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Developing Academic Language In The Context Of A Fourth Grade Mathematics Geometry Curriculum Unit

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DEVELOPING ACADEMIC LANGUAGE IN THE CONTEXT OF A FOURTH GRADE MATHEMATICS GEOMETRY CURRICULUM UNIT

by

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CHAPTER ONE

Introduction

My students need to develop a strong understanding of functional academic vocabulary in order to access grade-level mathematics content. Using a variety of learning styles, and engaging tasks that utilize reading, writing, speaking, and listening skills, my goal is for students to understand and use specific vocabulary and functional language. In this capstone, I am pursuing the question: *How can a mathematics geometry curriculum unit be organized to support and develop academic language for fourth grade students through meaningful engagement strategies?*

Personal History

From a young age, I had an interest and talent for mathematics. In third grade, I loved the logic puzzles and problem-solving challenges my teacher posed as a warm-up every day. Starting in fourth grade, I participated in a pull-out group that was part of the district gifted education program called Challenge Math, that presented advanced math in open-ended, visual, and otherwise different forms. In middle school and high school, I was always ahead in math courses, and within my classes I often acted as a peer tutor for my classmates. As a student, I liked learning math through small-group work, rather than teacher lecture and individual work. In college, I decided to major in Elementary Education with a Teaching Math minor, thinking I would teach math with a focus on differentiation for gifted learners like myself through problem solving, algebraic thinking, and logic based strategies. Over the next several years as a new teacher, this thinking would change.

When I first began my teaching career, I had very little experience working with students who were English learners. My first teaching job was teaching fifth grade in a first-ring suburb of the Minneapolis/St.Paul area. I had several students who were ELL students with varying proficiency levels, including three students who I was told were Level 2 EL's, and a few others at Levels 3, 4, and 5. As a new teacher, I did not have much frame of reference as to what these levels meant, other than knowing that my students were learning English as a second language. I've since developed a better understanding of the six levels of language proficiency as defined by WIDA, the multi-state group that defines the standards and assessments used for English Language Learners cooperative ("WIDA FAQ's"). I didn't know how much or how little English the ELL students in my classroom knew, and I wasn't really sure what, if anything, I needed to do as their teacher to help them learn. The first few weeks of school, I observed and got to know my students, hoping to figure out what it meant to teach students with limited English proficiency. I noticed that my ELL students followed directions, most of the time, and although they were very quiet, I didn't immediately see a need to change my teaching methods or lesson plans. Their scores were low, but my curriculum didn't really offer much help to me as a new teacher for differentiation.

As that first year continued, I found out my Level 2 ELL students needed small-group guided reading instruction at a first-grade reading level. This was new to me, since I had student-taught in fifth grade and was used to teaching guided reading using chapter books and other fluent-reader texts. The school literacy coach met with me after school, talked me through the key components of an emergent or early guided reading lesson, and gave me enough resources to get started. It was a little bumpy at first, but teaching that low-level guided reading group soon became one of my favorite lessons of the day. In the small group setting, my otherwise timid and quiet students opened up, using the simple non-fiction texts about animals, seasons, and other topics as a starting point for making connections and sharing in conversation about their summer visit to the zoo, the pet dog they left behind in the Thai refugee camp, and more. Mid-year, following a staff development session, I started tracking my students' fluency scores and reading level. One day a week I would conduct a quick running record, and the resulting graph showed the students' improving reading level. It was motivating for them and for me to see the improved score, though I think our connections and conversations were more beneficial for overall language learning.

My principal was in the process of implementing SIOP (Sheltered Instruction Observation Protocol) strategies for all teachers in mainstream classrooms, and I had the opportunity to attend the four-day SIOP training in the summer of 2012. Sheltered Instruction Observation Protocol (SIOP) is a "research-based measurement tool designed to measure the quality of instruction delivered in multilingual contexts" (Freeman & Crawford, 2008). SIOP helps teachers organize their instruction in ways that systematically develop academic language and literacy skills (Freeman & Crawford, 2008). The eight components of SIOP provide scaffolding and support within the mainstream classroom for English learners to access grade-level academic content. One of my biggest take-aways from SIOP is the philosophy that what is good for English learners is good for all learners. This teaching practice can be applied in many settings- what is good for special education students is good for all students, what is good for gifted learners is good for all learners. To me, best practice teaching has come to mean teaching that gets students moving, interacting, using creativity, and accessing new skills in scaffolded settings.

At first thought, teaching math to ELL students seems like it should be easier than teaching reading. Numbers are the same in any language, right? Upon closer analysis, I saw that there is an immense amount of language embedded in even the simplest math lesson. Teaching and doing math involves a lot of steps, and the functional language needed to comprehend is a process in and of itself. The academic language of math is specific and requires explicit instruction. I spent two years specializing in teaching math to a cluster group of ELL students, often completely re-writing homework assignments and creating content when the Everyday Math curriculum used in the district was not accessible for my students' levels.

The following year I took a teaching position in another district. The ESL population at my current school isn't quite as large as my previous building, and I spent my first two years with a small cluster of gifted-identified students in my classroom, so my focus for differentiation shifted. However, even though I was no longer teaching as many ELL students, I found that my passion for helping ELL students continued, and that the teaching strategies that I had learned through SIOP training truly benefit all students, not only ELL students. Many native-English speaking students struggle with academic vocabulary, and need direct instruction and meaningful practice with reading, writing, speaking, and listening skills around specific and functional academic language. Lesson activities are meaningful when students are engaged and using language, collaborating with each other, and thinking critically.

Capstone Focus

I spent a long time reflecting and thinking about an area in which to focus for this capstone project. My experiences in the classroom as a teacher over the past several years have shaped my teaching philosophy to what it is today. My own experiences as a student in elementary, high school, college, and professional life have also influenced my passions and worldview. Classrooms are not what they were 20 or 30 years ago, and teachers need curriculum and tools that reflect the need for 21st century learning skills for all students. All students need opportunities to interact with content and build skills of inquiry and collaboration that prepare them for an information-rich world. In the age of Google and Smartphones, memorization is less important than application, and my student's future employers will be looking for strong communication skills. Also, all students need a strong foundation in mathematics to be college and career ready. It is my hope that through this capstone project, I will develop curriculum that addresses these needs.

In the following chapters, I will describe, rationalize, and develop a fourth grade geometry math curriculum unit. Chapter 2 reviews the literature that supports best practice for curriculum design and development as well as research-based strategies for vocabulary development, language acquisition, and student engagement. Chapter 3 discusses the methodology used in developing the curriculum unit following the Understanding by Design framework developed by Wiggins and McTighe (2005). Chapter 4 narrates the resulting curriculum unit and individual lesson plans. Chapter 5 summarizes the project and my final reflections on the process and outcome. Teaching academic language is a complex process, and as an effective teacher, my hope is to make learning engaging and meaningful. This project is the result of my exploration and curriculum unit design surrounding the question: *How can a mathematics geometry curriculum unit be organized to support and develop academic language for fourth grade students through meaningful engagement strategies?*

CHAPTER TWO

Literature Review

Overview

What are the components of an effective curriculum? Who benefits from lesson activities designed to increase student engagement and develop academic language? How should curriculum be planned to meet the needs of diverse student populations, and specifically the needs of English Language Learners (ELL)? These are some of the many questions I considered as the topic of this research was developed. This chapter discusses themes and research necessary to support the exploration of the research question: *How can a mathematics geometry curriculum unit be organized to support and develop academic language for fourth grade students through meaningful engagement strategies?*

The first area of the literature that will be discussed is English as a Second Language. How are ELL students unique in their learning needs? My research topic focuses on academic language and engagement, which are both important topics for ELL students. I will then go on to further discuss academic language, the various components of language use and development that must be considered and why attention to academic language is crucial for student success. Mathematics teaching methods are driven by today's academic standards, but attention must also be given to the research that supports proven methods and the skills, including language skills, students need to access the mathematics content. Next, this chapter will give attention to discussing the importance of student engagement and the impact that effective engagement strategies have shown on academic performance. Finally, I give attention to mathematics teaching methods that support best practice instruction as illustrated by research studies and experts in the field, specifically noting those practices that show meaningful impact on the geometry and measurement strand of mathematics.

English as a Second Language

This section will examine the unique challenges and learning needs of students who are not yet proficient in the language of instruction. Around the world, students who are minority language speakers struggle in schools because "they lack the valued skills of school literacy and language use" (Zwiers, 2008, p. xv). In the United States, students who are not yet proficient in academic English are usually identified and labeled as English Language Learners, or ELLs. English as a Second Language (ESL) programs support the academic needs and English language development of students learning English (Bardak, 2010). Many resources are available to teachers and schools through the WIDA Consortium, which is made up of 27 states that share a framework of English language proficiency standards as well as assessments, professional development resources, and current research (Cook, Boals, & Lundberg, 2011). WIDA formerly stood for World-Class Instructional Design and Assessment, however, the consortium has since determined that the acronym does not adequately describe its mission, and now, WIDA simply stands for WIDA ("WIDA FAQs"). The WIDA framework is an excellent resource for both mainstream academic content teachers and ESL teachers alike. Additionally, many states use the WIDA assessments to measure and track the progress of English learners as mandated by the federal No Child Left Behind Act (Cook, et al., 2011).

In Minnesota, all students are assessed in reading and math from third grade through high school using the Minnesota Comprehensive Assessments (MCA) standardized tests that were implemented to meet the requirements of the federal No Child Left Behind Act. Standardized testing has come to exert a significant influence on the instructional decisions and practices in schools since the passing of the No Child Left Behind Act in 2001 (Bielenberg & Fillmore, 2004). Standardized testing is especially challenging for ESL students. When students are not proficient in a language, yet are given standardized tests in that language, the test is often not a valid measure of the students' knowledge and skills in that content area (Haag, Heppt, Stanat, Kuhl, & Pant, 2013). Yet, the results of high-stakes standardized tests can "undermine English Language learners' opportunities for high school graduation and education beyond high school" (Bielenberg & Fillmore, 2004, p. 45). Mathematics standardized assessments are linguistically complex. Martiniello (2009) analyzed this linguistic complexity in a study of the performance of English Language Learners on a state fourth-grade mathematics test. Test items that contain complex grammatical structures and specific terms whose meaning cannot be derived from context are considered most linguistically complex and have the lowest expected item score for ELL students, while non-ELL students do not demonstrate the same difficulty with the test items (Martiniello, 2009). Similar results have been found in other studies, indicating that word problems and other mathematics items containing varying amounts of language negatively affect the overall mathematics test performance of ELL students (Haag, Heppt, Stanat, Kuhl, & Pant, 2013). When further analyzed by strand, Martiniello (2009) found that ELL students were at highest disadvantage with data analysis, statistics, and probability standards, but also had difficulties with the language barriers in number sense and operations strands and geometry and measurement strands. Test items in the algebra, patterns, and functions strands often include more visual schematic representations, making them more easily comprehensible to ELL students (Martiniello, 2009).

Students who speak dialects other than standard English also face challenges when it comes to academic performance. It is widely known that African American students underperform when compared to white students, even when factors such as socioeconomic status are removed from comparison. Recognizing and validating students' cultures plays an important role in effective teaching (Blake & Van Sickle, 2001). The Oakland School Board in California formally recognized Ebonics as a primary home language in the 1990's, sparking controversy ever since (Blake & Van Sickle, 2001). In the classroom, students are often able to code-switch between their social dialect and more formal academic language when provided adequate support structures, but standardized tests do not typically provide linguistic support. Code-switching, or being able to seamlessly navigate between cultures and language dialects, is a learned skill, and is necessary for both ELL students and for native English speaking students who do not identify with the mainstream culture of schools. Mastering academic language means that students can "negotiate multiple academic environments, make sense of complex content, articulate their understanding of that content in academic forms, and assess their own growing understanding" (Cook et al., 2011, p. 66). In the next section, academic language development and its implications for English language learning and academic content learning will be explored.

Academic Language

This section will explore the complexities of what it means to learn language through academic content and language implications for content instruction. The American Educational Research Association defines Academic English as "the ability to read, write, and engage in substantive conversations about math, science, history, and other subjects" (cited in Freeman & Crawford, 2008). Academic language differs from everyday language in all subject areas. "English used in informal settings has less complex grammatical forms, few uses of technical vocabulary, frequent use of slang and idioms, frequent cultural and contextual references, and a much more personal sense" (Cook et al., 2011, p. 67).

Cummins (cited in Zwiers, 2008), a well-known researcher of bilingualism, was the first to define academic language using the terms basic interpersonal communicative skills (BICS) and cognitive academic language proficiency (CALP). The terms have become an important foundation for teachers in understanding ELL students' language development needs. BICS encompasses the less complex language used in everyday social situations. Often, social language used in conversation includes other helpful comprehensible input such as picture clues, gestures, facial expressions, real objects, or shared background knowledge (Zwiers, 2008). Regardless of the language spoken at home, most students possess the linguistic skills and resources for everyday communication when they enter school, or at least can quickly transfer their social skills from their home language to the target language used at school (Bielenberg & Fillmore, 2004). On the other hand, academic language (CALP) is more abstract, formal, and usually lacks such supports and comprehensible input. Zwiers (2008) asserts that ideas surrounding the study of academic language have shifted over the years, and various researchers have penned specific definitions. Zwiers' (2008) own definition identifies academic language as the "the set of words, grammar, and organizational strategies used to describe complex ideas, higher-order thinking processes, and abstract concepts" (p. 20).

In order to be successful in core content areas, students must learn and utilize academic language. Academic language is the vehicle through which students acquire new content knowledge and communicate their understanding (Haag, et. al., 2013). Leung (2005) describes two interpretations of the usefulness of specific and technical academic language: some view language as a sign of expertise and valued knowledge, and others see it as unnecessary jargon. A very common misconception among teachers when considering academic language is thinking that academic language is just a "long list of key content words" (Zwiers, 2008, p. xiii). However, academic language is much more complex than just teaching vocabulary terms. Recognizing the complexities of academic language is challenging for teachers, who have spent years studying and teaching their content, to the point that "academic language for most teachers is our everyday language, which makes it hard to notice and, therefore, hard to teach" (Zwiers, 2008, p. 39).

All students enter school with a foundation of language and thinking skills that represent their home culture and community (Zwiers, 2008). At school, students construct varying levels of general and specialized language to access the culture and content of different academic disciplines. This is easier for students whose home and community language and culture significantly overlaps with the mainstream language and culture of school. Particularly important to note are the general language skills for knowing, thinking, reading, and writing that are used across the disciplines. Students from diverse language and cultural backgrounds "need rich classroom experiences that accelerate the language that supports their content knowledge, thinking skills, and literacy skills" (Zwiers, 2008, p. xiv).

In mathematics, demonstrating knowledge and expertise through the understanding and use of language is an important part of making meaning (Leung, 2005). Students draw upon subject-specific vocabulary, discourse, and grammar in communicating their understanding of academic content (Cook et al., 2011). When students are limited to informal everyday language, they are not always able to access or accurately explain knowledge specific to the subject of mathematics. Students often might know and understand more than they are able to show through typical assessments, because they lack the language skills needed to demonstrate and explain their understanding (Blake & Van Sickle, 2001). In writing, students will omit words or choose simpler language when they do not know the vocabulary terms needed. When responding to a problem in context, students might miss what the problem is asking because they are unfamiliar with the context or misunderstand embedded figurative language.

Understanding and utilizing the academic language of mathematics, or any subject, is not simply learning a set of vocabulary words and their meanings. Zwiers (2008) uses bricklaying as a metaphor to explain the multi-faceted process of supporting language development, stating, “Students need to learn not only the big words (bricks) but also how to explain and link these bricks together with more subtle expressions (mortar) and grammar” (p. 39). In planning lessons to meet the academic language needs of learners, teachers must consider the language needed for full participation in lesson content, “including vocabulary and language that teachers would use during instruction, as well as language that students would need to use to let us know if they had met our mathematical goals” (Bresser, Melanese, & Sphar, 2009). The following sections will further discuss components and strategies for academic language development alongside content instruction in the mathematics classroom.

Vocabulary. It is important for students to learn vocabulary within context so they can connect new understandings and meaning with prior knowledge. Students must also be given the opportunity to recognize and reflect upon differences between everyday definitions of words and the mathematical application of the same term or concept (Chen & Li, 2008). Many vocabulary terms do not have fixed meanings, but are open to interpretation dependent on context (Leung, 2005). Freeman and Crawford (2008) describe the language of mathematics as “deceptively familiar” (p. 11). This is because many words used in mathematics have meaning specific to the content of mathematics that differs greatly from the word's more common everyday definition. Barrow (2014) gives examples of math vocabulary terms such as "chord, foot, and volume" that can cause confusion for ELL students because they have multiple meanings in both everyday contexts and academic content (p. 36). Moschkovich describes mathematical vocabulary learning as constructing multiple meanings for words, not just learning a list (as cited in Chen & Li, 2008). As summarized by Leung (2005), vocabulary instruction is most effective when it is a tool for exploring and expanding content knowledge, not a fixed endpoint in instruction. According to Sheffield and Cruikshank, "terms are most effectively understood when taught concurrently with hands-on experiences" (cited in Sherman & Randolph, 2004, p. 28). When students are given multiple opportunities to explore and apply technical vocabulary terms and their meanings, they develop a deeper understanding and are able to apply their knowledge in new ways.

One common support that can be used after explicitly teaching vocabulary or developing working definitions is to provide sentence frames to guide student dialogue (Bresser et al., 2009). Other common teaching strategies include teacher-student discussion and questioning that allows for clarification and expansion of meaning and ideas (Chen & Li, 2008). Moschkovich (1999) describes a lesson in which the teacher uses the instructional strategies of interpreting, clarifying, and rephrasing student responses. Building vocabulary does not need to be the focus of the lesson, but is developed through academic content when the teacher uses strategies to uncover content in student talk and bring different points of view and meanings into working definitions

(Moschkovich, 1999). Teaching strategies include "1) using several expressions for the same concept; 2) using gestures and objects to clarify meaning; 3) accepting and building on student responses; 4) revoicing student statements using more technical terms; and 5) focusing not only on vocabulary development but also on mathematical content and argumentation practices" (Moschkovich, 1999, p. 11). Students also benefit from using their native language in defining and making meaning of vocabulary terms in addition to speaking and writing in English (Chen & Li, 2008). Often, students might be unfamiliar with a context in English, but when they are given the opportunity to blend understanding in their native language with new English language learning, comprehension is enhanced and the previous knowledge is used to build new understanding (Barrow, 2014). Barrow (2014) describes the strategies of chunking, which allows students to learn new concepts in connection with background knowledge in context, and journaling, which provides opportunities for students to reflect upon and expand their understanding. Ultimately, providing language support allows students, especially ESL students, the opportunity to participate in their learning more than they would otherwise.

Language Form and Function. As stated earlier, academic language is composed of both linguistic "bricks" and "mortar," that is to say the specific content vocabulary and "the general but sophisticated words used across a variety of domains that mature users use to communicate complex thoughts" (cited in Zwiers, 2008, p. 22). "Mortar" language is often abstract, often overlooked, and yet integral to the "tasks, test, and texts of school" shared across content areas (Zwiers, 2008, p. 22). Academic language is used to describe complex concepts clearly, facilitate higher-order thinking, and describe abstract concepts (Zwiers, 2008). One of the challenges of guiding students towards mastery of language function and grammar is the complexity involved. Fillmore (2014) asserts that teaching academic discourse in isolation is not possible. Instead, teachers should expose students to text rich in academic content and complex language, and through carefully planned discussions, unpack the meaning that contributes to enhanced student understanding (Fillmore, 2014).

Grammatical competence is essential to understanding, expressing, and participating in classroom activities and greater academic fields. Zwiers (2008) defines grammar as “the set of rules that govern language in a community” (p. 34). An especially important component is syntax, “which is the set of conventions for putting words and phrases together into sentences” (Zwiers, 2008, p. 34). Students who are native speakers of the mainstream language often don’t notice or consciously know the rules of grammar, but rather use correct grammar due to natural immersion in rich language contexts. However, directly teaching grammar to ESL students is necessary, because teaching highly important grammar rules and patterns in context allows students to apply them without waiting many years for them to sink in (Zwiers, 2008). Modeling language through strategies such as sentence starters, emphasis, teacher repetition, and think alouds gives students opportunities to isolate important language functions and practice producing increasingly complex structures with scaffolded support, repeated practice, and immediate feedback (Zwiers, 2008). Learning a new language involves learning to navigate and utilize the language within a social context, which in turn means that grammar and function cannot be taught in isolation (Fleming, Bangou, & Fellus, 2011).

"Educators must set up learning environments in which students feel safe to take risks with their evolving academic language" (Zwiers, 2005, p. 62). Fillmore's (2014) strategy centralizes around a guided discussion of a text selection rich in complex language and related content. Teachers select interesting and informative passages, and carefully plan questions to guide students in unpacking the language structures, forms, and functions in the process of understanding the meaning being expressed (Fillmore, 2014).

Bielenberg and Fillmore (2004) describe the benefits of planning and communicating language objectives alongside content objectives in daily lesson plans.

> These language objectives remind teachers and students alike to pay attention to features of academic English, such as those illustrated here. Language objectives may focus on academic English vocabulary, common academic English structures, or such language functions such as explaining, defending, and discussing. Highlighting academic language—however briefly—as an objective in every lesson enhances student awareness of academic English and promotes student achievement (Bielenberg & Fillmore, 2004, p. 49).

Language objectives are then translated into intentional learning activities that engage students in comprehending and producing language in increasingly complex forms. Through meaningful content and engaging activities rich in linguistic interactions, students build the capacity to interact with both the content and language at increasingly complex levels.

As described above, explicit teaching of academic language is beneficial to all students, and is critical for ELL's and students who speak nonstandard dialects of

English. ELL students can appear to understand English when their social language is fluent, but mastering academic language requires carefully planned instructional activities that focus on building language form and function as well as specific vocabulary terms. In the next section, methods that contribute to best practice mathematics learning will be discussed.

Mathematics Teaching Methods

A wide variety of teaching methods contribute to successful mathematics instruction in the classroom, however, an emphasis on standards to guide instruction ensures that students are held to high achievement goals regardless of the textbook or curriculum available. Standards-based instruction started with mathematics standards created by the National Council of Teachers of Mathematics (NCTM). The NCTM first published the *Professional Standards for Teaching Mathematics* in 1991 (Firmender, Gavin, & McCoach, 2014). Prior to these standards, teachers used textbooks as the primary curriculum, but teaching to instructional content standards has now become the norm expectation in teaching. Over 40 states have adopted the Common Core State Standards (CCSS) (Firmender et al., 2014), which include eight Standards for Mathematical Practices that “describe how students should interact with and engage in learning mathematical content” (Firmender et al., 2014). The “how” of learning mathematics is often just as critical in instruction as the “what” of mathematics content. According to Freeman and Crawford (2008), the NCTM standards and principles are widely accepted without much debate, and many states, including Minnesota, have developed state mathematics standards based on the NCTM principles, including a "critical emphasis on principles and processes and promotes exploratory [discovery] learning through 'real-world' issues" (Freeman & Crawford, 2008, p. 10).

In 2000, the NCTM published an updated Principles and Standards for School Mathematics (PSSM) that further identifies and emphasizes six fundamental principles for "creating a mathematics learning community that accentuates problem solving, reasoning, and conceptual understandings" (McKinney, Chappell, & Berry, 2009, p. 278). These principles are similar to the mathematical practices in the Common Core State Standards, and promote mathematics instructional activities that help students develop conceptual understanding, flexible thinking, problem-solving abilities, and communication skills (Neumann, 2014). However, traditional mathematics teaching methods that do not align with the NCTM principles and standards continue to be commonplace in elementary classrooms (McKinney et al., 2009). According to Hiebert (1999), the majority of students learn basic arithmetic by eighth grade, as evidenced by the National Assessment of Educational Progress (NAEP). However, students' knowledge and skills lack depth and conceptual understanding, as evidenced by performance on any tasks that "require students to extend their skills, reason about them, or explain why they work" (Hiebert, 1999, p. 12). Many teachers continue to use lecture, rote memorization, drill and practice methods, and a set curriculum to teach math because it is how they were taught and it is what they know. Further, in high poverty and urban settings, where the achievement gap is most prominent, instruction that focuses on basic skills without attention to problem solving is even more frequently found as standard practice (McKinney et al., 2009).

What does current research say are the best practice teaching methods for teaching mathematics, especially mathematics that prepares students for complex problem-solving and application-based projects? Major theories agree that "mathematical ideas must be personally constructed by students as they intentionally try to make sense of situations, and that to be effective, mathematics teaching must carefully guide and support students' construction of personally meaningful mathematical ideas" (Battista, 2012, p. xv). Most sources support student problem solving and teaching through mathematical reasoning and critical thinking as means of facilitating understanding (Anhalt, Farias, Farias, Olivas, & Ulliman, 2009; Firmender, Gavin, & McCoach, 2014). Additionally, it is beneficial for teachers to use students' knowledge and ideas as a starting point for new instruction (Battista, 2012). It is important to note that it is not possible to draw an explicit connection between research and standards. This is because standards are ultimately value statements about the priorities and goals determined as "best" (Hiebert, 1999). Research can inform standards, but human judgment places value (Sriraman & Pizzulli, 2005). Hiebert (1999) and Sriraman and Pizzulli (2005) offer a discussion of the relationship between the NCTM standards and mathematics research, and the planning and self-reflection required of teachers committed to including both as the basis for instruction. The process standards offered by the NCTM promote problem solving, reasoning, communication, and other cognitive skills that are much more rigorous than what traditional mathematics instruction approaches encompass (McKinney et al., 2009). Best-practice pedagogy often is cross-categorical and the Principles and Standards of Mathematical Practice set forth by the

NCTM are interconnected and equally essential to high-quality instruction. In the following sections, several components of the principles and process standards will be further explained as they apply to quality mathematics instruction in the elementary classroom.

Representations.

The NCTM Process Standards state that students should be given opportunities to represent mathematics in a "variety of ways: pictures, concrete materials, tables, graphs, number and letter symbols, spreadsheet displays, and so on" (*Executive summary: Principles and standards for school mathematics,* 2000). Clements, Battista, Sarama, and Swamintathan (1997) assert the theory that "mathematical understanding is constructed to a large extent in images, many of which are spatial in nature" (p. 172). Developing spatial abilities is considered a valuable skill by the NCTM, is related to mathematical competencies, and contributes to the development of flexible thinking (Clements et al., 1997). Math manipulatives give students hands-on practice in the formation of basic mathematical understanding, and are an important instructional method in the introduction of new concepts. Children at the elementary school level primarily reason at the concrete operational stage, making hands-on learning opportunities especially vital in developing new mathematical concepts (Sherman & Randolph, 2004). As noted in the previous section, utilizing hands-on materials is also an important strategy for building EL students' academic language. Manipulating shapes also plays a role in deductive reasoning at all levels of spatial understanding (Shannon, 2002). Students can start with describing what they know, and use gestures, familiar words, and written drawings symbols to communicate their understanding and reasoning. The NCTM recommends that students are actively involved in measuring objects and space in familiar surroundings (Sherman & Randolph, 2004). It is also important to note the benefits of students creating and using multiple representations, recognizing that "the term representation refers both to process and to product—in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself" (Boss, Lee, Swinson, & Faulconer, 2010, p. 264). Representations are used to organize, record, and communicate mathematical ideas (Boss, Lee, Swinson, & Faulconer, 2010).

Representations also include the context for problems and mathematical reasoning. Capraro and Capraro (2006) studied the effects of utilizing content literature books to create dynamic and interactive learning environments that help students make sense of mathematical vocabulary. The study results indicate improved performance in geometry (Capraro & Capraro, 2006) for middle grades students. Within the context of the literature, students interact with mathematical ideas using the book as a starting point for representing content and vocabulary (Capraro & Capraro, 2006). Martiniello (2009) describes the effects of utilizing pictorial and schematic representations to help ESL students make sense of linguistically complex text. Connecting to what students already know helps all students build upon their knowledge as they develop more sophisticated mathematical and linguistic understandings. In schematic representations, relationships between elements or parts are used to make connections and instruct meaning (Martiniello, 2009).

Communication.

Many important mathematics instructional strategies utilize language and communication tools. Math talks, teacher-student discussion, and small-group student discussion all can be used to help students develop and further process mathematical understanding (Bresser et al., 2009). According to the NCTM, mathematical communication allows students to further their understanding by sharing their own ideas and analyzing the ideas of others (Firmender, Gavin, & McCoach, 2014). Student talk as a mathematics instructional activity leads to increased understanding in two ways: the teacher can formatively assess students' mathematical thinking and the act of talking itself can help develop deeper understanding (Franke et al., 2009). To elaborate on the latter, as students describe, explain, and justify their thinking, they "internalize principles, construct specific inference rules for solving problems, [and] become aware of misunderstandings and lack of understanding" (Franke et al., 2009, p. 381). Using classroom discussion and small group talk is also a primary strategy in best practice recommendations for teaching ELL's, as discussed in the academic language section previously. Students learn from each other as they talk through challenging mathematical problems. "Thinking together" encourages students to strive for "clarity and justification of ideas that push them to think about the quality and nature of abstract ideas" (Zwiers, 2008, p. 139). Often, these abstract ideas are the very essence of academic language skills students must develop in order to master academic content.

Despite the many benefits of student voice in the mathematics classroom, there are also challenges to consider. When students are expected to be able to "communicate mathematically, both orally and in writing, and to participate in mathematical practices such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments" (Chen & Li, 2008), students with limited English language skills are at a disadvantage and require additional instructional support. Many mathematics word problems are made up of complex sentences with multiple subordinate and independent clauses (Barrow, 2014). Students must be able to navigate both the context of the problem, which may be unfamiliar, as well as comprehend the language used. Multiple opportunities to practice comprehending such problems with scaffolded strategies such as acting out the problem, creating visual support diagrams, and identifying key terms are just some of the ways teachers can help ESL students and all students navigate mathematical discourse (Barrow, 2014).

When students are asked to explain their thinking rather than just give an answer or repeat a formula or procedure, they are developing the cognitive skills needed to learn new information (Neumann, 2014). "Sharing strategies also enables other students in the classroom to become flexible thinkers because they are now aware of other ways to solve a problem. These alternative strategies may be more efficient, easier to perform, or simply present a different method than the student had first considered" (Neumann, 2014, p. 3). Additionally, listening to others' explanations gives students the opportunity to deepen their own understanding, and multiple perspectives contribute to sharper thinking and connected ideas (Boss et al., 2010). Speaking in groups and listening to peers is authentic and rich language practice for ESL students. Teachers can clarify, but the explanations and ideas are constructed in the students' own words. By providing support and instruction for all students to meet rigorous expectations, students learn to be "clear, convincing, and precise in their use of mathematical language" (*Executive summary: Principles and standards for school mathematics,* 2000). Precision is important when eliciting student responses, so that the teacher and other students can understand their ideas and support or clarify mathematical understanding as necessary (Franke et al., 2009).

One commonly used strategy in teaching mathematics is having students work together in small groups to discuss and solve problems. Edwards (2003) conducted a study on the effects of collaborative problem solving within fifth and sixth grade classrooms consisting of native English speaking and ELL students. The results were mixed, showing mostly positive improvement in students' overall problem solving skill, but also noting that not all students could gain maximum benefit from the setting (Edwards, 2003). English Language Learners must have appropriate support and feel comfortable participating with their peers. Cohen suggests the use of assigned roles, such as summarizer, recorder, and "checker," to encourage active participation (cited in Edwards, 2003). When organized to maximize student engagement, small group learning can give students opportunities to learn using several of the methods described in this section, including hands-on learning, real-world problem solving, and discussion of academic content.

Engagement.

Engagement is a measure of the student's "involvement in learning tasks, or the extent to which behavior aligns with teacher expectations" and includes active behaviors such as asking and answering questions as well as passive behaviors such as listening and writing (Lan et al., 2009, p. 200). High levels of student engagement can be achieved in both large-group and small-group settings, though the success of each is dependent on several factors, including teacher organization. When teachers organize instruction with proactive strategies towards student behavior and self-regulation, students are more engaged in learning (Lan et al., 2009, p. 199). The socio-cultural theories initiated by Vygotsky argue that significant learning takes place through social interaction, and in the mathematics classroom, this means engaging students in teacher-student interaction or student-to-student interaction (Firmender, Gavin, & McCoach, 2014). Engagement strategies also help students utilize and develop language in the content areas. In the study by Hwang, Lin, Ochirbat, Shih, and Kumara (2015), students engaged in giving and receiving peer-to-peer feedback during a technology-integrated project. Peer assessment enhanced higher-level thinking and promoted high student motivation. Students were also more receptive to peer suggestions when compared with single instructor assessment (Hwang, Lin, Ochirbat, Shih, & Kumara, 2015). The research study conducted by Sherman and Randolph (2004) showed how classroom discussion can be used as a quick and effective tool for correcting student misunderstandings by sharing and analyzing correct and incorrect responses. In mathematics, engaging students effectively leads to learning that extends beyond rote memorization to applied understanding.

Equity.

The NCTM Principles and Standards for School Mathematics (*Executive summary: Principles and standards for school mathematics,* 2000) state that all students can learn mathematics when they have access to high-quality mathematics instruction. This includes setting rigorous expectations for all students and using alternative methods of instruction that meet students' differentiated needs. McKinney, Chappell, and Berry (2009) recommend that teachers who promote the equity principle in their classrooms "strive to address students' learning profiles, learning preferences, readiness levels, and cultural differences so as to tap into all students' capabilities and unique strengths that they bring to mathematics understandings." The equity principle is also a key component of best practice for academic language instruction to support ELLs. Many research studies connect the significant influence of students' oral and literacy experiences outside of school with access to learning and success at school (Zwiers, 2008). By taking a critical look at not only the mathematics content but also the reading, writing, speaking, and thinking skills expected to meet the criteria and expectations in the classroom, and then making those expectations explicit and clear, the gap between non-mainstream and mainstream students narrows (Zwiers, 2008).

Geometry

"The word geometry was derived from the Greek words with the original meaning of measuring the land." (Hwang, Lin, Orchirbat, Shih, & Kumara, 2015, p. 27). Geometry was a key component in the study of mathematics from the time of the ancient Greeks until about the 1960's (Shannon, 2002). This study of geometry was based in deductive reasoning, but has gradually been replaced in recent years by a heavy emphasis on numerical reasoning and less on spatial reasoning (Shannon, 2002). Today, when students learn geometry they study shapes, space, and the tools used to measure and define them.

Geometry is an essential building block towards advanced math and science (Hwang, et al., 2015). As such, the traditional and typical methods in elementary school mathematics of memorizing and calculating formulas are not enough in teaching geometry. Students are much more motivated when geometry is taught through methods that “enhance children’s imagination, critical thinking, and spatial reasoning” (Hwang et al., 2015, p. 27). Furthermore, a strong spatial awareness is applicable in many problem-solving situations outside of the classroom, such as parking cars, playing tennis, putting up shelves, and in vocations such as brick-laying, dress-making, and drafting (Shannon, 2002). Burns (2007) also emphasizes the real-world importance of spatial reasoning skills, offering examples that adults encounter such as “when having to figure quantities for wallpaper, floor covering, paint, fabric, lawn needs, or a myriad of other home projects” (p. 108) and the vocational industries of building trades, interior design, and architecture.

When students are only taught to memorize formulas, their understanding lacks a foundation in concepts of shape and physical awareness. When geometry is taught with an understanding of how students construct knowledge, the resulting student learning has meaning and a foundation in spatial structure (Battista, 1999). The National Council of Teachers of Mathematics (NCTM) also recommends that students be given opportunities to develop understandings and procedures through investigation rather than memorize prescribed formulas (Capraro & Capraro, 2006). Geometry is full of formulas and procedures, such as calculating the area of a rectangle. Hwang, Lin, Ochirbat, Shih, and Kumara (2015) give examples of having students find the surface area of various blocks and boxes, study and make observations about shapes in the world around them, and other real-world applications of basic geometric principles. When students are guided through learning activities that lead to the construction of mathematical ideas, the resulting knowledge is personally meaningful and less fragile when applied to new problems (Battista, 2012).

One important concept that is part of the MN State Standards for Mathematics in fourth grade is the concept of angles. Devichi and Munier (2013) summarize previous research on children's construction of the angle concept following a historically Piagetian approach. "The same steps are taken to build representational space as those taken for perceptual-motor space" (cited in Devichi and Munier, 2013, p. 2). A common misconception for students when comparing angle size is to focus on the length of line segments rather than recognizing the two-dimensional space in relation to lines (Devichi & Munier, 2013). Students also "frequently fail to recognize that two angles are the same measure if they are oriented in non-standard directions" (cited in Smith, King, & Hoyte, 2014, p. 96). Typical classroom activities such as worksheets or identifying and classifying angle examples on the board make it difficult for students to jump straight to abstract thinking and understanding without any concrete understanding to build upon (Smith, King, & Hoyte, 2014). Activities that allow students to manipulate and explore relationships with concrete objects, as well as activities with dynamic elements, have been shown to have the most positive impact in teaching angle concepts to young learners (Devichi & Munier, 2013). Body-based movement activities in which students act out angle movements provide the opportunity for students to draw connections in the development of the mathematical concepts (Smith et al., 2014). Visual representations also play an important role, providing documentation for what students do and see and facilitating connections between the concrete and abstract representations (Smith et al., 2014). Keeping this research in mind along with possible implications for other geometry and measurement concepts will be important in the construction of a successful curriculum plan.

Another geometry and measurement concept included at the fourth grade level is understanding and calculating area and perimeter for rectangles and geometric figures that can be divided up into rectangular shapes. This can be very challenging for students, as argued by Sherman and Randolph (2004), citing statistics from the National Assessment of Educational Progress (NAEP) and other sources that show “that fourth and eighth grade students sometimes confuse area and perimeter” and “that this lack of understanding continued to affect children in older grades” (Sherman & Randolph, 2004), p. 26). It is important that students develop an understanding beyond simply memorizing formulas in order minimize student misconceptions such as these. There are many drawbacks to not taking the time to build conceptual understanding of geometric concepts such as area and perimeter. Sherman and Randolph (2004) argue that “memorizing misunderstood formulas is a short term solution that does not provide for long term retention, conceptual understanding or procedural skills” (p. 35).

Conclusion

Freeman and Crawford (2008) state, “To understand mathematics, a student needs to be able to read, solve problems, and communicate using technical language in a specialized context (p. 12). For English Language Learners, this is no easy task. Teachers of ESL students must utilize specific and engaging strategies to facilitate student learning of both academic language skills and mathematics content. Chapter 2 has outlined several important components necessary for teaching mathematics to ESL students. Academic language considerations are vital for making academic content accessible for ESL students and benefit other students as well. Mathematics teaching methods are rooted in the standards that guide best-practice mathematics instruction and teaching strategies that have been proven to work well.

It is important to rely on evidence-based strategies when designing effective learning activities, which in this case will meet the goal of exploring the research question: *How can a mathematics geometry curriculum unit be organized to support and develop academic language for fourth grade students through meaningful engagement strategies?* The learning and themes gathered in this literature review will now be used in the creation of an effective and engaging curriculum unit for a fourth grade mathematics class in the area of geometry and measurement. Chapter 3 will describe the curriculum design process that will be guide and shape the learning plans and curriculum unit.

CHAPTER 3

Methods

Overview

Previous chapters have discussed the need for and benefits of a mathematics curriculum that develops students' academic language through academic content. This is a need within my current school setting, as I will describe further in this chapter. It is also a need in diverse classrooms everywhere, and it is my hope that the geometry curriculum developed through this capstone project will be a helpful resource for others teaching fourth grade mathematics as well. I am exploring the research question: *How can a mathematics geometry curriculum unit be organized to support and develop academic language for fourth grade students through meaningful engagement strategies?*

This chapter will begin by detailing the rationale and outline by which I have organized the unit and lessons, based on the Understanding by Design framework by Wiggins and McTighe (2005). I also give attention to the work of Battista (2012) that outlines mathematical understanding as a cognitive process rooted in logical reasoning and the WIDA Standards Framework ("WIDA ELD Standards"), which organizes the necessary supports for ELL students at all proficiency levels to access the academic content and cognitive thinking processes. The next section describes the setting and participants for which the curriculum unit is designed. More detail regarding the planning for language and cognition is included in the desired results, assessment evidence, and learning plan sections. Finally, Chapter 3 will conclude by describing in detail a plan for developing each element of the mathematics geometry curriculum unit that will support and develop academic language for fourth grade students through meaningful engagement strategies.

Curricular Framework

The curriculum unit is designed using the Understanding by Design framework as developed by Wiggins and McTighe (2005). Understanding by Design calls for planning with the end in mind and ensuring that all elements of the lessons planned align with the overall goals for student learning. The first step in backwards design is to determine goals (Wiggins & McTighe, 2005). In this unit, goals are derived from the Minnesota State Standards in Mathematics for fourth grade. The planned unit goals also consider levels of language and levels of mathematical understanding, as informed by the WIDA Standards for Language Acquisition ("WIDA ELD Standards"), the Cognition Based Assessment (CBA) Levels of Understanding of Geometric Shapes (Battista, 2012), and other literature sources as referenced in Chapter 2. In the Understanding by Design process, enduring understandings, essential questions, knowledge, and skills are identified in alignment with the standards-based unit goals (Wiggins & McTighe, 2005). Mathematical knowledge and skills are paired with needed language skills in the areas of reading, writing, speaking, and listening and communicated to teachers and students through content and language objectives. Needs across all language dimensions are considered, including discourse, sentence, and word/phrase, and will be included in the unit goals. Keeping the focus on the "big ideas" or enduring understandings is important in order to promote academic talk and thinking in deep and connected ways (Zwiers, 2008). Specific attention to the less obvious and more general academic terms, grammatical structures, and content-specific vocabulary will require attention as language and content outcomes are identified.

The next step in the design process is to select assessments that will accurately and logically measure the identified goals (Wiggins & McTighe, 2005). The final step in backwards design is to select or create engaging and appropriate learning activities (Wiggins & McTighe, 2005). The learning activities included in the proposed curriculum unit align with the several of the teaching best practices for academic language and mathematics instruction highlighted in Chapter 2, including math talks, collaboration, math manipulative use, and technology integration.

It is important to note that the design process following the Understanding by Design framework is not a linear plan (Wiggins & McTighe, 2005). The elements of goals, assessments, and learning activities are carefully developed with the mindset of always prioritizing the overall goal of aligning all elements to maximize student learning through concurrent engagement with academic content and language skills. The end product is an organized unit consisting of carefully chosen goals, assessments, and learning activities that other teachers and I can follow for teaching in a fourth grade classroom setting. More specific details about the school and students for which this curriculum unit is designed are described in the next section.

Setting and Participants

This curriculum unit is designed for fourth grade students at an elementary school in a large suburban district of the Twin Cities area in Minnesota. According to the Minnesota Department of Education website's "Minnesota Report Card", in 2016, this elementary school has a total enrollment of 702 students in pre-kindergarten through fifth grade. The school is racially diverse, with 38.5% of students identifying as Black, not Hispanic origin, 31.6% of students identifying as Asian/Pacific Islander, 20.7% of students identifying as White, not Hispanic origin, 8.4% of students identifying as Hispanic, and 0.8% of students identifying as American Indian ("Minnesota Report Card," 2016). At the school, 23.4% of all students are English Learners, and 12.0% of all students are Special Education students. 64.5% of students participate in the Free/Reduced Price Lunch program and 1.9% of students reported as homeless. ("Minnesota Report Card," 2016). This information about student demographics plays an important role in the planning and creation of curriculum. Mathematics content lessons that focus on building academic language will benefit all students, but will especially benefit English Learners, who make up nearly one-fourth of the student population at the school.

There are four sections of fourth grade for the 2016-2017 school year, each with an average of 28 students. Each class is a heterogeneous mix of gender, race, and ability. English Language Learners at the school come from a variety of home language backgrounds, including Hmong, Vietnamese, Lao/Laotian, Russian, Arabic, and Spanish. Most are first or second generation Americans, and were born in the United States, but a few students have moved here more recently. Students use their home language for speaking and listening in social and everyday situations at home and sometimes among friends. About half of the parents of English Language Learners have some English language understanding, and some students have older siblings or other family members who are fluent in English who provide academic support at home such as helping with homework.

Fourth grade students learn the core subjects of math, language arts, science, and social studies within their homeroom classroom. Sixty minutes each day are scheduled for math instruction. As of the 2016-2017 school year, every fourth grade student in the district has use of a district iPad for school and home use. Utilizing this technology in the development of new curriculum offers many opportunities to support and extend learning for all students. The curriculum unit developed will teach the Minnesota state standards in Mathematics from the geometry and measurement strand. The district and school currently use *Math Expressions* (Fuson, 2008) curriculum; however, many students struggle with this curriculum so teachers often use additional resources to supplement.

It is my hope to not only create an organized geometry curriculum unit that will be used year after year without piecemealing together various activities, but also differentiate to meet the specific needs of English Learners and other students who are developing academic vocabulary and language skills for classroom discourse. Keeping in mind the mathematics content standards and the academic language skills necessary for accessing the content will be the focus that drives the planning of this curricular unit. The next section will outline the steps of the planning process within the Understanding by Design framework.

Curriculum Elements

Desired results.

The first stage in the Understanding by Design process is to identify curricular goals. There are many possible approaches for beginning this stage of the design process, including studying the essential language features at the discourse, sentence, and word/phrase levels, analyzing the ideas in state content standards, considering real-world applications, beginning with an existing resource or favorite activity, reflecting around a key skill, focusing on an important assessment, or starting with an existing unit for refinement (Wiggins & McTighe, 2005). I have chosen to focus on identifying goals from the standards, which in this setting are the Minnesota State Standards in Mathematics. First, the mathematics content standards from the geometry strand were analyzed in consideration of unit goals, including necessary knowledge and skills. The WIDA Framework for Language Development was then consulted to align and identify language skills that students will need in order to successfully access the mathematics content. Figure 3.1 below shows the fourth grade geometry and measurement benchmarks from the Minnesota State Standards in Mathematics that are used as the starting point for developing the lesson and unit goals.

4.3.1.1 Triangles
Describe, classify and sketch triangles, including equilateral, right, obtuse and acute triangles. Recognize triangles in various contexts.

4.3.1.2 Quadrilaterals
Describe, classify and draw quadrilaterals, including squares, rectangles, trapezoids, rhombuses, parallelograms and kites. Recognize quadrilaterals in various contexts.

4.3.2.2 Compare & Classify Angles
Compare angles according to size. Classify angles as acute, right and obtuse.

Figure 3.1. 4th Grade Geometry and Measurement Benchmarks

It is important to note that the Mathematics Standards provide benchmarks for defining what fourth grade students are expected to know and be able to do at the end of the curricular unit. What the benchmarks lack is a description of the developmental levels that lead to students obtaining the desired knowledge and skills. In the next section, a more detailed geometric framework as supported by the work of Battista (2012), Clements and Sarama (2009), and others will be described as it applies to the goals of this curricular setting.

The process of determining desired results and setting unit goals is carried out in a fluid process along with the two stages that follow: the assessment evidence and the learning plan. As noted in the Curriculum Framework section, the Understanding by Design framework is not intended to be a strict sequence but rather a flexible process that results in an organized and logical product (Wiggins & McTighe, 2005). Battista's (2012) levels of reasoning of Geometric Shapes are situated within the Cognition Based Assessment (CBA) system, so references to assessment and cognition level are correlated.

Cognition Based Assessment of Geometric Reasoning.

Before specific levels of geometric reasoning are detailed in the following sub-sections, the distinction must be made that the defined levels are designed to highlight and assess thinking and reasoning, not levels of students (Battista, 2012, p. 47). That is to say, some students may operate at more or less advanced levels of reasoning when presented with different tasks, dependent on background knowledge, availability of physical manipulatives, connections to other problems, or a variety of other factors (Battista, 2012). The following sub-sections will describe characteristics of each level of reasoning, as well as implications for lesson planning and classroom discussion.

Level 1: Visual-Holistic Reasoning. The most basic level of geometric reasoning within the context of a Cognition Based Assessment (CBA), as described by Battista (2012), is Visual-Holistic Reasoning. Students at this level see and identify shapes as whole objects, base their understanding in what an object "looks like," and use familiar objects to define and make connections (Battista, 2012). Shapes are recognized as wholes, but the student can't yet define attributes or properties of shapes (Clements & Sarama, 2008). Orientation of shapes greatly affects students' reasoning at Level 1 (Battista, 2012). For example, students commonly misidentify shapes if the figure is "upside down," or use rotation to justify an incorrect shape name. While the majority of fourth grade students are capable of reasoning beyond Level 1 of the CBA system (Battista, 2012), misconceptions that stem from viewing shapes as wholes are common and must be addressed in developing geometric understanding that meets the standards and benchmarks outlined previously.

Level 2: Analytic-Componential Reasoning. The next Cognition Based Asesessement (CBA) level as defined by Battista (2012) is Analytic-Componential Reasoning. At this stage, students can “attend to, conceptualize, and specify shapes by describing their parts and the spatial relationships between the parts” (Battista, 2012, p. 2). Initially, students use informal and everyday language to describe shape properties and parts, such as “pointy” or “square corners.” There is an inherent imprecision in these informal descriptions, and as students move towards more accurate mathematical terms to define and talk about geometric concepts, their understanding becomes more complete and transferrable to other topics (Battista, 2012). Students’ reasoning within Level 2 of Battista’s (2012) CBA system varies greatly in sophistication, ranging from simple, visual, and imprecise descriptions to complete and correct descriptions that use formal geometric terms. The precision of language is often dependent on student’s prior experiences with shapes in more formal academic settings, and can be built upon with explicit instruction that builds upon and increases student definitions and understanding. For EL students, focusing on the language that allows students to clearly express their reasoning gives voice to their cognitive understanding. Making connections to students’ home language is an often-used strategy for linking new understandings of shape properties to prior knowledge. Within the range of understanding at CBA Level 2, students increase their ability to analyze interrelated parts and use formal geometric concepts to specify relationships between parts of shapes (Battista, 2012).

Level 3: Relational-Inferential Property-Based Reasoning. As students develop the capacity to reason and classify interrelated shapes, Battista (2012) defines

CBA Level 3 as Relational-Inferential Property-Based Reasoning. At the more basic understanding of CBA Level 2, student definitions of shapes encompass all properties and features. At Level 3, students can interrelate properties and use justification in increasingly sophisticated ways. The language and cognition skills at this level are also important to note, as students' justifications "start with empirical associations (when Property X occurs, Property Y occurs), progress to construction-based explanations for why one property "causes" another property to occur, move to logically inferring one property to another, and end with using inference to organize shapes into a hierarchical classification system" (Battista, 2012, p. 37). Students need to be able to communicate their observations, understanding of cause and effect, inferential thinking, and how concepts and objects are interrelated. The academic language skills required for formal discussions and written explanations will need to be modeled and taught for both EL students and native English speakers in the fourth grade classroom.

Level 4: Formal Deductive Proof. At the most advanced level of Battista's (2012) Cognition Based Assessment (CBA) levels, students understand and can construct formal deductive proofs. This is a skill required in traditional high school geometry courses, and is included here to give a complete picture of the range of cognitive development as it relates to spatial reasoning and geometric shapes. The student reasoning at CBA Level 4 "recognizes differences among undefined terms, definitions, axioms, and theorems" (Battista, 2012, p. 3). The system of axiomatic thinking is at an advanced level of academic language use, and is formed through significant practice and math talk in academic settings.

The WIDA Standards Framework

This section will explain the organizational tool used in this curriculum unit to address the needs of ELL students in order to meet the desired results of accessing mathematics content and improving academic language skills. The WIDA Standards Framework is a tool personalized by the teacher, school, or district with the intent of planning for specific language supports for ELL students to successfully access academic content and meet state content standards ("WIDA ELD Standards"). Its use in this curriculum unit will identify and guide some of the language forms and functions as students achieve greater cognitive reasoning skills as they relate to geometry concepts of shape. The Standards Framework also fits into the desired results stage of the Understanding by Design process (Wiggins & McTighe, 2005), because identifying goals for language learning alongside content learning is an intentional and essential part of helping ELL students succeed in the classroom.

The first component of the WIDA Standards Framework identifies the English Language Development Standard, which in this case is the third standard, pertaining to the Language of Mathematics. WIDA English Language Development Standard 3 states, "English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics" ("WIDA ELD Standards," p. 3). The second component of the framework lists the connection, meaning the state content standard, and example topic. In this mathematics geometry curriculum unit, the connecting benchmark standards are from the Minnesota State Standards in Mathematics. The next component of the WIDA Standards Framework lists one or more example contexts for language use. In the course of an entire lesson, students will utilize language through various learning tasks, and the teacher can plan for activities in the speaking, listening, reading, and writing domains. Making strategic decisions regarding these activities within the context of the example will help guide student advancement in both language and content knowledge and skills ("WIDA ELD Standards").

In the "Cognitive Function" component of the framework, the particular cognitive demand for the lesson activity is expressed using verbiage from Bloom's revised taxonomy (Anderson, Krathwohl, & Bloom, 2001). Bloom's revised taxonomy provides a framework for consistency across language levels and across content areas. It is important to note that all students, even those at the most basic language proficiency levels, can think and process at the highest cognitive levels, however, those with limited English language skills may not yet be able to access or communicate the linguistic materials. Planning appropriate tasks for what students at each language proficiency level can do enables students to construct meaning and express complex ideas within the content and cognitive task ("WIDA ELD Standards").

WIDA's standards framework distinguishes five levels of language proficiency, defined by specific criteria, with Level 6, Reaching, signifying the end of the continuum, where language performance meets all criteria ("WIDA ELD Standards"). A Model Performance Indicator (MPI) is written as an example of how language is produced or processed within the identified academic context. The MPI consists of three elements: language function, content stem, and instructional support ("WIDA ELD Standards"). Displaying the MPIs together in a table as a strand shows the progression between language proficiency levels, and teachers can see the language development in the example context ("WIDA ELD Standards"). MPIs are used to differentiate learning for individuals and groups of students, and matching students to their level of proficiency within the strand allows the teacher to challenge the student beyond their current independent proficiency level. In this unit, students will stretch both their language proficiency and their cognitive understanding as they contemplate and express their reasoning of geometric concepts. Figure 3.2 below shows the components of the WIDA Standards Framework and Model Performance Indicators across the spectrum of language proficiency levels as they apply to the mathematics topic of triangles as included in the planned curriculum unit. Throughout the unit, similar cognitive thinking skills, content stems, and language supports will be utilized for all the mathematics geometry benchmarks addressed as noted in the previous desired results section.

<table>
<tr><td colspan="7">English Language Development Standard 3 English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
Content Connection: Minnesota State Standards in Mathematics Benchmark 4.3.1.1: Describe, classify and sketch triangles, including equilateral, right, obtuse and acute triangles. Recognize triangles in various contexts.
Example Context for Language Use: Students will classify examples and non-examples of types of triangles and provide justification.</td></tr>
<tr><td colspan="7">Cognitive Function: All students will classify types of triangles and justify their reasoning.</td></tr>
<tr><td rowspan="2">Language Domains: Speaking</td><td>Level 1 – Entering</td><td>Level 2 – Beginning</td><td>Level 3 – Developing</td><td>Level 4 – Expanding</td><td>Level 5 – Bridging</td><td rowspan="2">Level 6 - Reaching</td></tr>
<tr><td>Match word cards (sides, angles) and triangle types (acute, right, obtuse, scalene,</td><td>Identify and define types of triangles from triangle sort cards using an anchor chart</td><td>Categorize triangle sort cards and explain categories using a Venn Diagram</td><td>Compare and Contrast examples and non-examples of types of triangles</td><td>Justify classification of triangle types in a small group discussion using</td></tr>
</table>

	isosceles, equilateral) and triangle sort cards using an anchor chart with pronunciation and simplified definition recordings on the iPad	with pronunciation recordings on the iPad	graphic organizer, sentence stems and teacher-modeled language.	with a partner using a Venn Diagram graphic organizer	teacher-modeled language.	
Topic Related Language: Triangle, acute triangle, right triangle, obtuse triangle, angle, acute angle, right angle, obtuse angle, degrees, sides, congruent, equivalent, length, scalene triangle, isosceles triangle, equilateral triangle						

Figure 3.2 Model Performance Indicators for the Topic of Triangles

Assessment evidence

Once desired results for the curriculum have been identified, the next stage in planning using the Understanding by Design process is to determine appropriate assessments by which to measure the unit goals. Wiggins and McTighe (2005) stress the importance of crafting assessments in which students demonstrate their knowledge beyond simply giving quizzes and short-answer tests. Traditional tests and quizzes do not measure the complete spectrum of student performance nor do they promote enduring understandings. Effective and meaningful assessments are about much more than just generating grades (Wiggins & McTighe, 2005). While there is a responsibility to record and report student progress following district expectations for standards-based grading, first and foremost assessments are selected and designed to measure student learning in the hopes of assisting all students towards achieving proficiency in the essential knowledge and skills of the unit. Wiggins and McTighe (2005) suggest building assessments from six facets that show deeper understanding. The six facets are explanation, interpretation, application, perspective, empathy, and self-knowledge. All of these require sophisticated language skills and promote higher order thinking. It will be important to consider language skills within the assessments, both what students can do as well as what scaffolds English Learners will need in order to fairly assess their mathematics content understanding. Anticipating and identifying student misconceptions about mathematics content is part of the formative assessment process, and is explained as it applies to the planned fourth grade geometry unit in the next section.

Planning for Misconceptions. Students often can get "stuck" on incorrect thinking, or understandings and prior knowledge that limit the construction of new mathematical ideas. When teachers are aware of common misconceptions within a unit of study, appropriate steps can be taken to correct erroneous thinking. In this section, various common misconceptions are explored along with suggested methods for guiding students to build understandings that support sound geometric understanding.

Students often believe that regular polygons are the only "real" shapes. For example, students might say that any triangle other than an equilateral triangle is not a "real" triangle, or are unable to recognize trapezoids that differ from the common pattern block manipulative shape (Minnesota STEM Teacher Center Frameworks, 2017). The misconception of regular polygons develops at an early age. With few exceptions, early childhood books, toys, and learning materials introduce basic shapes in rigid ways with few irregular or real-world examples (Clements & Sarama, 2008). Variations of the typical closed and symmetrical shapes, or "exemplars" as Clements and Sarama (2008) call them, can be used to challenge and expand student definitions. Non-examples are also useful in getting students to recognize shapes at increasingly sophisticated levels.

Another common misconception is believing that changing the orientation of a shape changes its name or classification. Students are confused when asked to identify a shape that is turned "the wrong way." Children are less likely to get stuck on this misconception when the lesson activities utilize manipulatives, or if they can walk around a large shape placed on the floor (Clements & Sarama, 2008). Rich discussion that leads students to justify their ideas and providing many examples and non-examples in various contexts can also help students correct misconceptions about shape orientation.

Understanding how angles determine the definition and classification of many shapes is often a challenging concept for students to master. Students often view angles as stand-alone objects, and specifically at CBA Level 1 reasoning (Battista, 2012), students can only view shapes as whole objects. Thinking that angle size is dependent upon the length of the rays is a typical misconception (Minnesota STEM Teacher Center Frameworks, 2017). Students may mix up geometric terms such as acute and obtuse. Students may also fail to see concave angles as defining parts of a shape. Several strategies can be used to correct misconceptions related to angles. Multiple experiences with examples and non-examples will expose students to angles as they form shapes and enrich student understanding.

Learning plan

The third and final stage of the Understanding by Design framework is the learning plan, which primarily consists of lesson activities. In this curricular unit, many of the lesson activities are adapted from the existing district curriculum, as well as supplemental learning activities already being used by teachers in my school setting. The learning plan relies heavily on the research gathered in the literature review phase of this capstone project and uses evidence based practices for optimal student learning. Utilizing learning activities that reach multiple modalities ensures that all students can learn using their preferred learning style.

When teaching fourth grade geometry lessons in the past, I have had students sort shapes cut from construction paper and justify their categories in small group discussion. I have also had students play games that promote movement and student talk, such as quiz-quiz-trade and four corners, for reviewing and reinforcing developed vocabulary definitions and other key concepts. The classroom interactive whiteboard is another excellent teaching resource when available. Protractor tools available in computer software can be used for teacher modeling as well as allow students to manipulate and measure angles on a larger scale. Alternatively, iPad apps for geometric tools can be projected onto the board or screen as available for whole-class instruction and used by students during small group work and independent practice. Geoboards and other tactile manipulatives can help students both explore new learning about triangles and quadrilaterals as well as cement essential understandings. Tangrams present many rich opportunities for problem-solving challenges and beneficial experiences in part-whole relationships of shapes (Clements & Sarama, 2008). Real-world application through projects and problems from students daily lives have the benefit of engaging and motivating students as well as integrating language skills authentically into lesson activity. These are just some possible lesson activities I have considered for inclusion in the completed unit plan. Other activities not described in this section are also included in the final plans, as the planning process was conducted and all aspects necessary for student learning were developed.

Word and Phrase Dimension. Developing and using common language terms are important for both the teacher and students as they engage in dialogue around geometric concepts. Students enter the classroom with a wide range of prior knowledge related to shapes, spatial awareness, and geometry vocabulary. As noted in Chapter 2, it is beneficial for students to play an active role in constructing definitions for vocabulary terms, rather than simply be given a list of terms and definitions from the teacher (Sherman & Randolph, 2004). When students use their own words, they are able to make connections to prior knowledge and show a deeper understanding of the word and how it applies to the mathematical context. Of course, planning for language definitions for vocabulary words is just one dimension of academic language planning and instruction that is essential to the learning plan.

Sentence Dimension. Providing sentence frames for students as they define and discuss the target geometric shape names and categories is an essential instructional support for all students. Differentiating sentence frames for the range of language proficiency levels will enable all students to produce spoken and written language to express their understanding and reasoning. Figure 3.3 details some sentence frames that students will use as they identify and classify various types of triangles and quadrilaterals.

Beginning	This is a _________. It is/has ________. This is not a _________. It is/has ________.
Intermediate	This is a ________ because _________. This is not a _________ because _________.
Advanced	This shape has ________, ________, and ________. This shape has _________, _________, and _________; therefore it is/is not a ________.

Figure 3.3 Sentence Frames for Naming Shapes

Students will also use language to communicate their reasoning as they draw conclusions about properties that define specific shapes. Sentence patterns for stating causational relationship will need to be modeled and practiced. An example of one sentence frame that could be used is: "If ______ has ______, then I know it is a ______."

Sentence frames for naming and justifying shapes alone are not sufficient for collaborative student discussion that builds upon and deepens understanding towards a central goal. The next section will discuss the discourse dimension of developing academic language, with specific focus on building a classroom culture rich in authentic and meaningful discussion.

Discourse Dimension. Many lesson activities will begin or cumulate with teacher-led large-group discussion. Facilitating effective whole-class discussion is a skill that requires foresight and planning. Too much teacher talk limits student thinking and can inhibit opportunities for deep understanding. However, an appropriate level of prompting, planning, and guidance is necessary to avoid too much unrelated, misdirected, or erroneous student talk (Zwiers, 2008). Goldenberg (cited in Zwiers, 2008) qualifies effective classroom discussions as those that are "engaging and relevant, maintain a discernible topic throughout, not be dominated by any one student or teacher, and have all students engaged in extended conversations" (p. 114). The classroom should be a challenging yet non threatening environment, promoting positive support of others rather than combative discourse, and requires both students and teachers to develop an attitude of humility, flexibility, and a willingness to modify or even abandon ideas when new evidence is presented (Zwiers, 2008). Beyond developing a positive classroom environment, proper planning for class discussion should include predicting "possible tangents, elaborations, and connections to student lives" (Zwiers, 2008, p. 114). Teachers need to anticipate and pre-teach background knowledge and language needed to access the big ideas and thoughts of the topic. Zwiers (2008) recommends paying attention to pacing, and slowing down discussion to allow for wait time and student think time enables students to mentally piece together new concepts using what is often complex language. In discussion environments, listening is an active and challenging skill, and supporting student success by providing appropriate think time, as well as activities to clarify and reinforce key objectives will ensure that students do not tune out or lose track of what is being said.

A mix of both whole-class discussions and small-group work will provide opportunities for all students, especially ELL students, to engage in content-rich speaking and listening. Despite the teacher's best planning and intentions, not all students will speak in whole-class discussions, usually due to shyness and feeling intimidated in a large group. Large class sizes can also make whole-group discussions difficult for teachers to effectively manage. Mixing in opportunities for directed academic conversation in small-groups or pairs provides an alternative that, when properly structured, offers the same rich language and content understanding benefits (Zwiers, 2008). However, students can easily get off task or not adequately explore the lesson objectives if small group work is not properly set up. The purpose and type of discussion must be clear in order for group talk to be productive. When the focus or form of discussion is vague, students waste time, either in confusion or unrelated talk (Zwiers, 2008). Discussion skills for various modalities must be taught and modeled, and supported through listening and ongoing feedback (Zwiers, 2008).

Students at the lower levels of geometric reasoning as defined by Battista's (2012) Cognition Based Assessement (CBA) will rely on visual and empirical thinking, which should be allowed as it lays the foundation for higher levels of geometric reasoning. Student talk at the visual thinking level will include describing what something "looks like" and will include comparisons to other shapes, examples, and real-world objects. Teacher questions can help students see the limitations of relying on visual information only, and students will begin to move toward more logical deductive explanations (Battista, 2012). Logic statements follow an "If, then" structure, and students can use schematic visual diagrams such as a tree map to organize their deductions.

Building a culture of meaningful academic discussions that students can engage in independently is a process that is cultivated throughout the school year. It is worth noting that while hands-on activities and use of manipulatives is one way to promote student engagement, "Just because an activity is engaging or 'hands-on' doesn't mean it will automatically cultivate academic talk" (Zwiers, 2008, p. 137). Meta-discussions with students, for example asking questions like “Why do we talk in class?” and “What happens in good group discussions?” helps build students’ capacity for monitoring and engaging in effective group talk (Zwiers, 2008).

As discussed in the Chapter 2 literature review, students build academic language through rich discussion, so the planned curriculum unit incorporates a structure for building student capacity for sharing and building upon each other’s ideas. The same concept is true for constructing strong mathematical understanding. “Primary responsibility for establishing the validity, or ‘truth’ of mathematical ideas should lie with students, not teachers or textbooks” (Battista, 2012, p. 65). Students are given the responsibility of solving mathematical problems by making conjectures and then using reasoning and justification to explain how and why the solution is valid (Battista, 2012). Because the teacher is not giving out the correct answers, but rather student voice is prominent, student explanations must be detailed enough for other students to be able to follow and understand the reasoning (Battista, 2012). This will be challenging for ELL students, who typically lack confidence and may be hesitant to speak at length. Providing sentence frames for students to model their ideas, practicing collaborative discussion structures, and organizing students into deliberate groupings for small group discussion will help alleviate stress and promote participation by all students. Student discussions will become increasingly collaborative as they build upon one another's ideas and use language to clarify, disagree, and elaborate. Students need language to connect what their classmates propose with their own knowledge and ideas. The collaborative language supports table shown in Figure 3.4 acts as a resource for students as they seek to clarify, disagree, and elaborate in rich academic discussion with their peers.

Clarify	**Disagree**	**Elaborate**
Will you explain that again? I have a question about what you said about ____. Could you give an example of what you mean by _____.	Another way to look at it is _____. I do agree with what you said about _____, but I think _____. I have a different answer. I wrote down that _____.	You made a good point when you said ____. I see what you are saying. I agree because _____. My idea builds on ____'s idea. I think _____.

Figure 3.4 Collaborative Language Supports

Conclusion

This chapter has described the three stages of the Understanding by Design framework that provide the structure for the developed fourth grade geometry curriculum unit. The information about student population of the elementary school and classroom for which this unit is designed give important context for the instructional decisions made. Chapter 3 has also outlined the rationale behind the chosen curricular framework, Understanding by Design (Wiggins & McTighe, 2005). The Understanding by Design process focuses on desired results, assessment evidence, and a learning plan that supports cohesion. Specifically, Battista's (2012) Cognition-Based Assessment and levels of geometric reasoning are utilized to inform assessment evidence, as supported by the WIDA Standards Framework ("WIDA ELD Standards") tool for purposeful language supports. All of these elements make up the method that supports the curriculum development process around the research question: *How can a mathematics geometry* *curriculum unit be organized to support and develop academic language for fourth grade students through meaningful engagement strategies?*

The geometry curriculum unit developed through this capstone project is the result of blending evidence-based strategies from the literature as well as my own experience of what works in my classroom setting. Creating curriculum is a complicated process, weaving together many elements of teaching and learning to meet set goals. In the next chapter the results are shared in the form of a completed curriculum unit that integrates meaningful engagement strategies, academic language development, and standards-based mathematics geometry content. Chapter 4 also provides a narrative of the unit plan and individual lesson modules, organized using the three stages of the Understanding by Design framework, as planned in Chapter 3. Each lesson consists of desired results, assessment evidence, and learning activities that align with the unit goals. Resources and materials are described and shared, as well as a rationale that explains the instructional decisions of the unit design.

CHAPTER 4

Curriculum Plan

Overview

Chapter four outlines in detail four lesson modules that make up the curriculum plan addressing the research question: *How can a mathematics geometry curriculum unit be organized to support and develop academic language for fourth grade students through meaningful engagement strategies?* Following the method plan from Chapter three, each lesson module was designed with desired results, assessment evidence, and a learning plan for a cohesive and intentional curriculum unit that will meet the needs of all fourth grade students in the classroom. Specific attention is given to the needs of ELL students, with academic language goals and supports incorporated throughout the unit. Activities in the learning plan engage students in cognitive reasoning designed to build greater geometric understanding, with students constructing and explaining their own knowledge rather than simply receiving facts and definitions from the teacher.

This unit is designed to work best with students organized into intentional pairings and small groups to maximize meaningful conversation and structured language practice so all students can demonstrate and build their cognitive reasoning skills.

Considerations for student groupings include, but are not limited to: English language proficiency, mathematics proficiency, home language, behavior, and learning style. Students build knowledge from the input they receive from their classmates. Ideal student groupings allow all participants to benefit from their role as both a learner and as a peer coach or tutor. Additionally, student groups should be varied throughout the school day to give all students opportunities to hear from different perspectives, build relationship and community within the classroom, and build upon student strengths in various curricular and non-curricular areas. One common situation is to pair a student with a lower English language proficiency level who also has a good mathematics foundation with a student at an intermediate language proficiency level who has gaps in their mathematics understandings. Another pairing might match a middle level EL student with a student at a bridging or reaching language proficiency level. Pairing a fluent or native English speaker with a high level EL student will provide a peer model of advanced language. Students with the same home language background who will collaborate well with minimal behavior distractions have the added benefit of being able to discuss academic concepts in both English and their home language, which adds opportunities to clarify and expand their understanding. When student groupings are not closely matched, such as if a student who is a native English speaker and high-performing in mathematics were paired with a newcomer EL student, both students miss out on the opportunity to give and receive feedback. The student with more skills may resent the peer tutor role, and the student developing from a lower level will have a hard time connecting, contributing, or keeping up with the pace of group conversation and academic work. Social interactions should not be ignored either, and teacher understanding of each individual student and relationships between students is an ongoing and dynamic process over the course of the school year.

The teacher role throughout the curriculum plan is as a guide, seeking to challenge and advance students' independent inquiry into the cognitive reasoning and mathematical understanding goals of the unit. Battista (2008) advises against giving answers, and instead says, "Asking probing questions can be critically important in encouraging students to use more sophisticated descriptions of shape properties" (p. 82). The collaborative language supports included in the lessons provide a framework for students to question, challenge, and build upon one another's thinking as they talk about mathematical ideas and move towards more advanced levels. Many of the activities can also be repeated as independent stations in a guided math setting for additional practice and reinforcement once initially taught. Additionally, the modules are not designed to be covered in a single lesson, and should be taught over the course of several days, utilizing ongoing formative assessment evidence and student performance to determine student acquisition of geometric reasoning and academic language skills and responding accordingly.

Module 1: Angles

Desired Results

Minnesota State Standards in Mathematics Benchmark 4.3.2.2: Compare angles according to size. Classify angles as acute, right and obtuse.

Content Objective: Students will recognize and classify various angles in real-world contexts.

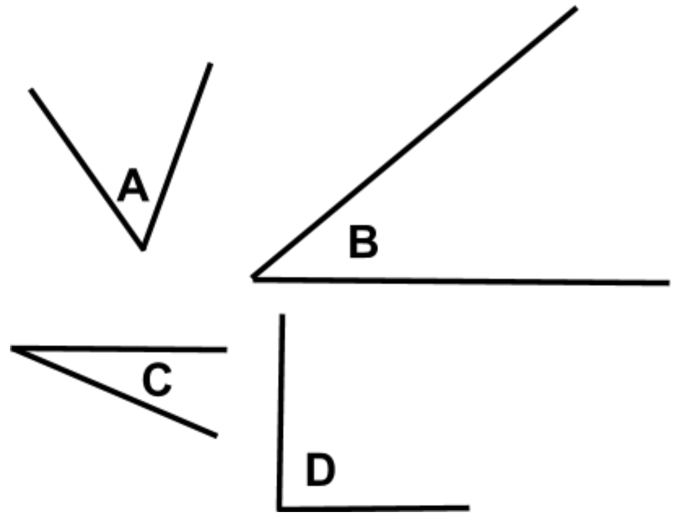
Language Objective: Students will describe angles as acute, obtuse, and right by comparing the size of the angle turn.

Assessment Evidence

Students will share the results of their learning by sharing the angles they find around the classroom (real-world context) as a result of the photo scavenger hunt activity with the rest of the class. The brief informal presentations will provide the teacher with a formative assessment of both the content and language objectives above.

Learning Plan

Which one doesn't belong? The anticipatory set of the lesson seeks to engage all students in the cognitive thinking skills of comparing and contrasting, as well as using language supports to justify and explain their thinking. In the style of "Which One Doesn't Belong," by Danielson (2016), students are presented with a set of four pictures, shown in Figure 4.1.1., along with the question prompt, "Which one doesn't belong?" and the sentence frames for language support. There are no wrong answers, as each picture can be defended as unique from the other three in some way. Linking to the lesson topic, angles, the pictures presented here are all angles.



I think ____ doesn’t belong. It is different because _________.

I think ____ doesn’t belong because ____, ____, and ____ all are/have ______.

Figure 4.1.1. Which One Doesn’t Belong?

In Figure 4.1.1. above, pictures A, B, and C, are all acute angles, whereas Picture D is a right angle. Pictures B, C, and D, contain a line segment that is parallel to the horizontal plane, and Picture A is “pointy.” The line segments in Pictures A, C, and D, are of similar length, and Picture B has longer line segments. These are just some potential student responses. The teacher can expand upon and clarify student responses with appropriate technical vocabulary, probing questions, and affirmations of geometric reasoning.

A common misconception with angles that students think an angle is bigger or smaller based on the length of the line segments or rays of the angle, rather than the inner rotation. The teacher can probe for this misconception by asking students which of the pictures in Figure 4.1.2. is the “biggest” angle:

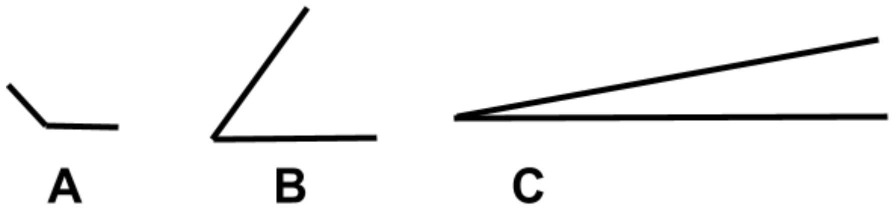


Figure 4.1.2. Which is the Biggest Angle?

Angle A has the largest angle measure; it is an obtuse angle and greater than 90 degrees. Angles B and C are acute angles. However, as the angle measure decreases in the figure, the length of the line segments in each angle increases. Students may state that angle C is the largest angle because of the line segment lengths. As a follow-up, the teacher can direct students to use their pencil and ruler to extend the line segments in angles A and B to the same length as angle C, then compare again.

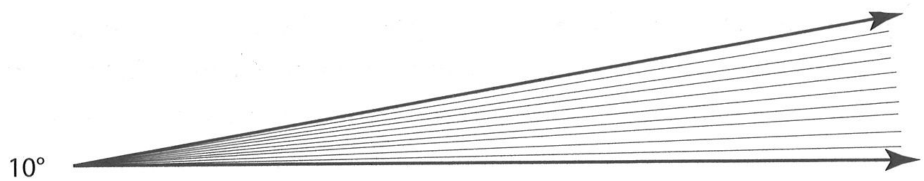
Battista (2012) provides a good example for building understanding of angle measurement, as shown in Figure 4.1.3. below. Students can connect rotational movement to partial turns of a 360° circle. Showing an image of a one-degree (1°) angle and a ten-degree (10°) angle divided into one-degree increments will guide students to focus on the inner rotation of an angle, rather than the lines, rays, or segments that frame an angle.

Understanding Angle Measurement[1]

Angles are measured in degrees. One degree is the amount of rotation it takes to move the bottom ray onto the top ray as shown below.

It takes 360 one-degree (1°) rotations to make one complete revolution, all the way around a circle.

It takes 10 one-degree rotations to make a 10° angle.



[1] See Battista, 2003, for more suggestions for teaching angle measurement.

Figure 4.1.3. Angles as Rotational Measurements (Battista, 2012, p. 77)

Angle Sort. Each group of students is given a set of angle cards (see Appendix A) and directions to sort the angles in an open sort. Students are to work together and come to a consensus on which angles fit together, and how to define or describe each category. Students will benefit from having the sentence frames for collaborative language support available, which can be printed on a card for each group (see Figure 4.1.4. and Appendix A).

Clarify	Disagree	Elaborate
Will you explain that again? I have a question about what you said about ____. Could you give an example of what you mean by _____.	Another way to look at it is _____. I do agree with what you said about _____, but I think _____. I have a different answer. I wrote down that _____.	You made a good point when you said ____. I see what you are saying. I agree because _____. My idea builds on ____'s idea. I think _____.

Figure 4.1.4. Collaborative Language Supports

As students work, the teacher circulates groups to listen in, reinforce, and clarify as needed. Some student groups may recall the terms acute angle, obtuse angle, and right angle from previous geometry units in second or third grade, and others will need to be taught the vocabulary terms. Using one or more group's sorted angles as an example for whole class discussion will wrap up the activity and ensure that everyone has common terminology and the target vocabulary for the lesson. As a class, practice naming and providing justification for the classification of angle types using the sentence frames below in Figure 4.1.5. Also, point out for ELL students the modification of the article "a" to "an" as they complete the sentence frames, as in "an acute angle," "an obtuse angle," and "a right angle" where the first letter is either a vowel or consonant.

_______ is a ________. It has _________.
_______ is a ________. I know because _________.

Figure 4.1.5. Sentence Frames for Justification

The sorted groups of acute, obtuse, and right angles with vocabulary labels and definitions should be posted as an anchor chart for students to refer to throughout the lesson and unit. Students can then take a picture of the anchor chart with an iPads, if available, to have the anchor chart with vocabulary, definition, and visual accessible. Students could also add additional annotations, written notes, and audio recordings with pronunciations using their iPads to refer back to when working independently.

Hidden Shape Angles. The next exercise of the lesson introduces students to angles as components of shapes. Two shapes, a trapezoid and a square, are shown to students, as in Figure 4.1.6. below. The obtuse, acute, and right angles of the shapes are identified and labeled using a color key (see Appendix A). If students have a hard time seeing the angles, drawing the angles next to the corners of the shape, or even cutting out the angles may help students see the components.

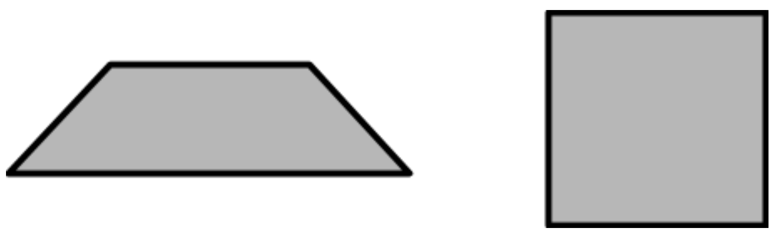


Figure 4.1.6. Hidden Shape Angles

After students have marked the angles independently, they can check their work in pairs or small groups, using language to explain and justify their thinking, such as, "I labeled these as ____ angles because ______. Do you agree?" and "Yes, I also labeled those as ______," or "No, I think those are _____, because _____. Let me show you." As needed, the sentence frames can be modeled and posted as a visual for students. Students who have correctly identified the angles will have marked two acute angles and two obtuse angles on the trapezoid and four right angles on the square. If students respond differently, discuss to clear up any misconceptions or extend student thinking. In the scope of this unit, reflex angles are not explicitly taught. However, if students bring them up, teachers should use their discretion to go beyond the lesson objectives if it will not cause students to be confused.

Photo Scavenger Hunt. Next, students will work in pairs using technology to record their work. iPads are preferred, as the camera can be paired with a drawing or annotating application to photograph and identify the real world angle examples students find. If students do not have access to technology, they can participate by sketching and describing the angles they find.

Before students are dismissed to work, showing a teacher example as a model will help students understand their task. Students can also reference the anchor chart of acute, obtuse and right angles and some may need the language support of the sentence stems for identifying and justifying shape definitions used in the earlier activity.

Lesson Conclusion Students will share their photos with the rest of the class. Each group will select one photo to share, and describe the angle type. Groups first practice sharing with one other pair before presenting in front of the class. Students at lower levels of language proficiency can use the sentence frames from earlier lesson activities (see Appendix A). More proficient students will not need the sentence frames after multiple practice opportunities. As students share with the class, the teacher can reinforce the lesson objectives, and also begin to casually point out other geometric elements that will be explored in the curricular unit.

Module 2: Triangles

Desired Results

Minnesota State Standards in Mathematics Benchmark 4.3.1.1: Describe, classify and sketch triangles, including equilateral, right, obtuse and acute triangles. Recognize triangles in various contexts.

Content Objective: Students will categorize examples and non-examples of each type of triangle by sides and angles properties and justify how examples are sorted.

Language Objective: Students will use sentence frames to classify examples and non-examples of types of triangles and provide justification.

Assessment Evidence

Students complete several sorts of triangles, including a compare and contrast sort using a Venn Diagram graphic organizer. The results of this student work are recorded by taking a photo on the students' iPads, which can be sent to the teacher for later review and evidence of learning. Students' written comments during the gallery walk provide evidence of language use, and participation in discussion in the lesson conclusion allows teachers to measure student success with using language to justify their geometric reasoning.

Learning Plan

Open Sort. Students are given a set of triangle cards to sort (see Appendix B). It is an open sort, so students may sort the shapes in any way, but must be able to explain their reasoning. After a few minutes, students share their sorts with their small groups. They talk about similarities and differences in how the triangles are sorted. The teacher circulates among the groups, listening for evidence of the range of levels of geometric reasoning (Battista, 2012) as discussed in Chapter 3. Students at a higher level of geometric reasoning for this activity will describe the components of triangles with increasingly sophisticated and mathematical language, and use what Battista (2012) describes as deductive and inferential reasoning to justify their work, indicating Cognition Based Assessment (CBA) levels 3 and 4. ELL students can use their home language as well as new mathematical vocabulary in English to demonstrate their cognitive reasoning about the geometric shapes, and through continued practice, learn the academic language for effectively communicating the mathematical content in English.

Teachers need to be mindful of common student misconceptions, such as only recognizing equilateral triangles as valid examples or discounting any examples that do not lie flat on a base (Minnesota STEM Teacher Center Frameworks, 2017). To correct these and other misconceptions, the triangle sort cards in Appendix B were designed to represent a wide variety of triangle types and orientations. As needed, additional examples can be added by drawing on blank cards. Students at CBA level 1 (Battista, 2012) will have a difficult time further sorting triangles, or will rely only on sorting by what the shape as a whole looks like rather than identifying components and properties, for example, identifying triangles as pointy, fat, tall, etc. Teachers can show students examples and non-examples in efforts to prompt increased geometric reasoning and correct misconceptions.

Students use language to classify and categorize as they place shapes together in groups. Figure 4.2.1. below shows sentence frames that students can use as they talk about their sorting process. When introducing the sentence frames, the teacher may need to go over some of the terms and clarify any that students do not understand. Multiple meanings of the word "like" should be discussed; in this situation, "like" means "similar," not showing preference.

Similar and Grouped together	____ and ____ both have _____. ____ and ____ belong together because _________. ____ and ____ are similar because _____. The shapes in this group all are/have ______. ____ is/has _____. Likewise, ____ also is/has ______, so they are in the same group.
Different and separate groups	_____ is different from _____. It does not have ______. _____ is not like ______. It is not ______. _____ is separate from _____ because ______.

Figure 4.2.1. Sorting Sentence Frames

Angles and Sides Sorts. Next, students are directed to sort their triangle cards into groups by the types of angles each triangle has. Give students a blank sorting mat like the three-category sort graphic organizer in Appendix B to organize their work. As a class, review what acute, right, and obtuse angles look like. After students have sorted the triangles, label the groups with the vocabulary: acute triangles, right triangles, and obtuse triangles. Teacher questioning is useful for formative assessment of students' cognitive reasoning around components of geometric shapes, and guided inquiry in response can guide students towards higher levels of geometric reasoning. Students may still hold misconceptions about triangle examples placed in various orientations, so direct students to rotate the triangle sort cards as necessary to recognize acute, obtuse, and right angles.

The same activity is repeated with the same triangle sort cards, but this time students are directed to sort their triangle cards into groups by the number of sides of equal length. Students will need a ruler to measure sides they cannot estimate to determine the equivalence of. After students have sorted the triangles, label the groups with the vocabulary: scalene triangles, isosceles triangles, and equilateral triangles. Again, teacher questioning reinforces and challenges students within the frame of cognitive based assessment for geometric reasoning (Battista, 2012).

As a class, make an anchor chart to display, or alternatively have students record using their iPads as available, the six types of triangles as defined by angles and sides. Students can voice record the pronunciation and definition of each vocabulary word in addition to or as an alternative to writing the definition. At least one example of a triangle that fits that category should be included. Students will refer back to this vocabulary often. As needed, students can be provided with sentence frames for naming shapes such as the ones presented in Figure 3.2 in the previous chapter. At the end of this activity, some students will be able to identify triangles as belonging to more than one definitive category, for example, “Triangle d is both acute and isosceles because all the angles are less than 90 degrees and two of the sides are the same length.” The next activity will help to challenge students’ possible misconception that each triangle can only be categorized in one way.

Venn Diagram and Gallery Walk. Understanding that triangles can be named in more than one way is one of the more challenging concepts for students to master. This activity helps students identify triangles by more than one property. Depending on students’ level of experience with Venn Diagrams, practicing with one or more examples with familiar everyday content will prepare students for utilizing the tool to organize their geometric understanding. Specifically, point out the overlapping section in the middle that indicates an item belongs to both groups, and an item that belongs in neither group is placed around the outside of the circles. An example is provided in Figure 4.2.2., but can be tailored to meet the needs and interests of particular students, perhaps by eliciting student suggestions for categories to compare.

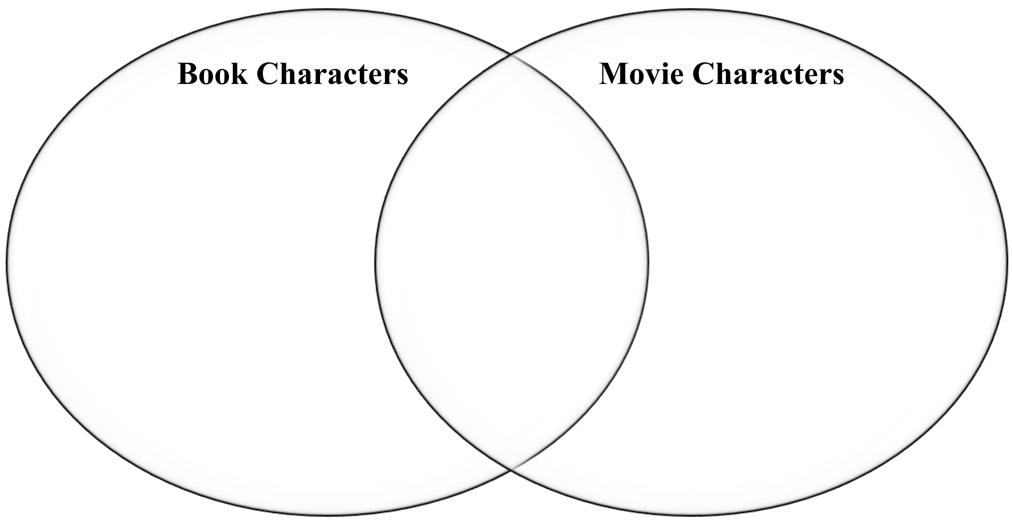


Figure 4.2.2. Venn Diagram with Familiar Context

Students work with their partners to place their triangle sort cards from the previous activity onto a Venn Diagram mat with the following category labels as shown in Figure 4.2.3. Each pair of students is given a different category, and once groups have sorted their triangles onto the Venn Diagram mats, they will do a “gallery walk” to view other students’ categories. The activity is completed twice, if time allows, so students practice comparing several types of triangles. The gallery walk is done after the second round of sorting. After the first sort, students share with one other group to practice giving and receiving feedback and to practice the geometry vocabulary terms and definitions.

First Round (assigned categories for Venn Diagram)	**Second Round**
acute and scalene	right and isosceles
acute and isosceles	right and equilateral*
acute and equilateral*	obtuse and scalene
right and scalene	obtuse and isosceles
right and isosceles	obtuse and equilateral*
right and equilateral*	acute and scalene
obtuse and scalene	acute and isosceles
obtuse and isosceles	acute and equilateral*
obtuse and equilateral*	right and scalene
*see note below	*see note below

**All equilateral triangles are acute, so the Venn Diagram for this pair will have an "empty" category. Additionally, there are no right triangles or obtuse triangles that are also equilateral, so those pairs will be empty in the overlapping "both" category in the Venn Diagram.*

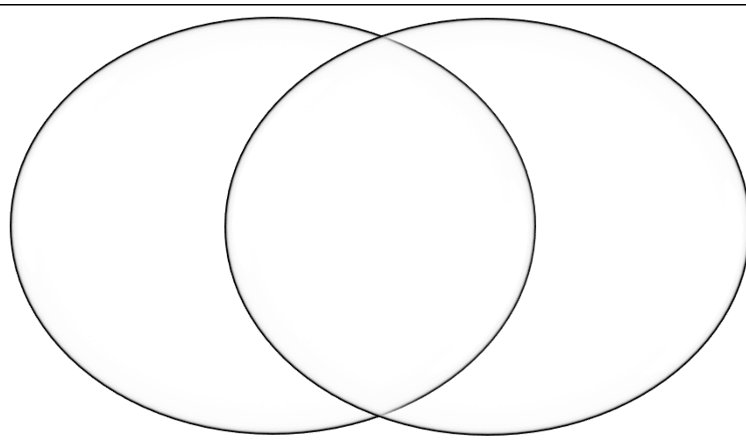


Figure 4.2.3. Assigned Categories for Triangles Venn Diagram Activity

Students will use language to compare and contrast the triangles as they place the sort cards on the Venn Diagram mat and as they analyze the work of others in the gallery walk. Reviewing the sentence frames from the previous open sort activity (see Figure 4.2.1) will provide language support for ELL students at lower proficiency levels and help all students connect the cognitive skills of both activities.

To conduct the gallery walk, all students finish working and stand by their workspace. On the teacher signal, students rotate around the room to view and discuss others' work. At each workstation, students can verbally share a comment or question, or write their response on a sticky note and attach it to the work. Students at higher language proficiency levels can help their peers write. If students do not have much experience with providing constructive peer feedback, it would be important to discuss how to make specific and positive comments that focus on the work, not on the students who did the work. Thinking stems and examples of comments that students at a range of language proficiency levels could make during the gallery walk are given in Figure 4.2.4. below. Once students have viewed the work of a few other groups, students return to their original workstations to read the comments left by others. Students can then decide if they want to change anything about their original work, and take a photo with their iPads to save their work for assessment.

Thinking Stems for Gallery Walk:

We see…
We notice…
We think…
We agree with…
We disagree with…
We wonder…

Beginning	Intermediate	Advanced
• The equilateral triangles all have equal sides. • We agree that these are acute triangles. They all have acute angles.	• We notice all the triangles in the middle section of the Venn Diagram have right angles and 2 equal sides. • We wonder why the triangles with obtuse angles can't also have right angles.	• We disagree with triangle b, and think it should be moved to the right triangle only section of the Venn Diagram because 1 side looks like it is a different length. • We notice that the equilateral triangles all have equivalent angles but the acute triangles are not all equilateral triangles because not all the angles are the same.

Figure 4.2.4. Thinking Frames and Example Statements for Galley Walk

Toothpick Investigation. Extending students' abilities to classify triangles in more than one way is continued in the next part of the lesson. Throughout this activity, it will be helpful for students to refer back to the anchor charts created in the sides and angles sorting activity as well as utilizing the Venn Diagram activity as a graphic support. Students will construct various triangles out of toothpicks, using a toothpick to represent one measurement unit of length. Students can easily see side length by counting the number of toothpicks used to construct each side of the triangle, but will need to use estimation to determine approximate angle size and type. This will be a good time to review and reinforce types of angles as practiced in the previous lesson. Students use a recording table to keep track of their work, recording triangle side lengths, a sketch of the angles, and the classification names. The recording table is shown in part below in Figure 4.2.5. The full activity chart can be found in Appendix B.

Triangle Sides	Sketch of Angles	Type of Triangle (angles: acute, right, obtuse)	Type of Triangle (sides: equilateral, isosceles, scalene)

Figure 4.2.5. Toothpick Investigation

Lesson Conclusion. Students will share the results of the Toothpick Investigation in small groups, providing justification for how they classified each created triangle. As needed, students will discuss, clarify, and come to consensus on any divergent results. Students can refer to the collaborative language supports in Appendix A from Module 1 to assist with respectful academic content discussion that goes deeper than back and forth reading of answers. Students who have met the lesson objectives can classify a target triangle in two ways, and provide justification that explains the angle and side properties of the shape. A sentence frame can be provided for students at lower language proficiency levels, such as "This is a(n) _______ triangle, because it has _________. It is also a(n) ________ triangle, because it has _________."

Module 3: Quadrilaterals

Desired Results

Minnesota State Standards in Mathematics Benchmark 4.3.1.2.: Describe, classify and draw quadrilaterals, including squares, rectangles, trapezoids, rhombuses, parallelograms and kites. Recognize quadrilaterals in various contexts.

Content Objective: Students will classify shapes as quadrilaterals, parallelograms, trapezoids, rectangles, rhombuses, squares, and kites.

Language Objective: Students will name and describe defining properties of different types of quadrilaterals.

Assessment Evidence

All students should be able to use definitions to name and organize quadrilaterals and other polygons by their properties. Students at the higher levels of geometric reasoning will demonstrate an understanding of hierarchical classification, which is a much more advanced skill requiring logical inferring skills. Students should also be led to distinguish defining properties of distinct shapes from other descriptive but non-essential characteristics. Teacher questioning will guide and inform formative assessment throughout the lesson activities.

Learning Plan

Constructing Shapes. This activity is adapted from the Four Triangles Problem developed by Burns (2007) as a way for students to explore polygons with 3, 4, 5, and 6 sides made from construction paper triangles. It is an open-ended activity that makes use of physical manipulatives. Language structures and supports have been added for the purpose of this curriculum unit. Students will need several 3-inch construction paper squares, so they can save and record their work constructing multiple variations and varieties of polygons. The first stage of the activity starts with "showing children how to cut a square in half on the diagonal to make two triangles. The teacher can ask the students what they notice about the properties of the two triangles. Drawing upon their knowledge from the previous triangles module, students should be able to identify that the triangles for the activity are right isosceles triangles, with one right angle and two congruent sides as defining properties.

Have them explore the different ways to put the triangles together, following the rule that two edges the same size must be matched" (Burns, 2007, p. 122). Figure 4.3.1 below provides a visual to help students understand the expectation for matching triangle sides. Using two triangles, students will be able to construct the following shapes: triangle, square, parallelogram.

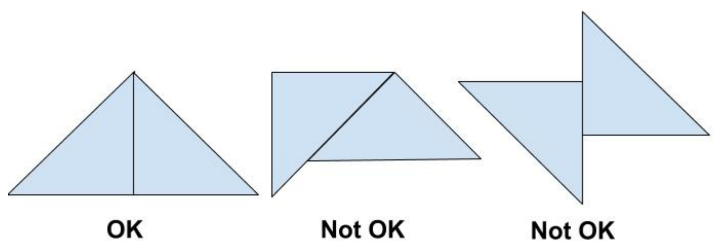


Figure 4.3.1. Matching Triangle Sides

Next, have students combine their two triangles with their assigned partner to investigate the shapes that can be constructed using four triangles. If students are not already doing so naturally, demonstrate the various transformations possible, rotating and flipping the triangles to create new possibilities. At the same time, rotation and reflection do not always result in a unique shape, and students should be guided towards recognizing transformations of congruent constructions. As students work, they use a new pair of 3-inch construction paper squares for each shape construction and save their work by sorting the constructed polygons on large piece of paper in the center of the group workstation. Alternatively, students can be provided with a sorting mat that has four categories, such as the blank graphic organizer in Appendix C. As they work, students can collaborate with others, sharing shapes that are similar to and differ from those constructed by other groups.

Once student groups have constructed as many shapes as possible with four triangles, come together to discuss student findings. The teacher should look for group work that has sorted the constructed shapes by number of sides. All possible arrangements of the four congruent right triangles include a larger triangle, five different quadrilaterals, two different pentagons, and six different hexagons (Burns, 2007). As a class, label the categories: triangles, quadrilaterals, pentagons, and hexagons (see Appendix C). This is an opportunity to point out the derivation of the word parts tri-, quad-, penta-, and hexa- as ordinal prefixes. Next, separate out just the quadrilaterals. Ask students to further sort these shapes with their groups.

If students have trouble isolating properties and components of the shapes to make sub-categories, direct them to notice angles and parallel lines. The anchor charts of angles and triangles developed in the earlier modules as well as other classroom references, such as word walls containing mathematics vocabulary, or online or printed math glossaries, are helpful for students. Students should use grouping language in this sorting activity as practiced in previous modules, and can again be provided with sentence frames as in Figure 4.2.1 in Module 2. Once students have sufficiently sorted the quadrilaterals, come together as a class to create an anchor chart to share what students came up with. Discuss and write a definition for each category of quadrilateral, listing the properties true of the shapes in that category. The teacher also leads a discussion into the distinction between defining properties and other characteristics that are true of a category of shapes, but do are not necessary to the definition of the shape (see graphic organizer in Appendix C). Specifically, the vocabulary terms to be defined as required by the Minnesota State Standards in Mathematics are: quadrilateral, parallelogram, trapezoid, kite, rhombus, square, and rectangle.

Battista (2012) makes the distinction in the Cognitive Based Assessment (CBA) levels of geometric reasoning that at Level 2, student definitions of shapes are inclusive of all properties, whereas students reasoning at CBA Level 3 are able to use deductive and inferential reasoning (Battista, 2012) to give a minimal definition of the properties that must be present to identify a specific shape, allowing other characteristics that can be inferred through logical deduction to be left unstated in the definition. An example of this distinction in geometric reasoning is given below in Figure 4.3.2. Parallel and congruent sides of a rectangle are inherent characteristics given the four right angles. Advanced students at the highest CBA level may be able to partially or fully explain the dependent characteristics using formal mathematical proof, though this is not expected at the fourth grade level. Teachers can guide students towards clearing up their misconceptions by highlighting the word parts “rect,” from the Latin *rectus* meaning right, and “angle,” from the Latin *angulus*, that make up the vocabulary word “rectangle.” In name, rectangles are not defined by their sides, though having four sides and square corners is a very common definition for rectangles given in primary grades.

CBA Level 2: Analytic-Componential Reasoning	**CBA Level 3: Relational-Inferential Property-Based Reasoning**
A rectangle is a quadrilateral that has two pairs of parallel sides, two pairs of congruent sides, and four right angles.	A rectangle is a quadrilateral that has four right angles.

Figure 4.3.2. Rectangle Definitions

There are several additional common misconceptions about quadrilaterals that the teacher needs to anticipate and address as they arise during discussion and creation of the class anchor chart. Most notably, students think that each four-sided shape can only be classified in one way based on its attributes (Minnesota STEM Teacher Center Frameworks, 2017). For example, many students will argue that squares and rectangles are distinctly different objects. Or, those who can accept that squares are a special type of rectangles may not recognize that squares also meet the defining requirements of rhombuses. Clarifying the defining properties of each type of quadrilateral as well as using graphic supports such as Venn Diagrams and hierarchical classification charts are useful tools for guiding students towards clearing up these misconceptions. Students may also have misconceptions in defining quadrilaterals only by their sides, not recognizing angles as the defining property for rectangles, and angles are important characteristics to notice in other types of quadrilaterals. Another important misconception to correct is students thinking that regular polygons are the only “real” shapes (Minnesota STEM Teacher Center Frameworks, 2017). Pointing out various examples of non-regular quadrilaterals that meet the defining criteria for trapezoids, kites, parallelograms, rectangles, etc., and drawing additional examples helps students expand their definitions. Students at the lowest levels of CBA geometric reasoning (Battista, 2008) will especially get “stuck” on examples of regular quadrilaterals, and want to classify as non-examples any shapes that are “too skinny/fat,” “not even,” or otherwise unlike the most common examples of squares, rectangles, rhombuses, parallelograms, trapezoids, and kites. Students may also want to use non-technical terms to name shapes, such as diamond instead of rhombus (Minnesota STEM Teacher Center Frameworks, 2017). Students need to see shapes presented in a wide variety of orientations, as viewing shapes as valid only when situated flat on a base is a common misconception. Addressing student misconceptions through examples, questioning, an overall atmosphere of inquiry and non-judgment will guide all students towards more in-depth cognitive thinking and geometric reasoning.

When discussing the results of the four triangles investigation, provide additional visual examples of any quadrilateral categories not present. Have students use rulers to construct additional quadrilaterals, drawing them on the chart or onto small paper cards to be moved around into different classification categories. Review the defining property that all quadrilaterals have exactly four sides and four angles.

Leave up the display as an anchor chart for students to refer back to in the next activity, Name that Quadrilateral. ELL students at lower proficiency levels can take a picture of the anchor chart with their iPads as available, and annotate with a voice recording of the pronunciation of the terms and the definition read aloud and clarified in simpler language as needed.

Name that Quadrilateral. Students will use the quadrilaterals constructed in the previous activity, The Four Triangles Problem, and the class anchor chart as examples to help complete the chart in Figure 4.3.3 (see blank student chart in Appendix C). Students sketch an example shape under the “shape” column, which then affects their answers in the corresponding columns of each row. As needed, discuss and define the following terms: sides, angles, parallel, congruent and equal length. Real-world examples and connections to previous lived experience will help ELL students and all students make connections to the vocabulary. As they work to sketch at least one example for each distinct type of quadrilateral, students are encouraged to provide more than one name for each quadrilateral as often as possible.

Shape	How many pairs of parallel sides?	How many congruent sides?	How many right angles?	Shape name(s)
trapezoid	1	0, 2, or 3	0 or 2	quadrilateral
parallelogram	2	2 or 4	0 or 4	quadrilateral
rhombus	2	4	0 or 4	quadrilateral parallolgram
square	2	4	4	quadrilateral parallelogram rhombus rectangle
rectangle	2	2 or 4	4	quadrilateral parallelogram
kite	0 or 2	2 or 4	0, 1, 2, or 4	quadrilateral rhombus
quadrilateral	0 or 2	0, 2, 3, or 4	0, 1, 2, or 4	

Figure 4.3.3. Name That Quadrilateral

Students go over their completed charts with others in a small group, adding other possible shape names they may have missed when working independently. The teacher circulates among groups and offers additional questions and challenges, asking, for example, “Does this shape fit in this group? Why or why not?” Have students check to see that they have included at least one example for each quadrilateral category: quadrilateral, trapezoid, parallelogram, kite, rectangle, rhombus, and square. For students who have only included the expected examples, for example a trapezoid with 0 right angles and 2 congruent sides or a kite with 0 right angles, offer sketches of other non-typical examples to challenge possible student misconceptions and expand their definitions. In the next lesson activity, students will further investigate the hierarchical classification of quadrilaterals, but for now, students should at least be able to recognize that a shape can be included in more than one classification category.

True or False, and Why? Students will answer a series of true/false questions such as, "All rhombuses are parallelograms." This type of tiered classification is an advanced form of classification, so students will benefit from practicing the cognitive skill with familiar content before engaging in the activity with the geometry knowledge. Some easy statements for students to connect with are listed as examples in Figure 4.3.4. below.

All students in Mr./Ms. _____'s class are fourth grade students. (True)
All fourth grade students are in Mr./Ms. ______'s class. (False)
All students in Mr./Ms. _____'s class are boys. (False)
All apples are fruit. (True)
All dogs are also mammals. (True—will likely need to define mammal)

Figure 4.3.4. Shared Experience Statements

Students defend their response as to whether the statement is true or false using justification language as in Figure 4.3.5. As needed, make sure all students understand the terms true and false as factual/correct and not fact/incorrect.

The statement is true. An example is _________. The statement is true, because _________.	The statement is false. A non-example is _________. The statement is false, because ________.

Figure 4.3.5. True/False Justification Sentence Stems

Students could also practice coming up with their own empirical statements, using any context they have knowledge about. Once students are comfortable reasoning with the true/false statements with familiar content, move on to presenting the geometric statements in Figure 4.3.6. for students to hypothesize and investigate (Battista, 2008).

Pair 1	All rhombuses are parallelograms. All parallelograms are rhombuses.
Pair 2	All kites are rhombuses. All rhombuses are kites.
Pair 3	All squares are rectangles. All rectangles are squares.

Figure 4.3.6. Quadrilateral True/False Statements

Battista (2008) advises that while all students can participate in the class discussion around the true/false statements above and advance their geometric reasoning, some students will not yet be ready to accept the conclusions about the hierarchal classification properties, depending on their level of geometric reasoning. Using visual examples and constructing quadrilaterals will provide concrete examples and non examples to reinforce student reasoning and arguments. Students can use iPad applications, other technology resources, or physical manipulatives such as toothpicks or geoboards.

Lesson Conclusion: Hierarchal Classification. Students will use their definitions of quadrilaterals to build a hierarchal classification chart for quadrilaterals, building upon students' conclusions from the True/False activity completed previously. It is helpful to practice completing a tree map classifying other shared knowledge, and discuss how the graphic records relationship information. Possible examples are shown in Figure 4.3.7. (adapted from Battista, 2008, p. 108). The examples should be personalized to the group of students; drawing from the students' lived experience and background knowledge, with particular attention to ELL students at lower proficiency levels. Another helpful language support is to create a chart with pictures of familiar objects.

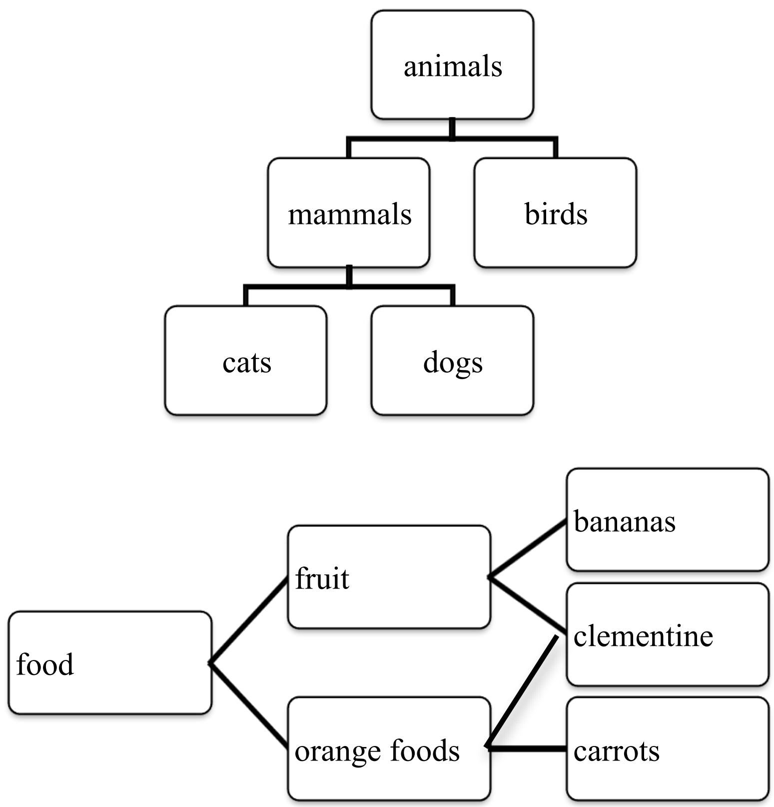


Figure 4.3.7. Examples of Hierarchical Classification

Students can practice drawing logical conclusions about the relationships shown in the hierarchical charts using the sentence frames in Figure 4.3.8. Drawing upon the collaborative language supports from Module 1 (see Figure 4.1.5. and Appendix A) will help students take their discussion deeper, challenging and building upon one another's statements.

Beginning	A _____ is also a ____. Both have _____.
Intermediate	A ____ is also a ____ because ______.
Advanced	_____ and _____ both have _____, therefore they are also _____. A ____ has _____, which means it is a _____, but also is a ______.

Figure 4.3.8. Classification Sentence Frames

Finally, students will build a hierarchal classification graphic to represent the relationships between types of quadrilaterals, including parallelograms, trapezoids, kites, rectangles, rhombuses, and squares. Some students will need to use teacher-provided examples and definitions, and those more proficient can rely on their geometric reasoning independently to place the shape names in the chart. Figure 4.3.9. shows an example of a hierarchical classification chart of quadrilaterals that acts as a tool for leading a discussion in deductive reasoning around the relational definitions of quadrilaterals based on their properties. The common misconceptions noted in the previous activity, The Four Triangles Problem, should be reviewed and readdressed as necessary in the discussion about relationships between types of quadrilaterals.

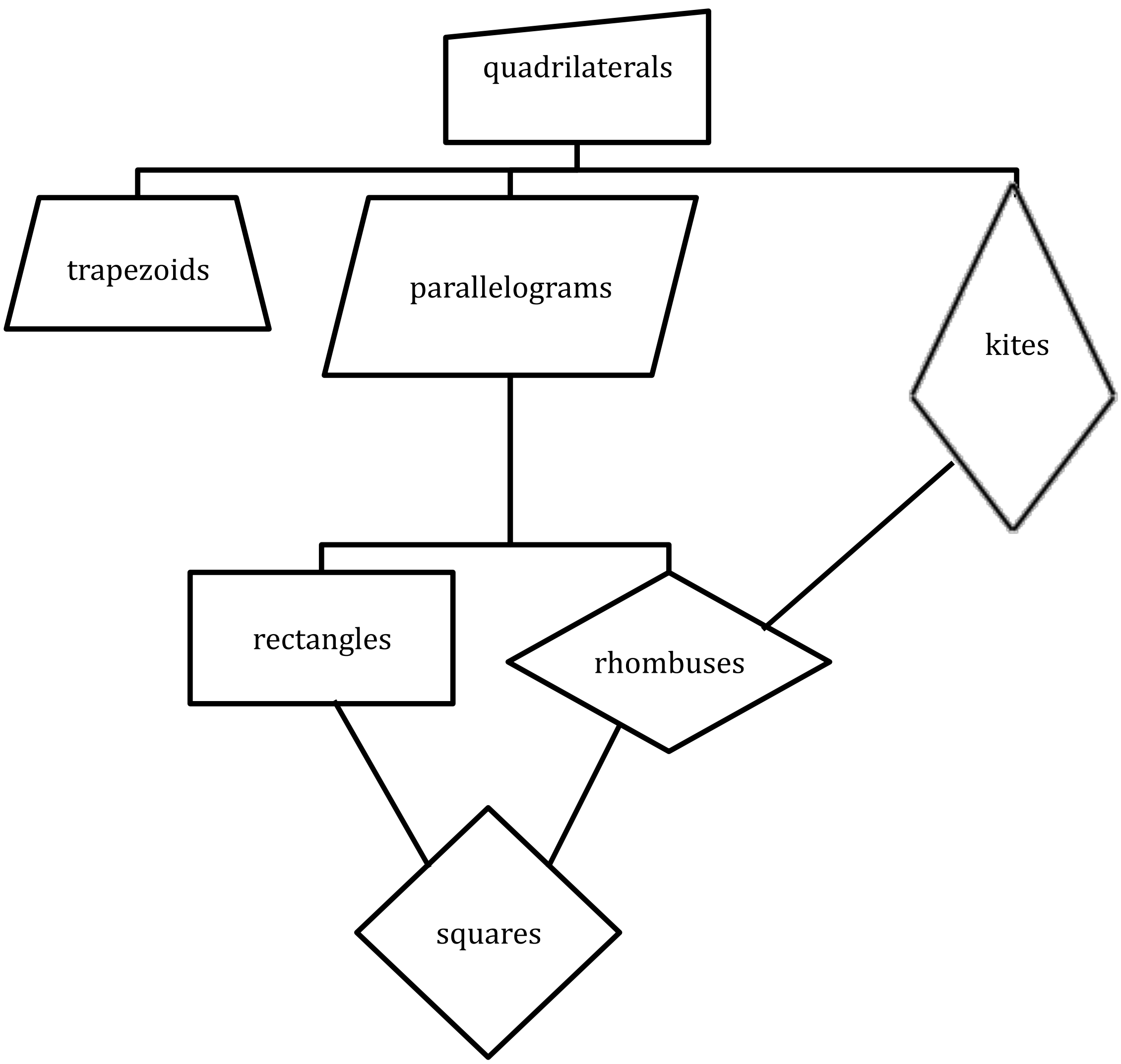


Figure 4.3.9. Quadrilaterals Organized in a Hierarchical Classification Chart

Module 4: "Guess My Rule" Game

Desired Results

Minnesota State Standards in Mathematics Benchmark 4.3.1.1: Describe, classify and sketch triangles, including equilateral, right, obtuse and acute triangles. Recognize triangles in various contexts.

Minnesota State Standards in Mathematics Benchmark 4.3.1.2: Describe, classify and draw quadrilaterals, including squares, rectangles, trapezoids, rhombuses, parallelograms and kites. Recognize quadrilaterals in various contexts.

Content Objective: Students can describe, classify, and sketch various types of triangles and quadrilaterals by components such as angles, parallelism, and side length.

Language Objective: Students will justify each classification for groups of shapes.

Assessment Evidence

At the conclusion of the lesson, students will share a written paragraph or verbal description defining several shapes that do and do not fit into a classification category. Throughout the lesson, students are presented with a wide variety of both triangles and quadrilaterals and encouraged to analyze components and properties of each shape as they pertain to its geometric definition and classification. In the paragraph or description, students use justification language to provide reasoning for the placement of each example and non-example shape.

Learning Plan

Guess My Rule Game: Whole Class. The activity in this section is adapted from Battista (2008) and is designed to encourage geometric reasoning at increasingly advanced cognitive-based assessment levels as described in Chapter 3 (Battista, 2008). Both triangles and quadrilaterals shape sort cards (See Appendix D) are used in this activity, but the number of shape examples may be reduced as needed so as not to overwhelm struggling students. The activity can be modeled in two phases, first as a fishbowl activity led by the teacher with a small group of students, and then a second time with all students in the class participating. The teacher should select students to participate in the fishbowl who have shown strong proficiency with the mathematics content and academic language in the previous modules. Modeling the activity with a fishbowl helps build confidence for less proficient students, who benefit from seeing the activity in action and then participating the second time. In the fishbowl, the participating students sit with the teacher in a central location, and other students circle around to observe the lesson activity, listening to the discussion and watching what the teacher and students do. The teacher explains the activity as it is acted out.

To begin the activity, the teacher shows students the complete group of shapes. Then, a select group of shapes is separated, and the following statements are made:

> I'm thinking about a special group of shapes.
>
> There is a rule for belonging to the group.
>
> Your job is to figure out the rule.
>
> I will tell you if the shapes belong to the group or not.
>
> (Battista, 2008, p. 83)

The teacher shows a small group of 2-4 shapes that belong in the group, and 1-2 non-examples that do not belong to the group. Then, the teacher selects another shape, and students show thumbs up if they think it belongs and thumbs down if it does not belong. The teacher moves the shape into the group or to the side and says, “This shape does/doesn’t belong in the group.” This is repeated several times, until many students are guessing correctly thumbs up or thumbs down. Students turn to their partner to guess the rule for the group, using the sentence frames in Figure 4.4.1.

A shape is part of the group if it is/has _____. I know because _____.
All the shapes in the group are similar because _______, therefore the rule is ______.

Figure 4.4.1. “Guess My Rule” Sentence Frames

Students then share their conjectures with the class. Several conjectures are offered, and the teacher leads a discussion to narrow down student suggestions and reach a group consensus. Non-examples and the properties that exclude them from the group are also pointed out.

Guess My Rule Game: Small Group Practice. Students work in collaborative groups of 4-5 students and take turns creating a “Guess my Rule” group for other students to figure out. Students follow the same procedure as modeled by the teacher, starting with 3 shapes that belong in the group and 1 non-example shape that does not belong. Then, other shape cards are sorted as examples and non-examples of the rule, until students in the group think they can identify the rule. ELL students at lower language proficiency levels can use thumbs up and thumbs down to participate non-verbally, and sentence frames as provided in the whole class game in Figure 4.4.1 provide additional language support. Students can record their results by taking photos and videos using their iPads as available.

Lesson Conclusion. To wrap up the "Guess My Rule" activity, students can either write a paragraph description of their rule and sorted shapes, or verbally share the results with another group. ELL students at lower proficiency levels will benefit from a sentence frame for organizing their response with transition language, such as the example below in Figure 4.4.2.

All the shapes in my group follow a rule. The rule is _____. The first shape, _____, is a _____, so it follows the rule. The second shape, ______, also has _____, so it follows the rule. The third shape, _____, is a _______, but does not have ______, so it does not follow the rule.

Figure 4.4.2. Sentence Frame with Transition Language

Conclusion

Chapter 4 has defined the desired results and assessment evidence for the planned mathematics geometry curriculum unit. Each of the four modules addresses the Minnesota State Standards in Mathematics for fourth grade. Content and language objectives describe what students are expected to know and be able to do at the conclusion of the curriculum unit, and the assessment evidence sections provide a plan for measuring whether students have met the desired results or if further teaching is necessary. Each of the four modules details a learning plan that incorporates several engaging activities for cognitive reasoning and opportunities to practice academic language within the classroom setting. Activities within the learning plan may be repeated, extended, or modified as needed in response to student assessment evidence to ensure that all students further their cognitive reasoning skills around geometry and spatial awareness as well as develop academic language within the context of mathematics geometry.

Chapter 5 will further discuss possible options for expanding the scope of the curriculum unit as presented, and review all previous chapters as they pertain to the research question: *How can a mathematics geometry curriculum unit be organized to support and develop academic language for fourth grade students through meaningful engagement strategies?*

CHAPTER 5

Conclusion

The preceding chapters have explored in depth the research question: *How can a mathematics geometry curriculum unit be organized to support and develop academic language for fourth grade students through meaningful engagement strategies?* Chapter 1 introduced my personal history, teaching experience, and motivation for selecting the particular focus of this curriculum writing capstone project. As a fourth grade teacher of a culturally and linguistically diverse student population, I have seen firsthand the need for a curriculum that provides tools to support and build academic language alongside meaningful mathematics context rich in real-world applications. In Chapter 2, I reviewed current research literature as it pertained to the research question and best practice implications for academic language and mathematics teaching methods. Many parallels and similar themes among best practice recommendations were found within the two disciplines, and the planned curriculum unit is designed with the goal of simultaneously supporting mathematics content learning and academic language acquisition. Chapter 3 outlined the methods used to structure the developed curriculum unit, giving background to the Understanding by Design model developed by Wiggins and McTighe (2005), Battista's (2012) Cognitive Based Assessment for Geometric Levels of Reasoning, and the best practice recommendations for academic language instruction from Zwiers (2008), among other sources and influences. Chapter 3 also described the desired results and plan for assessment evidence, and the rationale used in developing activities within the learning plan. I then detailed a narrative in Chapter 4 of a curriculum unit that took into account the findings of the literature review and methods studied in pursuit of the research question, *How can a mathematics geometry curriculum unit be organized to support and develop academic language for fourth grade students through meaningful engagement strategies?* Now, in Chapter 5, I revisit the process and consider next steps. The first section is a personal reflection of the capstone. Next, I review the literature review and highlight its most significant influences. The following section considers the limitations of the curriculum unit as written, and is followed by suggestions for further study. The final section provides a conclusion summary of this and previous chapters.

Capstone Process Reflection

As a teacher in a large school district, the instructional decisions I make in my classroom are often heavily influenced by district policy and provided curriculum. Taking on the role of researcher and opening myself up to allowing literature and best practice findings to influence the development of a curriculum unit separated my work from the politics and policies and instead focused on evidenced-based strategies proven to meet the needs of diverse learners. The process of creating curriculum became much more detail-oriented than I originally anticipated, as I sought to analyze and provide support and justification for learning activities that aligned with standards-based curriculum unit goals.

I began this process hoping to create a curriculum unit that addressed what was lacking in my district's current curriculum: hands-on activities and mathematics instruction rooted in real-world application. From my previous experience with Sheltered Instruction Observation Protocol (SIOP) and collaboration with ESL teachers, I also knew that the ESL students I teach need specific and intentional language supports embedded within academic content instruction. However, as I read more into the research and discovered various expert recommendations for instructional considerations in both mathematics and language methods, I found myself realizing that the familiar saying, "You don't know what you don't know," very much applied to my capstone journey.

Even now, as I reflect upon the process and culminating product, I find myself with the desire to further study and practice the methods for teaching mathematics and language with a base in cognitive thinking and student-led inquiry. In the Further Study section of this chapter, I go into more detail about the possible areas I wish to research and develop as I continue my journey as a teacher and lifelong scholar. First, I will look back on my learning with a review of the literature review conducted in Chapter 2.

Literature Review Revisited

My literature review consisted of study into two major discipline areas: Academic Language for English as a Second Language Instruction and Mathematics Teaching Methods. As I synthesized my research findings, I sought out commonalities between the two disciplines applicable for design of the planned curriculum unit, and each area of study gave me insight to inform my instructional decision-making.

In my initial research to develop the literature review, I found it challenging to find references that specifically focused on geometry instruction methods. Number Sense and Operations with attention to arithmetic and problem solving seem to dominate the field of study and practice in mathematics for the elementary school level. However, eventually I found significant research to support the importance of attention to geometry and spatial awareness at the elementary level, and sources to provide recommendations for instructional practices as utilized in the development of the curricular unit.

My research into the field of methods for teaching English as a Second Language only skims the surface of possible study and analysis of supporting language development. As a mainstream classroom teacher, my own knowledge and background into the many facets and dimensions of language is limited. Through this literature review, I was able to learn and apply new understanding in the development of a learning plan that provides supports for students to practice their developing academic language skills alongside academic content. I chose to focus on geometry for this capstone project partially because geometry is heavily dependent on students' understanding of vocabulary specific to the academic domain. My review of the literature helped me see the importance of considering all dimensions of language, including word, sentence, and discourse, to enable students at all levels of language proficiency to access content and increase their academic language skills. The research conducted into academic discourse provided many examples of how failing to anticipate and support specific language skills leaves many students unable to fully engage in content learning within the traditional classroom setting. I noted many parallels between recommendations for engaging students in academic language discussions and engaging students to construct mathematics knowledge through cognitive reasoning processes. One important component of pedagogy explored in both areas of the literature review is the role that equity plays for teachers and students. ELL students are provided equitable access to learning when academic experiences are both comprehensible and meaningful. In the mathematics classroom, considerations for equity include addressing cultural differences, diverse learning styles, and many other factors. In Chapters 3 and 4, I sought to produce a curriculum unit that would reflect the findings of the literature reviewed.

Limitations

The curriculum unit plan designed as a result of this capstone research assumes opportunities for flexibility of instructional time and number of days allowed for the mathematics geometry unit. The teacher must also have the freedom to professionally interpret the district or school curriculum materials as the activities in the learning plan for this curriculum unit are used to augment existing materials. The unit plan assumes the need to differentiate for ELL students at a range of proficiency levels, yet likely will need to be adapted as a result of formative assessment evidence, adjusting pacing, and adjusting language supports as necessary. Additionally, the curriculum was designed with a particular school setting in mind, with 60 minutes for mathematics instruction daily and technology such as iPads for students readily available. Even within the planned school setting, the particular student makeup of the class can vary greatly from year to year.

Teachers adapt curriculum all the time to address the particular needs of each group of students, including the overall personality of the class, presence of students with disabilities, range of language proficiency levels, influence of social factors, and more. This curriculum unit may not work for every class, every student, or every teacher.

The capstone project addresses a narrow range of instructional benchmarks from the Minnesota State Standards in Mathematics for fourth grade. The unit does not go into detail a plan for addressing students with very limited previous mathematics instruction, as most students will enter fourth grade with some prior knowledge in the strand of geometry from previous geometry and spatial awareness instruction in earlier grades. I also did not write complete Model Performance Indicators for the entire unit, but instead incorporated some of the WIDA Standards Framework elements in planning for language practice and supports ("WIDA ELD Standards") throughout the curriculum.

Further Study

This capstone project focused on developing a curriculum unit that addressed academic language needs within the context of engaging mathematics geometry content. When it came time to write the unit plan, I chose to focus on three of the benchmarks from the Minnesota State Standards in Mathematics Geometry and Measurement Strand in the development of the curriculum topics of angles, triangles, and quadrilaterals, which all utilize the cognitive skills of classification, description, and justification. In accordance with the standards, fourth grade students are also expected to meet benchmarks in the topic areas of angle measurement, area and perimeter measurement and calculations, and shape transformations. Each of these could be developed using a similar process as carried out in this capstone project. The other major strands of mathematics study, including number sense, operations, algebra, data analysis, and probability, are also possible topics for further research and curriculum development that addresses and incorporates academic language development.

Conclusion

In conclusion, the process of completing a capstone project has been a valuable experience that has helped me grow as a teacher. My research findings, particularly those pertaining to teaching academic language alongside academic content, will enable me to better serve the needs of linguistically diverse learners in my classroom now and in the future. As described in Chapter 1, I began my career with little knowledge and few skills related to the teaching of ELL students, and even now I feel as though this is an area in which I am a novice teacher. In Chapter 2, I conducted research into best practice mathematics teaching methods as I sought to plan instruction that goes beyond basic arithmetic and trains students in problem solving and cognitive thinking skills that are applicable to real world situations. Chapter 2 also included study in the area of English as a Second Language and found that academic language requires attention across the vocabulary, sentence, and discourse dimensions of language. In Chapter 3, I outlined the methods for developing curriculum, and in Chapter 4, the resulting curriculum unit plan was narrated with attention to the desired results, assessment evidence, and learning plan, including anticipated misconceptions and academic language supports to communicate mathematical thinking. Chapter 5 summarized the previous chapters and included a reflection about the process of exploring the research question: *How can a mathematics* *geometry curriculum unit be organized to support and develop academic language for fourth grade students through meaningful engagement strategies?*

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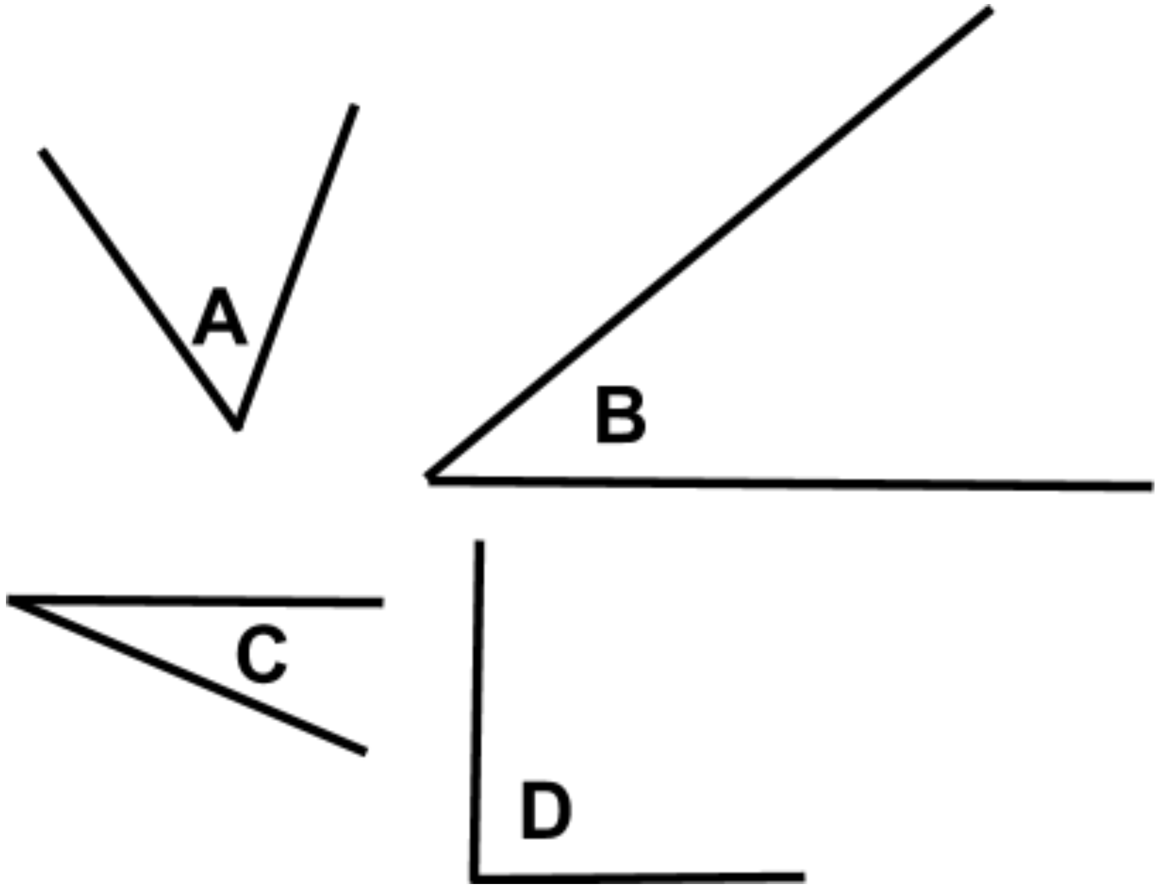
Appendix A

Module 1 Blackline Masters

Which One Doesn't Belong?

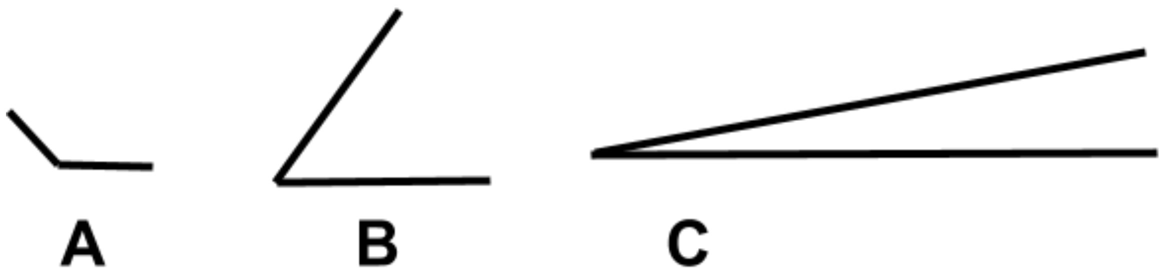


I think ____ doesn't belong. It is different because _________.

I think ____ doesn't belong because ____, ____, and ____ all

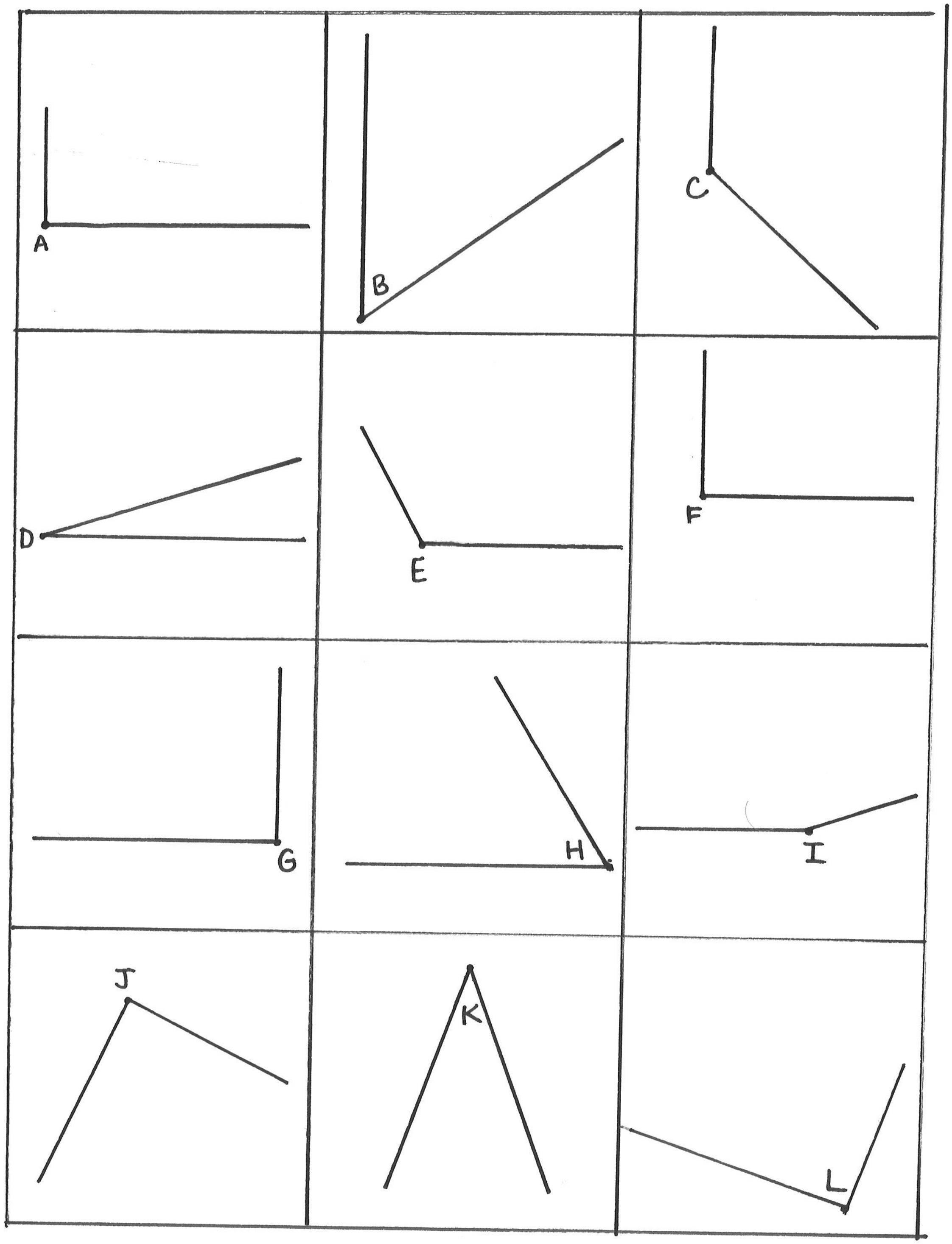
are/have ______.

Which is the biggest angle?



I think angle ___ is the biggest angle because __________.

Angle Sort Cards



Collaborative Language Supports

Clarify	Disagree	Elaborate
Will you explain that again? I have a question about what you said about ____. Could you give an example of what you mean by _____.	Another way to look at it is _____. I do agree with what you said about _____, but I think _____. I have a different answer. I wrote down that _____.	You made a good point when you said ____. I see what you are saying. I agree because _____. My idea builds on ____'s idea. I think _____.

Sentence Frames for Justification

________ is a _________. It has __________.
________ is a _________. I know because __________.

Name: ________________

Hidden Shape Angles

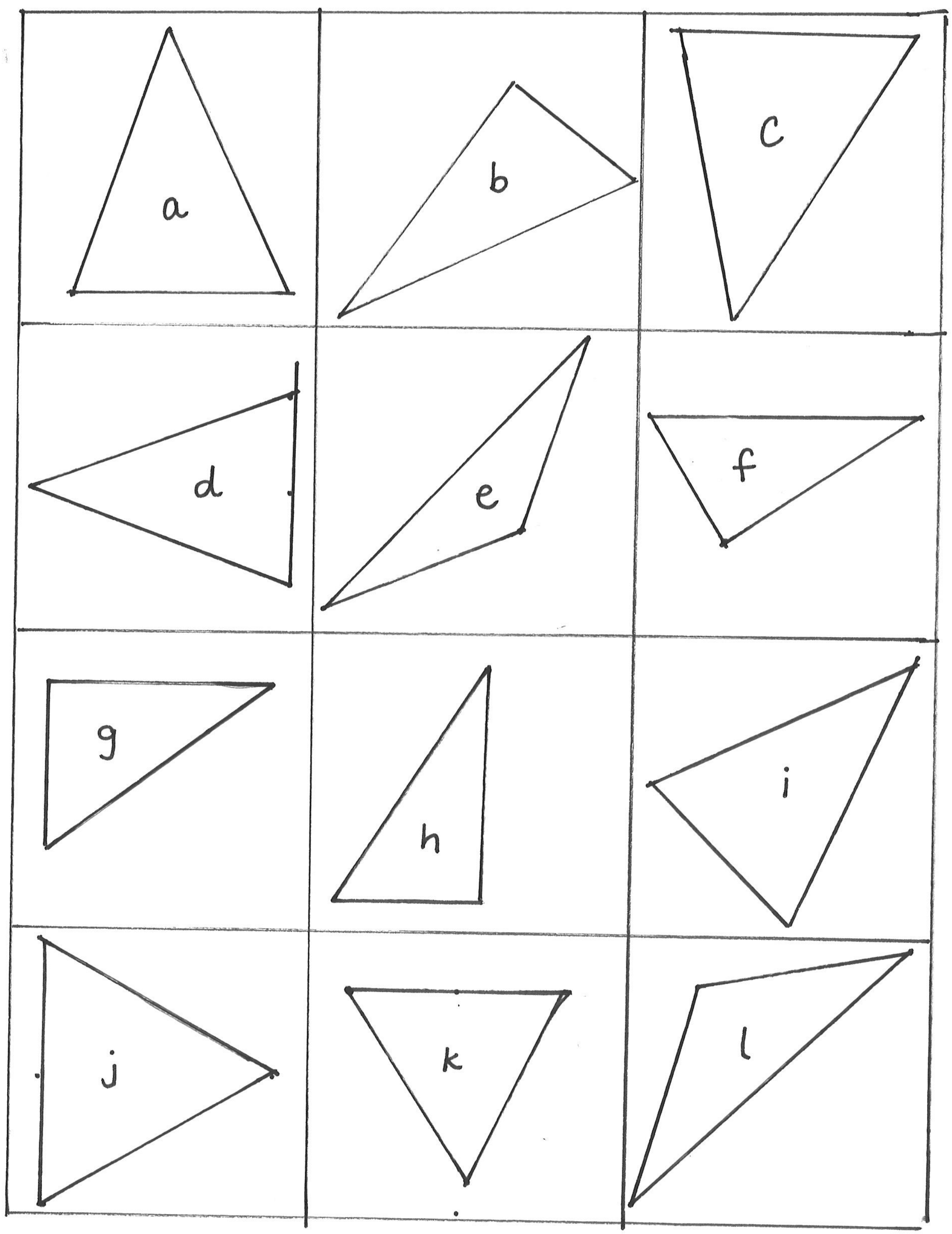
Directions: Color the angles in the shapes using the key below.

Key	
obtuse angle	orange
right angle	red
acute angle	blue

Appendix B

Module 2 Blackline Masters

Triangle Sort Cards



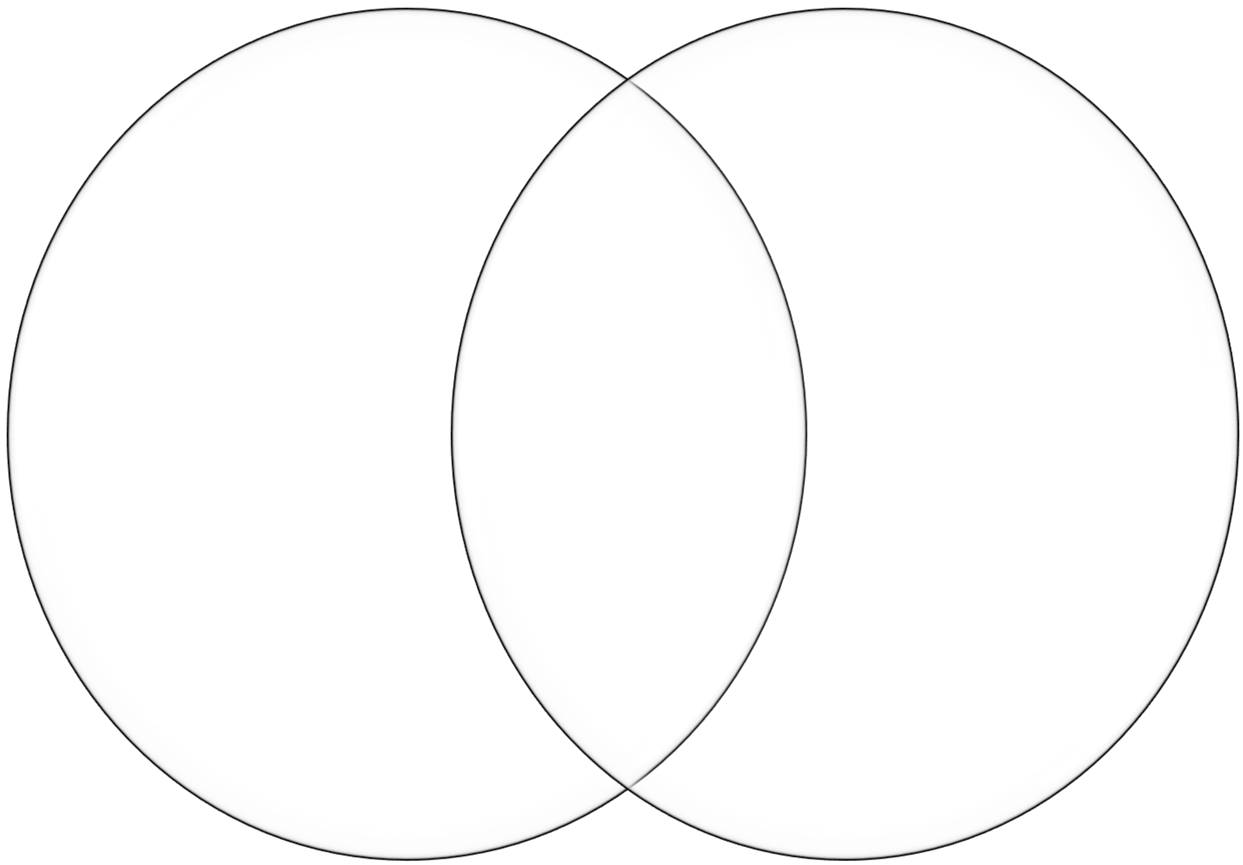
Sorting Sentence Frames

Similar and Grouped together	____ and ____ both have _____. ____ and ____ belong together because ________. ____ and ____ are similar because _____. The shapes in this group all are/have ______. ____ is/has _____. Likewise, ____ also is/has ______, so they are in the same group.
Different and separate groups	_____ is different from _____. It does not have ______. _____ is not like ______. It is not ______. _____ is separate from _____ because ______.

3 Category Sorting Mat
Graphic Organizer

Category 1: ________________	Category 2: ________________	Category 3: ________________

Venn Diagram
Graphic Organizer



Thinking Stems for Gallery Walk:

We see…

We notice…

We think…

We agree with…

We disagree with…

We wonder…

Example Comments:		
Beginning	**Intermediate**	**Advanced**
• The equilateral triangles all have equal sides. • We agree that these are acute triangles. They all have acute angles.	• We notice all the triangles in the middle section of the Venn Diagram have right angles and 2 equal sides. • We wonder why the triangles with obtuse angles can't also have right angles.	• We disagree with triangle b, and think it should be moved to the right triangle only section of the Venn Diagram because 1 side looks like it is a different length. • We notice that the equilateral triangles all have equivalent angles but the acute triangles are not all equilateral triangles because not all the angles are the same.

Name: ________________

Toothpick Investigation

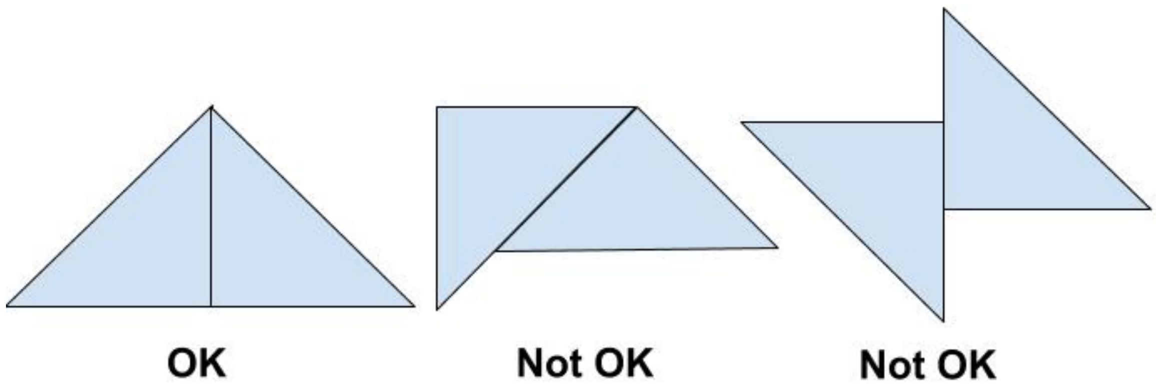
Triangle Sides	Sketch of Angles	Type of Triangle (angles: acute, right, obtuse)	Type of Triangle (sides: equilateral, isosceles, scalene)

Appendix C

Module 3 Blackline Masters

The Four Triangles Problem:

Rule for Matching Sides



4 Category Sorting Mat

Category 1: ______________	Category 2: ______________	Category 3: ______________	Category 4: ______________

**The Four Triangles Problem:
Polygon Categories**

Triangle	Quadrilaterals	Pentagons	Hexagons
1.	1.	1. 2.	1.
	2.		2.
	3.		3.
	4.		4.
	5.		5.
			6.

Types of Quadrilaterals

quadrilaterals
Defining Properties:
Other Characteristics:
Examples:

parallelograms	**trapezoids**	**kites**
Defining Properties:	Defining Properties:	Defining Properties:
Other Characteristics:	Other Characteristics:	Other Characteristics:
Examples:	Examples:	Examples:
rectangles	**rhombuses**	**squares**
Defining Properties:	Defining Properties:	Defining Properties:
Other Characteristics:	Other Characteristics:	Other Characteristics:
Examples:	Examples:	Examples:

Name: _______________

Name That Quadrilateral

Shape	How many parallel sides?	How many congruent sides?	How many right angles?	Shape name(s)

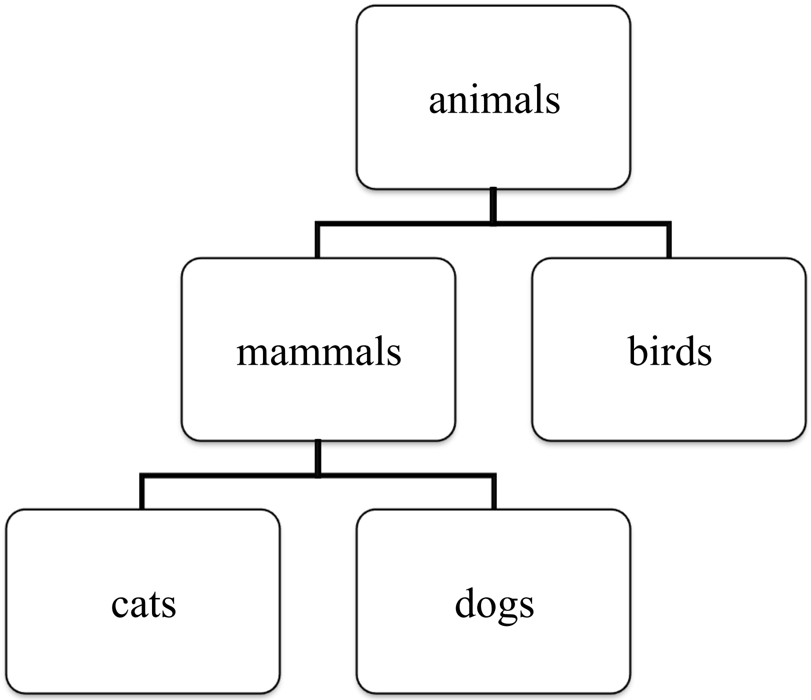
Quadrilateral True/False Statements

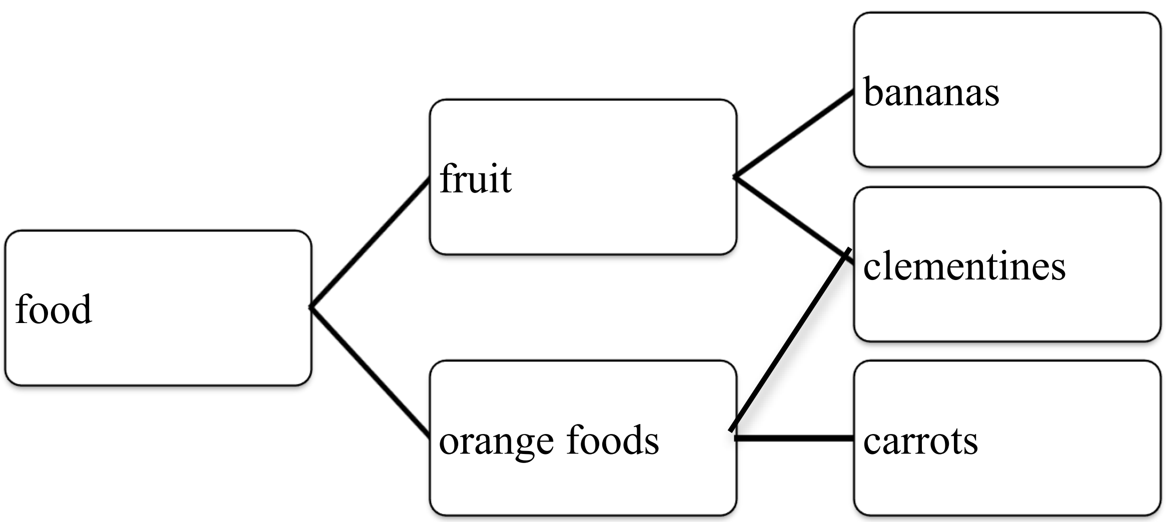
Pair 1	All rhombuses are parallelograms. All parallelograms are rhombuses.
Pair 2	All kites are rhombuses. All rhombuses are kites.
Pair 3	All squares are rectangles. All rectangles are squares.

True/False Justification Sentence Stems

The statement is true. An example is __________. The statement is true, because __________.	The statement is false. A non-example is __________. The statement is false, because _________.

Examples of Hierarchical Classification





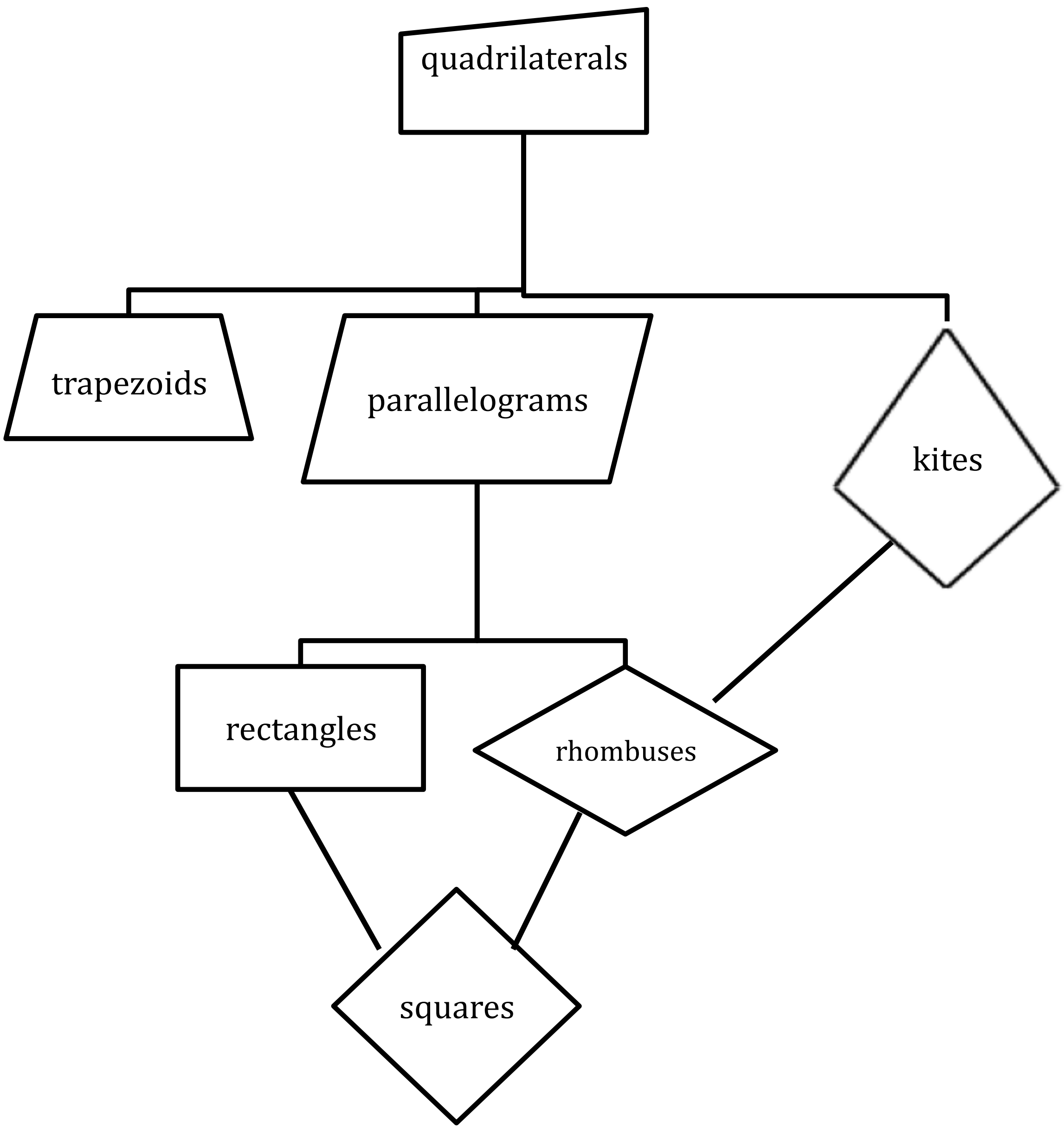
Classification Sentence Frames

A __________ is also a __________ because ____________.

_________ and _________ both have _______________, therefore they are also _________.

A _________ has _______________, which means it is a _________, but also is a ___________.

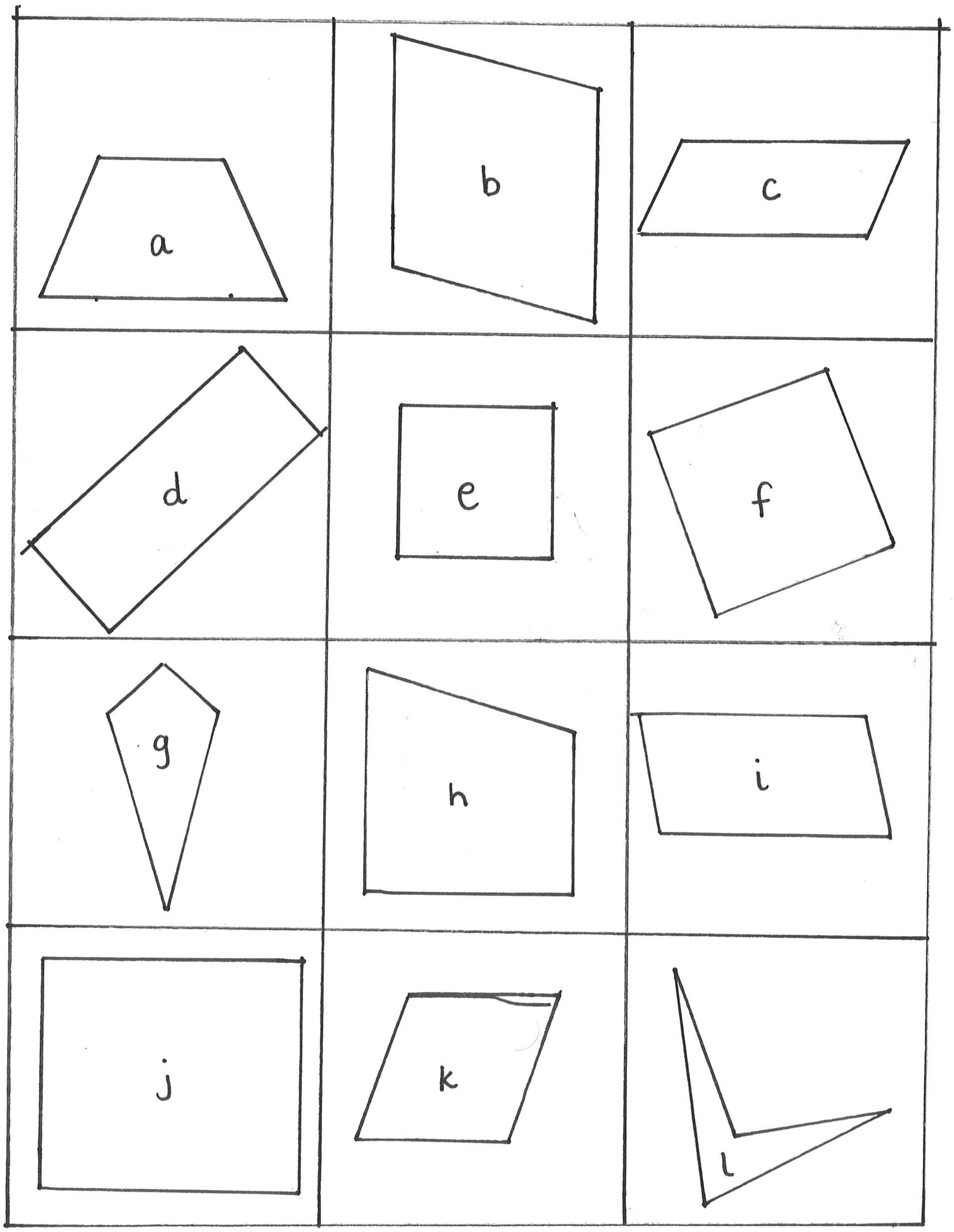
Hierarchical Classification of Quadrilaterals



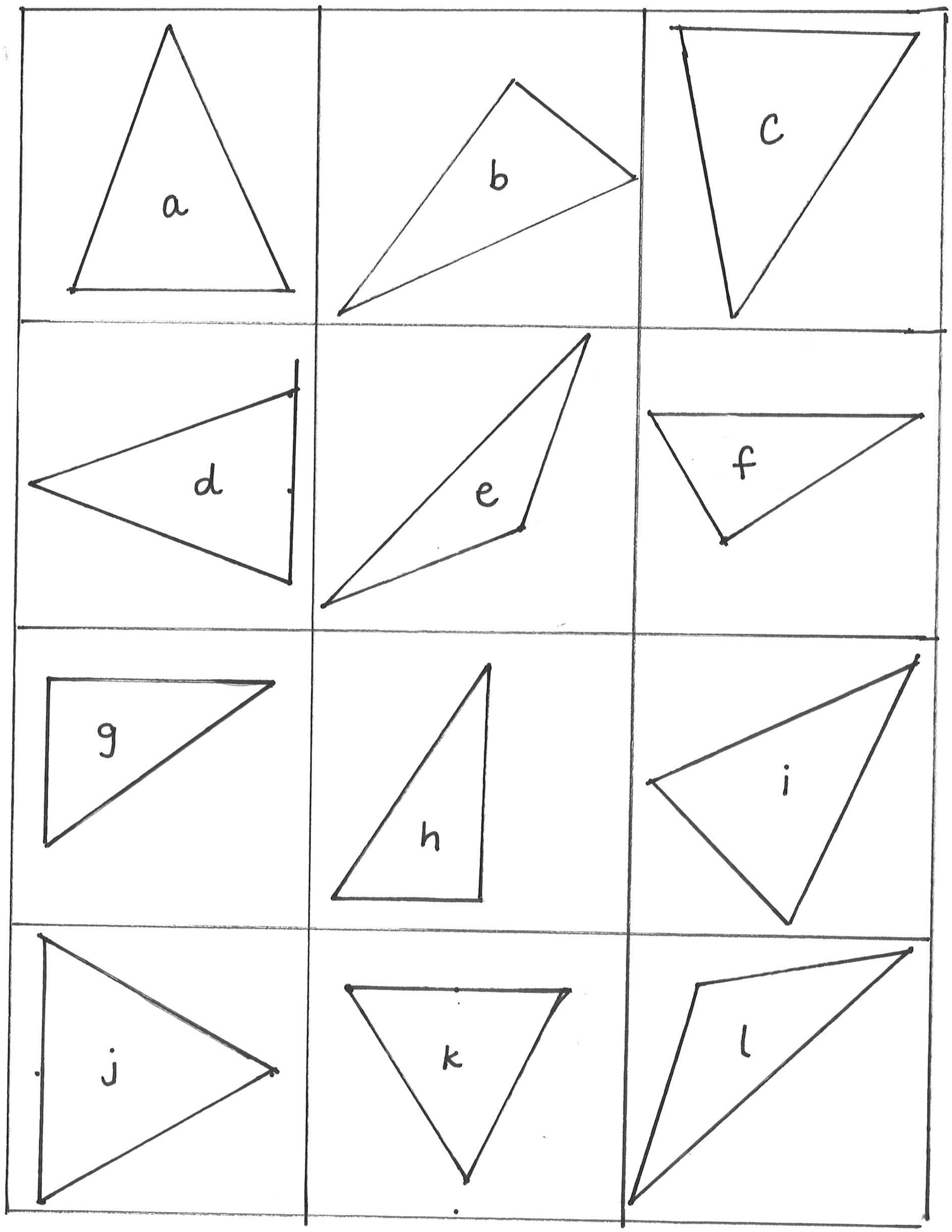
Appendix D

Module 4 Blackline Masters

Quadrilateral Sort Cards



Triangle Sort Cards



"Guess My Rule" Sentence Frames

A shape is part of the group if it is/has ________. I know because

______________.

All the shapes in the group are similar because _________,

therefore the rule is ______________.

"Guess My Rule" Game

1) Sort 3 shapes as examples into a group that follow the same rule. Sort 1 shape into another group as a non-example that does not belong.
2) Sort more shapes into the example and non-example groups until other students think they can guess the rule.
3) Students show thumbs up or thumbs down to guess if a shape will be sorted into the example or non-example group. Use the sentence frames to guess the rule.

“Guess My Rule” Conclusion Sentence Frame

All the shapes in my group follow a rule. The rule is _____. The first shape, _____, is a _____, so it follows the rule. The second shape, ______, also has _____, so it follows the rule. The third shape, _____, is a _______, but does not have ______, so it does not follow the rule.