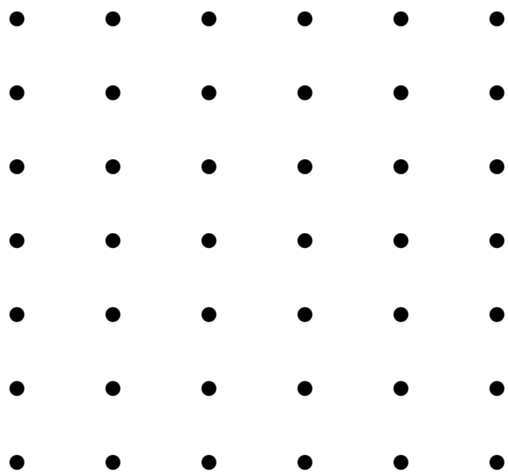


How many rows? ________ How many columns? _________

How many total squares? __________________

Show your work here:

For stages 1-3 - Taken from: Grade 3 - Brickwedde, 2016 #P9

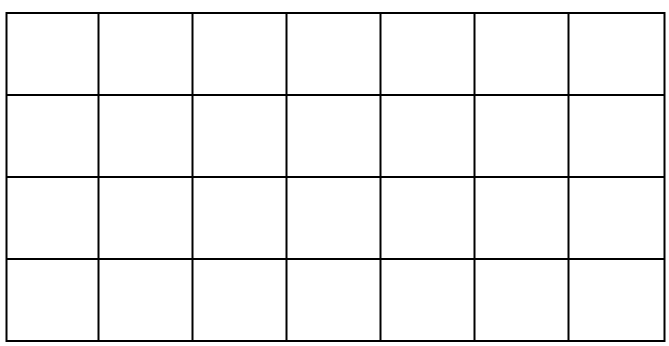


How many rows? ______

How many columns? _______

How many total dots? ______

Show your work here:



How many rows? ________ **How many columns? _________**

How many total squares? __________________

Show your work here:

For stages 2-3 - Taken from: Grade 3 - Brickwedde, 2016 #P10

Name: ________________________ Date: ____________________

George had the job of placing apples into clear plastic bags and tying them shut to get the apples ready for sale. He started with a big crate of apples. When he was done he had tied shut ___ bags with ____ apples inside each bag. He still had ___ apples in the crate. How many apples were in the crate to begin with?

(7, 10, 4) (12, 10, 7) (24, 10, 8)

Answer with label: __________________________________

Write a complete sentence to explain your answer

____

Celia has the same job. She knows ahead of time how many apples are in her crate. If she puts ten apples inside each bag, how many full plastic bags will she have filled if she starts with _____ apples in the crate? (217) (812) (1,243)

Answer with label: __________________________________

Write a complete sentence to explain your answer:

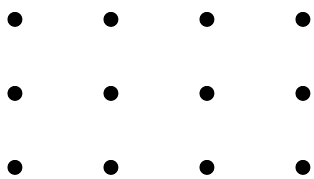
____

For stages 2-3 - Taken from: Grade 3 - Brickwedde, 2016 #P11

Name: ________________________ Date: ____________________

***Directions:* How can you find the number of dots or boxes in the figure using chunks of numbers?**

Do not count by ones!

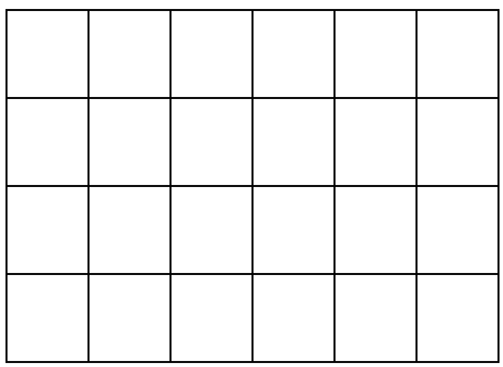


How many rows? ________

How many columns? _________

How many total dots? __________________

Show your work here:

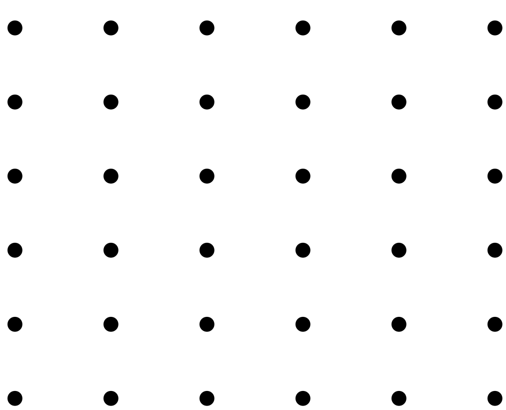


How many rows? ________

How many columns? _________

How many total squares? ____________

Show your work here:



How many rows? ______

How many columns? _______

How many total dots? ______

Show your work here:

For stages 2-3 - Taken from: Grade 3 - Brickwedde, 2016 #P12

Name: _________________________ Date: ____________________

Gisselle is making ___ gift bags for guests coming to her birthday party. If she puts 4 candies in each bag, how many pieces of candy will she give away in all of the bags?

(6) (9) (14)

____

Answer with a label

Jordan was helping to bake cookies in the kitchen. He puts ____ rows of cookies with ____ cookies in each row. If he did that on ____ cookie sheets, how many cookies will he have made?

(4, 7, 3) (4, 8, 4) (5, 7, 6)

____

Answer with a label

For stages 2-3 - Taken from: Grade 3 - Brickwedde, 2016 #P13

Name: ________________________ Date: ____________________

Solve the problems. Think about how to do it in as few steps as possible!

Alexis bought _____ pairs of sox at the store. Each pair cost ______. How much money did she spend?

(3, $4.50) (3, $4.99) (6, $4.55)

____

Answer with a label

Ronald read ____ pages in his book. Jacqueline read ____ times as many. How many pages did Jacqueline read?

(25, 4) (14, 6) (36, 4)

____

Answer with a label

For stages 2-3 - Taken from: Grade 3 - Brickwedde, 2016 #P14

Name: ________________________ Date: ____________________

Solve the problems. Think about how to do it in as few steps as possible!

The cafeteria has ____ apples. If each apple is cut into six slices, how many slices will they have to serve for students to eat?

(10) (25) (36)

____

Answer with a label

Jared was helping to sort _____ individual grapes into cups for the students to eat for snacks. If he puts the ___ grapes into a cup, how many cups can he fill?

(35, 5) (48, 6) (72, 6)

____

Answer with a label

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P15

Name: ________________________ Date: ____________________

The art teacher needed to buy some new boxes of crayons for the class to use on projects. Crayons come in different size boxes with some sizes having more colors than others. If she bought ___ boxes with six crayons inside each and bought ____ boxes with eight crayons inside each, how many individual new crayons would she have for the class to use?

(4, 5) (6, 8)

Answer with label: ___________________________________

Write a complete sentence to explain your answer

____

The art teacher has ____ crayons. She has ____ baskets into which she can place the crayons. If she place the same number of crayons into each basket, how many crayons will she end up having in each?

(48, 4) (64, 8) (54, 6)

Answer with label: ___________________________________

Write a complete sentence to explain your answer:

____

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P16

Name: ________________________ Date: ____________________

The shop owner of a clothing store is doing inventory. She is counting the number of packages of sox that are on the shelf for customers to buy. She notes that there are ____ packages with ____ pairs of sock inside each package. There is also another brand of socks next to those on the shelf. There are ____ packages with only ____ pairs of sock inside each one. How many pairs of sox are inside all of those packages on the shelf?

(3, 9, 3, 6) (5,9,7,4) (10,8,5,4)

Answer with label: ____________________________________

Write a complete sentence to explain your answer

____

Imagine that you are working at the factory that sorts individual sox into pairs of sox, then places them on the small hangers to then ship to stores to be sold to customers. If you have ____ sox in front of you, how many pairs of sox can you place onto the plastic hangers?

(32) (54) (136)

Answer with label: ____________________________________

Write a complete sentence to explain your answer:

____

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P17

Name: ________________________ **Date: ____________________**

Hot dogs are sold 8 to a package. Hot dog buns are sold 10 to a package. This makes having everything come out even hard to do. If I bought ____ packages of hot dogs and ___ packages of buns, would I have more hot dogs or buns? And how many extra would I have of the one more than the other?

(8, 6) (12, 10)

Answer with label: **__________________________________**

Write a complete sentence to explain your answer:

____

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P18

Name: ________________________ Date: ____________________

Teachers use more blue ink pens than red ink pens over the course of the school year. Blue pens are sold with 10 pens in each box; red pens come 8 to a box. In the supply room down by the office, there is a drawer that has ___ boxes of blue pens and ____ boxes of red pens. Are there more individual blue pens then red, or the opposite? How many more of the one color pen is there than the other pen?

(5, 6) (7, 8)

Answer with label: __________________________________

Write a complete sentence to explain your answer:

____

____

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P19

Name: ___________________ Date: ____________________

There are ____ markers to be placed into 4 baskets so that there are the same number of markers in each basket. How many markers, therefore, will be in each basket?

(32) (48) (56)

***Answer with label:* __________________________________**

Write a complete sentence to explain your answer

____

There are ____ markers to be placed into cups. If ____ markers go into each cup, how many cups will be needed for all those markers?

(27, 3) (48, 6) (56, 8)

***Answer with label:* __________________________________**

Write a complete sentence to explain your answer:

____

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P20

Name: ___________________ Date: ____________________

The bakery at the grocery store sells packages of sugar cookies with 10 cookies inside each package. How many individual cookies are there if I opened up ____ packages and put those cookies on a tray?

(13) (22) (45)

Answer with label: ___________________________________

Write a complete sentence to explain your answer

The baker has a tray filled with ____ chocolate chocolate-chip cookies. If ten cookies go into a box, how many full boxes can be placed out on the shelf for sale?

(132) (431) (1256)

Answer with label: ___________________________________

Write a complete sentence to explain your answer:

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P21

Name: ___________________ Date: ____________________

Jars of tomato sauce come in cartons with 10 jars inside one carton. If a grocery store receives a delivery of ___ cartons, how many individual jars of tomato sauce does the store now have to sell?

(16) (45) (123)

Answer with label: ___________________________________

Write a complete sentence to explain your answer

____

At the bakery where they make fortune cookies to give away as a treat at restaurants, there are ____ individually wrapped fortune cookies. One hundred fortune cookies fill a carton that is then sealed and ready for shipment to restaurants. How many cartons can be filled with that many fortune cookies?

(512) (1014) (2604)

Answer with label: ___________________________________

Write a complete sentence to explain your answer:

____

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P22

Name: ___________________ Date: ____________________

Ten cookies are inside each box. Ten boxes fill up a carton. How many individual cookies would be in 16 cartons?

Answer with label: ___________________________________

Write a complete sentence to explain your answer

____

How many boxes of cookies can be filled with 4,321 cookies if 10 cookies are inside each box?

Answer with label: ___________________________________

Write a complete sentence to explain your answer:

____

A pair of sox costs $4.99. How much would five pairs of sox cost?

Answer with label: ___________________________________

Write a complete sentence to explain your answer:

____

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P23

Name: ____________________ Date: _____________________

Ahmed had the job today of placing apples into clear plastic bags and tying them shut to get the apples ready for sale. Just like his friend the day before, he started with a big crate of apples. When he was done he had tied shut ___ bags with ____ apples inside each bag. He still had ___ apples in the crate. How many apples were in the crate to begin with? Can he fill any more bags with 10 in each bag? When he is done, what would be the total number of bags that he has filled?

(12 10, 4) (29, 10, 2) (54, 10, 36)

Answer with label: ____________________________________

Write a complete sentence to explain your answer

Tatia has the same job today as her friend had yesterday as well. She took the time to find out ahead of time how many apples are in her crate before she started to work. If she puts ten apples inside each bag, how many full plastic bags will she have filled if she starts with _____ apples in the crate?

(341) (1,114) (2,513)

Answer with label: ____________________________________

Write a complete sentence to explain your answer:

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P24

Name: ___________________ **Date:** ____________________

Fact Combinations

Directions: Use the "groups of language" when reading out loud a fact such as 5 x 3 (Say, "five groups of three")
Use what you know about easier combinations to help figure out the harder combinations. Example: If you know 3 x 3 is 9, then 6 x 3 = 9 + 9 because six threes are three threes plus another group of three threes.

10 x 6 = ___	___ *= 4 x 7*	*6 x 3 =* ___	___ *= 3 x 7*	*3 x 3 =* ___
___ *= 5 x 5*	*4 x 4 =* ___	___ *= 7 x 5*	*6 x 2 =* ___	___ *= 5 x 8*
4 x 6 = ___	___ *= 9 x 2*	*2 x 8 =* ___	___ *= 3 x 8*	*8 x 10 =* ___

Open Number Sentences

Directions: What goes on the line to make the number sentence true? Use the "groups of language" when you read the number sentences to help you find the missing piece of information.

9 x 10 = ___	*9 x 10 = 8 x 10 +* ___
9 x 5 = ___	*9 x 8 = 10 x 8 –* ___
9 x 15 = 9 x ___ *+ 9 x 5*	*5 x 9 =* ___ *x 9 + 2 x 9*
3 x 9 = ___	*5 x 9 =* ___

More – Less

Directions: Write the number that is more or less then the number the question asks.

________, 34,627 _________,
10 less *10 more*

________, 16,076 _________,
1000 less *1000 more*

________, 274,426 _________,
100 less *100 more*

_________, 760,804 _________,
1000 less *1000 more*

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P25

Name: ___________________ **Date:** ____________________

One hundred fortune cookies go in a carton. How many individual fortune cookies would there be in 75 cartons?

Answer with label: ___________________________________

Write a complete sentence to explain your answer

____

A thousand paper clips come in a carton. Inside that carton the paper clips are in smaller boxes with 100 paper clips inside each box. How many smaller boxes of paper clips are there in 5 cartons? How many individual paper clips are in those 5 cartons?

Answer with label: ___________________________________

Write a complete sentence to explain your answer:

____

A pair of sox costs $5.95. How much would seven pairs of sox cost?

Answer with label: ___________________________________

Write a complete sentence to explain your answer:

____

For stages 3-5 - Taken from: Grade 4 - Brickwedde, 2016 #P26

Name: ___________________ Date: ____________________

Fact Combinations

Directions: Use the "groups of language" when reading out loud a fact such as 5 x 3 (Say, "five groups of three")
Use what you know about easier combinations to help figure out the harder combinations. Example: If you know 3 x 3 is 9, then 6 x 3 = 9 + 9 because six threes are three threes plus another group of three threes.

6 x 3 = ___	___ = 3 x 6	3 x 12 = ___	___ = 12 x 3	10 x 10 = ___
___ = 5 x 5	4 x 4 = ___	___ = 3 x 3	12 x 1 = ___	___ = 0 x 5
3 x 7 = ___	___ = 7 x 3	4 x 8 = ___	___ = 7 x 4	3 x 4 = ___

Open Number Sentences

Directions: What goes in the box to make the number sentence true? Use the "groups of language" when you read the number sentences to help you find the missing piece of information.

9 x 8 = ___ x 8 – 1 x 8	9 x 9 = 9 x 10 – 9 x ___	7 x 7 = 7 x ___ + 7 x 5
11 x 6 = ___ x 6 + 1 x 6	12 x 8 = ___ x 8 + 2 x 8	6 x 8 = 5 x 8 + ___ x 8

Order & Compare

Directions: Which one is bigger or are the numbers the same? Use <, >, or = on the line to show your answer.
Remember: x is less then (<) y; x is greater than (>) y.

5,000 + 400,000 + 5 + 70,000 + 10 __________ 400,000 + 70 + 600 + 1,000 + 5 + 60,000

700 + 50,000 + 3,000 + 20 __________ 9,000 + 50,000 + 600 + 4 + 30

Round to the Nearest 10 or 100 or 1,000

Directions: The mathematician's rule for rounding is if the number in the place you are ask to round is 5 (50) (500) or higher, you go up to the next ten (or hundred, or thousand). If the number in the place is you are ask to round is 4 (49) (499) or less, then go back to the ten (or hundred or thousand).

1,403, Round to the nearest 10. ______	8,676, Round to the nearest 10. __________
5,101, Round to the nearest 100. __________	7,468, Round to the nearest 100. __________
4,165, Round to the nearest 1000. __________	6,283. Round to the nearest 1000. __________

Appendix K
Practice Station

Practice Station - This station is to be completed in small groups or working independently without teacher assistance. It can be used for homework if students do not finish. Materials in the practice station should be district developed curriculum that directly correlates with developing multiplicative number sense. Materials are provided in Everyday Math Curriculum for students to use. Materials are provided if needed in the following pages. Materials in Appendix K are taken from Brickwedde Grade 3 Unit 9 (2016).

Taken from: Unit 9 Grade 3 - Brickwedde, 2016

Name: ________________________ **Date:** ____________________

Directions: *Read the story. Fill in the sheet with your answers. Show how you solved the problem.*
The school carnival sells tickets for various games at $.25 a ticket. Tickets are sold in a sheet of 10 tickets. Your task is to organize a sheet that the ticket sellers will use that organizes how much money to ask customers based on the number of ticket sheets they buy.

Number of Sheets	Number of Tickets	Cost of Tickets
1	10	
2		
3		
4		
5		
10		

True/False Questions

Directions: *Read the numbers sentences below. Decide if it is true or false. Write a sentence to say why it is true or why it is false. HINT: It will be useful to use your "groups of language." Example: Read 2 x 3 as "two groups of three."*

8 x 8 = 4 x 8 + 4 x 8 True/False
Why true or why false?

7 X 6 = 3 x 6 + 3 x 6 True/False
Why true or why false?

Taken from: Unit 9 Grade 3 - Brickwedde, 2016

Name: __________________________ **Date:** _____________________

Directions: *Read the story. Fill in the sheet with your answers. Show how you solved the problem.*

The clerk in the ticket office at the movie theater is selling tickets for $5.00 for the matinee shows. The table below helps that person know how much money to ask people to pay. Fill out the chart then answer the question below it.

Number of People	Number of Tickets	Cost of Tickets
1		
2		
3		
5		

How can the ticket seller use this chart to quickly figure out the price for a group of 4 friends and 6 friends? Explain your thinking.

True/False Questions

Directions: *Read the numbers sentences below. Decide if it is true or false. Write a sentence to say why it is true or why it is false. HINT: It will be useful to use your "groups of language." Example: Read 2 x 3 as "two groups of three."*

7 x 8 = 4 x 8 + 4 x 8 True/False

Why true or why false?

8 X 6 = 3 x 6 + 3 x 6 True/False

Why true or why false?

Taken from: Unit 9 Grade 3 - Brickwedde, 2016

Name: ________________________ **Date:** ____________________

Directions: *Read the story. Fill in the sheet with your answers. Show how you solved the problem.*

The coaches of the Little League baseball teams are making plans to order T-shirts for the players on their teams. Each T-shirt costs $2. There are 12 players on each team roster. Fill in the chart below to show how much money will be spent buying T-shirts.

Number of Teams	Number of T-Shirts	Cost of T-Shirts
1		
2		
3		
5		
7		
8		

How can the coaches use this chart to quickly figure out the price for 4 teams and 6 teams? Explain your thinking.

True/False Questions

Directions: *Read the numbers sentences below. Decide if it is true or false. Write a sentence to say why it is true or why it is false. HINT: It will be useful to use your "groups of language." Example: Read 2 x 3 as "two groups of three."*

6 x 8 = 5 x 8 + 1 x 8 True/False

Why true or why false?

7 X 7 = 5x 7 + 3 x 7 True/False

Why true or why false?

Taken from: Unit 9 Grade 3 - Brickwedde, 2016

Name: ________________________ **Date: ____________________**

Fact Combinations

***Directions**: Use the "groups of language" when reading out loud a fact such as 5 x 3 (Say, "five groups of three") Use what you know about easier combinations to help figure out the harder combinations. Example: If you know 3 x 3 is 9, then 6 x 3 = 9 + 9 because six threes are three threes plus another group of three threes.*

What are…

Two twos? ____	*Three threes? ____*	*Four fours? ____*	*Five fives? ___*
4 x 4 = ___	*2 x 2 = ___*	*10 x 10 = ___*	*3 x 3 = ___*
2 x 5 = ___	*4 x 5 = ___*	*8 x 5 = ___*	*5 x 5 = ___*
3 x 5 = ___	*6 x 5 = ___*	*12 x 5 = ___*	*5 x 10 = ___*
3 x 4 = ___	*6 x 4 = ___*	*12 x 4 = ___*	*4 x 10 = ___*
2 x 4 = ___	*4 x 4 = ___*	*8 x 4 = ___*	*10 x 4 = ___*

Open Number Sentences

***Directions:** What goes in the box to make the number sentence true? Use the "groups of language" when you read the number sentences to help you find the missing piece of information.*

6 x 6 = (3 x 6) + (___ x 6)

6 x 6 = (6 x 5) + (6 x ___)

6 x 6 = (___ x 6) + (4 x 6)

___ x 6 = (4 x 6) + (4 x 6)

More – Less

Directions: Write the number that is more or less than the number the question asks.

_________, 5,030 _________, — *10 less*, *10 more*

_________, 90,184 _________, — *1000 less*, *1000 more*

_________, 43,715 _________, — *100 less*, *100 more*

_________, 37,712_________, — *10 less*, *10 more*

Taken from: Unit 9 Grade 3 - Brickwedde, 2016

Name: ________________________ **Date: ____________________**

Fact Combinations

Directions: Use the "groups of language" when reading out loud a fact such as 5 x 3 (Say, "five groups of three")
Use what you know about easier combinations to help figure out the harder combinations. Example: If you know 3 x 3 is 9, then 6 x 3 = 9 + 9 because six threes are three threes plus another group of three threes.

___ = 4 x 4	***3 x 4 = ___***	***___ = 4 x 6***	***3 x 8 = ___***	***___ = 2 x 6***
5 x 2 = ___	***___ = 6 x 3***	***3 x 7 = ___***	***___ = 4 x 3***	***10 x 7 = ___***
___ = 7 x 0	***9 x 2 = ___***	***___ = 10 x 5***	***5 x 5 = ___***	***___ = 8 x 5***

Open Number Sentences

Directions: What goes in the box to make the number sentence true? Use the "groups of language" when you read the number sentences to help you find the missing piece of information.

6 x 6 = 4 x 6 + ___ x 6	***___ x 6 = 6 x 6 + 1 x 6***
___ x 6 = 5 x 6 + 1 x 6	***7 x 6 = ___ x 6 + 3 x 6***
6 x 6 = 3 x 6 + ___ x 6	***7 x 6 = 5 x 6 + ___ x 6***
___ = 6 x 6	***7 x 6 = ___***

More – Less

Directions: Write the number that is more or less than the number the question asks.

_________, 9,789	***_________,***	***_________, 9,789***	***_________***
10 less	***10 more***	***100 less***	***100 more***
________, 50,856	***_________,***	***________, 50,856***	***_________***
100 less	***100 more***	***10 less***	***10 more***

Taken from: Unit 9 Grade 3 - Brickwedde, 2016

Name: ________________________ **Date: ____________________**

Fact Combinations

Directions: Use the "groups of language" when reading out loud a fact such as 5 x 3 (Say, "five groups of three")
Use what you know about easier combinations to help figure out the harder combinations. Example: If you know 3 x 3 is 9, then 6 x 3 = 9 + 9 because six threes are three threes plus another group of three threes.

3 x 3 = ___ ***___ = 6 x 2*** ***5 x 6 = ___*** ***___ = 4 x 3*** ***4 x 6 = ___***

___ = 5 x 4 ***2 x 9 = ___*** ***___ = 3 x 8*** ***9 x 5 = ___*** ***___ = 3 x 9***

6 x 3 = ___ ***___ = 3 x 7*** ***7 x 5 = ___*** ***___ = 10 x 4*** ***4 x 4 = ___***

Open Number Sentences

Directions: What goes in the box to make the number sentence true? Use the "groups of language" when you read the number sentences to help you find the missing piece of information.

___ x 7 = 4 x 7 + 3 x 7 ***6 x ___ = 6 x 5 + 6 x 3***

___ x 7 = 4 x 7 + 4 x 7 ***6 x ___ = 6 x 5 + 6 x 1***

7 x 7 = 5 x 7 + ___ x 7 ***6 x ___ = 6 x 5 + 6 x 2***

7 x 7 = 10 x 7 – ___ x 7 ***6 x ___ = 6 x 5 – 6 x 1***

___ = 7 x 7 ***___ = 6 x 5***

More – Less

Directions: Write the number that is more or less than the number the question asks.

_________, 2,771 _________, ***_________, 2,771 _________***
10 less 10 more ***1000 less 1000 more***

_________, 61,587 _________ ***_________, 61,587 _________***
100 less 100 more ***10 less 10 more***

Taken from: Unit 9 Grade 3 - Brickwedde, 2016

Name: ________________________ **Date: ____________________**

Fact Combinations

***Directions**: Use the "groups of language" when reading out loud a fact such as 5 x 3 (Say, "five groups of three")*

Use what you know about easier combinations to help figure out the harder combinations. Example: If you know 3 x 3 is 9, then 6 x 3 = 9 + 9 because six threes are three threes plus another group of three threes.

10 x 10 = ___	*___ = 4 x 6*	*8 x 3 = ___*	*___ = 4 x 7*	*3 x 4 = ___*
___ = 5 x 5	*3 x 7 = ___*	*___ = 5 x 7*	*6 x 3 = ___*	*___ = 5 x 8*
4 x 4 = ___	*___ = 9 x 2*	*4 x 5 = ___*	*___ = 2 x 8*	*6 x 10 = ___*

Open Number Sentences

***Directions:** What goes in the box to make the number sentence true? Use the "groups of language" when you read the number sentences to help you find the missing piece of information.*

9 x 10 = ___	*9 x 3 = 10 x 3 – ___*
9 x 5 = ___	*9 x 8 = 10 x 8 – ___*
4 x 9 = 4 x ___ – 4 x 1	*7 x 9 = 7 x ___ – 7 x 1*
6 x 9 = 6 x ___ – 6 x 1	*7 x 9 = ___*

More – Less

Directions: Write the number that is more or less then the number the question asks.

_________, 3,409 _________, _________ 4,270 _________

10 less *10 more* *100 less* *100 more*

_________, 13,444 _________, _________, 39,985 _________

100 less *100 more* *1000 less* *1000 more*

Taken from: Unit 9 Grade 3 - Brickwedde, 2016

Name: ________________________ **Date:** ____________________

Fact Combinations

***Directions**: Use the "groups of language" when reading out loud a fact such as 5 x 3 (Say, "five groups of three")*
Use what you know about easier combinations to help figure out the harder combinations. Example: If you know 3 x 3 is 9, then 6 x 3 = 9 + 9 because six threes are three threes plus another group of three threes.

10 x 6 = ___	*___ = 4 x 7*	*6 x 3 = ___*	*___ = 3 x 7*	*3 x 3 = ___*
___ = 5 x 5	*4 x 4 = ___*	*___ = 7 x 5*	*6 x 2 = ___*	*___ = 5 x 8*
4 x 6 = ___	*___ = 9 x 2*	*2 x 8 = ___*	*___ = 3 x 8*	*8 x 10 = ___*

Open Number Sentences

***Directions:** What goes in the box to make the number sentence true? Use the "groups of language" when you read the number sentences to help you find the missing piece of information.*

9 x 10 = ___	*9 x 10 = 8 x 10 + ___*
9 x 5 = ___	*9 x 8 = 10 x 8 – ___*
9 x 15 = 9 x ___ + 9 x 5	*5 x 9 = ___ x 9 + 2 x 9*
3 x 9 = ___	*5 x 9 = ___*

More – Less

***Directions:** Write the number that is more or less then the number the question asks.*

_________, 34,627 _________,
10 less 10 more

_________, 16,076 _________
100 less 100 more

_________, 274,426 _________,
100 less 100 more

_________, 760,804 _________
1000 less 1000 more

Appendix L
Website Resources

1. A website that focuses on cognitively guided instruction research and responding to children's thinking in mathematics which provides resources for teachers to use in the form of curriculum, research, games, other website resources.

http://www.projectmath.net/

2. Resource for subitizing cards for teacher stations. Also is a resource for math games and research.

http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/resourcelibrary.aspx#4

3. A website which has variety of electronic games that students can do independently.

www.gamebaseded.com

Appendix M
Reference List for Project

Anoka-Hennepin Independent School District #11. (2013). *Conceptual Place Value Intervention Guide*: *Assessments and Resources*. Anoka, MN: Author.

Anoka-Hennepin Independent School District #11. (2014). *Anoka Hennepin K-12 Curriculum Unit Plan*. Anoka, MN: Author.

Brickwedde, J. (2012a). *Developing base ten understanding: Working with tens, the difference between numbers, doubling, tripling…, splitting, sharing & scaling up*. Retrieved from http://www.projectmath.net/wp-content/uploads/2017/06/Developing-Base-Ten-Understanding.pdf.

Brickwedde, J. (2012b). *Windows into children's mathematical thinking: Instructional tools focusing on multiplication and division in the intermediate grades.* Retrieved from http://www.projectmath.net/professional-development-resources.html.

Brickwedde, J. (2016). *Curriculum Units*. Retrieved from http://www.projectmath.net/publications/curriculum-units/.

Carpenter, T. P., Fennema, E., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic & algebra in elementary school.* Portsmouth, NH: Heinemann.

State Government of Victoria. (2012). *Resource Library*. Retrieved from http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/resourcelibrary.aspx#4

Wright, R. J., Stanger, G., Stafford, A. K., & Martland, J. (2006). *Teaching number in the classroom with 4-8 year-olds*. Los Angeles, CA: Sage Publications Ltd.