Optimization of Quantity Discounts Using JIT Technique under Alternate Cost Policies

Rudresh Pandey[#1], Shradha Goyal[*2]

[#]*Department of management, ABES (EC)*
Ghaziabad, U.P, India

[1]rudreshpandey@gmail.com

[*]*Jagannath International Management School*
Kalkaji, New Delhi, India

[2]bansal.shradha@gmail.com

***Abstract* - In traditional economic order quantity modeling technique, as per the storage in a warehouse, the rate of demand is considered to be fixed, whereas in real world practice rate of demand may be dependent on time, price and stock. This paper studies problems based on allocation of order quantity under quantity discounts by revising mathematical models already studied in this area. For example, in a multi warehouse system like a super departmental store, the rate of demand is mostly subjective on the basis of stock demand. In industry, the maintenance of large stock of goods in warehouses has a higher probability of consumers as compared to an industry with small quantity of stock. Such procedures implied in single warehouses systems may be logical for level of stock that is dependent on demand. Hence, a good and large stock level mostly results in a higher profits and larger sales. The objective is to optimize profit under the effect of price variations in the form of quantity discounts based on an alternative cost functions, with the help of JIT inventory technique and analyzing a mathematical model based on it.**

Key words: - Stock, EOQ model, Depreciation, warehouse, inventory.

1. Introduction

In traditional economic order quantity modelling technique, as per the storage in a warehouse, the rate of demand is considered to be fixed, whereas in real world practice rate of demand may be dependent on time, price and stock. This paper studies problems based on allocation of order quantity under quantity discounts by revising mathematical models already studied in this area. For example, in a multi warehouse system like a super departmental store, the rate of demand is mostly subjective on the basis of stock demand. In industry, the maintenance of large stock of goods in warehouses has a higher probability of consumers as compared to an industry with small quantity of stock. Such procedures implied in single warehouses systems may be logical for level of stock that is dependent on demand. Hence, a good and large stock level mostly results in a higher profits and larger sales.

2. Literature Review

Ref. [8] studied “a model in which the demand rate is affected by both level of stock and market price”. Ref. [1] refers to “an inventory model for depreciation of goods with variable stock, varying time and rate of demand over a limited domain of planning”. Ref. [7] analyzed “an EPQ model for deterioration of stock when the rate of demand varies not only with level of display-stock level but as well as with the market price per unit”. “An EOQ model for delicate substance under stock reliant promotion rate and time reliant part backlogging with constant worsening was studied” by [2]. [9] Worked on an inventory model considering the amelioration on goods in store.

Ref. [9]; [10] investigated a deteriorating stock sculpt with demand varying with time and backlogging happening because of shortages. [4] Studied a model for deteriorating substance with assumptions that holding costs depends on time and demand is dependent on price. Ref. [3]; [5] formulated a most favourable market price and batch volume with time varying decline and partial backlogging. Ref. [6] offered “a supply chain model for fresh matter with sloped rate of

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demand". A worldwide optimizing strategy for decomposing things with inclined type rate of demand under two-staged trade credit finance policy in relation of preserving of technology.

3. Objective and Research Gap

The objective of this problem is to optimize profit under the effect of price variations in the form of quantity discounts based on an alternative cost functions, with the help of JIT inventory technique and analyzing a mathematical model based on it. Also considering quantity discounts in the system.

Earlier work of research on quantity discounts mainly considered partial blurriness in the decision problems, that is, when both the objectives of the decision maker and the constraints are indistinct has not been studied up to now. This paper works on combining both aspects into a single model and hence filling the gap found from earlier research work. Subsequently, a mathematical modelling method for solving the above stated problem is presented. The model considers advertising, marketing and selling costs to obtain the optimized profits and other related costs. Finally, the performance of the models is computed by using a numerical illustration separately for both linear and non-linear cost functions.

4. Problem Formulation

Starting with the assumptions of advertising, marketing, storage and selling price the respective cost functions are considered and the net profit is calculated by taking difference of total revenue and total cost.

4.1 Notations

The model is developed under the following notations, where,

D is the rate of demand per year

K_r is the initial replenishment cost

K_c is the dispatch cost / truck

h is the per unit cost of carrying inventory / time (units)

n is the number of lots dispatched

Q is the quantity dispatched

C_r is the cost of purchase / unit

λ is the fixed

W is the cost of waiting per unit / unit time

α is the constant $\left(0<\alpha<1\right)$

P is the selling price / unit

ν is the fraction of defectives in a lot

$C(Q)$ is the per unit cost

λ' is the rate of production

4.2 Assumptions

Model is derived with following assumptions:

1. The rate of demand D (unit / year) is dependent of selling price Rs.P per unit.
2. Occurrence of shortages is not allowed.
3. Real time single order supply is there.
4. Rate of production λ' (units / year) is finite and $D>\lambda'$.
5. Constant cost of replenishment C_s (Rs/ order) and $C_s=\frac{K_r\lambda}{n}+K_c\lambda$
6. Holding cost of inventory is given by C_h, $C_h=h\left(n-1\right)+w\left(1+\frac{D}{\lambda}\right)$
7. Cost of advertisement in a small fraction of revenue and is αPD where $\left(0<\alpha<1\right)$.
8. Same part of lot that gets damaged is called let fraction defective.

   Number of units damaged in lot = M / Total lot size = Q

   i.e. $=M/Q$ and $\left(0<\nu<1\right)$

   Also ν is assumed to be a random variable and distributed by Beta and $\left(1,h\right)$ parameters, sot that,

$$E\left(\nu\right)=\frac{1}{1+h}g>0$$

9. Advertisement occurs *f* times. This f is a random variable with expectations E (*f*) = *N*. Also frequency of advertisement is dependent on objectives of management, market nature, resources, nature & types of customers etc.

10. The annual total expected profit is Z = E [F (Q)]. Z is found to be unique, real and continuous function of Q, where Q is the size of the lot.

(Q^* represents the optimized value of Q and therefore helps to calculated respective Z^*i.e. optimized value of total annual profit Z).

4.3 Methodology

Following the above mentioned assumptions, the cost involved in advertising is taken as,

$$=\alpha PD$$

The damage cost $=\upsilon C\left(QD\right)$

Therefore, total annual system cost is,

$$=\alpha PD+vC\left(QD\right)+\frac{Q}{2}h\left(n-1\right)+w\left(1+\frac{D}{\lambda}\right)+\left[\frac{K\lambda}{\eta}+K_c\lambda\frac{D}{Q}\right]+\left[C\lambda-\right.$$

(This total annual cost includes cost of marketing and storage as well)

Total annual revenue = *D* [P – C(Q)]

Therefore,

Annual profit = Total annual revenue – Total annual cost

$$\Rightarrow \quad F\left(Q\right)=DP-C\left(Q\right)-\alpha PD-vC\left(QD\right)-\frac{Q}{2}h\left(n-1\right)+w\left(1+\frac{D}{\lambda}\right)$$

$$-\left(\frac{K\lambda}{n}+K_c\lambda\left(\frac{D}{Q}\right)-C\lambda-\frac{w}{2}\right)$$

To find the total net revenue, determine the per unit selling price P.

The selling price includes the mark up price, is calculated by,

$$P=\theta CQ$$

Where and $\theta>1$ is parameter of mark-up.

Using sensitivity analysis, the variation in θ can show the effect of price on profit.

Therefore eqn. (5) and (6) gives,

$$F\left(Q\right)=D\left(\theta-1\right)C\left(Q-D\right)\theta+v\,CQ-\frac{Q}{2}h\left(n-1\right)+w\left(1+\frac{D}{\lambda}\right) \quad \ldots\ldots(7)$$

From eqn. (7), special cases of D and C (Q) functions.

The study is given as:

$$D=\phi\left(P\right)=\frac{Kf}{P^n} \quad \ldots\ldots(1)$$

Where, ……(2)

f is the advertisement frequency,

$\eta\left(>0\right)$ is the elasticity of demand,

$K>0$ known constant.

Therefore, the expression (6) and (8) gives, ….(3)

$$D=Kf\ \theta C\left(Q\right)^{\eta}$$

C(Q) is a single valued, real, unit cost function. It is dependent on quantity of order (Q), hence the quantity discounts are taken as:- …. (4)

$$C\left(Q\right)=b+\frac{d}{Q^r},$$

In this *b*, *d* and *r* are given constants, with either $b>0, d>0$ and $r>0$ or $b>0, d<0$ and $r<0\ \forall$ Q.

Equation (7), (9), (10), gives the profit function as:- …. (5)

$$F\left(Q\right)=Kf+\theta\left(b+\frac{d}{Q^{r}}\right)^{1-\eta}-Kf\theta^{-\eta}\left(b+\frac{d}{Q^{r}}\right)^{1-\eta}-\alpha Kf\,Q\left(b+\frac{d}{Q^{r}}\right)^{1-\eta}$$

$$-vKf\theta^{-\eta}\left(b+\frac{d}{Q^{r}}\right)^{1-\eta}-\frac{Q}{2}h\left(n-1\right)+w\left[1+\frac{Kf}{\lambda'}\theta\left(b+\frac{d}{Q^{r}}\right)^{1-\eta}\right]$$

$$-\frac{K\lambda}{\eta}+K_{c}\,\lambda+\theta\left(b+\frac{d}{Q^{r}}\right)^{1-\eta}-\frac{1}{Q}-C\lambda-\frac{w}{2}$$

…. (11)

Also,

$$Ev=\frac{1}{1+h}=g$$

$E\,f = N$

(12)

Equations (11), (12) give the expected net profit function on as $Z=E\left[F\left(Q\right)\right]$

$$=K\gamma N\theta^{-\eta}\left(b+\frac{d}{Q^{r}}\right)^{1-\eta}-\frac{Q}{2}\left(h\,n-1\right)+w+\frac{KN\theta^{-\eta}}{2\lambda}\left(h\,n-1\right)+w\,Q\left(b+\frac{\ }{(}\right.$$

$$-\frac{K_{c}\lambda}{\eta}+K_{c}\lambda\left[\frac{KN\theta^{-\eta}}{Q}\right]\left(b+\frac{d}{Q^{r}}\right)^{1-\eta}+\left(C\,\lambda-\frac{w}{2}\right)$$

(13)

Where $\gamma=\theta\left(1-\alpha\right)-\left(1+g\right)$

The objective is to optimize expected net profit by finding the maximum quantity or order size.

The values Q^{*} (optimum value of Q) and Z^{*} (optimum value of Z) can be found by given derivatives,

$$\frac{\partial Z}{\partial Q}=0 \text{ (Which gives } Q^{*}\text{)}$$

and $$\left|\frac{\partial Z}{\partial Q}\right|_{Q=Q^{*}}<0$$

The general solution of the given conditions, also considers the solutions of n^{th} degree polynomial equation with variable Q.

Taking $\lambda'\to\infty$, makes Economics Lot Size (ELS) model revised as Economics Order Quantity (EOQ) model and hence eqn. (13) can be written as

$$Z=KN\gamma\theta^{-\eta}\left(b+\frac{d}{Q^{r}}\right)^{1-\eta}-\frac{Q}{2}h\left(n-1\right)+w-\frac{K\lambda}{\eta}+K_{c}\lambda\left[\frac{KN\theta^{-\eta}}{Q}\left(b+\frac{d}{Q^{r}}\right)^{-\eta}-C\lambda-\frac{w}{2}\right]$$

4.4 Particular Cases

To further explain the model, cost function for quantity discount is taken in 2 different cases, as defined below:-

4.4.1 Case A Linear Form of Cost Discount Function

The derived model is taken in the particular cases with elasticity of demand $\eta=1$ and r = –1 in the quantity discounts in linear form, given as.

$$C\left(Q\right)=b+dQ$$

Where $b>0$ and $d<0$

Therefore, model in equation (13) gives,

$$Z=\frac{KN\gamma}{\theta}-\frac{Q}{2}\left(hn-1\right)+w\left[1+\frac{KN}{\lambda'\theta\left(b+dQ\right)}\right]+\frac{K_{c}\lambda}{\eta}+K_{c}\lambda-\frac{KN}{\theta Q\left(b+dQ\right)}-C\lambda-\frac{w}{2}$$

…. (13)

Hence Q* can be calculated by:-

$$Q^{4}+A_{1}Q^{3}+A_{2}Q^{2}-A_{3}Q-A_{4}=0$$

Where

$$A_{1}=\frac{2b}{d}$$

$$A_{2}=\frac{b\left(b\lambda'\theta-KN\right)}{\lambda'\theta d^{2}}$$

$$A_3 = \frac{4KN\left(K_r\lambda + nK_c\lambda\right)}{n\theta\left(hn-1\right)+w}$$

$$A_4 = \frac{2NKb\left(K_r\lambda + nK_c\lambda\right)}{n\theta d^2\left(hn-1\right)+w}$$

Finally,

$$2\lambda'\left(K_r\lambda + nK_c\lambda\right)\ 3dQ\left(b+dQ\right)+b^2 > nbd\ h$$

This case helps to optimize expected net profit and inventory cost simultaneously.

Numerical Illustration

Considering a hypothetical problem for illustration of the above developed linear case of cost function,

Consider

$\lambda' = 100$ Units per year, $K_r = 5$ Rs., $K_c = 7$ Rs., $C_R = 0.20$ Rs., $h = 1$ Rs.

$n=2, b=1, d=-2, \theta=4, K=20, N=10, w=0.03, \alpha$

$E = \frac{3}{3+2} = 0.06 = g$ and $\gamma = 0.4$

Then the values of A_1, A_2, A_3 and A_4 are as under:

$A_1 = -1, A_2 = 0.125, A_3 = 3689.32$ and $A_4 = 461.17$

Using these values,

$Q^* = 15.8303331$

and hence the expected net profit $Z^* = 13.28670478$ Rs.

4.4.2 Case B Non Linear Cost Discount Function

Consider the constant outlay curve (that is with $\eta = 1$) and the simple hyperbolic form of quantity discount case (with $r = 1$) that is the hyperbolic quantity discount cost function is represented by, $C\left(Q\right) = b + \frac{d}{Q}$

Where *b* and *d* are positive constants. Hence, the expected profit function of (13) reduces to,

$$Z = \frac{KNr}{\theta} - \frac{Q}{2}h\left(n-1\right)+w+\frac{KN}{2\lambda'\theta}h\left[n-1\right]+wQ\left(b+\frac{d}{Q}\right)^{-1}$$

$$-\frac{K_r\lambda}{\eta}+K_c\lambda\left[\frac{KN}{\theta Q}\left(b+\frac{d}{Q}\right)^{-1}-C\lambda-\frac{w}{2}\right]$$

…. (17)

$$Z = \frac{KN\gamma}{\theta} - C\lambda - \frac{w}{2} + CQh\left(n-1\right)+w+\frac{KN}{2\lambda'\theta}h\left(n-1\right)+w+\frac{Q^2}{d+bQ}$$

$$-\frac{K_r\lambda}{\eta}+K_c\eta+\frac{KN}{\theta\left(d+bQ\right)}$$

Hence, given below polynomial in *Q* helps to calculate the optimized lost size *Q*. i.e.

Following quadratic equation in *Q* (that is $\frac{\partial Z}{\partial Q} = 0$) gives:

$$\Rightarrow \quad Q^2 + H_1Q + H_2 = 0$$

Where

$$H_1 = \frac{2d}{b}$$

and

$$H_2 = \frac{\lambda' d^2 n\theta h\left(n-1\right)+w-2KNb\left(K_r\lambda+\eta K_c\lambda\right)}{bnh\left(n-1\right)+w+\left[b\lambda'\theta-KN\right]}$$

Where $\frac{b^2}{d^2} < \frac{n\,h\left(n-1\right)+w}{2\lambda'\left(K_r\lambda+nK_c\lambda\right)}$ satisfies the sufficient condition

Numerical Illustration

Using a hypothetical problem, to illustrate the above found hyperbolic cost discount function.

Consider

$\lambda' = 100$ Units per year, $K_r = 5Rs.\quad K_c = 7\ Rs.\quad C_r = 0.20\ Rs.\quad h = 1\ Rs.$

$n=2,\ b=5,\ d=2,\ \theta=4,\ K=20,\ N=10,\ w=0.03,\ \alpha=0.5,\ \lambda=2$ …. (18)

$$E = \frac{3}{3+2} = 0.6 = g \text{ and } \gamma = 0.4$$

Then the values of H_1 and H_2 are as under:

$$H_1 = 0.8 \text{ and } H_2 = -409.7467097$$

Using these values,

$$Q^* = 19.846153$$

and hence the expected net profit $Z^* = 1.011616198$ Rs.

5. Conclusion

This paper compares the values of Q^* and Z^* in linear and hyperbolic cases and shows that these values are inversely propositional to each other. To justify two cases have been discussed for two different functions under consideration. In case B it is proved that Q^* is higher in case of hyperbolic cost function whereas Z^* is lower and in case A low Q^* in linear cost function and high Z^* in linear function is proved.

5.1 Future Scope of Study

Above conclusions are based on the result achieved by sensitivity analysis done using Matlab software. This result can be further tested in future in real world SME scale industry related to medicine, raw and processed food and chemicals, with more of real time assumptions implied and further developing the given results.

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