Mathematical Modelling Solutions for Stock and Cost Dependent Inventory in a Limited Display Space Warehouse

Shradha Goyal[#1], Rudresh Pandey[*2]

[#] *Department of Management*

Jagannath International Management School
Kalkaji, New Delhi, India

[2]bansal.shradha@gmail.com
[*] *ABES (EC)*
Ghaziabad, U.P, India

[1]rudreshpandey@gmail.com

***Abstract*— Study in this paper is concerned with optimization of both quantity of order and selling price together, considering EOQ model for items with depreciating nature. It is based on the few assumptions like rate of demand is dependent on level of stock displayed on shelf as well as per unit selling rate, also, the space for stock display is finite. Two mathematical models are studied to investigate the further revised EOQ modelling for obtaining maximum profits and also develop models for such optimized solutions. Justification and analysis of the work developed and studied is done through sensitivity analysis and numerical examples.**

***Keywords*—** *Inventory management, cost, demand, stock dependent, depreciation.*

1. Introduction

In the traditional models for stock management, the rate of demand is frequently understood to be either invariable or dependent on time but not dependent on the levels of stock. However, sensibly higher shelf space of a product invites higher sales of the goods. This depends on quality, quantity and demand of the product. On the other hand, lower quantity on display implies the assumption of less demand of the product. Hence, we can conclude that the occupied shelf space and visibility of certain common consumable goods influence its rate of demand. With the increase in purchasing trend, in past few years, the marketing analyst and experts have observed the parallel direction relation between demand and amount of shelf space for a product.

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2. Literature Review

Ref [1] marked the fact that huge quantity of user commodities displayed in a superstore would draw more demand. Ref [13] also studied the direct proportionality between demand at the retail store and the quantity of displayed stock. He recognized an EOQ based algorithm for patter on demand dependent on inventory level in a power from of equation. He worked on a inventory model for multi-units stock with property of depreciation and demand dependent on quantity using non linear goal programming algorithm with resources as constraints. Ref [2] Offered a model based on rate of demand dependent on the instant replenishment of stock levels till the optimum level is achieved, and assumes that after this level the rate of demand becomes stable. He dismissed the idea of complete use of stock in one cycle time which was imposed. Further studied the work of [3] for delicate goods that decline at a regular pace. Ref [7] Worked on expansion of an inventory model where demand is dependent on level of inventory by adding to it casual yield. Ref [14] Studied Urban's model for steadily depreciating things. [8] Further worked on the EOQ model in which the order is a function of more than one variable i.e. cost, instance, and stock level. Ref [15] further researched on the EOQ model by considering a nonlinear holding cost. Ref [5] Studied multi-item stock models for depreciating goods with demand dependent on stock in an assumed environment. Ref [9] gave a summarized and combined model of existing inventory-control model, product assortment model, and display - space availability models. Ref [6] Developed an EOQ model for multi-period with stock-dependent, and rate of demand sensitive with

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variations in price. He suggested a model for selling depleting stock from multi outlets, under uni - management with restrictions of stock and net available display space. Other papers related to this area are [4], [10], [12] and others.

Ref [11] showed, "Probability of high sales with reference to the high amount of display quantity. However, this policy followed in extreme may leave an adverse effect as well". Hence, this paper primarily studies optimum level of inventory, to show the facts that the majority trade outlets have restricted display area and secondly studies how to avoid an unconstructive notion on customer because of extremely accumulated stock customer's way. As the rate of demand is influenced by level of inventory as well as selling price, these factors are thus considered to establish an inventory model in which the rate of demand on quantity of stock displayed and the selling price. Then, the essential assumptions for the proposed model and the notations used in the paper are provided. Further, algorithm for mathematical model is set up. The properties of the optimized result are discussed along with presentation and justification of solution algorithm and numerical. A simpler algorithm for optimized cycle time, economic order quantity and re - ordering level is discussed. Then the last section is to discuss the conclusion and future scope of the study.

3. Problem Formulation

This problem is to establish an optimized value of *t*, P and Y so that the net mean profit in a process of replenishment is optimal.

4. Notations And Assumptions

Following assumptions and notations are used for studying, a single-item deterministic inventory model for depleting stock with rate of demand dependent on price and stock.

1) No Shortages allowed.
2) Upper limit of display quantity is B as per available space and requirement.
3) Instant and infinite rate of Replenishment.
4) Known and fixed procurement cost K per order.
5) Known and fixed purchase cost c per unit and the holding cost h per unit per unit time. The fixed selling price p per unit is independent variable within the replacement rounds, where P > C.
6) The continuous rate of depreciating θ ($0 \leq \theta < 1$) is only applicable to current in hand stock. Two cases are possible for the cost of depreciating goods (1) there is a non positive scrap value that is, value is either negative or zero; (2) There is a positive disposal cost, that is, value which is positive. Note that C > s (or = s).
7) All replacement trends are similar. Therefore, a usual cycle of scheduling with length = t (i.e., the range of schedulling is [0, T]).
8) The rate of demand R(I(t), P) can be calculated by:-
$R(I(t), P) = \alpha\ (P) + \beta\ I\ (t)$, where I(t) is the level of inventory level at any time t, $\beta \geq 0$, and α (P) is a non-negative function of P with α ' (P) = d α (P) /dP < 0.
9) Urban (1992) gave a theory which said, "it may be desirable to order large quantities, resulting in stock remaining at the end of the cycle, due to the potential profits resulting from the increased demand." Accordingly, the starting and final points in levels of inventory Y may not be zero (i.e., Y ? 0). If, order quantity Q enters the inventory system at time t = 0. Therefore, I(0) = q + Y. In the interval [0, t], the depreciation in inventory is cumulative of demand and depreciation. At any time t, the inventory level falls to y, i.e., I(t) = Y. The starting and final points in inventory level y are known as ordering point.

4.1 Mathematical Model and Analysis

At an instant $t = 0$, the level of inventory $I(t)$ has its maximum value (with $I \leq B$) because of replenishing of economic order quantity Q. Then the level of inventory reduces slowly to Y till the last day of the cycle time at $t = T$ mainly because of demand and secondly due to depletion. A graphical representation of such system is shown in Figure 1.

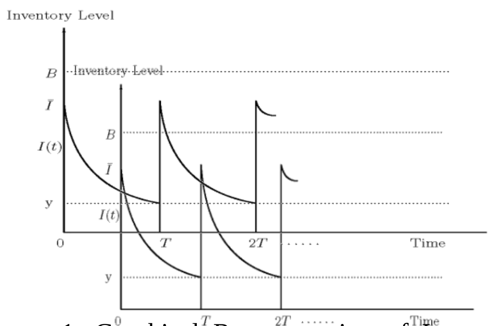

Figure 1. Graphical Representation of Inventory System

Level of inventory at any time t can be represented by:-

$I'(t) + \theta I(t) = -R(I(t), P)$, $0 \le t \le T$, (1)

Considering the boundary condition I (t) = Y. Therefore, solution of (1) is given by,

$$I(t)=Ye^{(\theta+\beta)(T-t)}+\frac{\alpha(P)}{(\theta+\beta)}\left(e^{(\theta+\beta)(T-t)}-1\right) \quad (2)$$

using (2), the net profit tP over the period [0, t] is denoted by,

$$tP = (P-C)\int_0^t R(I(t),P)\,dt - K - [h+\theta(C+s)]\int_0^t I(t)\,dt$$

$$= (P-C)\alpha(P)\,t - K + [(P-C)\beta - h - \theta(C+s)] \times \left[\int_0^T Ye^{(\theta+\beta)(T-t)}+\frac{\alpha(P)}{(\theta+\beta)}\left(e^{(\theta+\beta)(T-t)}-1\right)dt\right]$$

$$= (P-C)\alpha(P)\,t - K + [(P-C)\beta - h - \theta(C+s)] \times \left[\frac{1}{\theta+\beta}\left(Y+\frac{\alpha(P)}{\theta+\beta}\right)\left(e^{(\theta+\beta)t}-1\right)-\frac{\alpha(P)}{\theta+\beta}t\right] \quad (3)$$

Thus, the average profit per unit time is

AP = tP / t

$$=\left\{(P-C)\alpha(P)\,t - K + [(P-C)\beta - h - \theta(C+s)] \times \left[\frac{1}{\theta+\beta}\left(Y+\frac{\alpha(P)}{\theta+\beta}\right)\left(e^{(\theta+\beta)t}-1\right)-\frac{\alpha(P)}{\theta+\beta}t\right]\right\}/t \quad (4)$$

4.1.1 Necessary conditions:

Taking the first derivative in (4) w.r.t "t", we get,

$= \partial AP / \partial t$

$$=\frac{1}{t^2}\left\{K + [(P-C)\beta - h - \theta(C+s)] \times \left(\frac{1}{\theta+\beta}\right)\left(y+\frac{\alpha(P)}{\theta+\beta}\right)\left[(\theta+\beta)te^{(\theta+\beta)t} - e^{(\theta+\beta)t}+1)\right]\right\} \quad (5)$$

Using Appendix 1, we get that $[(\theta+\beta)te^{(\theta+\beta)t} - e^{(\theta+\beta)t} + 1] > 0$.

And, $(P-C)\beta$ is the profit per unit of inventory and $[h + \theta(C + s)]$ is the total of holding cost and cost of depreciation per unit inventory.

If, $\Delta_1 = (P-C)\beta$ and $\Delta_2 = h + \theta(C + s)$ and based on the values of Δ_1 and Δ_2 , two cases are discussed for finding the optimal value of t* = t :

Case 1) $\Delta_1 \ge \Delta_2$ (profit from inventory)

$\Delta_1 \ge \Delta_2$ is that the profit per unit inventory is greater than the total of carrying and depreciation costs per unit inventory. Implies, inventory is a profit giving factor. Hence from Appendix 1, $\partial AP / \partial t > 0$, if $\Delta_1 \ge \Delta_2$, (4) becomes an increasing function of t with $I(t) \le B$.

Hence, we can say that piling up of inventory up to the optimum level B of inventory can be exhibited in shop shelves without creating a adverse perception of consumers.

Hence I(0) = B, and this implies,

$t = 1/(\theta+\beta)\ \ln((B(\theta+\beta)+\alpha(P))/(Y(\theta+\beta)+\alpha(P)))$

This means that t is dependent on P and Y. Putting (6) in (4), it shows that AP is a function of Y and P. The necessary conditions for optimization of AP are $\partial AP / \partial Y = 0$ and $\partial AP / \partial P = 0$. Hence, two conditions can be derived:

$$\frac{-K(\theta+\beta)^2}{\Delta_1-\Delta_2} = (\theta+\beta)(Y-B) + [Y(\theta+\beta)+\alpha(P)]\ln\left(\frac{B(\theta+\beta)+\alpha(P)}{Y(\theta+\beta)+\alpha(P)}\right) \quad (7)$$

$$\frac{\alpha(P)\theta+[\theta(P+s)+h]\alpha'(P)}{\theta+\beta}t^2 + \frac{\left(e^{(\theta+\beta)t}-1\right)}{\theta+\beta}\left\{\beta\left(Y+\frac{\alpha(P)}{\theta+\beta}\right) + (\Delta_1-\Delta_2)\frac{\alpha'(P)}{\theta+\beta}\right\}t$$

$$=-\left\{K+(\Delta_1-\Delta_2)\left(\frac{1}{\theta+\beta}\right)\left(Y+\frac{\alpha(P)}{\theta+\beta}\right)\times\left[(\theta+\beta)te^{(\theta+\beta)t}-e^{(\theta+\beta)t}+1\right]\right\}\frac{\partial t}{\partial P} \quad (4)$$

(8)

And,

(1) $$\frac{\partial t}{\partial P} = \frac{(y-B)\alpha'(P)}{[\alpha(P)+(\theta+\beta)B][\alpha(P)+(\theta+\beta)Y]} \quad (9)$$

Thus, equations (7) and (8) gives optimized values P* and Y*of P and Y respectively. Putting P* and Y* in (6), the optimized value of t* is obtained.

Validity of sufficient conditions cannot be clearly justified analytically as AP(Y, P) is a complex function. However, the optimal solution is possible by putting numerical values in illustrations. Firstly, it was assumed in case1 that building inventory is profitable. Secondly, AP is a continuous function in Y and P over a compact set [0, B] X [0, L], for a sufficiently large number L. Hence, AP will have a maximum value. Also, the solution obtained satisfies (7) and (8). If this solution obtained is unique in nature, then it is the optimized value, otherwise, we will have to put y and p in (4) and find the optimized value.

Case 2) $\Delta_1 < \Delta_2$ (no profit from inventory)

Initially, differentiating AP partially w.r.t Y, to get

$\partial AP/\partial Y= 1/t\ [(\Delta_1-\Delta_2)\ 1/(\theta+\beta)\ (e^{(\theta+\beta)t}-1)]<0$ (10)

Next, substitute Y*=0 in (4) to get AP as a function of P and t.

Hence, the necessary condition for values of AP to be optimum is $\partial AP/\partial P=0$ and $\partial AP/\partial t=0$, giving the following cases:

(i)

$$\frac{-K(\theta+\beta)^2}{\alpha(P)(\Delta_1-\Delta_2)} = (\theta+\beta)te^{(\theta+\beta)t} - e^{(\theta+\beta)t} + 1 \quad (11)$$

(ii)

$$[\alpha(P)\theta + (P\theta + h + \theta s)\alpha'(P)]t = \frac{-(e^{(\theta+\beta)t}-1)}{\theta+\beta}[\beta\alpha(P) + (\Delta_1-\Delta_2)\alpha'(P)] \quad (12)$$

Values of t and P can be obtained from (11) and (12). Putting Y*= 0 and the values of t and P obtained from (11) and (12) in (2), we get, either I(0) < B or I(0) ≥ B. If I(0) < B, then optimum values are t*= t, P*=P and I(0) = q*. If I(0) ≥ B, then for I(0) = B, we get,

$$t = 1/(\theta+\beta)\ \ln((B(\theta+\beta)+\alpha(P))/(\alpha(P))) \quad (13)$$

This is a function of P. Substituting Y* = 0 and (13) in (4), to get AP as a function of P. Hence, the necessary conditions for AP to have maximum value is dAP / dP = 0. Therefore,

$$\frac{\alpha(P)\theta + [\theta(P+s)+h]\alpha'(P)}{\theta+\beta}T^2 + \frac{(e^{(\theta+\beta)t}-1)}{\theta+\beta}\left[\beta\frac{\alpha(P)}{\theta+\beta} + (\Delta_1-\Delta_2)\frac{\alpha'(P)}{\theta+\beta}\right]t = -\left\{K + (\Delta_1-\Delta_2)\frac{\alpha(p)}{(\theta+\beta)^2}\left[(\theta+\beta)te^{t} - e^{(\theta+\beta)t} + 1\right]\right\}\frac{dt}{dP} \quad (14)$$

Where t is as defined in (13) and in

$$\frac{dT}{dP} = \frac{-B\alpha'(P)}{[\alpha(P)+(\theta+\beta)B]\alpha(P)} \quad (15)$$

The optimum value P* is calculated from (14), putting P* in (13), the optimum value t* is determined.

5. Results

The above solved model for finding an optimized value of selling price (P*), ordering point (Y*), cycle time (t*), and economic order quantity (q*) can be summarized as given:

1. Solving equations (7) and (8), to get the values of P and Y.

2. When $\Delta_1 \geq \Delta_2$, then P*=P, Y*=Y, q*= B – Y* and the optimal value T* can be obtained by putting p and y in (6).

3. When $\Delta_1 < \Delta_2$, then take y*=0. Solving (11) and (12) to get the values of T and p. Substituting y * = 0, p and T in (2) and find I(0). If I (0) < B, then the optimized values are as; T* = T, p* = p and Q* = I(0).

4. I (0) > B is given by solving equations (11) and (12) simultaneously for solutions t and P, and the optimized values of P* can be obtained by (14) and t* is obtained by putting P* in (13), and q* = I (0) by putting P* and t* in (2).

5.1 Numerical examples

To illustrate the above discussed model, numerical example given below are solved. .Initially, take the function α (P) = xP−r, where x, r are negative constants. That is, it is reflected that demand is a constant elasticity s of the price.

Example 1. Let K = Rs.10 per cycle, x = 1500 units/ time, h = Rs.0.5 per unit /time, s = Rs.10 /unit, r = 3.5 and θ = 0.15. Using the above discussed methodology, the optimized solution thus obtained in the following example. Since (4), (6), (7), (8), (9) nonlinear in nature, and are solved using software. The computed optimized values of P, Y, t, q and AP with respect to different values of β, B, C are shown in Table 1(given in the last)

Table 1 shows when $\Delta_1 \geq \Delta_2$ and the following conclusions are made,

1) An increasing value of β results in increased values of q* and AP*, and lower values of Y*, P * and t*. It shows that increase in rate of demand will result in increased optimal EOQ and Average contribution, and decrease in optimum point of ordering, cycle time and selling price.

2) An increasing value of β results in increased values of q*, t* and AP*, but decreasing values of Y*and P*. Hence, an addition in display space will give increase in optimized EOQ, cycle time and average contribution, but decrease in

optimal point of ordering point as well as selling price.

3) Higher value of C gives higher values of q* and t*, and lower values of Y* and AP*. Hence, the higher cost of purchasing gives higher optimal cycle time and economic order quantity, but a lower optimal point of ordering and mean profit.

Example 2 Let K = \$15/cycle, x = 1500 units / time, h = \$0.5 per unit / time, c = \$1.5 /unit, s = \$10 / unit, r = 3.8, θ = 0.15 and B = 350. Using Step 3 of the discussed method, the optimum point of ordering (Y*) = 0. Solving equations (2), (4), (11) and (12) in software. The computed results for the optimal values of P, q, t and AP with respect to different values of β are shown in Table 2 (given in the end).

Table 2 shows that an increasing value of β results in increased values of q*, P*, t* and AP*. Which implies increased rate of demand results in increased optimum EOQ, selling cost, phase time and mean profit when $\Delta_1 < \Delta_2$.phase time and mean profit when $\Delta_1 < \Delta_2$.

6 Conclusions

This paper is summarized about an inventory model for depreciation of goods considering the demand as dependent variable on selling price and inventory on shelf. Also, a upper limit of stock on a shelf in a retail shop, is imposed, so as not to leave a negative impression on consumers. Under such situation, a model is discussed and proposed for maximum profits. Following which, the characteristics of the optimized solutions obtained are mentioned. Also, a numerical example and its solution algorithm is solved to elaborate the application and usefulness of the model. A simpler methodology is established to evaluate the optimum series time, economic quantity of order and re order point. Further, we study few spontaneously rational supervisory outcomes. Like, if per unit profit from inventory is greater than per unit cost of inventory, then the building up stock is cost-effective and therefore the establishment of stock can reach to the optimum allowed limit. Otherwise, not. Also, the closing stock must be nil. Furthermore, application of the discussed model is illustrated further using few numerical examples. The results helps to analyze the importance of outcome of selling price based on stock on the perception of system, and hence is important factor while working on development of inventory models. The effects of various factors on decision parameters are shown by using sensitivity analysis.

4.1 Scope for Future Study

Discussed models can be enhanced on considering price increases, capacity benefits, and credit of trade along with others. Along with, the concerned to expand the planned form of multi-unit stock with finite display space or with consideration of the rate of demand in a polynomial function with respect to demand dependent on in-hand inventory. Finally, in future this study can be extended to variable and stochastic demand pattern from the deterministic function.

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Appendix

Case 1:- $\Delta_1 \geq \Delta_2$

AP is an increasing with respect to T.

To show $\partial AP/\partial T>0$ take,

$F(x) = xe^x - e^x + 1$, for strictly positive x (A.1)

From A.1, we have $f'(x) = xe^x > 0$.

⇨ f(x) is an increasing function with respect to $x \geq 0$.

⇨ $f(x) > f(0) = 0$ (A.2)

Assume $x = (\theta + \beta)T$, from A.1 and A.2, to get,

$(\theta + \beta)Te^{(\theta + \beta)T} - e^{((\theta + \beta)T)} + 1 > 0$, for strictly positive T. (A.3)

Using (5) and (A.3), we have $\partial AP/\partial T>0$

Table 1 ($\Delta_1 \geq \Delta_2$)

β	*B*	*C*	*Y**	q*	P*	t*	*AP**
0.20	100	1.0	29.7176	70.2923	6.036369	2.995083	53.8080
0.25			27.5519	72.4580	5.057348	2.228933	65.6780
0.30			21.6559	78.3540	4.401510	1.874283	74.6845
0.35			12.9293	87.0806	3.916533	1.700831	81.5774
0.40			1.5186	98.4913	3.542568	1.626914	86.6178
0.25	100	1.0	27.5519	72.4580	5.057348	2.228933	65.6780
	110		25.7993	84.2106	4.916374	2.437729	66.5223
	130		19.8742	110.1357	4.727227	2.927701	67.8531
	150		12.1958	137.8141	4.618074	3.478271	68.6822
	170		3.9875	166.0224	4.552949	4.059206	69.1926
0.25	100	1.2	47.2088	52.7021	5.192384	1.538303	79.0717
		1.4	38.7816	61.2283	5.099465	1.811719	72.2745
		1.6	27.5519	72.4580	5.057348	2.228933	65.6780
		1.8	14.7001	85.2009	5.09415	2.827995	59.3962

Table 2 ($\Delta_1 < \Delta_2$)

β	q^*	P^*	t^*	AP^*
0.12	162.1660	1.586133	0.666560	131.4941
0.14	169.6222	1.986959	0.396068	132.4196
0.17	181.8270	1.896776	0.147557	134.1735
0.19	191.5231	1.607169	0.287270	135.3116
0.22	211.5050	1.127088	0.486686	137.3604