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How Can Inquiry-Based Learning Be Successfully Implemented In The Secondary Mathematics Classroom

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HOW CAN INQUIRY-BASED LEARNING BE SUCCESSFULLY IMPLEMENTED

IN THE SECONDARY MATHEMATICS CLASSROOM

by

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ABSTRACT

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The research question addressed in this project was, how can inquiry-based learning by successfully integrated in the secondary mathematics classroom? It documents one teacher’s creation of a set of inquiry-based curriculum modules intended for use in AP Calculus AB and Precalculus mathematics courses. The curriculum created by the author aims to guide students to self-construct new knowledge through a process of settling doubt within a community of learners; a process known as an inquiry cycle. He analyzes both the intended use and flexibility the curriculum modules, discusses how to measure their effectiveness, and how to adopt different strategies in the case of failure.

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CHAPTER ONE

Introduction

By the time I finished two student teaching placements in 2009, I had spent more than 16 weeks as an observer in mathematics and science classrooms. Now, with 7 years of teaching experience in both subjects, there is one observation I made as a student teacher that I still approach with fresh curiosity: students retain scientific concepts for longer and with better clarity than concepts in mathematics. As a teacher of both subjects, I try to inform my practice from both subject areas. The math teacher inside me becomes jealous of how enjoyable it is to teach through the scientific method and becomes frustrated when it is revealed, later in the year, that my math students have retained few prior techniques. I used the word "techniques" because in the daily reality of teaching mathematics to all levels of students, we math teachers eventually find ourselves teaching techniques instead of concepts. Math teachers tend to demonstrate procedures whereas science teachers show how a truth came to be. That may be an overstatement, but over the years, I would assert the typical mathematics classroom is less experiential, has fewer heated academic debates, and presents less evidence of concepts learned than the science classroom. It is my opinion these qualities, more readily found in the science classroom, are responsible for the better retention of scientific concepts I observe in my students. If you, the reader, accept my assertion that scientific concepts are more easily retained because of the difference in instruction techniques, then wouldn't it be prudent to incorporate more inquiry-based instruction in non-science subject areas? Specifically, *how can inquiry-based learning be successfully implemented in a secondary mathematics classroom?*

My Purpose, Pursuit, and Legacy

I consider myself a lifelong learner and my intention when first beginning my teacher education was to produce a hoard of inspired lifelong learners, ultimately adding some depth to the local culture. As I reflect upon my career, in its adolescence, I do not find my original intentions grandiose, unattainable, or unreasonable, but I feel I have produced fewer students of the quality described above than I expected. I feel the students our district produces are too dependent on procedures prescribed by instructors and lack the drive, curiosity, and grit that are characteristics of lifelong learners. This is of no fault of our students. While none of our teachers would admit to wanting to teach a mathematics course in a purely procedural fashion, in the reality of our daily practices as math teachers it seems there is often a legitimate need to shortcut the time allotted for critical thinking. Sometimes students miss school and need to be caught up. Sometimes the unit ends too quickly and we need to expedite the schedule. Sometimes students are lazy or disengaged for one of a thousand reasons. Eventually, every teacher will connect the dots for a student where the student could have connected them by his or her self, depriving the student of the practice of critical thought. If this pattern persists too long, our students learn to be dependent on the shortcut procedure rather than the inquiry portion of the process. Too often, this is the result.

Maybe I feel this way because the majority of courses I have taught have been 9^{th} grade level and do not regularly have the opportunity to interact with my former students as they become seniors and graduate. Maybe I do not see all the wonderful things my students have achieved after they have left my care. Maybe my perception is wrong and all of my former students are rock stars. I have no data to support any of these claims, but the truth is surely some place in the middle. I would like my perception to be more positive. I would like to feel more confident turning over my students to the world. I would like to feel confident in leaving an annual class of eager young minds. By the time I leave teaching, I want to have contributed to a generation of learners.

How I Learn

When I think of the most profound and influential learning moments from my education – the moments when an enduring understanding was formed – I think about my best teachers and how they engaged me with curiosity. My best teachers did rely on using a trick, game, pre-packaged curriculum module, or web-based content. They made me think more than they made me do. They spent a great deal of time working out a single complex problem. They chose to spend the time to fully investigate. They asked tough questions and did not let up until they had teased out a solution from our ideas. When it came time for mechanics, my best teachers fired through a variety of small, simple, but distinct problems to show how the theory was applied. These are the types of teachers who were most effective in teaching me.

The teachers I remember as greats still taught with a "teacher-centered" pedagogy. In the end, it was not the students who were creating new knowledge for themselves; it was still being dispensed in some form from the front of the class. Even so, I was able to have my private moments of eureka at my desk because my teachers were able to guide me to think my way to a solution before it was demonstrated outright. It was enough for me because, as a math student, I was a "big picture" type student: as long as I had a firm grasp of the main concept, I could flesh out the details. If my teachers were able to guide me to discover for myself the main concept, the sensation of discovery combined with pride of mastering a concept made the learning experience highly profound. Without the space in the conversation for thought, without the room for reflection, if the answer had simply been stated, these experiences would not have been as meaningful.

When I entered college as an undergraduate, working on bachelors of the arts in chemistry and mathematics, the use of inquiry by instructors increased. It was the natural progression of our learning. After a set of scaffolding courses as freshmen and sophomores, the path within each field became choice-driven rather than a linear sequence of prerequisites. We had covered the basics and now had enough base knowledge to pursue topics of our choice in any order. As such, the courses became more student-centered and we were expected to drive our own education by completing research projects – both of an academic and application nature. Up until this point, I was still highly dependent on direction from the instructor and I began to struggle more than in the past. My grades began to suffer. A's and B's were now B's and C's. The absence of an "A" on my report card was entirely new to me. In spite of this, I feel the work I did as a junior and senior was my most profound work as an undergraduate. It was certainly the most engaged I had ever been.

From a teacher's perspective, considering my experience, I wonder which experience is better for students and adult learners. Is it better to build a person's confidence by having them achieve a set of clearly defined goals using prescribed techniques or is it more useful for that student to experience a high level of engagement and discomfort – perhaps at the cost of his or her perception of success? Which is more beneficial in the long term? Which will encourage more lifelong learning? In my experience, whenever I want to learn something new as an adult, I find myself being driven by curiosity rather than being directed by an advisor or authority figure. As an adult lifelong learner, I more often apply the methods used in an inquiry-based classroom than the methods used with direct instruction.

How I Teach Science

So far, my experience as a science teacher is limited. I have taught just four sections of Physical Science at a 9th grade level over the course of two years. Despite my lack of experience and despite the curriculum of each course being new to me each time – the two years I taught were each taught in different buildings – I look back on my work and recognize it as some of my most successful teaching. Teaching science is different than teaching math. It is more relaxed. The curriculum is less precise. There is more time. There are fewer standards to hit. The subject material is more conversational. The truths are more fluid and ever-changing. All this encourages argument, disagreement, investigation, and curiosity. I consider my time spent teaching science as highly successful because by embracing these tenants, I am able to structure higher level thinking and, thus, leave the student with a more enduring understanding of the material.

When I teach science, I present uncertainty alongside evidence. We do labs to provide evidence of a theory. It is common for students to be required to correctly conclude the theory, based on their evidence, through their work instead of having me disclose it to them. We examine uncertainty, discuss error, and decide what to dismiss and what to keep. Sometimes we make mistakes and our results do not lead us to accepted scientific fact, so we have a conversation about how we arrived at our conclusion compared to that of the rest of the world. I find the scientific method, the foundation of teaching and learning in science, has a dramatic effect on the way teaching can proceed in a classroom. As Neil deGrasse Tyson, one of the greatest science communicators of our time, explained recently, the scientific method exists "to minimize the chance you will misinterpret truth… The only purpose of the scientific method is to make sure you are not fooled into thinking something is true that is not or thinking something is not true that is," (Rogan & deGrasse Tyson, 2017).

The acquisition of knowledge through the scientific method, in my classroom, is designed to be experiential and students are encouraged to embrace uncertainty. Evidence is presented, investigated, and created to produce a truth and these truths are often softer, less absolute, than truths in mathematics.

When I think about the strength of scientific vs math truth, there is a difference. In my second year teaching I came across a colleague of mine, a science teacher, who had a large 1-10 number line posted along one side of his classroom labeled, "The Spectrum of Truth". He explained to me he used it debate with his class to determine the strength of evidence behind a scientific concept. I pointed out all scientific disciplines are based on human observations and, thus, are fallible, whereas mathematics can be derived from pure theory. We had a conversation about this, posing that one could order the different scientific disciplines based on how purely they were based on indisputable evidence. On the lower end, we decided things like psychology and sociology should reside, while physics and engineering ranked somewhere near the top of the scale. But we found ourselves comparing the different disciplines to the purest subject area we could think of: mathematics. The purity of mathematics may be its strength when ranking it on my colleague's truth spectrum, but, from a teacher's perspective, it introduces a high level of rigidity in how it can be learned and makes learning by inquiry more challenging. I find teaching science, by contrast, is much more conducive to inquiry-based learning.

How I Teach Math

Most days in my math classroom involve some direct instruction. I rely on it on a daily basis because I find it to be the most efficient manner to present new material. I consider it the easy part of the job – students are trained to learn in this manner and it is also the way I was taught. There is a level of familiarity that is beneficial to getting the job done. Also, due to our district's organization, I have fewer choices about how information is presented, how long I can spend on a topic, and how I can assess my students' skills. With a large number of students being transferred between any of five teachers at the end of each trimester, it is appropriately critical for our department to teach in a similar manner on a similar pacing schedule. This approach often means we have to agree on common instructional strategies and that often means direct instruction. I agree with our department's approach and I believe we do good work together. Still, I try to find less defined areas in our curriculum so I can expand the variety of what my students experience.

In general, I like to minimize time spent in direct instruction so I can give my students something more interesting to do. On any given day, if it is possible to present the material without direct instruction, I try my best to pursue that opportunity. If I use direct instruction during a lesson, after I am finished, I transition from being the focal point of their interest to supporting them in their work. Ideally, I have a meaningful example or investigation for them to pursue. Any sort of independent practice, group work, or student-centered activity provides me the opportunity to interview and informally assess what they know and build relationships. Since I am a "big picture" learner, I tend to lay out the larger concepts early and quickly during a lesson and use additional activities to practice the mechanics or procedures of solving whatever problems are posed that day. When I have the opportunity to teach more creatively, the amount of new content presented is often less than with direct instruction. In most cases, student-centered lessons are used to reinforce an idea rather than to introduce a new one.

Inquiry-based learning does exist in my classroom, albeit less frequent than I would like. If I use and inquiry-based approach, typically the lesson will be veiled as an experiment or investigation in the style of the scientific method. The inquiry lessons I do are usually 1-2 days in length and are rarely successive. I find the lessons to be challenging for both teacher and student. As a teacher, I am never sure how much guidance in enough or too much. Keeping students in an extended state of discomfort is stressful. Students become frustrated. I hear complaints. Students may even lose faith that I know what I'm doing or question whether I'm supporting them appropriately. I think the students' reactions are perfectly appropriate. They are not used to being taught this way and they can't yet see the relief that will eventually come. In the meantime, they are accumulating far more questions than answers. The cycle of frustration feeds back upon itself. Eventually though, the end does come and their experience and evidence is formalized into the language and symbols of mathematics. After the experience, I see my students forming strong opinions on this style of learning. As their coach, I want them to understand they will experience high levels of frustration and relief when they complete an inquiry cycle. My eventual goal with inquiry-based learning is to incorporate enough activities so my students become familiar with the process, hopefully increasing the likelihood they enjoy such activities in the future.

How I Would Like To Teach Math

I would like my math classroom to feel more like my science classroom. I would like to produce curiosity and I would like my students to gain comfort with investigative learning. I want my students to experience the clarity that comes with building and fully understanding a concept. For that to happen; for my teaching style to move forward from where I am, I think it is necessary to make a significant commitment towards inquiry-based learning.

The biggest hurdle I see in doing so is time pressure. In my experience, teaching a concept through inquiry is a less efficient use of time than direct instruction. I think the outcomes are better for my students, but, in my experience, committing to teaching through inquiry takes more time. Even a pedestrian viewing of the Minnesota state math standards will reveal one indisputable truth: there are a lot of standards. Compared with the science standards, for example, the list of math standards is several times the length and each math standard is written with a higher level of specificity than the average science standard (Minnesota Department of Education, 2007, 2009). This introduces a layer of rigidity in what can be done if only considering the reduced time that can be dedicated to each topic. Add to it a requirement that I adhere to district standards and the pacing set by my course-alike professional teams, and time pressure becomes a real challenge.

Another drawback I have noticed is the opportunity for students to discover an untruth and use it, unchecked, as fact. If they discover misleading pattern and it leads to a misconception, I find it extremely difficult to erase what they think they know and replace it with correct theory. I don't take this possibility lightly. If a student leaves my care having learned a false truth, I think that is worse than learning nothing.

Despite the challenges presented by inquiry-based learning, I find the style the most satisfying as a teacher. When it works, it works well and resonates with students. I look at my day to day activities as a science teacher and think it must be possible to include more possibility for a more open-ended approach in math class. I am of the opinion that teaching mathematics and teaching science have sufficient commonalities to be combined. I would like to include more inquiry-based learning in my math classroom, but I need more ideas and safeguards to implement it properly. As such, this topic deserves more investigation and research before I can commit to using inquiry more frequently in my classroom. In this project, I hope to determine the best practices of inquiry-based learning in a secondary mathematics classroom and use these key elements to produce viable inquiry modules that can be inserted into a variety of courses I teach. To develop these modules, I will focus my research on this question: *How can inquiry-based learning be successfully implemented in a secondary mathematics classroom?*

Summary

I hope to answer the question: *How can inquiry-based learning be successfully implemented in a secondary mathematics classroom?* In Chapter One, I described how my experiences as a student and teacher impact how I teach today. I expressed interest in, as well as concerns associated with implementing more inquiry-based learning in my classroom. In Chapter Two, I will define inquiry-based learning, explore best practices, and examine benefits and challenges of implementing inquiry-based learning in a mathematics classroom. In Chapter Three, I will describe the basic theory used to help me develop curriculum modules based on what I learned in the literature review. In Chapter Four I will summarize what I have produced as my project and describe how I intend to implement my curriculum in my professional life.

CHAPTER TWO

Literature Review

Introduction

In Chapter One I discussed what I would like to be as a teacher and how my experiences, both as a student and a teacher, have left me wanting to modify my teaching style, specifically in mathematics, to include more authentic styles of learning in order to benefit my students. I also shared my belief that incorporating more inquiry-based instruction should allow me to achieve my goals as an educator. I realize however, this belief I hold is just that – a belief. If I am to commit more heavily to inquiry-based learning in my practice, I need to feel confident the instructional strategies of inquiry-based learning will actually make a positive difference and benefit my students. Lastly, I have concerns that committing heavily to inquiry-based instruction may have a negative effect on the learning of my students rather than a positive one. More research is needed. I would like to know *how can inquiry-based learning be successfully implemented in a secondary mathematics classroom?*

In this chapter, I first examine the recent movement towards inquiry-based learning and offer a modern definition of inquiry-based learning. I then discuss the key elements of what inquiry-based instruction looks like in a classroom and describe some tools used to measure the degree of inquiry present in a classroom. Next, I describe the role of the teacher and the role of the student and how a teacher can optimize the use of inquiry-based learning, comparing the best practices I find with aspects of scientific inquiry. Finally, I will discuss the potential benefits and challenges associated with implementing inquiry-based learning.

A Call to Action

Reform in mathematics education has been ongoing for decades, more recently called to action by the National Council of Teachers of Mathematics (NCTM) and the National Research Council (NRC). In their respective reports, each organization proposed instructional guidelines for high school mathematics and science education that recommend an increase in student-centered instruction, authentic learning, and higher level thinking (National Council of Teachers of Mathematics (NCTM),1989, 2000; National Research Council, 2000). More specifically, the NCTM has proposed students should ideally learn the value of mathematics, become better problem solvers, improve their confidence, and be able to better communicate and reason mathematically (NCTM, 1989) and most recently advocated the use of inquiry-based instruction to improve mathematics education (NCTM, 2000). As a result, many math educators have begun to slowly reject transmission ideology in favor of a constructivist position (Siegel & Fonzi, 1995). While most modern math teachers would agree a student-centered approach is best, teacher-centered transmission techniques still widely persist, particularly in the mathematics classroom. In 2000, NCTM contended that the typical math lesson is still highly predictable and teacher-centered: the typical lesson starts with a short review of the previous day's work or homework, the teacher will give instruction on a new technique, procedures are modeled, and finally, students are directed towards individual practice (Goos, 2004; NCTM, 2000). In general, teachers know their techniques should be more student-centered. Teachers of mathematics identify a significant gap between the amount of time they would ideally spend using student-centered approaches and the amount of time they actually spend using these techniques (Marshall, Horton, & Switzer, 2009). With the recent directives coming from the NCTM and NRC, there is currently a clear push to facilitate student-centered learning and improve critical thinking amongst our students (Marshall & Horton, 2011).

Inquiry based learning (IBL) and inquiry-based instruction (IBI) techniques align strongly with the recommendations of the NCTM and NRC. The same aspects of teaching and learning being pedaled by these national organizations play to the strengths of IBL. I began researching IBI techniques in 2009 and I was unable to find consensus on what exactly constituted IBI at the time. Since then, however, it is clear to me significant effort has been spent within the research community to offer better descriptions of IBL in order to distinguish it from similar pedagogies like project-based learning, scientific inquiry, and self-regulated learning. There is a lack of a clear definition of IBL or IBI within the early literature and some noise persists. Specifically, the definitions of the pedagogies listed above are increasingly encompassing as one travels back in time through the literature. As such, the next section of this paper will seek to clarify and offer a working definition for IBL and IBI. After defining IBI and IBL, I will list some key elements of an inquiry-based classroom, describe an inquiry cycle, and describe the roles of teacher and students. Additionally, I will present several rubrics used to measure the degree of inquiry in a classroom and, finally, I will discuss the positive effects and challenges of running an inquiry-based classroom.

A Definition of Inquiry.

Inquiry-based learning is not a new concept in education (Harlen, 2013). It has strong roots in the constructivist pedagogies, based most directly on the work of Dewey and Vygotsky. Both of these educational psychologists cited the importance of igniting a student's curiosity and imagination as a vehicle to drive a student to inquire and investigate (Dewey, 1938; Vygotsky, 1978). Dewey sought to ignite curiosity by making education directly applicable to real life. These philosophies lead to the development of project-based learning, where students take on a real problem and solve it through academic means; ideally aligned with the curriculum.

Typically, these projects were student-driven with less direction offered from the instructor than was common at the time (Dewey, 1938). The individual learning side of project-based learning evolved into “self-regulated learning,” where it is left to the individual to discover new knowledge and to decide which learning strategies to pursue (Pajares, 1996; Pape, Bell, & Yetkin, 2003). Vygotsky took up the social aspects of project-based learning. As it pertains to IBL, Vygotsky’s work was important in establishing recommendations to create comfortable learning communities within classrooms and developing social learning cultures in which scaffolding was possible and encouraged amongst peers, rather than using direct transmission from an instructor (Vygotsky, 1978). Once this culture is established in a classroom it can be used to engage in reflective discourse, settling doubt and correcting misunderstandings within the community of learners (Harpaz, 2005). These aspects of curiosity, social learning, and reflective discourse are central pillars of IBL and IBI.

As stated previously, there is no clear consensus definition for inquiry-based learning, but the literature review I have conducted has produced a large number of good descriptors and key elements of an inquiry-based classroom. Many authors have good contributions to make, so I will attempt to define IBL as inclusively as possible. In the most general terms, in an inquiry-based classroom, the active construction of new knowledge is done by the individual through investigative techniques and utilizing the community of learners. Students are motivated by ambiguity, curiosity, and by the resolution of doubt or frustration (Borasi, 1992; Vygotsky, 1978). *IBL is a movement away from authoritarian, teacher-centered techniques* (Adler, 1997) *where an understanding is developed through an iterative process of settling doubt by collecting evidence, openly reflecting upon conclusions, and justifying a position until a consensus is reached* (Borasi, 1992; Dewey, 1938; Marshall, Horton, Smart, & Llewellyn, 2008).

In an inquiry-based lesson, the traditional roles of student and teacher are likely to change. The teacher, who is traditionally thought of as the expert, takes a less central role and new experts appear amongst the student body. To provide scaffolding, the teacher poses a question, then provides an autonomous activity along with resources to support an investigation, but rarely explicitly demands a particular strategy (Pape et al., 2003; Pape & Smith, 2002). The student and teacher both spend more time listening to the members of their learning community (Aulls, Magon, & Shore, 2015). The goal is to have students actively construct personal understandings with an authentic experience of discovery; ultimately improving their math literacy (Siegel & Fonzi, 1995). From an instructional standpoint, the instructor's focus should be more on growing an idea rather than the problem solving strategy or procedure (Goos, 2004). Additionally, the teacher must recognize when a group has gone astray and guide them back to a viable strategy. When an idea within a group has fully formed, it is the teacher's responsibility to recognize the end of the cycle and formalize the group's conclusion, often formalizing conventional terms and symbols associated with the topic (Marshall et al., 2009).

As one can imagine, the implementation of IBL can be challenging and so the next sections will be focused on the essential elements of an inquiry-driven classroom (including a description of an inquiry cycle), the best practices for implementation, and further exploration of the roles of teachers and students. The remaining sections of the paper will discuss the specific benefits and challenges associated with implementing IBI.

Essential Elements of an Inquiry-driven Classroom

As mentioned before, this literature review has uncovered a string of evolving terms to describe IBL. With that comes a variety of authors offering their nuances to the definition. In the scope of this paper it is ultimately left to me to decide which elements are crucial to the IBL process. The following is a list compiled from several authors describing what I have come to believe are the most essential elements of an inquiry-based lesson:

- New knowledge is co-constructed between students through experience, investigative techniques, and academic discourse (Borasi, 1992; Jarrett, 1997; Pape et al., 2003),
- At least some of the curriculum is co-constructed between students and teacher (Aulls et al., 2015),
- An essential question is asked to begin the lesson or unit (Minner, Levy, & Century, 2010; Staples, 2007),
- Students are required to explore a topic using a diverse set of resources or are required to perform experiments (Jarrett, 1997; Minner et al., 2010; Seigel et al., 1998),
- Mathematical and inductive reasoning are used to analyze observations (Blair, 2014; Pape et al., 2003)
- An academic discourse ensues to diversify, exchange, evaluate, and adopt or discard an idea or answer to the question (Cobb, Boufi, & McClain, 1997; Pape et al., 2003; Staples, 2007),
- Students report their findings (Aulls et al., 2015; Minner et al., 2010),
- Through discussion or direct instruction, the instructor formalizes the findings of the class, including introducing conventional notation and definitions (Jarrett, 1997; Minner et al., 2010, Staples, 2007).

Likewise, it important to understand what inquiry is not. Too often, superficial features of IBL pass as reform, but fail to engage students in sustained instances of critical thought (Staples, 2007). Teachers commonly create student-centered activities that are strictly procedural in nature and do not engage students in high level thought. If the lesson does not contain a driving research question, it is not inquiry (Bunterm et al., 2014). Instead, to achieve successful inquiry-based learning, teachers need to approach lesson creation through the lens of their students' curiosity (Borasi, 1992).

As a science teacher, it is easy for me to see the framework of the scientific method in the essential elements listed above. In my science classes, the scientific method is presented as a cycle – one that may need to be repeated before a satisfactory end is reached. Likewise it is possible an inquiry lesson taught in a mathematics classroom may need several iterations before a satisfactory conclusion is achieved. For that reason, IBL and IBI are often presented within the context of an "inquiry cycle." Notably, Staples (2007), Harlen (2013), and Li, Moorman, and Dyjur (2010) have all offered detailed descriptions of stages of a typical inquiry cycle. I have consolidated their descriptions into a single graphic. See Figure 1.

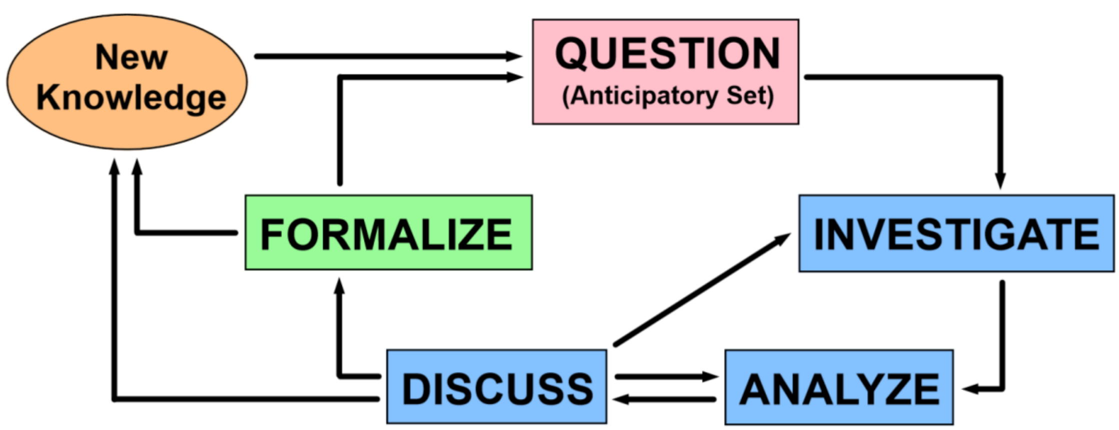

Figure 1. A Typical Inquiry Cycle.

The authors above would agree an inquiry cycle begins by appropriately framing and asking a guiding question. The frame and question should be specific enough to keep students focused and within the scope of the course, but also leave open the opportunity for creative problem solving (Borasi, 1992). The question also must be big enough to encompass all learning objectives and drive curiosity for the entirety of the unit (Smith, 2007). With such emphasis put on the question, I imagine a great deal of thought goes into framing the topic before any specific activities are planned.

Once the question has been posed, the class enters into a mini-loop where social learning is the mechanism by which new knowledge is constructed (Pape & Smith, 2002). Students are provided resources to explore and research the topic and then allowed to gather evidence, identify patterns, and develop a working theory (Li et al., 2010). In this mini-loop of investigation, analyzing, and discussion, students are forced to synthesize and communicate their ideas to the group (Seigel, Borasi, & Fonzi, 1998). At this time, the instructor fulfills the role of advisor – someone with whom students can present their most recent idea. The traditional roles of students and teacher may entirely switch (Aulls et al., 2015). In this stage, students present their ideas to each other and to the teacher. They should be able to scrutinize the ideas of the group with valid mathematical reasoning and by presenting evidence to support or debunk a claim (Cobb et al., 1997). They should argue with each other, articulating their ideas to each other and developing the structure of the underlying mathematics (Cobb et al., 1997; Pape & Smith, 2002). It is during this stage in which the highest level of critical thinking occurs. The level of frustration amongst students will be high because they are attempting to do something they have not done before. They lack the tools of convention traditionally offered up at the onset of a lesson taught using direct instruction. Instead, students discover the limits of their current mathematic literacy, are forced to forge their own solutions, and decide to move ahead with their own conventions (Staples, 2007).

Eventually, the community settles on a solution to the posed question. In some cases, a group may have a fully developed solution and is ready to move on, but it is also likely, through the course of investigation, a solution is incomplete or partially inaccurate. In either case, it is appropriate for the instructor to offer a final summary to clear up any misconceptions and to ensure proper respect has been paid to the conventions of mathematics. During the formalization stage in a mathematics classroom, a teacher may, for example, explicitly state conventional definitions for mathematics terms and/or present symbols and notation used with each topic (Adler, 1997). The formalization stage closes the inquiry cycle and can be viewed as the end of the unit if a new cycle is not to begin. At the end of a cycle, a student's new knowledge has likely been constructed from a combination of investigation, academic discourse, and the formalization stage of the cycle. Staples (2007), Harlen (2013), and Li, Moorman, and Dyjur (2010) have each described an inquiry cycle as a process that can immediately feed into a new cycle. The new knowledge constructed can serve as the basis for a new question, which then begins a new cycle. Harlen (2013) also offers the unique idea of running several shorter cycles to inform a larger, over-arching idea of the course. Using Harlen's technique, I think it would be easy to incorporate aspects of IBL in any classroom or develop a hybrid unit that uses both direct instruction and IBI.

It is apparent, after conducting this portion of this literature review that authors differ in their preferred degree of guidance given to the student during an inquiry cycle. I have more questions. To what level does the teacher provide support? Does the teacher want students to pursue a specific problem? Does the teacher dictate specific strategies? How much background is given? What resources are allowed? How thorough is the instructor's formalization stage? How much is driven by what the student wants compared to what the teacher wants? These are major questions left up to the teacher to decide. Thankfully, there is some guidance available in the literature in the way of measurement tools and suggested best practices.

The next section of this paper will provide tools that measure and rate the degree of inquiry present in a classroom and then provide some suggested roles for both teacher and student. This chapter will conclude with some expected results of implementing an inquiry-based classroom.

Inquiry Rubrics

Since inquiry based instruction is largely subjective, a number of authors have attempted to quantify the level of inquiry present in a classroom. Often, this takes the form of qualitative interview-based research, relying on the instructors to self-report and reflect upon their practices. Kogan and Laursen (2014), Goodnough and Cashion (2006), and Butler (2002) all performed qualitative research based on teacher responses to attempt to determine the effectiveness of IBI and how it affects the attitudes of teachers and students. In doing this research, the authors categorize and count the verbal responses of interviewees under key topics like "student-centered" and "investigation." The qualitative research in the studies above is great for exploratory purposes, but these methods are open-ended and fall short of developing a standardized measurement.

From my perspective, a well-designed rubric could serve as a guide for curriculum development. I would like to stimulate inquiry at a high level but realize the lack of structure associated with an open-ended inquiry probably cannot be sustained in successive cycles. I imagine students would be clamoring for structure. The following rubrics define levels of inquiry, offering varying levels of pre-defined structure to students. Using them will allow me to teach through inquiry as often as possible at varying levels while offering the support my students need at any given stage.

In my review of the literature, I have found three simplistic four-tier rating systems and three extremely detailed rubrics all aimed at assigning a number or rating to the level of inquiry present in an activity. The four-tier rating systems are extremely succinct, but do a good job of distilling the essential elements of an inquiry based classroom in well-organized tables. The detailed rubrics are impractical to apply on a regular basis, but I could see a use for them as an evaluative component of a teacher observation. First, I will present a rubric put forth by Harpaz in 2005:

	problem	means	solution
level 1	given	given	given
level 2	given	given	to be found
level 3	given	to be found	to be found
level 4	to be found	to be found	to be found

Figure 2. A table organizing four levels of inquiry (Harpaz, 2005, p. 18).

Harpaz developed his table so that educators can rate a lesson by quickly answering three yes or no questions. The intention for his rubric is to allow teachers to track their progression from low to high levels of inquiry-based instruction. His hope is that the table is used as a motivational tool to help develop a strong community of thinking (Harpaz, 2005). Of the rubrics to be presented in this section, this is a strong candidate to be used as a reflection tool by a teacher on a daily basis.

Next up, Rezba, Auldridge, and Rhea (1999) offers another four-level rubric:

Inquiry Level	Description and examples
1	**Confirmation**—Students confirm a principle through an activity in which the results are known in advance. "In this investigation you will confirm that the rate of a chemical reaction increases as the temperature of the reacting materials increases. You will use effervescent antacid tablets to verify this principle. Using the following procedure, record the results as indicated, and answer the questions at the end of the activity."
2	**Structure inquiry**—Students investigate a teacher-presented question through a prescribed procedure. "In this investigation you will determine the relationship between temperature and the reaction rate of effervescent antacid tablets and water. You will use effervescent antacid tablets and water of varying temperatures. Using the following procedure, record the results as indicated, and answer the questions at the end of the activity."
3	**Guided inquiry**—Students investigate a teacher-presented question using student designed/selected procedures. "Design an investigation to answer the question: What effect will water temperature have on the rate at which an effervescent antacid tablet will react? Develop each component of the investigation including a hypothesis, procedures, data analysis, and conclusions. Implement your procedure only when it has been approved by your teacher."
4	**Open inquiry**—Students investigate topic-related questions that are student formulated through student designed/selected procedures. "Design an investigation to explore and research a chemistry topic related to the concepts we have been studying during the current unit on chemical reactions. Implement your procedure only when it has been approved by your teacher."

Figure 3. A table defining four levels of inquiry (Rezba, Auldridge, & Rhea 1999, p. 5).

Rezba et al. (1999) developed this rubric for use in science classrooms, but it is equally at home in a mathematics class. All four levels have excellent descriptors, which could replace the numerical designations and each level has a short example of an inquiry activity that fits the level. This is helpful for educators who want a little bit more detail in determining where the lesson currently is on the spectrum of inquiry and what would be necessary to elevate it. As a professional development tool, this rubric could also serve as a quick reference guide for teachers new to IBI.

The last of the simple rubrics is from Marshall and Horton (2011):

Inquiry and Student Thinking

Construct measured	*Pre-inquiry (Level 1)*	*Developing inquiry (Level 2)*	*Proficient inquiry (Level 3)*	*Exemplary inquiry (Level 4)*
Order of Instruction	Teacher explained concepts. Students either did not explore concepts or did so only after explanation.	Teacher asked students to explore concept before receiving explanation. Teacher explained.	Teacher asked students to explore before explanation. Teacher and students explained.	Teacher asked students to explore concept before explanation occurred. Though perhaps prompted by the teacher, students provided the explanation.

Figure 4. A portion of a descriptive rubric detailing inquiry-based instruction (Marshall & Horton, 2011, p. 95)

This rubric is a summarized portion of Marshall, Horton, Smart, and Llewellyn’s 2008 Electronic Quality of Inquiry Protocol (EQUIP) rubric. The complete EQUIP rubric is much more extensive and encompasses all aspects of running an inquiry-based classroom, but this snippet focusses on the roles of teachers and students in an inquiry-based lesson (Marshall et al., 2008; Marshall & Horton, 2011). I see this rubric as a guiding document to assist a teacher while in the midst of a lesson. Determining how much support to offer students is a difficult decision to make and having a resource like this available during the investigation phase of an inquiry cycle may prove useful. If hypothetically, a teacher wanted to use an inquiry cycle that operated with a high level of inquiry, this rubric could offer guidance to the teacher as to what support to offer.

For a more encompassing measure of all aspects of running and inquiry-based lesson, Marshall, Horton, Smart, and Llewellyn (2008) developed the Electronic Quality of Inquiry Protocol (EQUIP) rubric. The EQUIP rubric is designed to be used by a 3rd party observer to the lesson. The rubric breaks down a lesson into 19 assessable standards. These 19 standards are rated on a 3, 4, or 5 point scale, depending on the subcategory, with a higher score indicating a higher level of inquiry. Each standard belongs to one of the following four larger categories: instruction, discourse, assessment, and curriculum choice. See Appendix A for the complete EQUIP rubric.

I think the EQUIP rubric has more grounded expectations of what inquiry can look like in a classroom. Some of the more enthusiastic supporters of IBL can seem idealistic at times. When reading through the descriptors of the highest levels of inquiry on the EQUIP rubric, it is obvious the main goal behind the rubric is to encourage student engagement, proper academic discourse, and high-level thinking whenever possible. An example of this is toned-down version of open inquiry is apparent in the descriptor for “Exemplary Inquiry (4),” where it states, “Teacher occasionally lectured, but students were engaged in investigations.” (Marshall et al. 2008, p. 4). Some of the more idealist authors cited in this paper might argue that, in the highest level of inquiry, there is no direct instruction (Harpaz, 2005; Rezba et al., 1999). I, however, remain encouraged by the EQUIP rubric and will use its descriptors to inform decisions I make as I develop my curriculum project.

Even more detailed than the EQUIP rubric, Shore, Chichekian, Syer, Aulls, and Frederiksen, developed a 79-item Likert scale questionnaire in 2012 to determine the importance of every imaginable aspect of the roles of teachers and students in an inquiry-based classroom. The McGill Strategic Demands of Inquiry Questionnaire (MSDIQ) is designed to be completed by teachers. Each respondent evaluates the importance each of the 79 inquiry-based instructional themes. Each of the 79 items has at least one research citation associated with it. See Appendix B for the complete MSDIQ.

While the MSDIQ has some very thorough descriptions of IBI, it is more adept at determining teachers' attitudes towards inquiry-based learning (Alston, 2017). This would only be useful in determining how compatible IBI techniques are with a teacher. On the other hand, when compared to the other rubrics cited in this paper, the MSDIQ offers the most comprehensive description of the role of students in an inquiry setting. I would be curious to give the MSDIQ to the other members of my course-alike team, but realistically, the only thing for which I intend to use this rubric is to define the role of students.

The last rubric included in this paper is one centered on determining what curriculum to pursue. Originally, Fitzgerald and Byers (2002) created what has come to be known as the "Fitzgerald2002" to help science teachers select curriculum that best develop the ideas of scientific inquiry. As discussed already, there are many similarities between scientific inquiry and what math educators call inquiry-based learning, so using this guide to select or create mathematics curricula is valid. The Fitzgerald2002 dissects a potential activity into four categories. The first category ensures the activity contains accurate content that is aligned with national and state standards while the subsequent three categories ask specific questions to ensure an outcome can be reached using an inquiry cycle. See Appendix C for the complete Fitzgerald2002.

I intend to use the Fitzgerald2002 as a check to ensure what I am asking of my students is achievable through an inquiry cycle. Fitzgerald and Byers (2002) have produced a simple list of yes / no questions I can use as a test against my potential inquiry activities. The second use I have for the Fitzgerald2002 is its rather extensive list of descriptions of ways students can present a solution within an ongoing discourse.

Now that this paper has established a definition of IBL, key elements of IBL, and several means to measure it, it is now time to examine the role of the teacher and the role of the student in an inquiry-based classroom. After that, this chapter will conclude with some expected results of implementing an inquiry-based classroom.

The Role of a Teacher

The role of a teacher is critically important in an inquiry-based classroom. The success of the class is dependent on the ability of a teacher to smoothly transition between facilitator, observer, and direct instructor (Aulls, 2015). To an untrained observer, an inquiry-based classroom may appear as though it is students who are doing the work. However, as the expert in the room, the teacher is responsible for providing the guidance necessary for students to scaffold from one level to the next within the framework of the inquiry activity (Pape et al., 2003; Seigel & Fonzi, 1995).

At the beginning of an inquiry cycle, the teacher outlines the framework and introduces a question. It is the teacher's responsibility to ensure students have sufficient prior knowledge, so some direct instruction may be appropriate to provide basic information at the introduction of an inquiry cycle. I think most teachers would have high confidence implementing this portion of the cycle. Posing the guiding question, however, is not a trivial matter. The purpose of the guiding question is to drive the curiosity of the students, set the boundaries of the exploratory phase, and provide enough detail to focus their efforts on the desired outcome (Fitzgerald, 2002; Seigel & Fonzi, 1995). Effective inquiry-based instruction should provide systematic opportunities for inquiry (Goodnough & Cashion, 2006), and a good guiding question lays the foundation for those opportunities. Seigel and Fonzi (1995) contend a properly formed guiding question is the result of several iterations; before, during and after the creation of the curriculum, and may even be the result of revisions based on previous inquiry cycle experiences. This requires an immense amount of content knowledge as well a clearly thought out pedagogical strategy (Marshall et al., 2008).

Once the question is posed, students enter into the exploratory phase where they will hypothesize, investigate, and begin to formulate a solution. During this phase, the teacher assumes the role of observer and facilitator (Goodnough & Cashion, 2006). Prior to embarking on an inquiry cycle, the teacher has taken time to clearly set the norms of classroom discourse because it is during this phase that the construction of new knowledge is dependent on the communication of ideas (Pape et al., 2003). In this phase of the cycle, it is now the facilitator's job to elicit student responses, seize on a partial thought for discussion, and extend mathematical thinking and mathematical proficiency through discourse (Staples, 2007). The facilitator should ask for clarification, elaboration, and force students to justify a claim (Goos, 2004). He or she will have to carry the conversation back and forth between the specific and the general (Cobb et al., 1997).

While not explicit, the facilitator also has the power to nudge a conversation. He or she can offer confirmation of an idea or minimize incorrect information. At the same time, it is important to withhold unnecessary judgments in an effort to maintain a comfortable social environment (Aulls et al., 2015; Goos, 2004). The facilitator should avoid, as best as possible, wholly feeding a concept to a student. Authentic uncertainty amongst students should be maintained until they have made a conclusion (Goos, 2004). Once a conclusion has been properly established and justified, it is also the job of the facilitator to decide when to move on (Staples, 2007).

If students are struggling, supporting them emotionally can become more important than guiding them through the content. The facilitator should nurture the feeling of pride that comes along with self-efficacy and use it to encourage independence (Blair, 2014). He or she should take time with struggling students to emphasize the importance of any good work the group has done, especially any conceptual understandings the group has developed (Aulls et al., 2015; Seigel et al., 1998; Staples, 2007). If the teacher chooses, students can be directed to focus on more fruitful lines of thought (Staples, 2007). The facilitator should remind students that making mistakes and getting lost is part of the experience and overcoming these challenges will ultimately contribute positively to the experience (Goos, 2004).

The teacher is also responsible for providing the appropriate resources for the investigation phase. Seigel, Fonzi, and Borasi (1998) strongly encourage the use of secondary texts to facilitate inquiry. Without the appropriate resources available, student progress may stall (Seigel & Fonzi, 1995). Keeping in mind this research was done before hand-held internet connected devices, they offer several reasons why additional texts are useful within an inquiry cycle. Secondary texts can be used to challenge an initial conception, generate data, elicit reflections on their work, or texts can serve to verify or disprove a conclusion (Seigel et al., 1998).

From a 2017 perspective, I may not want my students to be able to have free access to the internet during an exploratory phase. As I tell my students, “Math is not a secret. The math we do here is not special. It is not hidden away in a secret part of the internet for only a few to know.” To my point, with the correct search phrase, detailed descriptions of any topic in mathematics is only a few keystrokes (or taps of a finger) away. The accessibility of solutions may spoil the sensation of discovery for my students or, worse yet, may shortcut their high-level thinking. As a teacher, will I allow free use of the internet as a resource? It is likely any question I could think of has already been asked on the internet. Or must my creativity transcend that of the internet? As I work on my curriculum project I will have to make appropriate decisions about whether I choose to ask a common question so they have support or whether I choose to take my students on a journey off the beaten path where they will have less support outside of my classroom.

The last significant responsibility of a teacher during an inquiry cycle is to formalize the concepts constructed by students. At some point, students will be asked to demonstrate their knowledge in a conventional way. As such, it is important for a teacher to discuss mathematical conventions, including agreed upon definitions and symbols or notation used with each concept (Goos, 2004; Li et al., 2010). The end of a cycle can also serve as a reflection or celebration. Ideally, students should feel a sense of accomplishment (Marshall & Horton, 2011) and the teacher should invite ownership of their achievement (Goos, 2004). Finally, the teacher can decide whether or not to use the students' new knowledge to develop a new guiding question, draw a connection to a previous topic, or simply move on to the next item in the curriculum (Harlen, 2013).

The Role of Students

An inquiry-driven classroom is student-centered, so students should have a great deal of choice when IBL methods are used. The core responsibility for a student in an inquiry-based classroom is to investigate a topic and collaborate with their classmates to produce a claim. They should be able to make an appropriate selection from their repertoire of problem solving strategies and engage in collaboration to move their agenda forward (Morrison & McDuffy, 2009; Pape & Smith, 2002). Students must take on more responsibility for their learning as compared with direct instruction techniques (Goodnough & Cashion, 2006). It is a desirable trait for the student to be self-directed (Pape & Smith, 2002; Goodnough & Cashion, 2006), but even unmotivated students should have the opportunity for success if the teacher produces an engaging guiding question (Marshall et al., 2009). In order for a student to be successful in an inquiry-based classroom, he or she must gain comfort with taking some risk and allowing others to make suggestions about their work (Cobb, et al., 1997). Staples (2007) adds that non-algorithmic thinking and the ability to manage frustration are also helpful traits. She also says the more curious a student becomes, the more pursuant they will be towards a solution.

During discourse, students should offer conjecture, justification, and initiate argument (Cobb et al., 1997; Goos, 2004). They should seize the chance to critique each other's work, ask for clarification, locate errors, or suggest more research is necessary (Goos, 2004). Students should know a question met with a question is not a dismissal of their work, but an invitation to proceed to the next step (Staples, 2007). With some practice, students should be able to articulate questions that drive inquiry (Harlen, 2013). Some of these skills will come naturally to students and, with some patience, all of them are coachable (Staples, 2007). With these strategies in place, the community of learners should become self-regulating, improving the chances of success and greatly reducing the chances of incorrect information being learned.

If student and teacher are able to work together, both adhering to these roles, the community of learners Vygotsky (1978) talked about should take root and allow inquiry to flourish. I cannot imagine a successful community such as the one described above appears on the first day of implementing inquiry. On the first attempt, both parties will have to be patient with each other as they learn their roles and learn what works and what does not. I imagine both teacher and student have a lot to learn. Goodnough and Cashion (2006) suggest implementing IBI in phases of decreasing structure (as defined by the rubrics) to minimize the growing pains.

As I get closer to implementing IBI in my classroom, I need to know if these new techniques will result in more student success. In the last sections of this paper, I examine the challenges and benefits associated with implementing IBI in a classroom.

Measuring the Effects of Inquiry-based Learning

When properly implemented, the hope is that IBI is more effective than direct instruction learning. NCTM (2000) encourages a certain level of inquiry, but always seems to connect inquiry to student engagement, implying perhaps the latter is more critical. So what are the actual outcomes of implementing a high level of inquiry in a mathematics classroom? It is possible to find quantitative studies that show both an improvement and no effect on student performance, but most of the studies I have found are qualitative. In a typical study investigating the outcomes of implementing inquiry, teachers and students are interviewed or given questionnaires about their experience. The literature is rich with studies like this, producing a large record of examinations of the attitudes and experiences of teachers and students. In this section I will examine both quantitative and qualitative studies that report on student achievement and student attitudes after experiencing inquiry. I will also present some studies that were ambivalent on the subject and I will discuss some of the areas of IBI where things can go wrong.

Quantitatively, the literature is very weak on the subject. Disappointingly, the largest, most inclusive studies I found also held more neutral/negative attitudes towards IBL. With that caveat given, I found two large scale quantitative examinations of implementing inquiry in a mathematics classroom worth writing about. Minner, Levy, and Century (2010) performed a meta study of 138 small scale studies of how inquiry effects student achievement. Disappointingly, they found that in just over half of the studies, using inquiry had a positive effect on student achievement. Their improvement in student performance they found was statistically significant, but they noted the difference observed between inquiry and non-inquiry groups was not substantial. The National Mathematics Advisory Panel (NMAP) conducted a similar study and again found very little difference in the achievement of inquiry and non-inquiry groups. They found so little difference, in fact, that their determination was that IBL is not a critical component to student success, even though they were well aware of NCTM's recommendations (National Mathematics Advisory Panel, 2008). While neither of these studies were a glowing review of IBL, they also showed IBL did not typically have a negative effect on student achievement. Both studies concluded with neutral views of inquiry.

The studies conducted by Minner, Levy, and Century and the NMAP might discourage a teacher from taking up an inquiry-based pedagogy, but both studies were also unable to show a negative effect. It may be a case of "do not harm." For example, Pajares and Miller (1997) found that students with high self-regulation skills – students who thrive under inquiry conditions – tend to be successful independent of whether they are or are not in an inquiry-based classroom.

There are studies much more enthusiastic about the implementation of IBL. One of the core aspects of IBL is that students carry more of the responsibility for learning than with direct instruction. When students complete and inquiry experience, the most commonly cited result of having increased responsibility is that students feel a sense of pride and a sense of ownership of the material (Goos, 2004; Morrison & McDuffy 2009; Zafra-Gómez, Román-Martínez, & Gómez-Miranda, 2015). This tends to contribute in a positive way to their overall attitude towards math. Generally, students learning through inquiry view the act of doing mathematics more favorably than students taught through direct instruction (Jarrett, 1997). Students report more enjoyment and satisfaction through inquiry, even amongst poorly performing students (Bunterm et al., 2014; Harlen, 2013; Zafra-Gómez et al., 2015). Results like these have a cumulative and sustained effect on student attitudes even when they leave and inquiry-based setting (Kogan & Laursen, 2014). The increased enjoyment leads to a substantial increase in students' confidence levels and increases student engagement (Pajares, 1996). The increased confidence and enjoyment creates a feedback loop. Students take up topics they otherwise would not. They debate things and insist on knowing why something is the way it is. Students start to view math as a fluid subject open for discussion (Goos, 2004). IBL instills curiosity early and it does not leave quickly (Harlen, 2013). Developing curiosity also develops creativity. Seigel, Borasi, and Fonzi had a student comment, "I could change the rules of math and explore the results," (Seigel, Borasi, Fonzi, 1998, p. 407). In general, the impression I received from this review of literature was that student response to IBL is overwhelmingly positive. As most of the authors of these studies are also advocates for IBL, I imagine the anecdotal responses they reported were perhaps biased, but there is no doubt the overall response from students is positive.

In addition to improving attitudes, IBL also improves the skills of students. With students spending a great deal of their time at high cognitive levels (Marshall & Horton, 2011), students tend to have a more thorough understanding of major concepts, they are able to explain concepts more thoroughly, and they tend to retain a high level of understanding for longer periods of time (Goos, 2004; Kogan & Laursen, 2014; Morrison & McDuffy 2009). Students learn how to construct personally effect strategies (Butler, 2002). More importantly, students tend to hold on to their problem solving strategies, making them more successful in the future (Kogan & Laursen, 2014).

While the quantitative side of the literature is mixed in concluding whether or not IBL has any positive effect on student performance, it is possible to find research that indicates IBL improves student achievement. Specifically, Richmond et al. (2015) noted performance improvement amongst low level students, particularly on high-level assessments. On the other end of the spectrum of students, Jones and Byrnes (2015) found students with strong self-regulation skills showed strong improvements in performance under inquiry conditions. In one of the more convincing studies I read, Zafra-Gomez, Román-Martínez, and Gómez-Miranda (2015) tracked an inquiry-based group and a non-inquiry based group of students through high school. When they examined the GPA and standardized test performance of the two groups, they observed a significant improvement in the inquiry-based group.

Given all this evidence, I personally believe it is likely that implementing inquiry will have a strong positive effect on student performance in my classroom. There are, of course, opportunities to make missteps. I view IBL as higher-risk pedagogy than direct instruction. I feel there is greater chance for success and a greater chance for failure with less middle ground. In this next section, I hope to address some of the possible pitfalls of IBL.

Challenges of the Inquiry-based Classroom

Success of an inquiry-based lesson is highly dependent on the student. It is largely up to the student to find his or her way. This can lead to assumptions being made without proper social vetting, which can lead to misinformation being adopted by the individual or group (Jarrett, 1997; Marshall et al., 2009). To combat this, Cobb et al. (1997) emphasize the importance observation by the teacher and creating a comfortable social environment.

Constructing new knowledge socially may be difficult for students new to inquiry because they will likely have under-developed social discourse skills (Staples, 2007). Commonly, pride gets in the way of asking a clarifying question or putting up a claim for debate (Goodnough & Cashion, 2006).

The sensation of anxiety is common amongst students participating in inquiry. Many of the responses given by students reveal the inquiry cycle was highly stressful for students (Pajares, 1996; Zafra-Gomez et al., 2015). The sensation of anxiety was the most common reason students gave when they had a negative experience (Marshall et al., 2008; Zafra-Gomez et al., 2015). Staples (2007) recommends coaching students so they understand feeling stressed is normal. To be fair, some students may be out of their depth and experience too much anxiety over the confusion they feel. As such, it is extremely important that the instructor make deliberate decisions about what a student is ready for and must spend time to determine what the level of rigor the activity should have (Shore et al., 2012). If a student shuts down due to the stress, it is fair to give them a break or offer them some tutoring. It is a struggle to sustain inquiry beyond a comfort point (Pape et al., 2003; Vygostsky, 1978). In my view, there is no harm in taking a break to perform some damage control.

Lastly, when students report negatively about their experience, they often make remarks about the lack of clarity in the activity. The NMAP railed on this in their report in 2008. They found high levels of confusion associated with IBL to be a strong predictor of poor performance. Pajares and Miller (1997) also stated that if an assessment or activity is too open-ended, the frustration and confusion can envelop any gains that might have otherwise been made.

In my view, the shortcomings of IBL can be overcome with a greater dedication to the craft. It is clear not all teachers are comfortable with teaching through inquiry. Not every member of a course-alike team may support IBI pedagogy. There is a built-in reluctance and unfamiliarity amongst teachers because teachers have less experience teaching with IBI and were likely not taught using student-centered techniques when they were in school (Marshall et al., 2008). Even when a teacher or a department adopts IBI pedagogy, the change may only be skin deep (Watts, 2005). To avoid many of the pitfalls listed above, I think it is important for a teacher to wholly embrace the idea and, especially on their first attempt, make every effort to have a plan for every outcome. It is natural for any professional to experience difficulty using a new technique and teachers are no different. I am convinced the mistakes I see above in the implementation of IBI can be addressed with further experience and reflective practice and, ideally, in combination with professional development.

Summary

There has been a call for incorporating more inquiry-based learning in mathematics classrooms by national organizations. Inquiry-based learning is defined as *a process in which an understanding is developed through an iterative process of settling doubt by collecting evidence, openly reflecting upon conclusions, and justifying a position until a consensus is reached*. This chapter produced a comprehensive list of the essential elements and provided several measurement tools capable of determining what level of inquiry is present in a classroom. In general, higher levels of inquiry require less guidance from the teacher during the investigation stage and rely more heavily on the skills of students in order to construct new knowledge. The roles of students and teachers were also examined in this chapter.

The literature review found there to be many potential benefits and challenges that could arise from implementing IBI. The research indicates the most consistent benefits to IBL include improvement of student attitudes, long term retention of content, superior depth of content knowledge, and improved student engagement. It was also shown that IBI had a small but positive effect on student achievement. These gains come alongside some risks, including high levels of stress and confusion amongst students. It was also noted that most teachers are not currently highly proficient in using IBI.

As a math and science teacher, I am always looking for ways to improve my practice. My original goal at the beginning of this project was to bring my math students up to the level of understanding and retention that I saw in my science students. After conducting this literature review, I am of the opinion that including more inquiry in my classroom has potential to significantly increase my students' understanding of mathematics. I am particularly optimistic that incorporating more inquiry will lead to better retention of concepts for longer periods of time. However, I remain skeptical that all inquiry all the time is the answer. I think, as with most techniques in teaching, a teacher needs to use discretion to determine the best techniques to use in a given situation.

In Chapter Three, I will provide a detailed description of the curriculum modules I will produce in Chapter Four. Chapter Three will show how the theory behind IBL, as described in the literature review, can be applied to secondary mathematics curriculum.

CHAPTER THREE

Project Description

My research question for this capstone project is *how can inquiry-based learning be successfully implemented in a secondary mathematics classroom?* In Chapter Three, I will provide descriptions of inquiry-based curriculum modules that should be exemplary answers to my question. I will show how each module is representative of IBI. Also in the chapter are descriptions of the intended audience, how my course-alike teams intend to use the modules, and a timeline for how to implement higher levels of inquiry. The chapter concludes with a description of the workbook format and information about how to access the materials. *My project consists of eight inquiry-based curriculum modules for the AP Calculus AB and Precalculus courses taught at my school*.

Audience

This upcoming academic year, I will be responsible for preparing 65 Calculus students for an AP Calculus AB exam in May. This is first time I have taught the course, so I have the opportunity to set a sound foundation of engaging curriculum for use in subsequent years. I have also taught Precalculus before and foresee myself teaching it again, so I have also used this project to enhance a unit of the Precalculus curriculum. Students enrolled in AP Calculus AB are typically in their third or fourth year of high school mathematics and were advanced ahead one or two years before they started 7^{th} grade based on their performance on an aptitude test taken in 6^{th} grade. These students have achieved success in challenging advance courses since they entered high school. By the time they reach their senior year, their numbers have dwindled. For example, this year our school taught eight sections of HP Geometry (the same group of students who are currently on track to enroll in AP Calculus as seniors), but next fall, we will only teach five sections of AP Calculus AB or BC. For this reason, AP Calculus students can be considered some of the best mathematics students our school has to offer. It should also be noted that AP Calculus students are taking their fourth year of high school math and only three years of mathematics are required for graduation. They take their work seriously. They are competitive, take their GPA seriously, and most take the course with the intention of earning college credit. My hope is, with this group, I will have the chance to use IBI to extend their knowledge after the AP exam in May.

The typical class of Precalculus students at my school is a 90/10 mix of mostly seniors with some juniors. The seniors in Precalculus are most likely students who took math as an elective their senior year or juniors who dropped out of the high performance track, but still chose to take math for a fourth year. Generally, their goals are to prepare themselves for a calculus course or to maintain their algebra skills for college. The skills and motivation in this group are less than that of the typical AP Calculus class. Also, a few students succumb to a bad case of "senioritis." The trigonometry module I have produced can replace some of the less interesting curriculum in the Precalculus program, increase student engagement, and provide a better base knowledge of trigonometry for these students prior to a college level calculus course.

I have also kept in mind my professional teams. I have included both of my colleagues on my Precalculus and Calculus teams as content reviewers for this paper. While completing this project, I have been corresponding with each of them so as to better meet our curriculum needs. The suggestions I received were helpful and I am encouraged by their willingness to experiment with new instructional techniques. That being said, I am certain not all of the curriculum modules I have produced will be used. I imagine some will be discarded and some will be adapted as we see fit. Within the course of teaching in my classroom, I have the freedom to go rogue and teach with as much inquiry as possible but I think a balanced approach is better. From a realist point of view, I am content in knowing my teams are thinking in new and different ways about instruction and are open to the idea of IBL.

Curriculum Rationale and Implementation

While developing the curriculum modules, the goal was to produce activities that score highly on the rubrics presented in Chapter Two. For the most part, they promote a high level of inquiry. Most of the rubrics from Chapter 2 described four levels of inquiry (Rezba, et al., 1999; Marshall & Horton, 2011). The activities I designed operate on an inquiry level of two, three, or four as measured by the rubrics cited in this paper. I did not design any level one inquiry activities because I found them to be substandard within the scope of the project's intentions. I was able to write two inquiry activities that would satisfy a rating of a four. All of my modules contain the majority of the essential elements of an inquiry-based classroom outlined in Chapter Two of this paper:

- New knowledge is co-constructed between students through experience, investigative techniques, and academic discourse (Borasi, 1992; Jarrett, 1997; Pape et al., 2003),
- At least some of the curriculum is co-constructed between students and between students and teacher (Aulls et al., 2015),
- An essential question is asked to begin the lesson or unit (Minner, Levy, & Century, 2010; Staples, 2007),
- Students are required to explore a topic using a diverse set of resources or are required to perform experiments (Jarrett, 1997; Minner et al., 2010; Seigel et al., 1998),

- Mathematical and inductive reasoning are used to analyze observations (Blair, 2014; Pape et al., 2003)
- An academic discourse ensues to diversify, exchange, evaluate, and adopt or discard an idea or answer to the question (Cobb, Boufi, & McClain, 1997; Pape et al., 2003; Staples, 2007),
- Students report their findings (Aulls et al., 2015; Minner et al., 2010),
- Through discussion or direct instruction, the instructor formalizes the findings of the class, including introducing conventional notation and definitions (Jarrett, 1997; Minner et al., 2010, Staples, 2007).

To assure each of the preceding elements became a part of the activity, I created a lesson plan template that explicitly breaks a lesson down into the stages of an inquiry cycle. This lesson plan template can be found in Appendix D and a completed lesson plan of and IBL activity can be found in Appendix E. The lesson plan template uses the basic structure of Understanding by Design (UBD) and adds to it two features: a quick reference of inquiry level near the title of the lesson and the learning sequence is described based on the structure of an inquiry cycle (Wiggins & McTighe, 2005). By including these two features, it becomes difficult for an instructor to neglect any portion of the lesson.

I also chose not to bombard my students with the highest levels of inquiry all the time. My natural tendency as a teacher is to teach in an investigatory manner, commonly registering as level one and level two by most rubrics, so my natural state is helpful preparation for my students. In order to get achieve a high level of inquiry, I need to first train my students by progressing them through lower levels of inquiry-based learning. Aspects of the classroom like setting up a comfortable environment for discourse and experimenting with highly structured inquiry can be practiced in my classroom on a daily or weekly basis, even if I am not using one of my official inquiry modules. Smith (2015) offers a realistic design for implementation fo IBL over time in her capstone. See Figure 5.

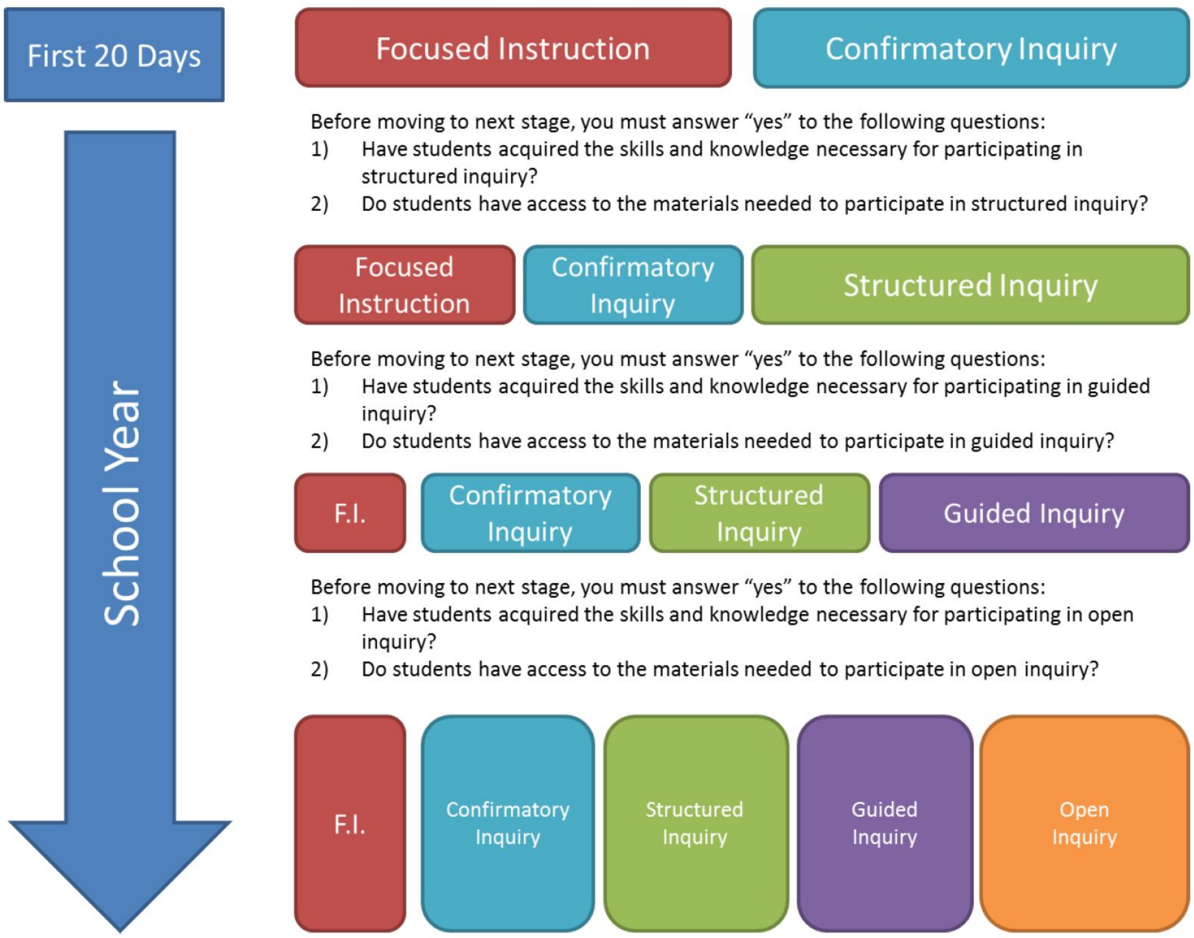

Figure 5. A model for implementing various levels of inquiry (Smith, 2015, p. 45)

I think the timeline of this progression is a little forced, but I assert students can already operate in a confirmatory inquiry manner without guidance beyond what a science teacher would give prior to a laboratory experiment. I predict the most effort will have to be spent moving the students from structured inquiry to guided or open inquiry. Given that my activities range in levels from two to four, it should be possible to organize the implementation of each activity so as to gradually decrease the amount of structure provided to students.

Content

The actual student inquiry activities are formatted in a workbook style. In essence, each activity is a short packet in which students are directed to participate in all the phases of an inquiry cycle. A typical packet is structured with a series of high-level guiding questions that, on their own, are probably beyond the capabilities of any one student but are approachable within the context of group learning. The questions in each guide were chosen with great care in order to guide a student's thinking process without giving too much away. Each packet directs them to write theorems and descriptions similar to what would appear in a textbook. Ideally they are constructing their knowledge and writing their own textbook. To combat the adoption of a misunderstanding, there are several "Critical Checkpoints" where a student is not allowed to move on until they have shared their work publicly with a teacher or other group. Another purpose of these checkpoints is to begin the transition from consensus to formalization.

Each curriculum module I produced is a self-contained inquiry cycle so each can be inserted during relevant areas in the established curriculum. My hope is these modules are more usable if it can be inserted into an existing pacing schedule without having to consider a huge amount of prerequisite knowledge. Based on a typical syllabus for AP Calculus AB, I am confident some of the modules I have produced could be presented to students at multiple points within the year. See Appendix F for an example AP Calculus AB syllabus. With others, there is a narrow window in which they will be effective because of the scaffolding nature of mathematics. On the onset of developing these modules, I was pessimistic that I would be able to connect one inquiry cycle to another, but the majority of my project ended up being a five-cycle long exploration of integral calculus. Here are the topics addressed:

AP Calculus AB modules (in suggested sequential order)

- The Power Rule: Students differentiate polynomials using the definition of the derivative in order to construct the general rule,
- The Area Problem Part I: Students are asked to find the area under various functions on their way towards constructing the idea of Riemann sums,
- The Area Problem Part II: Students are tasked with using software to examine and improve the accuracy of Riemann sum calculations,
- The Area Problem Part III: Students develop a method for a Riemann sum of discs as a way to find the volume of a rotational solid,
- The Area Problem Part IV: The exact area under a curve is connected to the antiderivative. Students construct The Fundamental Theorem of Calculus,
- The Area Problem Part V: Students try to break The Fundamental Theorem of Calculus. In doing so, they determine under what conditions it holds and under what conditions it fails,
- Polar Calculus: Near the end of the year, students are tasked with translating major one-variable calculus concepts into polar notation.

Precalculus module

- Next Level Trig: Students construct their first sine and cosine graphs. Students construct a complete unit circle for the first time.

Project Format

The purpose of this project was to create shareable materials for my entire team. As this capstone will also be published in the Hamline Digital Commons, it is reasonable to simply make all the materials I have created public. As such, I have created a public Google Drive folder. See Appendix G for the URL. With this capstone being published in the Hamline Digital

Commons and by providing my materials as open source, this small offering should have its best chance contribute in a larger sense to the educational community.

Inside the folder are the eight inquiry modules, their respective lesson plans, and all supplemental materials. Each inquiry module is available in *.pdf and *.docx format so a user can choose to edit the document or use as is. I included the *.pdf files in the event that the intended formatting is not maintained. Any physical artifacts for the curriculum are not able to be shared, but will be available during the final presentation. For the final presentation itself, I imagine I will use a guiding digital presentation (Google Slides) to present the implementation strategy, lesson plan templates, and a brief description of the content.

Summary

The eight curriculum modules I have produced for AP Calculus AB and Precalculus incorporate a high level of inquiry, including most of the essential elements outlined in Chapter Two. My modules include enough flexibility to allow me to train my students for higher levels of inquiry and I can choose to use or dismiss the modules, depending on the observed needs of my students. Yet, as this curriculum is new, I have only vague confidence IBI will prove to be a more effective instructional strategy. Since the modules can be added in at any time and there is an established curriculum already, my teams and I will have to evaluate the activities in real time to see if they have been effective. Specifically regarding AP Calculus AB, I will be highly dependent on the observations of my team members to assess the lessons' effectiveness because I have not taught the course before.

CHAPTER FOUR

Project Summary

Introduction

At the beginning of this capstone, I tasked myself with answering this question: *How can inquiry-based learning be successfully implemented in a secondary mathematics classroom?* I chose to pursue this question because the instruction I have given in the short time I have spent as a science teacher seemed more effective the instructional strategies I was using in my mathematics classroom. In Chapter One, I reflected on my experiences as a teacher and how I learned most effectively as a student. These reflections lead me to investigate inquiry-based learning. By attempting to answer my research question, I was confident I would develop some specific goals to be implemented in my future curriculum.

The literature review in Chapter Two produced the framework of how to successfully implement IBI in secondary mathematics classrooms and clearly defined the roles of teacher and student in an IBL classroom. Additionally, the literature review produced several measurement tools that proved to be instrumental in the creation of my curriculum project. The literature review did not, however, produce much in the way of specific inquiry activities appropriate for use in the higher levels of secondary mathematics. After identifying this need, I chose to create and share a series of workbook activities all based on the framework of IBL activities designed for upper level secondary mathematics. I also produced a lesson plan template along with completed lesson plans and supplemental materials for each activity.

Revisiting the Literature Review

I was happy to see in the literature calls to action from national organizations like NCTM and the NRC to include more inquiry-based learning (NCTM),1989, 2000; National Research

Council, 2000). When I started this project, I was not certain incorporating inquiry instructional strategies in secondary mathematics was a good or valid approach. As I uncovered more articles, the review gave me confidence that resources for implementing IBL were out there and supported by expert math teachers. The next step was to see if there was an agreed upon definition or pedagogy for IBI. I found the research to be slightly divergent on this topic, but there were a handful of authors who wrote about inquiry in similar terms. By combining their work, I decided to work with the following definition: *IBL is a movement away from authoritarian, teacher-centered techniques* (Adler, 1997) *where an understanding is developed through an iterative process of settling doubt by collecting evidence, openly reflecting upon conclusions, and justifying a position until a consensus is reached* (Borasi, 1992; Dewey, 1938; Marshall, Horton, Smart, & Llewellyn, 2008). It was refreshing to see these authors advocate for learning mathematics in a way I knew was possible, but was never sufficiently addressed in my teacher education.

In a similar way, most authors in the literature review of this capstone served as validation to my pedagogical beliefs, but it was authors Like Cobb, Boufi, and McClain (1997) along with Pape and Smith (2002) who invited me to a new level of instruction. These authors' focus was developing meaningful classroom discourse within a mathematics classroom. The description Cobb et al. (1997) offered of proper classroom discourse was on point towards developing critical thinking about mathematics and was ultimately the most influential source in deciding the format my project would take. After reading Cobb et al. (1997), it was clear to me the learning I was asking my students to do had to be done on an individual basis, but within a supportive group and classroom. As a result, I developed the workbooks with a series of high-level guiding questions that, on their own, are probably beyond the capabilities of any one student but are approachable with the support of a group.

The literature review also produced a learning process referred to as an inquiry cycle. Notably, Staples (2007), Harlen (2013), and Li, Moorman, and Dyjur (2010) all contributed to the structure of an inquiry cycle I used which included phases of questioning, investigation, analyzing, discussion, and formalization. Each lesson plan I produced for the project has a learning sequence modeled after the structure of an inquiry cycle as described above.

The next major influence the literature review had on my project was the rubrics created by some of the authors. The rubrics served to keep me well within the realms of IBI while writing a lesson. The rubrics listed in Appendices A through C provided excellent descriptions of the roles of teachers and students as well as descriptions of good inquiry lessons (Marshall et al., 2008; Shore, Chichekian, Syer, Aulls, & Frederiksen, 2012; Fitzgerald & Byers, 2002). While these descriptions were detailed and all-encompassing, I found myself gravitating towards the simpler rubrics by Harpaz (2005), Rezba, Auldridge, and Rhea (1999), and Marshall and Horton (2011) that gave simple descriptors of the level of autonomy expected at each level. These shorthand descriptors of the role of teacher and student were instrumental in keeping the design of my project focused and standardized. Since all these authors defined the level of inquiry in a lesson using four levels, I adopted a similar designation at the top of my lesson plan template. When writing a lesson, initially I thought I would set an inquiry level goal for each lesson, but I found it was much more appropriate to let the content drive each activity and assign an inquiry level after completing the module.

The literature review went on to clearly define the roles of teacher and student in an inquiry-based learning environment. While important information for this capstone, it only served me as background information while writing my curriculum modules. As I wrote, I had to assume the instructor was using instructional methods similar to the best practices described in this capstone. As I go on to teach using my material, the desriptions for teacher and student roles will be more important to me. Another piece of less useful information from the literature review was the analysis of the effectiveness of IBI. As best as I could determine, a review of the literature provided a luke warm assessment of the effectiveness of IBI. Qualitative survey-style reasearch tended to be strongly positive (Goos, 2004; Kogan & Laursen, 2014; Marshall & Horton, 2011; Morrison & McDuffy, 2009) and the results of quantitative meta studies were highly mixed (Minner et al., 2010; National Mathematics Advisory Panel, 2008). The studies listed above serve only to answer whether or not it is a good idea to pursue IBI curriculum development rather than how to go about doing it. Since my mind was already made up, I did not consider this information while producing my curriculum modules.

Finally, the literature concluded with some challenges of implementing IBI. Through the review of the literature, I was made aware of two common issues: students can develop a debilitating level of anxiety (Pajares, 1996; Zafra-Gomez et al., 2015) and, if left unchecked, there is a possibility for a misunderstanding to become permanent (Jarrett, 1997; Marshall et al., 2009). My strategy to combat these issues was twofold: gradually introduce students to higher levels of inquiry (Smith, 2015) and counter the possibility for misunderstandings by using strict observation and formalization (Cobb et al., 1997). Specifically, there is a guide for introducing students and teachers to the idea of IBL and what they can expect included in the supplementary materials. To combat misunderstandings, there are checkpoints built in to the workbook packets where the instructor or mastery-level group is expected to check in with a group's progress before the group moves on to the next topic. See Appendix G for examples of this work.

Future Work

This project was designed in preparation for my first year teaching AP Calculus AB. As I implement each module, it will be critical to continuously assess the effectiveness of the instructional methods outlined in this capstone. As a first year teacher of the course, I am missing a baseline assessment of my effectiveness as a teacher in this course, so I will be highly reliant upon my professional team to assess the effectiveness of the modules I have produced. I intend to use the curriculum modules I have developed as intended, but I also need to coordinate with my team to recognize when something is working well and not working. In the case of the latter, we can fall back on the established curriculum and try to adopt anything that worked well. I do not expect the transition to implementing more inquiry to be without issue. Currently, I view the teaching methods in our building to be traditional in nature, so there is much to learn amongst the staff if we feel IBI is something worth pursuing.

Conveniently, our teams are highly data-driven and we meet weekly to discuss instructional strategies. During these meetings, my fellow staff and I will discuss test performance and they will be able to inform me on whether or not they feel the modules are effective. I believe I have an ambitious and curious team to work with, but in the business of teaching, results matter. If difficulties arise, the modules I have produced might not get much use. However, if it turns out that this project is highly successful, it will be interesting to see what other ideas we can produce. While writing the modules, it started feeling like I was writing a textbook. If a large number of topics in the AP Calculus AB curriculum can be approached through inquiry, writing a complete textbook based on IBL might be an interesting project to hack away at over the course of my career. Regardless of the feasibility of that project, I would love for these techniques to spread across the entirety of the AP Calculus AB curriculum and all levels of math taught in our building. I would be proud to say I work in a progressive and successful department.

During the review process of this capstone, it has already been identified that our team is currently ignoring an opportunity to engage in some cross-curricular activities with our science department. A project like IBI may serve to close the gap. Currently, there is no coordination between our departments. The majority of AP Calculus AB students are also enrolled in Physics. As there is often a calculus and non-calculus perspective to physics, there are a lot of possibilities for extending what each course teaches.

Conclusion

At the beginning of this project, I sought to answer the question: *how can inquiry-based learning be successfully implemented in a secondary mathematics classroom?* I started with an instinct that it was possible to teach mathematics effectively through a method similar to the scientific method. To my delight, I found a community of experts that held the same belief despite the prevalence of direct instruction. After studying their work, I feel I have become a voice worthy of their community and this capstone might serve as a line in their conversation. One day I may make a more significant contribution.

As I look to implement to implement this curriculum in real life, I have to resolve the two worlds presented before me. From this project, I have a set of theoretically sound learning activities that by all accounts will serve as an excellent base on which to build a highly successful inquiry-based classroom environment. On the other hand, I face the reality of teaching professional students near the end of their career in a way that is unfamiliar to them, untested by me, and unproven within the AP Calculus AB course. I find myself at the end of this project viewing my next year of teaching as a high-risk, high-reward proposition. The gravity of a course like AP Calculus AB is not lost on me. Personally, I remain optimistic and the support of my professional teams gives me confidence that *we will, on some level, successfully implement inquiry-based learning in our secondary mathematics classrooms.*

APPENDIX A

The EQUIP Survey

EQUIP
(Electronic Quality of Inquiry Protocol)

Complete Sections I before and during observation, Sections II and III during the observation, and Sections IV-VII immediately after the observation. If a construct in Sections IV-VI absolutely cannot be coded based on the observation, then it is to be left blank.

Observation date: __________ Time start:________ Time end: _______ Observer: ________________________________

School: ___________________________ District: _____________________________ Teacher: ________________________________

Course: _________________________

I. Descriptive Information

A. Teacher Descriptive Information:

1. Teacher gender ____ Male (M), Female (F)

2. Teacher ethnicity ____ Caucasian (C), African-American (A), Latino (L), Other (O)

3. Grade level(s) observed _____________ 4. Subject/Course observed ________________________

5. Highest degree _____________________ 6. Number of years experience:____________ 7. Number of years teaching this content ______

B. Student/Class Descriptive Information

1. Number of students in class: _____________

2. Gender distribution: _____ Males _____Females

3. Ethnicity distribution ______Caucasian (C) ______ African-American (A) ______ Latino (L) ______Other

C. Lesson Descriptive Information

1. Is the lesson an exemplar that follows the 4E x 2 Instructional Model? (PDI exemplar, non-PDI exemplar, non-exemplar)

2. Working title for lesson:

3. Objectives/Purpose of lesson: Inferred (I), Explicit (E) ___:

4. Standards addressed: State (S), District (D), None Explicit (N) ___ :

II. Time Usage Analysis						
Time	**Activity Codes**	**Organization Codes**	**Student Attention to Lesson Codes**	**Cognitive Codes**	**Inquiry Instruction Component Codes**	**Assessment Codes**
0-5						
5-10						
10-15						
15-20						
20-25						
25-30						
30-35						
35-40						
40-45						
45-50						
50-55						
55-60						
60-65						
65-70						
70-75						
75-80						
80-85						
85-90						

Activity Codes—facilitated by teacher

0. **Non-instructional time**—administrative tasks, handing back/collecting papers, general announcements, time away from instruction
1. **Pre-inquiry**—teacher-centered, passive students, prescriptive, didactic discourse pattern, no inquiry attempted
2. **Developing inquiry**—teacher-centered with some active engagement of students, prescriptive though not entirely, mostly didactic with some open-ended discussions, teacher dominates the explain, teacher seen as both giver of knowledge and as a facilitator, beginning of class warm-ups
3. **Proficient inquiry**—largely student-centered, focus on students as active learners, inquiries are guided and include student input, discourse includes discussions that emphasize process as much as product, teacher facilitates learning and students active in all stages, including the explain phase
4. **Exemplary inquiry**—student-centered, students active in constructing understanding of content, rich teacher-student and student-student dialogue, teacher facilitates learning in effective ways to encourage student learning and conceptual development, assumptions and misconceptions are challenged by students and teacher

Organization Codes—led by teacher

W Whole class
S Small group
I Individual work

Student Attention to Lesson Code—displayed by students

L **Low attention**, 20% or fewer attending to the lesson. Most students are off-task – heads on desks, staring out of the window, chatting with neighbors, etc.
M **Medium attention**, between 20-80% of students are attending to the lesson.
H **High attention**, 80% or more of the students are attending to the lesson. Most students are taking notes or looking at the teacher during lecture, writing on the worksheet, most students are volunteering ideas during a discussion, most students are engaged in small group discussions even without the presence of the teacher.

Cognitive Code—displayed by students

0. Other-e.g. classroom disruption, non-instructional portion of lesson, administrative activity
1. Receipt of knowledge
2. Lower order (recall, remember, understand) and/or activities focused on completion exercises, computation
3. Apply (demonstrate, modify, compare) and/or activities focused on problem solving
4. Analyze/Evaluate (evidence, verify, analyze, justify, interpret)
5. Create (combine, construct, develop, formulate)

Inquiry Instructional Component Code—facilitated by teacher

0. **Non-inquiry**: activities with the purpose of skill automation; rote memorization of facts; drill and practice; checking answers on homework, quizzes, or classwork with little or no explanation
1. **Engage**: typically situated at the beginning of the lesson; assessing student prior knowledge and misconceptions; stimulating student interest
2. **Explore**: students investigate a new idea or concept
3. **Explain**: teacher or students making sense of an idea or concept

   Extend: [Extend is important but is not coded as such because it typically is a new Engage, Explore, or Explain]

Assessment Code—facilitated by teacher

0. **No assessment observed**
1. **Monitoring** (circulating around the room, probing for understanding, checking student progress, commenting as appropriate)
2. **Formative assessment** (assessing student progress, instruction modified to align with student ability) or **Diagnostic assessment** (checking for prior knowledge, misconceptions, abilities)
3. **Summative assessment** (assessing student learning, evaluative and not informing next instructional step)

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www.clemson.edu/iim

EQUIP--2009

III. Lesson Descriptive Details		
Time (mins into class)	Classroom Notes of Observation	Comments

IV. Instructional Factors					
Construct Measured		*Pre-Inquiry (Level 1)*	*Developing Inquiry (2)*	*Proficient Inquiry (3)*	*Exemplary Inquiry (4)*
I1.	Instructional Strategies	Teacher predominantly lectured to cover content.	Teacher **frequently lectured** and/or used demonstrations to explain content. Activities were **verification only**.	Teacher **occasionally lectured**, but **students were engaged in activities** that helped develop conceptual understanding.	Teacher occasionally lectured, but students were engaged in investigations that **promoted strong conceptual understanding**.
I2.	Order of Instruction	Teacher explained concepts. Students either did not explore concepts or did so only after explanation.	Teacher asked **students to explore concept before receiving explanation**. Teacher explained.	Teacher asked students to **explore before explanation**. **Teacher and students explained**.	Teacher asked students to explore concept before explanation occurred. Though perhaps prompted by the teacher, **students provided the explanation**.
I3.	Teacher Role	Teacher was center of lesson; rarely acted as facilitator.	**Teacher was center** of lesson; **occasionally** acted as **facilitator**.	Teacher **frequently** acted as facilitator.	Teacher **consistently and effectively** acted as a facilitator.
I4.	Student Role	Students were consistently passive as learners (taking notes, practicing on their own).	**Students were active to a small extent** as learners (highly engaged for very brief moments or to a small extent throughout lesson).	**Students were active as learners** (involved in discussions, investigations, or activities, but not consistently and clearly focused).	Students were **consistently and effectively active as learners** (highly engaged at multiple points during lesson and clearly focused on the task).
I5.	Knowledge Acquisition	Student learning focused solely on mastery of facts, information, and/or rote processes.	Student learning focused on **mastery of facts** and process skills without much focus on understanding of content.	Student learning required **application of concepts** and process skills in new situations.	Student learning required **depth of understanding** to be demonstrated relating to content and process skills.

V. *Discourse Factors*					
Construct Measured		*Pre-Inquiry (Level 1)*	*Developing Inquiry (2)*	*Proficient Inquiry (3)*	*Exemplary Inquiry (4)*
D1.	Questioning Level	Questioning rarely challenged students above the remembering level.	Questioning rarely challenged students above the **understanding level**.	Questioning challenged students **up to application or analysis levels**.	Questioning challenged students at various levels, **including at the analysis level or higher; level was varied to scaffold learning.**
D2.	Complexity of Questions	Questions focused on one correct answer; typically short answer responses.	Questions focused **mostly on one correct answer;** some open response opportunities.	Questions **challenged students to explain, reason, and/or justify**.	Questions required students to explain, reason, and/or justify. **Students were expected to critique others' responses**.
D3.	Questioning Ecology	Teacher lectured or engaged students in oral questioning that did not lead to discussion.	Teacher occasionally **attempted to engage students in discussions** or investigations but was not successful.	Teacher **successfully engaged students** in open-ended questions, discussions, and/or investigations.	Teacher **consistently and effectively engaged students** in open-ended questions, discussions, investigations, and/or reflections.
D4.	Communication Pattern	Communication was controlled and directed by teacher and followed a didactic pattern.	Communication was **typically controlled and directed by teacher** with occasional input from other students; mostly didactic pattern.	**Communication was often conversational** with some student questions guiding the discussion.	Communication was **consistently conversational with student questions often guiding the discussion**.
D5.	Classroom Interactions	Teacher accepted answers, correcting when necessary, but rarely followed-up with further probing.	Teacher or another student **occasionally followed-up student response** with further low-level probe.	Teacher or another student **often followed-up response** with engaging probe that required student to **justify reasoning or evidence**.	Teacher **consistently and effectively facilitated rich classroom dialogue** where evidence, assumptions, and reasoning were challenged by teacher or other students.

VI. *Assessment Factors*					
Construct Measured		*Pre-Inquiry (Level 1)*	*Developing Inquiry (2)*	*Proficient Inquiry (3)*	*Exemplary Inquiry (4)*
A1.	Prior Knowledge	Teacher did not assess student prior knowledge.	Teacher **assessed student prior knowledge but did not modify instruction** based on this knowledge.	Teacher assessed student prior knowledge and then **partially modified instruction** based on this knowledge.	Teacher assessed student prior knowledge and then **modified instruction** based on this knowledge.
A2.	Conceptual Development	Teacher encouraged learning by memorization and repetition.	Teacher encouraged **product- or answer-focused** learning activities that **lacked critical thinking**.	Teacher encouraged **process-focused** learning activities that **required critical thinking**.	Teacher encouraged process-focused learning activities that involved **critical thinking that connected learning with other concepts**.
A3.	Student Reflection	Teacher did not explicitly encourage students to reflect on their own learning.	Teacher explicitly encouraged students to **reflect** on their learning but only at a minimal **knowledge level**.	Teacher explicitly encouraged students to **reflect** on their learning at an **understanding level**.	Teacher consistently encouraged students to reflect on their learning at multiple times throughout the lesson; **encouraged students to think at higher levels**.
A4.	Assessment Type	Formal and informal assessments measured only factual, discrete knowledge.	Formal and informal assessments **measured mostly factual, discrete knowledge**.	Formal and informal assessments **used both factual, discrete knowledge and authentic measures**.	Formal and informal assessment methods **consistently and effectively used authentic measures**.
A5.	Role of Assessing	Teacher solicited predetermined answers from students requiring little explanation or justification.	Teacher **solicited information from students to assess understanding**.	Teacher solicited explanations from students to **assess understanding and then adjusted instruction accordingly**.	Teacher frequently and effectively assessed student understanding and adjusted instruction accordingly; **challenged evidence and claims made; encouraged curiosity and openness**.

VII. Curriculum Factors					
Construct Measured		*Pre-Inquiry (Level 1)*	*Developing Inquiry (2)*	*Proficient Inquiry (3)*	*Exemplary Inquiry (4)*
C1.	Content Depth	Lesson provided only superficial coverage of content.	Lesson provided **some depth** of content but with **no connections made to the big picture.**	Lesson provided depth of content with **some significant connection** to the big picture.	Lesson provided depth of content with **significant, clear, and explicit connections** made to the big picture.
C2.	Learner Centrality	Lesson did not engage learner in activities or investigations.	Lesson provided **prescribed activities** with anticipated results.	Lesson allowed for **some flexibility during investigation** for student-designed exploration.	Lesson provided **flexibility for students to design and carry out** their own investigations.
C3.	Integration of Content and Investigation	Lesson either content-focused or activity-focused but not both.	Lesson provided poor integration of content with activity or investigation.	Lesson **incorporated student investigation that linked well with content**.	Lesson **seamlessly integrated the content and the student investigation**.
C4.	Organizing & Recording Information	Students organized and recorded information in prescriptive ways.	**Students had only minor input as to how to organize and record information**.	Students regularly organized and recorded information in **non-prescriptive ways**.	Students organized and recorded information in **non-prescriptive** ways that allowed them to **effectively communicate their learning**.

*VIII. Summative Overviews**		*Comprehensive Score***
Summative view of Instruction		
Summative view of Discourse		
Summative view of Assessment		
Summative view of Curriculum		
Overall view of Lesson		

*Provide brief descriptive comments to justify score.
**Score for each component should be an integer from 1-4 that corresponds with the appropriate level of inquiry. Scores should reflect the essence of the lesson relative to that component, so they need not be an exact average of all sub-scores in a category.

Marshall, J. C., Horton, B., Smart, J., & Llewellyn, D. (2008). *EQUIP: Electronic Quality of Inquiry Protocol*: Retrieved from Clemson University's Inquiry in Motion Institute, www.clemson.edu/iim.

INQUIRY IN MOTION
www.clemson.edu/iim

EQUIP--2009

(Marshall, Horton, Smart, & Llewellyn, 2008)

APPENDIX B

The McGill Strategic Demands of Inquiry Questionaire

McGill Strategic Demands of Inquiry Questionnaire criterion referencing

Questionnaire item *Prefaced by the question "How important is it in inquiry-based learning and teaching . . . ?"*	*Theme*	*Sources (details in References)*
1—for the student and teacher to have co-ownership of the question	Have co-ownership	National Research Council (1989), Wills (1995)
2—for the student and teacher to share construction of the curriculum	Share construction of the curriculum	Chiappetta (1997), National Research Council (1989)
3—for the student and teacher to share decision making	Share decision making	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999), National Research Council (1989), Wills (1995)
4—for the teacher to listen as much as he or she speaks	Listening to others	National Research Council (1989)
5—for the student to work in a nurturing and creative environment	Work and interact with surroundings or environment	Driver (1983), National Research Council (1989)
6—for the student to extend inquiry beyond the classroom	Extend inquiry beyond the classroom	National Research Council (1989)
7—for the teacher to tap into the student's and his or her own interests	Address student's and teacher's needs	Chiappetta (1997), National Research Council (1989)
8—for the teacher to explore his or her interest	Address student's and teacher's needs	Chiappetta (1997), National Research Council (1989)
9—for the teacher to address his or her needs and student's needs	Address student's and teacher's needs	Chiappetta (1997), National Research Council (1989)

10—for the teacher to provide a mentor	Having a mentor	Germann (1991)
11—for the teacher to model skills needed for the inquiry	Modeling skills	Germann (1991)
12—for the teacher to give the amount of time needed, be flexible with time	Time needed to work	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
13—for the student to organize time and space	Time needed to work	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
14—for the student to understand the goal of the task	Understand the goal of the task	Driver (1983)
15—for the student to divide the task into a coherent sequence of do-able steps	Dividing the task	Driver (1983), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
16—for the student to make a concept map or web or cluster	Make a concept map	Jeroski, Booth & Dockendorf (1992)
17—for the student to foresee possible outcomes of the activity	Foresee outcomes of activity	National Research Council (1989)
18—for the student to understand key concepts	Understand key concepts	National Research Council (1989)
19—for the student to understand instructions	Understand instructions	National Research Council (1989)

Questionnaire item *Prefaced by the question "How important is it in inquiry-based learning and teaching . . . ?"*	*Theme*	*Sources (details in References)*
20—for the student to describe his or her own problem-solving strategies	Describe own problem-solving strategies	Driver (1983)
21—for the student to have previous experience with similar activities	Have previous experience with similar activities	Driver (1983), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
22—for the teacher to encourage honest criticism of ideas	Encourage honest criticism of ideas	National Research Council (1989)
23—for the teacher to encourage creative risk taking	Encourage creative risk taking	Germann (1991)
24—for the student to connect old and new knowledge	Connect old and new knowledge	National Research Council (1989)
25—for the student to set aside preparation time	Time needed to work	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
26—for the student to brainstorm his or her ideas	Brainstorming essential in inquiry	Llewellyn (2005)
27—for the student to make a plan	Plan	National Research Council (1989), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
28—for the student to have different plans in advance to accomplish the task	Plan	National Research Council (1989), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
29—for the student to have backup plans at the end should the project stall	Plan	National Research Council (1989), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)

30—for the student to feel free to use imagination	Use imagination	Driver (1983)
31—for the student to keep motivated	Motivation	Germann (1991)
32—for the student to have self-motivation	Motivation	Germann (1991)
33—for the student to get a high grade[a]	Get a high grade	Boisvert & Roumain (2000)
34—for the student to win a prize[a]	Win a prize	Boisvert & Roumain (2000)
35—for the teacher to give sensitive feedback, positive reinforcement, praise for persistence	Give sensitive feedback	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
36—for the student to ask questions	Ask questions	Cole (1989), Driver (1983), National Research Council (1989), Wills (1995)
37—for the student to restate or reformat the problem	Restate and reformat the problem	Driver (1983)
38—for the student to make suggestions	Make suggestions	Germann (1991)
39—for the student to share emotions, feelings, ideas, and opinions	Share emotions and feelings	Cole (1989), Germann (1991), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)

Questionnaire item *Prefaced by the question "How important is it in inquiry-based learning and teaching . . . ?"*	*Theme*	*Sources (details in References)*
40—for the student to develop expectations of what will happen next	Develop and test hypothesis about the outcomes of the project	Driver (1983), Germann (1991), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999), National Research Council (1989)
41—for the student to offer hypotheses about outcomes	Develop and test hypothesis about the outcomes of the project	Driver (1983), Germann (1991), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999), National Research Council (1989)
42—for the student to make careful observations	Make observations	Cole (1989), Driver (1983)
43—for the student to identify where to obtain data	Identify where to obtain data	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
44—for the student to recognize hidden meanings in data	Recognize hidden meanings in data	Driver (1983)
45—for the student to record data	Record data	Cole (1989), Driver (1983)
46—for the student to classify data	Classify data	
47—for the student to search for resources beyond textbooks	Search for resources beyond textbooks	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999), National Research Council (1989)
48—for the student to search the Internet and World Wide Web	Search for resources beyond textbooks	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999), National Research Council (1989)
49—for the student to separate relevant and irrelevant information	Separate relevant and irrelevant information	Driver (1983), Germann (1991)
50—for the student to apply previous knowledge to new concepts	Apply previous knowledge to new concepts	Driver (1983)

51—for the student to understand how preconceptions affect learning	Understand how preconceptions affect learning	Boisvert & Roumain (2000)
52—for the student to be aware of how the inquiry event affects him or her personally	Be aware of how the inquiry event affects you personally	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
53—for the student to keep an open mind to change	Keep an open mind to change	Driver (1983), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
54—for the student to address doubts directly	Address doubts directly	Driver (1983), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
55—for the student to assist others to make observations	Make observations	Cole (1989), Driver (1983)
56—for the student to find patterns in data	Find patterns in data	Driver (1983), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)

Questionnaire item *Prefaced by the question "How important is it in inquiry-based learning and teaching . . . ?"*	*Theme*	*Sources (details in References)*
57—for the student to value personal judgment	Value personal judgment	Driver (1983), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
58—for the student to verify data or information	Verify data	Driver (1983), Germann (1991)
59—for the student to compare and contrast data with someone else's	Communicate work with others (comparing and contrasting)	Cole (1989), Driver (1983), Germann (1991), National Research Council (1989)
60—for the student to anticipate and respond to arguments in opposition to one's view	Anticipate and respond to arguments in opposition to one's view	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
61—for the student to seek different viewpoints	Communicate work with others (comparing and contrasting)	Cole (1989), Driver (1983), Germann (1991), National Research Council (1989)
62—for the student to test ideas and hypotheses	Develop and test hypothesis about the outcomes of the project	Driver (1983), Germann (1991), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999), (National Research Council 1989)
63—for the student to have a mental representation of the task	Have a mental representation of the task	Driver (1983)
64—for the student to construct new knowledge	Construct new knowledge	Driver (1983), Germann (1991), National Research Council (1989)
65—for the student to interact with or manipulate his or her surroundings	Work and interact with surroundings or environment	Driver (1983), National Research Council (1989)

66—for the student to communicate one's learning with others	Communicate work with others (comparing and contrasting)	Cole (1989), Driver (1983), Germann (1991), National Research Council (1989)
67—for the student to consider diverse means of communication	Consider diverse means of communication	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
68—for the student to organize the presentation of the project	Organize the presentation of the project	Germann (1991)
69—for the student to present data in tables and graphs	Present data in tables and graphs	Germann (1991)
70—for the student to use vocabulary appropriate to the audience and topic	Use vocabulary appropriate to audience or topic	Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
71—for the student to accept that more than one solution might be appropriate	Accept that more than one solution might be appropriate	Driver (1983)
72—for the student to apply new knowledge to future experiences	Apply new knowledge to future experiences	Driver (1983)
73—for the student to record methods, results, and conclusions	Record methods, results, and conclusions	Driver (1983)
74—for the student to explain the results	Explain results	Driver (1983)

Questionnaire item *Prefaced by the question "How important is it in inquiry-based learning and teaching . . . ?"*	*Theme*	*Sources (details in References)*
75—for the student to question the findings	Reflection	Short & Burke (1996)
76—for the student to reflect upon his or her inquiry experience	Reflection	Short & Burke (1996)
77—for the student to discuss what has been learned compared to what was known before	Discuss what has been learned compared to what was known before	Driver (1983), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)
78—for the student to evaluate the inquiry experience	Evaluate the inquiry experience	Cole (1989), Driver (1983), National Research Council (1989)
79—for the student to follow-up the project with a new set of questions	Follow-up project with new set of questions	Driver (1983), Ministère de l'Éducation du Québec [Quebec Ministry of Education] (1999)

[a]Items acting as distractors

(Shore et al., 2012, pp. 321-325)

APPENDIX C

The Fitzgerald2002 Inquiry Curriculum Guide

A. Increase their understanding of the science subject matter investigated

A1. Content:

A1a. The material provides content aligned with national, state or local standards.

A1b. The material provides opportunity to develop enduring understanding of subject matter content.

A1c. The material contains accurate content.

B1. Understanding of how scientists work

B1a. The material provides an opportunity to learn how different kinds of questions based on prior scientific knowledge suggest different kinds of investigations.

B1b. The material provides an opportunity to learn that scientists conduct investigation for a variety of reasons.

B1c. The material provides an opportunity to learn that scientists use a variety of tools, technology, and methods to extend the senses.

B1d. The material provides an opportunity to learn that mathematics is essential in scientific inquiry.

B1e. The material provides an opportunity to learn that scientists use evidence, logic, and current scientific knowledge to propose explanations.

B1f. The material provides an opportunity to learn that scientists collaborate and communicate with each other in a variety of ways to reach well-accepted explanations.

C. Develop the ability to conduct investigations

C1. Posing scientifically oriented questions

C1a. The material provides an opportunity to ask questions that can be answered through scientific investigations.

C2. Designing and Conducting Investigations

C2a. The material engages learners in planning investigations to gather evidence in response to questions.

C2b. The material engages learners in conducting the investigation.

C2c. The material engages learners in the use of analytical skills.

C3. Proposing Answers

C3a. The material engages learners in proposing answers and explanations to questions.

C4. Comparing explanations with current scientific knowledge

C4a. The material engages learners in the consideration of alternative explanations.

C4b. The material engages learners in linking explanations with scientific knowledge.

C5. Communicating and justifying results

C5a. The material engages learners in communication of scientific procedures and explanations.

C5b. The material engages learners in appropriately responding to critical comments.

C5c. The material engages learners in raising additional questions.

D. Developing the habits of mind associated with science

D1. Developing the habits of mind associated with science

D1a. The material promotes the questioning of assumptions (skepticism).

D1b. The material presents science as open and subject to modification based on communication of new knowledge and methods (openness).

D1c. The material promotes longing to know and understand (curiosity).

D1d. The material promotes respect for data (honesty).

(Fitzgerald & Byers, 2002, pp. 89-90)

APPENDIX D

A Lesson Plan Template for Inquiry-based Learning

[Lesson Title]

[Course]	[School]	[Instructor]
[Length of Activity]	Inquiry Level: [1-4]	[Room Location]

Enduring Understandings:
[List enduring understandings here]

Prerequisite Skills:
[List prerequisite skills here]

[Course] Reporting Standards – SWBAT:
[List course standards here]

Additional Objectives – SWBAT:
[List any additional standards here]

Assessments:
[List assessment opportunities here]

Materials Needed:
[List materials needed for the activity here]

Learning Sequence:

Activity	Notes:
Framing	[Describe how you will activate your students curiosity. Frame the topic and ask a guiding question.]
Investigation	[Describe what activities students will engage in to produce evidence for their eventual claim.]
Analyze	[Describe the process by which students are expected to construct new knowledge.]
Consensus	[Describe the conclusion students are expected to reach after their investigation.]
Formalize	[Describe what information must be formally presented to adequately address the learning target.]

Essential Questions:
[List guiding or analytic questions here]

Reflection:
[This space is reserved for the instructor to make notes during and/or after the lesson]

APPENDIX E

An Example Lesson Plan

The Area Problem – Part III

AP Calculus AB	[School]	[Instructor]
Two Long Days	Inquiry Level: 4 (Open)	[Room Location]

Enduring Understandings:

The areas and volumes of curvy shapes can be found by using Riemann sums.

Prerequisite Skills:

Riemann sum concept, spreadsheet coding, piecewise function drawing

AP Calculus AB Reporting Standards – SWBAT:

Use a Riemann sum of discs to calculate the volume of a rotational solid (Section 7.2)

Additional Objectives – SWBAT:

Distinguish between highly accurate estimates and exact answers
Use spreadsheets to perform tedious calculations
Draw with piecewise functions

Assessments:

Spot checks, written responses, Schoology submissions, critical checkpoint (disc method)

Materials Needed:

The Area Problem Packet (Part III)	Graphing Calculators	iPads (Sheets, Desmos)
Presentation guide (*.notebook)	Schoology Resource Folder	A discarded beverage bottle

Learning Sequence:

Activity	Notes:
Framing	This is an application of the Part I and II activities that will demonstrate the difficulties of applying math to real life.
Investigation	Students are introduced to solids of rotation. They should be able to visualize and sketch the resulting shape of rotating a region. They are also asked to draw the profile of a beverage bottle using piecewise functions. They will probably spend a lot of time coming up with functions that accurately represent the shape of the bottle.
Analyze	Students are asked to consider the cross section of a solid of rotation. They should be able to determine it is a circle and that should lead them to approximate the area using discs.
Consensus	Students should use a spreadsheet to perform a Riemann sum of discs.
Formalize	The instructor should approve of each group's plan of action (critical checkpoint) and help debug any faulty code. Point out π is a constant that does not need to be involved in the calculation.

Essential Questions:

How can you improve the accuracy of your work?
Have you produced an exact value?
What challenges did you encounter?
How versatile is using discs?

Reflection:

APPENDIX F

A Sample AP Calculus AB Syllabus

AP Calculus AB
Syllabus

Calculus is the greatest invention of the human mind. In this course we will learn, through the concept of limits, to transform the mathematics you know into something more useful, dynamic, coherent, and beautiful.

In some ways, this is an easy course: there really are only a few main ideas applied over and over. In other ways, it is very tough: you need to be proficient in all of the math you've learned leading up to this course. We will do several things to ensure success:

- Students will use a TI-83, TI-84, or TI-89 calculator. We will use them in explorations, assignments, presentations, and projects. Most exams will have a portion in which calculators are banned and a portion where they are required.
- We will approach our topics three ways: numerically, graphically, and analytically. This will help us to understand the concepts and give us more tools. There is often more than one way to solve a problem.
- We will stress precision. Students will learn to communicate mathematics with proper terminology and notation. They will present their work in front of the class.
- We will apply concepts to real world situations. That is where Calculus came from.
- We will review. We usually cover all of the topics required for the AP exam a few weeks before the exam date. During that time, you will solve and present your solutions to problems from prior years' exams. You will also write a paper explaining the major ideas of Calculus and how they relate.

Our textbook is Larson, Hostetler, Edwards. *Calculus of a Single Variable.* 8th ed. Boston: Houghton Mifflin Company. 2006.

Course Outline

Chapter P Preparation for Calculus
- P.1 Graphs and Models
- P.2 Linear Models
- P.3 Functions and Graph
- Using Your Graphing Calculator

Chapter 1 Limits and Their Properties
- 1.1 A Preview of Calculus
- 1.2 Finding Limits Graphically and Numerically
- 1.3 Evaluating Limits Analytically
- 1.4 Continuity and One-Sided Limits
- 1.5 Infinite Limits

Chapter 2 Differentiation

2.1 The Derivative and the Tangent Line Problem
2.2 Basic Differentiation Rules and Rates of Change
2.3 Product and Quotient Rules and Higher-Order Derivatives
2.4 The Chain Rule
2.5 Implicit Differentiation
2.6 Related Rates

Chapter 3 Applications of Differentiation

3.1 Extrema on an Interval
3.2 Rolle's Theorem and the Mean Value Theorem
3.3 Increasing and Decreasing Functions and the First Derivative Test
3.4 Concavity and the Second Derivative Test
3.5 Limits at Infinity
3.6 A Summary of Curve Sketching
3.7 Optimization Problems
3.8 Differentials

Chapter 4 Integration

4.1 Antiderivatives and Indefinite Integration
4.2 Area
4.3 Riemann Sums and Definite Integrals
4.4 The Fundamental Theorems of Calculus
4.5 Integration by Substitution
4.6 Numerical Integration

Chapter 5 Logarithmic, Exponential, and Other Transcendental Functions

5.1 The Natural Logarithm: Differentiation
5.2 The Natural Logarithm: Integration
5.3 Inverse Functions
5.4 Exponential Functions: Differentiation and Integration
5.5 Bases Other Than *e* and Application
5.6 Inverse Trigonometric Functions: Differentiation
5.7 Inverse Trigonometric Functions: Integration

Chapter 6 Differential Equations

6.1 Slope Fields
6.2 Differential Equations: Growth and Decay
6.3 Separation of Variables

Chapter 7 Applications of Integration

7.1 Area of a Region Between Two Curves
7.2 Volume: The Disk Method

Review for AP Test

Student Assessments

Trimester grades are computed using homework, class activities, quizzes and tests. In the third trimester, students also write a paper where they explain the major ideas of Calculus and how they relate.

An Example of a Student Activity

Students are given a hollow toy bowling pin and a cloth tape measurer and are asked to predict how much water will fill the pin. Most students measure the circumference of the pin at regular intervals and use a numerical integration technique with the disk method to calculate. Some others trace the pin and use the regression capabilities of their calculators to try to derive a formula that models the pin.

(Jones & Peterson, 2016)

APPENDIX G

Link to Open Source Inquiry Modules

https://drive.google.com/open?id=0BwQap5LIEm12dVRDMkhkNWVyYmc

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