Supply Chain Coordination with Quantity Discount for Seasonal Demand

Mikhail Sher[#1], Seung-Lae Kim[#2]

[#1]*Ph.D. Candidate, Decision Sciences Department, Drexel University*
3220 Market Street, 7th floor, Philadelphia, PA 19104
[1]mms355@drexel.edu (corresponding author)
[#2]*Professor of Operations Management, Decision Sciences Department, Drexel University*
[2]kimsl@drexel.edu

***Abstract*— Coordination between manufacturers and multiple buyers represents an important problem in supply chain management. In this paper, we develop a supply chain coordination mechanism in a system with a dominant manufacturer that delivers seasonal products to a group of buyers. These buyers have common replenishment times and receive delivery through a common delivery channel. A twice-stage ordering and production system is introduced in which the first order is placed at some time in advance of the selling season and a second order is placed closer to the selling period. This reorder strategy allows the buyer to collect additional information about seasonal demand, thereby reducing demand forecast error and simultaneously smoothing out production time. This twice-stage model results in savings for both manufacturer and the buyers. Strategies for developing sustainable cooperation between manufacturers and buyers are discussed in light of the conclusions of this model.**

***Keywords*—** *Logistics, Inventory Management, Optimal Order Policies, Supply Chain Coordination, Twice-Stage Ordering*

1. Introduction

This paper discusses a supply chain coordination mechanism for items with seasonal demand and quantity discounts. For the purposes of this discussion, buyers are defined as item distributors (as opposed to end consumers). Buyers are assumed to be affected by seasonal demand for the items being ordered. Ref. [1] classify item ordering (i.e. inventory replenishment) as a logistics decision which is “postponed in a ‘wait-and-see’ mode to optimize in the face of uncertainty” (p. 1219). As such, buyers prefer to receive orders at uneven intervals according to their seasonal demand requirements given that such order characteristics reduce uncertainty and minimize storage costs for the buyers. Manufacturers, conversely, are assumed to prefer large order quantities spaced equally throughout the year given that such order characteristics minimize production and order (i.e. shipping, holding, and production facility fixed costs) costs for the manufacturer. Prior research has proposed a number of methods for supply chain coordination in the presence of these conflicting motivations, including the use of credit to induce larger orders from buyers, revenue sharing, and twice-stage (TS) ordering and production systems which improve systemic coordination between buyers and manufacturers. While prior research has explored TS systems in the context of a single buyer and manufacturer, the usefulness of such a system for coordinating multiple buyers has not been fully explored. This paper develops a TS model which identifies optimal common order replenishment times and associated savings for a supply chain with a single dominant manufacturer delivering seasonal products to multiple buyers.

The framework introduced in this paper is applicable to non-agricultural seasonal goods, particularly those produced in a manufacturing setting. There are a variety of items in common use which fit this description, including items associated with specific holidays (i.e. decorations), seasonal attire (i.e. snow jackets or swimwear), and other items with seasonal demand. The remainder of this paper will provide an overview of prior literature followed by a description of the mathematical model which underpins our framework and numerical examples which illustrate the optimal common order replenishment time within our framework. Opportunities for future extensions of this model are also proposed, followed by some concluding remarks.

International Journal of Supply Chain Management
IJSCM, ISSN: 2050-7399 (Online), 2051-3771 (Print)

2. Prior Literature

In their paper entitled "Coordination of a single-manufacturer/multi-buyer supply chain with credit option," Ref. [2] focus their attention on a supply chain model where a single manufacturer sells a product to multiple buyers. The authors cite the need for such a model given that cases of single manufacturers supplying a product to a single buyer are uncommon in the modern production environment. Using a two-stage supply chain, the manufacturer supplies a product to multiple buyers located in different geographic areas. Given that prior research has identified that approximately sixty three percent of annual logistics costs can be tied to transportation, it is not surprising that consolidation of deliveries results in significant savings [3].

Ref. [2] develop their model using two transportation cost scenarios. The first is an ex-site delivery condition, where transportation costs are included in the product price and each buyer's order is handled independently. The second case is an ex-factory case where the cost of transportation is borne by the buyers. In both scenarios, coordinated product delivery at fixed intervals to multiple buyers sharing a common carrier reduces associated manufacturer and customer costs. Manufacturer can induce buyers to accept deliveries at fixed, rather than the buyer-preferred uneven, intervals through the provision of credit.

Ref. [4] discusses the problem of supply chain coordination for specific cases of seasonal products. Seasonal products are affected by comparatively short life cycles and uncertain demand. Unsold inventory loses much of its value once the selling season has ended, making excess inventory costly to buyers. Insufficient inventory acquisition, on the other hand, directly results in buyer welfare losses associated with forgone sales. Thus, accurate forecasting of product demand is crucial for buyers with respect to seasonal products. Naturally, the ability of the buyer to forecast product demand improves as the selling season approaches. Therefore, a retailer's preference is to place orders as close to the beginning of the selling season (i.e. late) as possible. Placing an order late, however, necessitates reductions in production time and results in increased costs for the manufacturer. The authors propose improvements to the operating system which allow for coordination between retailers and manufacturers and profit compensation plans which lead supply chain coordination to Pareto improvement. Choi, Ref. [5] provide support for this approach by showing that suppliers generally benefit from sharing good information with a retailer.

Ref. [6] develop a model featuring a quantity discount mechanism useful for facilitating supply chain coordination. This model is developed within a single product, multi-period setting where customer demand is probabilistic. In contrast to the model developed in our paper, Ref. [6] model a supply chain with a single buyer and manufacturer. The authors identify bounds within which the quantity discount results in increased profit and supply chain coordination. Additionally, the authors develop a method for apportioning increased profits between the buyer and manufacturer and derive the optimal discount level under that method.

For a comprehensive review of prior literature on supply chain coordination mechanisms, see Ref. [7].

3. Model Description

This paper develops a twice-stage (TS) ordering and production system model which extends the work of Ref. [2] and Ref. [4] for use with a dominant manufacturer and multiple buyers. Buyers place their first order at some time in advance of the selling season. A second, "late" order is then made closer to the selling period. The use of multiple order periods (i.e. reordering late) allows the buyer to collect more information about seasonal demand, thereby reducing demand forecast error and simultaneously smoothing out production time [8]. Information distortion mitigation facilitated by the TS model results in cost savings for both the manufacturer and the buyers.

In the original model of Ref. [2], the authors calculate total manufacturer costs in two ways: manufacturer total cost with individual deliveries and manufacturer total cost with simultaneous deliveries to all buyers. The authors then calculate the savings realized through the use of simultaneous deliveries instead of individual deliveries. We extend their analysis by introducing the aforementioned TS order and production system to account for seasonal demand as described by Ref. [4].

The model developed in our paper introduces a scenario with two periods, hereafter referred to as seasons: Summer, the season with higher item demand, and Winter, the season with lower demand. In a scenario with multiple buyers, it is logical to assume that buyers will have different order replenishment times given their unequal demands. We simulate this type of scenario using parameters provided by Ref. [9] for a single manufacturer and multiple buyers. Our analysis will focus on conditions under which total system costs are lower when the manufacturer uses their optimal shipment frequency (which is the same for all periods) rather than that of the buyer (which would differ between the two periods). Based on those conditions, we then calculate a range (minimum and maximum) within which manufacturers can offer quantity discounts to buyers in order to induce compliance with the manufacturer preferred shipping schedule.

To ensure conformance with other works in this area, we utilize the following notation from Ref. [2]:

Notation:

D total annual demand, $D=\sum_{i=1}^{n} D_i$, where D_i is the demand of ith buyer

S_{bi} ordering cost of ith buyer, where i = 1,..,n

S_m setup cost of the manufacturer

P production rate of the manufacturer, $P > D$

h_{bi} holding cost of ith buyer, where i = 1,..,n

h_m holding cost of the manufacturer

t_i individual optimum order interval of ith buyer (decision variable)

t individual optimum production run length of the manufacturer (decision variable)

tc_i minimum credit time required by ith buyer (decision variable)

T common order replenishment time (decision variable)

K integer lot size multiplier (decision variable)

Q_v economic production quantity of the manufacturer

Q_i economic order quantity of ith buyer

TCM total relevant cost per unit time of the manufacturer

C_i individual transportation cost per delivery borne by the manufacturer

C_c common transportation cost per delivery borne by the manufacturer

β_m average inventory factor at manufacturer's side, i.e., $(K-1)-(K-2)\rho_m$

ρ_m utilization rate of the manufacturer, $\frac{D}{P}$, where $0<\rho_m\le 1$

The following results in Eqs. (1) - (4), from Ref. [2] are subsequently utilized in our computational model.

With no coordination, the manufacturer and the buyers optimize their costs independently and the manufacturer delivers the items to every buyer individually. Thus, the manufacturer's total relevant cost per unit of time, denoted TCM_{bc}, is equal to the sum of the manufacturer's setup cost, order processing cost (including individual transportation cost), and inventory holding cost calculated as follows:

$$TCM_{bc}=\frac{S_m}{t}+\sum_{i=1}^{n}\frac{C_i D_i}{Q_i}+\frac{1}{2}h_m DT(1-\rho_m) \quad (1)$$

When the coordination through the common order replenishment time (T) is implemented, the manufacturer's total relevant cost is obtained by Eq. (2) below, which is the sum of setup cost, common transportation cost including order processing cost, inventory holding cost and compensation cost. Notice that the manufacturer, with the coordination, incurs additional (compensation) cost - which will be given to buyers to offset their increased costs. Buyer's increased costs are mainly inventory costs that are incurred due to the change of order interval from the buyer's optimal t_i to the common replenishment time T. The amount of compensation is the difference between the total costs before and after the coordination as shown in the last term of Eq. (2). In this paper, the compensation is assumed to be given to the buyers in the form of quantity discount or other types of credit.

$$TCM(K,T) = \frac{S_m}{KT} + \frac{C_c}{T} + \frac{1}{2} h_m DT\{(K-1)-(K-2)\rho_m\} + \sum_{i=1}^{n} \{ \left(\frac{S_{bi}}{T} + \frac{1}{2} h_{bi} D_i T \right) - \sqrt{2 S_{bi} D_i h_{bi}} \} \quad (2)$$

The manufacturer's savings in this coordination mechanism are expected to be greater than the compensation costs given to all the buyers, given that the common order replenishment time is optimized. The optimal common order replenishment time T^*, is calculated using the following equation, which is obtained by differentiating Eq. (2) with respect to T, while keeping K fixed:

$$T^*(K) = \sqrt{\frac{2((S_m / K) + C_c + \sum_{i=1}^{n} S_{bi})}{h_m D \beta_m + \sum_{i=1}^{n} h_{bi} D_i}} \quad (3)$$

We now substitute the value of T^* in Eq. (3) to Eq. (2) and optimize with respect to K. The optimal integer value of lot size multiplier, denoted by K_0, is obtained by selecting $K = K_0$, such that $Z(K_0) \le Z(K_0 - 1) \cap Z(K_0) \le Z(K_0 + 1)$. The optimal integer value K_0 is the one that satisfies the inequality relationship in Eq. (4).

$$K_0(K_0 - 1) \le \frac{S_m\{((\sum_{i=1}^{n} h_{bi} D_i)/(h_m D)) + (2\rho_m - 1)\}}{(1-\rho_m)(C_c + \sum_{i=1}^{m} S_{bi})} \le K_0(K_0 + 1) \quad (4)$$

The optimal K is substituted to Eq. (3) to find the optimal T, which in return is substituted back to Eq. (2) to find the manufacturer's optimal total costs. In the following numerical example section, we apply the above results to the case where the demand shows typical seasonality patterns.

4. Numerical Analysis

Table 1 provides data from Ref. [9] related to the Summer Season case, consistent with the analysis of Ref. [2].

Table 1: Summer Season Case Parameters

Buyer	**Demand (D_i)**	**Ordering Cost (s_{bi})**	**Holding Cost (h_{bi})**	**Transport Cost**
1	8	20	0.008	40
2	15	15	0.009	40
3	10	6	0.01	40
4	5	10	0.01	40
5	20	18	0.007	40

Table 2 provides similar data for the Winter Season case, a second period where demand is lower than observed in the Summer Season case.

Table 2: Winter Season Case Parameters

Buyer	**Demand (D_i)**	**Ordering Cost (s_{bi})**	**Holding Cost (h_{bi})**	**Transport Cost**
1	6	20	0.008	40
2	11	15	0.009	40
3	7	6	0.01	40
4	3	10	0.01	40
5	12	18	0.007	40

We also consider parameters related to manufacturer costs and total item demand in Table 3.

Table 3: Other Relevant Model Parameters

P	193.333	Production Rate of the Manufacturer
S_m	250	Setup Cost of the Manufacturer
h_m	0.005	Holding Cost of the Manufacturer
C_c	100	Common Transportation Cost per Delivery (borne by manufacturer)
D	58	Total Demand
ρ_m	0.300	Utilization Rate (D/P)

In order to address the different demand sizes in two periods, we utilize the model developed by Ref. [2] to analyse the identified supply chain problem: a manufacturing firm facing two seasons, each with different demands. The model can be applied separately to the Summer (higher demand period) and Winter (lower demand period), with separate consideration yielding two different common replenishment times (T_1 and T_2, respectively for the Summer and Winter season). Such analysis will also yield different cost savings (i.e. the common replenishment time case vs. the no coordination case) for each demand period.

For the scenario utilizing the values given in Tables 1 through 3 above, analysis (Eqn. 3) yields T_1=27.47 and cost saving of $59.39 for the summer season. For the winter season, analysis yields T_2=33.43 and cost savings of $31.52. Costs are given as cost per unit of time. Appendix A and B provide supporting numerical calculations for the Summer and Winter season cases, respectively. Assuming that each period, summer and winter, is of equal length, the individual period costs savings can be averaged over both periods by calculating their sum and dividing by two. Average savings between the two periods, therefore, is $45.45.

Given the aforementioned manufacturer preference for the cost benefits of large order quantities at fixed, equally spaced intervals, higher average savings may be achieved by using the same common replenishment time (T) for both seasons rather than the season-specific values of T calculated previously. A detailed numerical analysis of the savings associated with using the same T for both periods is provided in the appendices. This analysis is performed using the value of T which optimizes manufacturer savings for both periods. Appendix C describes the effects of using that optimal value of T*(K) (calculated as 96.20 in our analysis) on Summer season savings for the two-period model, while Appendix D describes similar outcomes for the Winter season savings. The selection of an optimal value for T*(K) used in both seasons leads to average cost savings as high as $81.52, representing an improvement of 79.36% over the average savings previously noted across the two base seasonal cases.

5. Practical Applications and Proposed Extensions

The developed model highlights the desirability of supply chain coordination between a manufacturer and buyers for products with seasonal demand patterns. Such an approach is highly applicable to a number of industries including the replenishment of frozen goods for supermarket stores [10], as well as B2B electronic markets [11]. To make such coordination practicable, a coordination framework must be developed which specifies mutually beneficial methods of cooperation. Selecting a coordination mechanism is an important tactical-strategic decision [12]. In order to support the model developed in this paper, an attempt is made to suggest and analyse methodological approaches to supply chain coordination between a manufacturer and distributors which result in total system cost minimization for both the production and distribution processes. A cooperation framework which successfully promotes this type of cooperative behaviour must naturally involve appropriate incentive alignment between the parties [13]. Prior literature has highlighted the limitations of revenue-sharing contracts for incentive alignment and cooperation. Specifically, Ref. [14] demonstrate that supply chains within which the buyer has some influence over demand through their actions or where they compete with other companies on both price and quantity may not achieve coordination through revenue-sharing contracts. Mutual agreement based on concessions between delivery size and interval, as described in our model, may be more practicable given the aforementioned limitations of revenue-sharing. We provide, therefore, two approaches which might allow for a sustained cooperation framework between a manufacturer and buyers in a supply chain for seasonal goods.

Manufacturer discounts in exchange for manufacturer preferred delivery schedule

Cooperation between the manufacturer and buyers can be induced by the manufacturer providing both a quantity discount and a constant reorder interval discount. In exchange for these concessions, the buyers agree to receive seasonal goods in equal size batches throughout the year. The manufacturer benefits from such an arrangement through reduced production costs achieved by eliminating the usual spikes in demand associated with seasonal goods. Regular shipments also allow the manufacturer to better manage the production schedule, possibly leading to lower capacity requirements. Assuming the manufacturer sets discounts at appropriate levels, the buyers are able to benefit from such an arrangement by offsetting storage costs through the receipt of those discounts. Examples of seasonal goods which might benefit from such an arrangement are seasonal apparel (e.g. winter coats or swimwear), holiday paraphernalia (e.g. items used for Christmas, Chanukah, or Diwali), and seasonal sports goods (e.g. skis). We can assess the feasibility of cooperation in this scenario by comparing the highest amount the manufacturer is willing to pay and the lowest among the buyers are willing to accept. Sustainability of this cooperative solution requires the first amount to exceed the second.

Mutual agreement on concessions between delivery size and interval

Cooperation can also be sustained through bargaining between the manufacturer and the buyer with respect to delivery parameters. Given the manufacturer's stated preference for fixed and equally spaced delivery intervals with large order quantities, and the buyer's stated preference for unequally spaced delivery intervals with demand-specified order amounts, it is logical that delivery size and interval would be areas of negotiation between the two parties. Specifically, buyers could make concessions in the size of their orders while manufacturers could agree to concessions in the delivery schedule. Such a solution would be sustainable in the presence of mutually beneficial trade-offs between these delivery parameters.

It is important to recognize that individual firm characteristics have a significant impact on a cooperative solution such as this. Identification of delivery parameter specifications at which buyers and manufacturers are able to make mutually beneficial trade-offs between delivery size and schedule, for instance, requires an examination of relative cost structures between the buyer and manufacturer. It is clear that a manufacturer might have higher costs in certain areas as compared to buyers, whereas buyers might have cost advantages in other areas. Minimizing total system costs, therefore, will include shifting costs to the party which has a comparative cost advantage relative to that cost. Cost sharing, as with coordinated advertising, is an additional mechanism by which firms may maximize system profits while simultaneously minimizing system costs [15].

6. Conclusion

This paper provides analysis of a supply chain for items with seasonal demand and quantity discounts. The developed model analyses conditions under which total system costs are lower when the manufacturer uses their optimal shipment frequency rather than that of the buyers. First such condition is manufacturer's preference for large order quantities spaced equally throughout the year in order to minimize their production and order costs. The second condition is that the savings manufacturer realizes by using the same common replenishment time (T) for both seasons rather than the season-specific T's exceed the combined additional costs buyers incur by accepting deliveries in equal size batches throughout the year. The analysis clearly shows that such conditions exist and delivers mutual benefits to both the manufacturer and the buyers, provided that a cooperative solution can be reached which promotes the necessary supply chain coordination. A game theoretic approach (e.g. Stackelberg Equilibrium) could also be used for problems of our type [16].

The cooperative solutions discussed above illustrate the benefits which can be achieved through the coordination of production and purchase activity. Mutual benefits (in the form of higher profits) can be achieved for both manufacturers and buyers by utilizing comparative advantages to lower system costs.

References

[1] A. Gupta and C. D. Maranas, Managing demand uncertainty in supply chain planning, *Computers and Chemical Engineering, 27(8-9)*, 2003, 1219-1227.

[2] S.P. Sarmah, D. Acharya, and S.K. Goyal, Coordination of a single-manufacturer/multi-buyer supply chain with credit option, *International Journal of Production Economics, 111(2),* 2008, 676-685.

[3] T. Schaefer, Supply chain comment: Rising transportation costs in 2011, Retrieved from www.scdigest.com/assets/Experts/Guest_11-02-10.php

[4] J. Chen and L. Xu, Coordination of the supply chain of seasonal products, *IOSR IEE Transactions on Systems, Man and Cybernetics, 31(6),* 2001, 524-532.

[5] T.M. Choi, J. Li, and Y. Wei. "Will a supplier benefit from sharing good information with a retailer?" *Decision Support Systems,* 56, 2013, 131-139.

[6] J. Li and L. Liu, Supply chain coordination with quantity discounts, *International Journal of Production Economics, 101(1)*, 2006, 89-98.

[7] K. Arshinder, A. Kanda, and S.G. Deshmukh, A review on supply chain coordination: Coordination mechanisms, managing uncertainty, and research directions, in T.M. Choi and T.C. Edwin Cheng (eds.), *Supply Chain Coordination under Uncertainty,* (New York: Springer-Verlag, 2011) 39-82.

[8] S. Chopra and M.S. Sodhi, Managing risk to avoid supply-chain breakdown, *MIT Sloan Management Review, 46(1)*, 2004, 53-62.

[9] A. Banerjee and J.S. Burton, Coordinated vs. independent inventory replenishment policies for a vendor and multiple buyers, *International Journal of Production Economics, 35(1-3)*, 1994, 215-222.

[10] M. Hammer, The super efficient company. *Harvard Business Review, 79(8),* 2001, 82-91.

[11] C. Wang, M. Benaroch, Supply chain coordination in buyer centric B2B electronic markets, *International Journal of Production Economics, 92*, 2004, 113-124.

[12] L. Xu and B.M. Beamon, Supply Chain Coordination and Cooperation Mechanisms: An Attribute-Based Approach. *Journal of Supply Chain Management*, *42*(1), 2006, 4-12.

[13] J.M. Whipple and R. Frankel, Strategic alliance success factors. *Journal of Supply Chain Management*, *36*(3), 2000, 21-28.

[14] G. P. Cachon and M. A. Lariviere, Supply chain coordination with revenue-sharing contracts: strengths and limitations, *Management Science, 51(1)*, 2005, 30-44.

[15] J. Zhang, Q. Gou, L. Liang, and Z. Huang, Supply chain coordination through cooperative advertising with reference price effect, *Omega, 41(2),* 2013, 345-353.

[16] A. N. Sadigh, M. Mozafari, and B. Karimi, Manufacturer-retailer supply chain coordination: A bi-level programming approach, *Advances in Engineering Software, 45(1)*, 2012, 144-152.

Appendixes

Appendix A: Summer Season Base Case

Panel A: Economic Order Intervals of the ith Buyer (t_i) and Determination of K_0

t_1	25		C_1*D_1/Q_1	12.8
t_2	14.91		C_2*D2/Q_2	40.25
t_3	10.95		C_3*D_3/Q_3	36.51
t_4	20		C_4*D_4/Q_4	10
t_5	16.04		C_5*D_5/Q_5	49.89
t	49.63		Sum	149.45
Determine K_0				
k_0	2			
$k_0*(k_0-1)$	2	<=	2.72	
$k_0*(k_0+1)$	6	>=	2.72	

Panel B: Total Relevant Manufacturer Costs (TCM), total savings, and minimum credit times (tc)

Q_v		2878.49	
[Eq. 4]	$K_0(K_0-1)$<=	2.72	<=$K_0(K_0+1)$
	K	2	
	βm	1	
T*(K) [Eq. 3)		27.47	
TCM_{bc} [Eq. 1]		157.28	
$TCM_{common\ repl}$		97.89	
Total savings		59.39	
tc_1(min)		0.11	
tc_2(min)		2.87	
tc_3(min)		4.97	
tc_4(min)		1.02	
tc_5(min)		2.38	
Max of min credit times		4.97	

Appendix B: Winter Season Base Case

Panel A: Economic Order Intervals of the ith Buyer (t_i) and Determination of K_0

t_1	28.87		C_1*D_1/Q_1	8.31
t_2	17.41		C_2*D2/Q_2	25.28
t_3	13.09		C_3*D_3/Q_3	21.39
t_4	25.82		C_4*D_4/Q_4	4.65
t_5	20.70		C_5*D_5/Q_5	23.19
t	60.52		Sum	82.81
Determine K_0				
k_0	2			
$k_0*(k_0-1)$	2	<=	2.74	
$k_0*(k_0+1)$	6	>=	2.74	

Panel B: Total Relevant Manufacturer Costs (TCM), total savings, and minimum credit times (tc)

Q_v		2360.39	
[Eq. 4]	$K_0(K_0-1)$<=	2.74	<=$K_0(K_0+1)$
	K	2	
	βm	1	
T*(K) [Eq. 3)		33.43	
TCM_{bc} [Eq. 1]		89.22	
$TCM_{common\ repl}$		57.71	
Total savings		31.52	
tc_1(min)		0.31	
tc_2(min)		3.84	
tc_3(min)		6.19	
tc_4(min)		0.87	
tc_5(min)		2.42	
Max of min credit times		6.19	

Appendix C: Same T for two seasons (Summer)

Panel A: Economic Order Intervals of the ith Buyer (t_i) and Determination of K_0

t_1	25.00		C_1*D_1/Q_1	12.80
t_2	14.91		C_2*D2/Q_2	40.25
t_3	10.95		C_3*D_3/Q_3	36.51
t_4	20.00		C_4*D_4/Q_4	10.00
t_5	16.04		C_5*D_5/Q_5	49.89
t	49.63		Sum	149.45
Determine K_0				
k_0	2			
$k_0*(k_0-1)$	2	<=	2.72	
$k_0*(k_0+1)$	6	>=	2.72	

Panel B: Total Relevant Manufacturer Costs (TCM), total savings, and minimum credit times (tc)

Q_v		2878.49	
[Eq. 4]	$K_0(K_0-1)$<=	2.72	<=$K_0(K_0+1)$
	K	2	
	βm	1	
T*(K) [Eq. 3]		96.20	
TCM_{bc} [Eq. 1]		164.25	
$TCM_{common\ repl}$		56.69	
Total savings		107.56	
tc_1(min)		26.35	
tc_2(min)		34.35	
tc_3(min)		37.77	
tc_4(min)		30.18	
tc_5(min)		33.40	
Max of min credit times		37.77	

Appendix D: Same T for two seasons (Winter)

Panel A: Economic Order Intervals of the ith Buyer (t_i) and Determination of K_0

t_1	28.87		C_1*D_1/Q_1	8.31
t_2	17.41		C_2*D2/Q_2	25.28
t_3	13.09		C_3*D_3/Q_3	21.39
t_4	25.82		C_4*D_4/Q_4	4.65
t_5	20.70		C_5*D_5/Q_5	23.19
t	60.52		Sum	82.81
Determine K_0				
k_0	2			
$k_0*(k_0-1)$	2	<=	2.74	
$k_0*(k_0+1)$	6	>=	2.74	

Panel B: Total Relevant Manufacturer Costs (TCM), total savings, and minimum credit times (tc)

Q_v		2360.39	
[Eq. 4]	$K_0(K_0-1)$<=	2.74	<=$K_0(K_0+1)$
	K	2	
	βm	1	
T*(K) [Eq. 3]		96.20	
TCM_{bc} [Eq. 1]		93.51	
$TCM_{common\ repl}$		38.03	
Total savings		55.47	
Average savings for two seasons		**81.52**	
tc_1(min)		23.57	
tc_2(min)		32.27	
tc_3(min)		35.90	
tc_4(min)		25.75	
tc_5(min)		29.63	
Max of min credit times		35.90	