An Integration Model of Planned Maintenance and Spare Parts Inventory for Periodic Order Policy

Norhafiza MohdNoor [#1], Nerda Zura Zaibidi [#2], Zurina Hanafi [#3]

[#] *School of Quantitative Sciences, College of Arts and Science*
Universiti Utara Malaysia,
06010 Sintok Kedah, Malaysia.
[1]fiza@uum.edu.my
[2]nerda@uum.edu.my
[3]h.zurina@uum.edu.my

***Abstract*— The availability of spare parts is very crucial to enable the maintenance tasks to be performed when required. Although failures occur at random, a certain amount of spare parts should be available to minimize the downtime required for repairing or replacing the failed items. Particularly when preventive maintenance is involved, spare parts may be needed to replace the defective but still working plant items. Thus, spare parts and planned maintenance are significantly related and should be studied together. By implementing the delay time concept, this study focus on periodic order policy and simultaneously optimize three decisions problems, namely inspection interval, order interval and maximum stock level. A numerical example is presented to illustrate the ability of the model.**

***Keywords*—** *Planned Maintenance, Spare Parts, Inventory, Periodic Order Policy, Delay Time Concept*

1. Introduction

Spare part is any part that is reserved for the purpose of maintenance and repairs. It is different from other inventory in a way that its function is to keep the equipment in an operating state. One of the factors that affect the amount of spare parts needed is the type of maintenance involved [1]. At the same time, the interval between each inspection also affecting the demand for spare parts. Relatively more preventive-based spare parts might be needed if the gaps between inspections are shorter. The reason behind it is because we might need the spare parts once inspection identified defective parts. However, if we increase the interval between inspections, it may miss the defect identification, and resulting in relatively more failure-based repairs. These two conditions clearly show the relationship between spare parts and inspection interval.

Periodic order policy is a classic inventory control system. One of the advantages is it can handle the variability of demand.

Availability of the spare parts is very important to ensure that the repairs can be done immediately. Several papers have been addressing problems related to spare parts. These include [2] and [3]. Different techniques have been applied to tackle the problem. Bayesian approach has been used to estimate the demand for spare parts [4]. [5] and [6] classified spare parts demand based on the criticality of the equipment. [6] also proposed new inventory model that reserves stock for critical demand. [7] includes an element of condition-based monitoring to determine the ordering decision for the expensive and highly critical components. Most of the previous studies only focusing on how to forecast the spare parts demand and ignoring the relationship that occur between spare parts demand and planned maintenance.

This study addresses the planned maintenance and spare parts inventory together. According to [8], large demands for spare parts are arising from preventive inspection and replacement. [9] and [10] developed a model that combine maintenance inspection and spare parts together. The model in [10] was derived in terms of the order interval and maximum allowable stock level. In [9], the model optimizes three decision variables, namely ordering quantity, ordering interval and inspection interval. In this paper we seek to establish a model that integrates the spare part provision and the maintenance related decisions for periodic order policy. In particular we simultaneously optimize three decision problems, namely inspection interval, order interval and maximum stock level.

International Journal of Supply Chain Management
IJSCM, ISSN: 2050-7399 (Online), 2051-3771 (Print)

2. Framework

Consider a system with many components, each with its own lifetime distribution. Each component will be replaced upon failure and the whole system is inspected every T time units. Any defects identified during inspection are removed and the system is considered to be renewed (as good as new).

Delay time concept can capture the relationship of failures and defects found during inspection. The delay time concept is explain in more detail in [11], [12] and [13]. By implementing the concept, we can capture the two-stage failure process and estimate the number of failures and the number of identified defects during inspection. Each component is modelled individually and then pooled together to get the number of failures and defects. Then, using a crude search, we can estimate the expected total cost per unit time for a given inspection interval, order interval and maximum level of stock.

3. Modelling Assumptions

The following assumptions were considered in this study

i. A system with many identical components, the two stage failure process is described by an initial and delay time probability density functions.
ii. Arrival of defective items follow a Homogeneous Poisson Process (HPP).
iii. If there is a failure, it is replaced by a new one, regardless the state of the item failed.
iv. There are two scenarios for the cost of failure replacements, the first is the normal failure-based replacement cost when the spare part is available and the second is an emergency failure replacement cost when the spare part is not available.
v. Inspections are scheduled on a fixed plan, regardless of age and the lapsed time since the last renewal of individual components.
vi. All inspections are perfect, i.e all defective components can be identified and replaced with new ones.
vii. There are two scenarios for the cost of defective component replacements at inspections. The first is the normal inspection-based replacement cost when the spare part is available and the second is an emergency inspection-based replacement cost when the spare part is not available.
viii. There is a maximum limit for stock level and the ordering quantity will be based on the difference between the maximum stock level and stock currently on hand.
ix. Order interval is fixed and it is assumed that the length of interval is the same like the inspection interval.
x. Order lead time is assumed to be too small and negligible.

The followings are the notation used throughout the study

λ	rate of defect arrivals
$F(x)$	cumulative distribution function of the delay time of the item
$n_f(t)$	random number of failures over $[0,t)$
$n_d(t)$	random number of defective components identified at preventive maintenance time *t*
$E[n_f(t)]$	expected number of failures over $[0,t)$
$E[n_d(t)]$	expected number of defects identified at preventive maintenance time *t*
$P[n_f(t)]$	probability mass function for $n_f(t)$
$P[n_d(t)]$	probability mass function for $n_d(t)$
C_h	holding cost per unit time per item
C_o	ordering cost per order
C_p	inspection cost
C_f	failure cost
C_{f_o}	failure-based replacement cost with spares available
C_{f_e}	failure-based replacement cost with no spares available
C_d	inspection-based replacement
C_{d_o}	inspection-based replacement cost with spares available
C_{d_e}	inspection-based replacement cost with no spares available
s	current stock level
S_m	maximum stock level

q	ordering quantity
x_f	demand for failures
x_d	demand for defective components
t	inspection interval
t_o	ordering interval
$TC(t, t_0, S_m)$	expected total cost per unit time as a function of t, t_o and S_m
$M(t, S_m)$	expected maintenance related cost per unit time as a function of t, t_o and S_m
$I(t, S_m)$	expected inventory related cost per unit time as a function of t, t_o and S_m
$m_f(t, S_m)$	expected failure-based replacement cost per unit time as a function of t, t_o and S_m
$m_d(t, S_m)$	expected inspection-based replacement cost per unit time as a function of t, t_o and S_m

4. Model Development

Figure 1 illustrates the scenario at one particular interval.

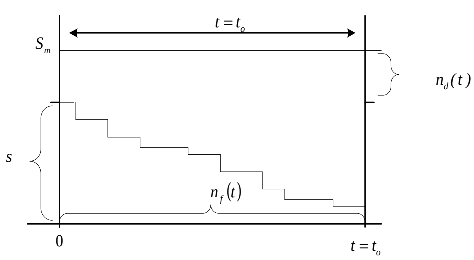

Figure 1. Illustration of inventory when $t = t_o$

It is assumed that the defective items' arrivals follow a homogeneous Poisson process (HPP). [12] has shown that both $n_f(t)$ and $n_d(t)$ are Poisson variables with means

$$E[n_f(t)] = \int_0^t \lambda F(x)dx \qquad (1)$$

and

$$E[n_d(t)] = \int_0^t \lambda[1 - F(x)]dx \qquad (2)$$

respectively. Substitute Eq. (1) and Eq. (2) in the Poisson distribution function leads to

$$P[x = n_f(t)] = \frac{e^{-\int_0^t \lambda F(x)dx}\left(\int_0^t \lambda F(x)dx\right)^{n_f(t)}}{n_f(t)!} \qquad (3)$$

and

$$P[x = n_d(t)] = \frac{e^{-\int_0^t \lambda(1-F(x))dx}\left(\int_0^t \lambda(1-F(x))dx\right)^{n_d(t)}}{n_d(t)!}$$

(4)

More information about Poisson process can be found in [14], [15], [16] and [17].

In this model, there are three important variables to be optimized, which are t, t_o and S_m. Since we want to integrate the inspection maintenance and the inventory for spare parts together, the model is optimized when the total cost $TC(t, t_o, S_m)$ associated with the maintenance and spare part is at its minimum. S_m is important because the order quantity, q is the difference between the maximum stock level and the current stock in hand, ($S_m - s$) every time we place an order. The process is similar to the re-order cycle policy in inventory stock control [18]. When determining optimal S_m, we set

$$S_m = E[n_f(t)] + E[n_d(t)] \qquad (5)$$

This is to ensure that we do not carry unnecessary stock at any particular time.

For the maintenance side, the costs involved are from the failure-based replacement, maintenance inspection and any inspection-based replacement occurred during inspection. Therefore

$$M(t, S_m) = \{C_f E[n_f(t)] + C_p + C_d E[n_d(t)]\}/t \qquad (6)$$

Since there is a constraint for the availability of the spare parts, we need to modify Eq. (6). For the failure-based replacement cost, whenever the demands for spare parts exceed S_m, emergency cost due to stock out C_{f_e} will occur. Thus,

$$m_f(t, S_m) = C_{fo}\sum_{x_f=0}^{S_m} x_f P[n_f(t) = x_f] + C_{fe}\sum_{x_f=S_m+1}^{\infty} x_f P[n_f(t) = x_f]$$

(7)

For the inspection-based replacement cost, the occurrence of emergency cost due to stock out C_{d_e} may come from two scenarios. One is when the current stock on hand cannot cover the number of defective items found during inspection. The other is when all the stocks are used up for failure-based replacements and none is left for the inspection-based replacement. Hence,

$$m_d(t,S_m)=C_{do}\sum_{x_f=0}^{S_m}\sum_{x_d=0}^{S_m-x_f}x_d P[n_f(t)=x_f]P[n_d(t)=x_d]+$$

$$C_{de}\sum_{x_f=0}^{S_m}\sum_{x_d=S_m-x_f+1}^{\infty}x_d P[n_f(t)=x_f]P[n_d(t)=x_d]+$$

$$C_{de}\sum_{x_f=S_m+1}^{\infty}P[n_f(t)=x_f]\sum_{x_d=0}^{\infty}x_d P[n_d(t)=x_d] \qquad (8)$$

Eq. (8) is based on the assumption set in Eq. (5) and two scenarios described earlier. Considering all these factors,

$$M(t,S_m)=C_{fo}\sum_{x_f=0}^{S_m}x_f P[n_f(t)=x_f]+C_{fe}\sum_{x_f=S_m+1}^{\infty}x_f P[n_f(t)=x_f]$$

$$+C_p+C_{do}\sum_{x_f=0}^{S_m}\sum_{x_d=0}^{S_m-x_f}x_d P[n_f(t)=x_f]\,P[n_d(t)=x_d]+$$

$$C_{de}\sum_{x_f=0}^{S_m}\sum_{x_d=S_m-x_f+1}^{\infty}x_d P[n_f(t)=x_f]\,P[n_d(t)=x_d]+$$

$$C_{de}\sum_{x_f=S_m+1}^{\infty}P[n_f(t)=x_f]\sum_{x_d=0}^{\infty}x_d P[n_d(t)=x_d]$$

(9)

For the spare parts inventory, the costs involved is holding cost and ordering cost. Since it is difficult to determine the exact duration of the spare parts remain in the stocks, we use an approximation of using half of the maximum inventory level as an average inventory in interval t_o. Therefore, the cost associated with holding cost is

$$(C_h\frac{S_m}{2}t_o)/t_o \text{ which leads to } \{C_h\frac{S_m}{2}\} \qquad (10)$$

On the other hand, the ordering cost equals

$$\frac{C_o}{t_o} \qquad (11)$$

However, since we assumed $t_o = t$, thus

$$I(t,S_m) = C_h\frac{S_m}{2}+\frac{C_o}{t} \qquad (12)$$

Combining Eq. (9) and Eq. (12) will set the expected total cost

$$TC(t,S_m)=$$

$$\{C_{fo}\sum_{x_f=0}^{S_m}x_f P[n_f(t)=x_f]+C_{fe}\sum_{x_f=S_m+1}^{\infty}x_f P[n_f(t)=x_f]+$$

$$C_p+C_{do}\sum_{x_f=0}^{S_m}\sum_{x_d=0}^{S_m-x_f}x_d P[n_f(t)=x_f]\,P[n_d(t)=x_d]+$$

$$C_{de}\sum_{x_f=0}^{S_m}\sum_{x_d=S_m-x_f+1}^{\infty}x_d P[n_f(t)=x_f]\,P[n_d(t)=x_d]+$$

$$C_{de}\sum_{x_f=S_m+1}^{\infty}P[n_f(t)=x_f]\sum_{x_d=0}^{\infty}x_d P[n_d(t)=x_d]\}/t+C_h\frac{S_m}{2}+$$

$$\frac{C_o}{t} \qquad (13)$$

5. Numerical Example

This section presents the illustration of numerical example. We assumed the stock out cost is twice the normal cost for the failure-based replacement and the inspection-based replacement. Table 1 shows the cost parameters used for the model. Scale parameter of 0.01 and 0.1 represents the initial and delay time function.

Table 1. Cost parameters

C_{fo}	C_{do}	C_{fe}	C_{de}	C_p	C_o	C_h
6	3	12	6	0.2	0.1	0.006

The result obtained is shown in figure 2. Based on the chosen parameters, we obtained the optimal solution at $t^* = 3$ and $Sm^* = 3$.

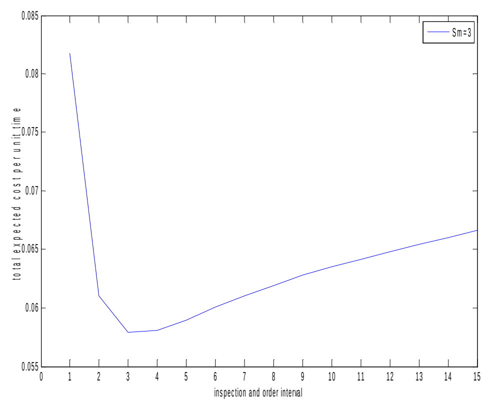

Figure 2. Expected total cost per unit time when inspection/order interval, $t = 1,2,3, \ldots 15$

6. Conclusion

Spare parts play an important role in ensuring smooth and efficient operation in a plant system. This study was conducted with the aim to find a joint optimized model between planned maintenance and spare parts inventory. To be specific, we are looking for the optimal solution with the inspection and order interval, and maximum level of stock to be held on hand act as the decision variables. The expected total cost per unit time obtained at the end are obviously depends on the assigned cost parameter and the scale parameter chosen for this study. This is a preliminary study and more studies will be carried out in the future. The first to consider is when we relax the assumption of ordering interval equals inspection interval. The second is to include a lead time into consideration. In this study, we ignore the lead time as we assumed that it is negligible and the stocks will arrive as soon as we order but at a different cost.

Acknowledgments

This research was supported by the Universiti Utara Malaysia (LEADS Grant / SO 12628).

References

[1] Kennedy, W. J., Wayne Patterson, J. & Fredendall, L. D., *An overview of recent literature on spare parts inventories,* International Journal of Production Economics, Vol 76, pp.201-215, 2002.

[2] Petrovic, R., Senborn, A. & Vujosevic, M., *A new adaptive algorithm for determination of stocks in spare parts inventory systems,* Engineering Costs and Production Economics, Vol 15, pp. 405-410, 1989.

[3] Rausch, M. & Haitao, L., *Joint Production and Spare Part Inventory Control Strategy Driven by Condition Based Maintenance,* Reliability, IEEE Transactions on, Vol 59, pp. 507-516, 2010.

[4] Aronis, K.-P., Magou, I., Dekker, R. & Tagaras, G.,*Inventory control of spare parts using a Bayesian approach: A case study*, European Journal of Operational Research, Vol 154, pp. 730-739, 2004.

[5] Dekker, R., Kleijn, M. J. & de Rooij, P. J., *A spare parts stocking policy based on equipment criticality*, International Journal of Production Economics, Vol 56-57, pp. 69-77, 1998.

[6] Chang, P.-L., Chou, Y.-C. & Huang, M.-G., *A (r,r,Q) inventory model for spare parts involving equipment criticality*, International Journal of Production Economics, Vol 97, pp. 66-74, 2005.

[7] Louit, D., Pascual, R., Banjevic, D. & K.S.Jardine, A., *Condition-based spares ordering for critical components*, Mechanical Systems and Signal Processing, 2011.

[8] Vaughan, T. S., *Failure replacement and preventive maintenance spare parts ordering policy,* European Journal of Operational Research, Vol 161, pp. 183-190, 2005.

[9] Wang, W., *A joint spare part and maintenance inspection optimization model using the Delay-Time concept*, Reliability Engineering and System Safety, pp.1535-1541, 2011.

[10] Wang, W. , Liu, Y. & Pecht, M., *A theoretical model to minimize the operational cost for canary-equipped electronic system's health management,* Proceedings of 2011 IEEE International Conference on Intelligence and Security Informatics, 2011.

[11] Christer, A. H., Wang, W., Baker, R. D. & Sharp, J., *Modelling maintenance practice of production plant using the delay-time concept,* IMA J Management Math, Vol 6, pp. 67-83, 1995.

[12] Wang, W., *Delay time modelling*, In Complex system maintenance handbook (Kobbacy, K. A. H. & Murthy, D. N. P.,Eds.), Springer, pp. 345-370, 2008.

[13] Christer, A. H., *Developments in Delay Time Analysis for Modelling Plant Maintenance*, The Journal of the Operational Research Society, Vol 50, pp. 1120-1137, 1999.

[14] Blischke, W. R. & Murthy, D. N. P., *Reliability Modeling, Prediction and Optimization,* John Wiley & Sons, 2000.

[15] Cox, D. R., *Renewal theory*, London, Chapman and Hall Ltd, 1982.

[16] Barlow, R. E. & Proschan, F., *Mathematical Theory of Reliability*, New York, John Wiley, 1965.

[17] C.Tijms, H.,*Stochastic Modelling and Analysis: a computational approach*, John Wiley & Sons, 1986.

[18] Wilkes, M., *Operational Research: Analysis and Applications,* London, McGraw-Hill Book Company, 1989.