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A Design Method of Robust Non-Fragile Guaranteed Cost Controllers for Linear Systems with Structured Uncertainties

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This paper deals with a design problem of robust non-fragile stabilizing controllers with guaranteed cost for linear systems with structured uncertainties. In this paper, we consider two classes of control gain perturbations and show that sufficient conditions for the existence of the robust non-fragile guaranteed cost controller are given in terms of linear matrix inequalities (LMIs). Additionally, a design method of optimal robust non-fragile guaranteed cost controllers which minimize an upper bound on a given quadratic cost function is discussed. Finally, numerical examples are presented to demonstrate the effectiveness of the proposed robust non-fragile controller.

key words robust non-fragile guaranteed cost controllers, control gain perturbations, LMIs

1. Introduction

Robustness of control systems to uncertainties has always been the central issue in feedback control and therefore for uncertain systems, a large number of design methods of robust controllers have been derived[1-3]. In particular, for so-called structured uncertainties in the form of $A(\Delta(t)) \triangleq A + \mathcal{D}\Delta(t)\mathcal{E}$ $(||\Delta(t)|| \le 1)$, a connection between quadratic stabilization and $\mathcal{H}^\infty$ control has also been established[4].

By the way in most practical situations, it is desirable to design robust control systems which achieve not only robust stability but also an adequate level of control performance. One approach to this problem is the guaranteed cost control approach first introduced by Chang and Peng[5]. This approach has the advantage of providing an upper bound on a given cost function and thus the system performance degradation incurred by the uncertainties is guaranteed to be less than this bound. Based on this idea, many significant results have been presented[6-9]. Petersen and McFarlane[6] adopted a parameter dependent Riccati equation approach and Yu and Chu[8),9] presented a controller design method based on linear matrix inequalities (LMIs).

On the other hand, there have been some efforts to tackle the design problem of robust non-fragile controllers[10-13]. Because, uncertainties in controllers appear for many reasons such as imprecision inherent in analog-digital and digital-analog conversion, finite word length, and finite resolution measuring instruments and roundoff errors in numerical computations, and any useful design procedure should generate a controller which also has sufficient room for readjustment of its coefficients in the final controller implementation[14),15)]. In practice, Keel and Bhattacharyya[14] have shown through a series of examples that the robust controller design without considering the relatively small uncertainties in controller implementation could lead to even closed-loop instability under small perturbations in controller parameters. Therefore, it is necessary to design controllers tolerating some uncertainties in their parameters. For linear systems with structured uncertainties existing only in the system matrix, a design method of a robust non-fragile LQ controller has been shown[11]. Also a non-fragile $\mathcal{H}^\infty$ controller for linear systems has been derived[12]. However, so far the LMI-based design method of robust non-fragile guaranteed cost controllers for linear systems with so-called structured uncertainties which are included in both the system matrix and the input one has little been considered as far as we know.

From this viewpoint, in this paper we present a design method of a robust non-fragile guaranteed cost controller for linear systems with structured uncertainties existing in both the system matrix and the input one. In this paper, we deal with plant uncertainties in the form of $A(\Delta(t)) \triangleq A+\mathcal{D}\Delta(t)\mathcal{E}$ $(||\Delta(t)|| \le 1)$ and additive and multiplicative control gain

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variations, and we show that sufficient conditions for the existence of the robust non-fragile guaranteed cost controller are given in terms of LMIs. Besides, a design method of optimal robust non-fragile guaranteed cost controllers which minimize an upper bound on a given cost function is considered.

This paper is organized as follows. In Sec. 2, notations and two useful lemmas which are used in this paper are shown and in Sec. 3, we introduce the classes of uncertain systems and the control gain perturbations under consideration. Sec. 4 and Sec. 5 contain the main results. The design method of the robust non-fragile guaranteed cost controller is presented. Finally, illustrative examples are included to illustrate the results developed in this paper.

2. Preliminaries

In this section, we show notations and two useful lemmas which are used in this paper.

In this paper, we use the following notations. For a matrix $\mathcal{A}$, the transpose of the matrix $\mathcal{A}$ and the inverse of one are denoted by $\mathcal{A}^T$ and $\mathcal{A}^{-1}$ respectively. Also $H_e\{\mathcal{A}\}$ means $\mathcal{A}+\mathcal{A}^T$ and I_n represents n-dimensional identity matrix. For real symmetric matrices $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A}>\mathcal{B}$ (resp. $\mathcal{A}\geq\mathcal{B}$) means that $\mathcal{A}-\mathcal{B}$ is positive (resp. nonnegative) definite matrix and $E\{\cdot\}$ and $Tr\{\cdot\}$ denote its expectation and its trace, respectively. Furthermore, the following two useful lemmas are used in this paper.

Lemma 1 For given constant real symmetric matrix Ξ, the following arguments are equivalent.

(i). $\Xi \triangleq \begin{pmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22} \end{pmatrix} > 0$

(ii). $\Xi_{11} > 0$ and $\Xi_{22} - \Xi_{12}^T\Xi_{11}^{-1}\Xi_{12} > 0$

(iii). $\Xi_{22} > 0$ and $\Xi_{11} - \Xi_{12}\Xi_{22}^{-1}\Xi_{12}^T > 0$

Proof: See Boyd. et al[16].

Lemma 2 For matrices $\mathcal{G}$ and $\mathcal{H}$ which have appropriate dimensions and a positive scalar γ, the following relation holds.

$$\mathcal{G}\mathcal{H}+\mathcal{H}^T\mathcal{G}^T \leq \gamma\mathcal{G}\mathcal{G}^T+\frac{1}{\gamma}\mathcal{H}^T\mathcal{H}$$

Proof: See Lemma 1 of Oya and Hagino[17].

3. Problem Formulation

Consider the uncertain linear system described by the following state equation (see **Remark 1**).

$$\frac{d}{dt}x(t) = A(t)x(t) + B(t)u(t) \tag{1}$$

where $x(t) \in \Re^n$ and $u(t) \in \Re^m$ are the vectors of the state (assumed to be available for feedback) and the control input, respectively. The matrices $A(t)$ and $B(t)$ are supposed to have appropriate dimensions and the following time-varying structure.

$$\begin{aligned} A(t) &= A + \mathcal{D}\Delta_{\mathcal{A}}(t)\mathcal{L} \\ B(t) &= B + \mathcal{E}\Delta_{\mathcal{B}}(t)\mathcal{M} \end{aligned} \tag{2}$$

In (2), the matrices A and B denote the known nominal values and the pair (A, B) is assumed to be stabilizable. The matrices $\mathcal{D}, \mathcal{E}, \mathcal{L}$ and $\mathcal{M}$ represent the structure of uncertainties. The matrices $\Delta_{\mathcal{A}}(t) \in \Re^{p\times q}$ and $\Delta_{\mathcal{B}}(t) \in \Re^{r\times s}$ denote uncertainties and satisfy $\Delta_{\mathcal{A}}(t)\Delta_{\mathcal{A}}^T(t) \leq I_p$ and $\Delta_{\mathcal{B}}(t)\Delta_{\mathcal{B}}^T(t) \leq I_r$, respectively. Furthermore, the quadratic cost function associated with the uncertain system (1) is given by

$$\mathcal{J}(0,\infty) = \int_0^\infty \left(x^T(t)\mathcal{Q}x(t) + u^T(t)\mathcal{R}u(t)\right) dt \tag{3}$$

where $\mathcal{Q} \in \Re^{n\times n}$ and $\mathcal{R} \in \Re^{m\times m}$ are positive definite symmetric matrices and can be adjusted by designers.

In order to consider control gain perturbations, the actual control input implemented is assumed to be

$$u(t) \triangleq K(t)x(t) \tag{4}$$

where $K(t) \in \Re^{m\times n}$ represent the control gain matrix and the following two classes of uncertainties for the control gain matrix $K(t)$ are considered[13].

- the multiplicative form:

$$K(t) \triangleq K + \mathcal{F}_{\mathcal{M}}\Delta_{\mathcal{K}_{\mathcal{M}}}(t)\mathcal{N}_{\mathcal{M}}K \tag{5}$$

- the additive form:

$$K(t) \triangleq K + \mathcal{F}_{\mathcal{A}}\Delta_{\mathcal{K}_{\mathcal{A}}}(t)\mathcal{N}_{\mathcal{A}} \tag{6}$$

where K is the nominal control gain matrix. In (5) and (6), $\mathcal{F}_{\mathcal{M}}, \mathcal{F}_{\mathcal{A}}, \mathcal{N}_{\mathcal{M}}$ and $\mathcal{N}_{\mathcal{A}}$ denote the structure of uncertainties for the control gain matrix and are known constant matrices with appropriate dimensions. Also, the matrices $\Delta_{\mathcal{K}_{\mathcal{M}}}(t) \in \Re^{k_{\mathcal{M}}\times l_{\mathcal{M}}}$ and $\Delta_{\mathcal{K}_{\mathcal{A}}}(t) \in \Re^{k_{\mathcal{A}}\times l_{\mathcal{A}}}$ represent the control gain variations and satisfy the relation $\Delta_{\mathcal{K}_{\mathcal{M}}}(t)\Delta_{\mathcal{K}_{\mathcal{M}}}^T(t) \leq \epsilon_{\mathcal{M}}I_{k_{\mathcal{M}}}$ and $\Delta_{\mathcal{K}_{\mathcal{A}}}(t)\Delta_{\mathcal{K}_{\mathcal{A}}}^T(t) \leq \epsilon_{\mathcal{A}}I_{k_{\mathcal{A}}}$ where $\epsilon_{\mathcal{M}}$ and $\epsilon_{\mathcal{A}}$ are known positive scalars.

Note that the manipulated control input for the uncertain system (1) is $u(t) \triangleq Kx(t)$, because the control gain variations $\Delta_{\mathcal{K}_{\mathcal{M}}}(t) \in \Re^{k_{\mathcal{M}}\times l_{\mathcal{M}}}$ and $\Delta_{\mathcal{K}_{\mathcal{A}}}(t) \in \Re^{k_{\mathcal{A}}\times l_{\mathcal{A}}}$

$$\begin{aligned}\frac{d}{dt}x^T(t)\mathcal{P}x(t) &= x^T(t)\left\{(A(t)+B(t)K(t))^T\mathcal{P}+\mathcal{P}(A(t)+B(t)K(t))\right\}x(t)\\ &< 0 \quad \text{for} \quad \forall x(t)\neq 0\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{d}{dt}x^T(t)\mathcal{P}x(t) &= x^T(t)\left\{(A(t)+B(t)K(t))^T\mathcal{P}+\mathcal{P}(A(t)+B(t)K(t))\right\}x(t)\\ &\leq -x^T(t)\left(\mathcal{Q}+K^T(t)\mathcal{R}K(t)\right)x(t)\end{aligned} \tag{11}$$

$$\begin{pmatrix} \Gamma_{\mathcal{M}}(\mathcal{S},\mathcal{W},\gamma,\delta,\mu) & \mathcal{S} & \mathcal{W}^T & \mathcal{S}\mathcal{L}^T & \mathcal{W}^T\mathcal{M}^T & \mathcal{W}^T\mathcal{N}_{\mathcal{M}}^T & 0 & \mathcal{E} & \mathcal{W}^T\mathcal{N}_{\mathcal{M}}^T \\ \mathcal{S} & -\mathcal{Q}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathcal{W} & 0 & -\mathcal{R}^{-1}+\nu\epsilon_{\mathcal{M}}\mathcal{F}_{\mathcal{M}}\mathcal{F}_{\mathcal{M}}^T & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathcal{L}\mathcal{S} & 0 & 0 & -\gamma I_q & 0 & 0 & 0 & 0 & 0 \\ \mathcal{M}\mathcal{W} & 0 & 0 & 0 & -\delta I_s & 0 & 0 & 0 & 0 \\ \mathcal{N}_{\mathcal{M}}\mathcal{W} & 0 & 0 & 0 & 0 & -\mu I_m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\Upsilon_{\mathcal{M}} & 0 & 0 \\ \mathcal{E}^T & 0 & 0 & 0 & 0 & 0 & 0 & -\nu I_r & 0 \\ \mathcal{N}_{\mathcal{M}}\mathcal{W} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\nu I_{l_{\mathcal{M}}} \end{pmatrix} \leq 0 \tag{13}$$

$$\Gamma_{\mathcal{M}}(\mathcal{S},\mathcal{W},\gamma,\delta,\mu) \triangleq H_e\{A\mathcal{S}+B\mathcal{W}\}+\gamma\mathcal{D}\mathcal{D}^T+\delta\mathcal{E}\mathcal{E}^T+\mu\epsilon_{\mathcal{M}}B\mathcal{F}_{\mathcal{M}}\mathcal{F}_{\mathcal{M}}^T B^T \tag{14}$$

cannot be handled. In this paper, we simply consider the actual control input $u(t)$ described by (4), (5) and (6) so as to design the guaranteed cost controller under control gain perturbations.

From (1) and (4), we get

$$\frac{d}{dt}x(t) = (A(t)+B(t)K(t))x(t) \tag{7}$$

For actual controller implemented, we shall give the following two definitions, which are similar to that in the work of Petersen and McFarlane[6].

Definition 1 The closed-loop uncertain system (7) is said to be quadratically stable if there exists a feedback gain matrix K and a matrix $\mathcal{P}>0$ which satisfy the matrix inequality (8) for all uncertainties $\Delta_{\mathcal{A}}(t)$ and $\Delta_{\mathcal{B}}(t)$ and all control gain perturbations $\Delta_{\mathcal{K}_{\mathcal{M}}}(t)$ or $\Delta_{\mathcal{K}_{\mathcal{A}}}(t)$.

Definition 2 The control law (4) is said to be a robust non-fragile guaranteed cost control with a cost matrix $\mathcal{P}>0$ for the uncertain system (1) and the quadratic cost function (3) if the closed-loop uncertain system (7) is quadratically stable and there exists the symmetric positive definite matrix $\mathcal{P}\in\Re^{n\times n}$ satisfying

$$H_e\left\{\mathcal{P}(A(t)+B(t)K(t))\right\}+\mathcal{Q}+K^T(t)\mathcal{R}K(t)\leq 0 \tag{9}$$

for all uncertainties $\Delta_{\mathcal{A}}(t)$ and $\Delta_{\mathcal{B}}(t)$ and all control gain perturbations $\Delta_{\mathcal{K}_{\mathcal{M}}}(t)$ or $\Delta_{\mathcal{K}_{\mathcal{A}}}(t)$.

The following lemma shows that a robust non-fragile guaranteed cost control will achieve quadratic stability of the uncertain closed-loop system (7) and define an upper bound on the cost function (3).

Lemma 3 Consider the uncertain system (1) with the quadratic cost function (3). Suppose that the control law (4) with control gain perturbations (5) or (6) is a robust non-fragile guaranteed cost control with the cost matrix $\mathcal{P}>0$. Then the uncertain closed-loop system (7) is quadratically stable and the following relation holds.

$$\begin{aligned}\mathcal{J}(0,\infty) &= \int_0^\infty\left(x^T(t)\mathcal{Q}x(t)+u^T(t)\mathcal{R}u(t)\right)dt\\ &\leq x^T(0)\mathcal{P}x(0)\triangleq\mathcal{J}^*\end{aligned} \tag{10}$$

Proof: From the **Definition 1** and **Definition 2**, the quadratic stability of the uncertain closed-loop system (7) is immediate. Let $\mathcal{V}(x,t)\triangleq x^T(t)\mathcal{P}x(t)$ be a Lyapunov function candidate. Then from the **Definition 2**, the time derivative of the quadratic function $\mathcal{V}(x,t)$ along the trajectory of the uncertain closed-loop system (7) can be computed as (11). By integrating both sides of the inequality (11) from 0 to T_f, we have

$$\begin{aligned}\mathcal{J}(0,T_f) &= \int_0^{T_f}x^T(t)\left(\mathcal{Q}+K^T(t)\mathcal{R}K(t)\right)x(t)dt\\ &\leq \mathcal{V}(x,0)-\mathcal{V}(x,T_f)\end{aligned} \tag{12}$$

Since the uncertain closed-loop system (7) is quadratically stable, that is, $x(T_f)\to 0$ when $T_f\to\infty$, we obtain $T_f\to\infty$. Thus we get the upper bound on the quadratic cost function (10).

It follows that the result of the lemma is true. The proof of **Lemma 3** is completed. □

From the above discussion, our control objective is to design the robust non-fragile guaranteed controller. That is to find the state feedback gain matrix

$$\frac{d}{dt}\mathcal{V}(x,t) = x^T(t)\big[H_e\{\mathcal{P}(A+BK+\mathcal{D}\Delta_{\mathcal{A}}(t)\mathcal{L}+\mathcal{E}\Delta_{\mathcal{B}}(t)\mathcal{M}K+B\mathcal{F}_{\mathcal{M}}\Delta_{\mathcal{K}_{\mathcal{M}}}(t)\mathcal{N}_{\mathcal{M}}K + \mathcal{E}\Delta_{\mathcal{B}}(t)\mathcal{M}\mathcal{F}_{\mathcal{M}}\Delta_{\mathcal{K}_{\mathcal{M}}}(t)\mathcal{N}_{\mathcal{M}}K)\}\big]x(t) \tag{16}$$

$$\begin{aligned}\Phi(\mathcal{S},\mathcal{W}) \triangleq\, & H_e\{\mathcal{P}(A+BK+\mathcal{D}\Delta_{\mathcal{A}}(t)\mathcal{L}+\mathcal{E}\Delta_{\mathcal{B}}(t)\mathcal{M}K+B\mathcal{F}_{\mathcal{M}}\Delta_{\mathcal{K}_{\mathcal{M}}}(t)\mathcal{N}_{\mathcal{M}}K+\mathcal{E}\Delta_{\mathcal{B}}(t)\mathcal{M}\mathcal{F}_{\mathcal{M}}\Delta_{\mathcal{K}_{\mathcal{M}}}(t)\mathcal{N}_{\mathcal{M}}K)\} \\ & + \mathcal{Q} + (K+\mathcal{F}_{\mathcal{M}}\Delta_{\mathcal{K}_{\mathcal{M}}}(t)\mathcal{N}_{\mathcal{M}}K)^T\mathcal{R}\,(K+\mathcal{F}_{\mathcal{M}}\Delta_{\mathcal{K}_{\mathcal{M}}}(t)\mathcal{N}_{\mathcal{M}}K) \\ & \le 0 \end{aligned} \tag{17}$$

$$\begin{aligned}\Psi(\mathcal{S},\mathcal{W},\gamma,\delta,\mu) \triangleq\, & \mathit{\Gamma}_{\mathcal{M}}(\mathcal{S},\mathcal{W},\gamma,\delta,\mu) + \frac{1}{\gamma}\mathcal{S}\mathcal{L}^T\mathcal{L}\mathcal{S} + \frac{1}{\delta}\mathcal{W}^T\mathcal{M}^T\mathcal{M}\mathcal{W} + \frac{1}{\mu}\mathcal{W}^T\mathcal{N}_{\mathcal{M}}^T\mathcal{N}_{\mathcal{M}}\mathcal{W} \\ & + \left(\mathcal{E}\Delta_{\mathcal{B}}(t) + \mathcal{W}^T\mathcal{N}_{\mathcal{M}}^T\Delta_{\mathcal{K}_{\mathcal{M}}}^T(t)\mathcal{F}_{\mathcal{M}}^T\mathcal{M}^T\right)\left(\mathcal{E}\Delta_{\mathcal{B}}(t) + \mathcal{W}^T\mathcal{N}_{\mathcal{M}}^T\Delta_{\mathcal{K}_{\mathcal{M}}}^T(t)\mathcal{F}_{\mathcal{M}}^T\mathcal{M}^T\right)^T \\ & + \mathcal{S}\mathcal{Q}\mathcal{S} + \mathcal{W}^T\left(I_m + \mathcal{F}_{\mathcal{M}}\Delta_{\mathcal{K}_{\mathcal{M}}}(t)\mathcal{N}_{\mathcal{M}}\right)^T\mathcal{R}\left(I_m + \mathcal{F}_{\mathcal{M}}\Delta_{\mathcal{K}_{\mathcal{M}}}(t)\mathcal{N}_{\mathcal{M}}\right)\mathcal{W} \\ & \le 0 \end{aligned} \tag{18}$$

$K \in \Re^{m\times n}$ and the symmetric positive definite matrix $\mathcal{P} \in \Re^{n\times n}$ satisfying the inequality (9).

4. Design of Robust Non-Fragile Guaranteed Cost Controllers

In this section, we show that the design method of the robust non-fragile guaranteed cost controller based on the LMI framework. Firstly, we give the following theorem for the guaranteed cost controller under multiplicative control gain perturbations of the form (5).

Theorem 1 Consider the uncertain system (1) with the quadratic cost function (3). There exists the state feedback gain matrix K such that the control law (4) with the multiplicative control gain perturbations of the form (5) is a robust non-fragile guaranteed cost control, if there exist $\mathcal{S} > 0, \mathcal{W}, \gamma > 0, \delta > 0, \mu > 0$ and $\nu > 0$ satisfying the LMI condition (13). In (13), $\mathit{\Upsilon}_{\mathcal{M}}$ is the matrix given by $\mathit{\Upsilon}_{\mathcal{M}} \triangleq I_s - \nu(I_s + \epsilon_{\mathcal{M}}\mathcal{M}\mathcal{F}_{\mathcal{M}}\mathcal{F}_{\mathcal{M}}^T\mathcal{M}^T)$ and $\mathit{\Gamma}_{\mathcal{M}}(\mathcal{S},\mathcal{W},\gamma,\delta,\mu)$ is the matrix given by (14).

If the solution $\mathcal{S} > 0, \mathcal{W}, \gamma > 0, \delta > 0, \mu > 0$ and $\nu > 0$ of the LMI (13) exists, then the state feedback gain matrix $K \in \Re^{m\times n}$ is obtained as

$$K = \mathcal{W}\mathcal{S}^{-1} \tag{15}$$

Proof: By using a symmetric positive definite matrix $\mathcal{P} \in \Re^{n\times n}$, we introduce the quadratic function $\mathcal{V}(x,t) \triangleq x^T(t)\mathcal{P}x(t)$ as a Lyapunov function candidate. From (2) and (5), the time derivative of the quadratic function $\mathcal{V}(x,t)$ along the trajectory of the closed-loop system (7) with uncertainties and control gain variations can be computed as (16).

Now we consider the condition (17) corresponding to the inequality (9). Note that if there exist the matrices $\mathcal{P} \in \Re^{n\times n}$ and $K \in \Re^{m\times n}$ which satisfy the condition (17), then the inequality (9) is also satisfied. If there exist the state feedback gain matrix $K \in \Re^{m\times n}$ and the symmetric positive definite matrix $\mathcal{P} \in \Re^{n\times n}$ which satisfy the condition (17), then the uncertain closed-loop system (7) is quadratically stable and the upper bound on the quadratic cost function (3) is given by (10), i.e. the control law (4) becomes a robust non-fragile guaranteed cost control (see **Lemma 3**).

Let us introduce the matrix $\mathcal{S} \triangleq \mathcal{P}^{-1}$ and consider the change of variable $\mathcal{W} \triangleq K\mathcal{S}$. Then pre- and post-multiplying (17) by $\mathcal{S}$ and using **Lemma 2**, we get the matrix inequality condition (18). If the inequality condition (18) holds, then the matrix inequality (17) is satisfied, because the following relation is obvious.

$$\Phi(\mathcal{S},\mathcal{W}) \le \Psi(\mathcal{S},\mathcal{W},\gamma,\delta,\mu) \tag{19}$$

Furthermore, applying **Lemma 1** to the condition (18), simple algebraic manipulation gives the matrix inequality (20) at the top of the next page. Also by using **Lemma 2**, we obtain the condition (21) at the top of the next page. One can see from the **Lemma 2** that if the matrix inequality (21) is satisfied then the inequality condition (20) holds. Note that the matrix $\mathit{\Lambda}(\mu,\mathcal{W})$ in (21) is given by

$$\mathit{\Lambda}(\nu,\mathcal{W}) \triangleq \frac{1}{\nu}\mathcal{E}\mathcal{E}^T + \frac{1}{\nu}\mathcal{W}^T\mathcal{N}_{\mathcal{M}}^T\mathcal{N}_{\mathcal{M}}\mathcal{W} \tag{22}$$

From **Lemma 1**, it is easy to verify that the condition (21) is equivalent to the LMI (13). If the solution $\mathcal{S},\mathcal{W},\gamma,\delta,\mu$ and ν of the LMI condition (13) exists, then from the relation $\mathcal{W} \triangleq K\mathcal{S}$, the state feedback gain matrix is given by (15).

It follows that the result of the theorem is true. Thus the proof of **Theorem 1** is completed. □

$$\begin{pmatrix} \varGamma_{\mathcal{M}}(\mathcal{S},\mathcal{W},\gamma,\delta,\mu) & \mathcal{S} & \mathcal{W}^T & \mathcal{S}\mathcal{L}^T & \mathcal{W}^T\mathcal{M}^T & \mathcal{W}^T\mathcal{N}_{\mathcal{M}}^T & 0 \\ \mathcal{S} & -\mathcal{Q}^{-1} & 0 & 0 & 0 & 0 & 0 \\ \mathcal{W} & 0 & -\mathcal{R}^{-1} & 0 & 0 & 0 & 0 \\ \mathcal{L}\mathcal{S} & 0 & 0 & -\gamma I_q & 0 & 0 & 0 \\ \mathcal{M}\mathcal{W} & 0 & 0 & 0 & -\delta I_s & 0 & 0 \\ \mathcal{N}_{\mathcal{M}}\mathcal{W} & 0 & 0 & 0 & 0 & -\mu I_m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -I_s \end{pmatrix}$$
$$+ H_e\left\{\begin{pmatrix} \mathcal{E}\Delta_{\mathcal{B}}(t) & \mathcal{W}^T\mathcal{N}_{\mathcal{M}}^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & I_s \\ 0 & 0 & \Delta_{\mathcal{K}_{\mathcal{M}}}^T(t)\mathcal{F}_{\mathcal{M}}^T & 0 & 0 & 0 & \Delta_{\mathcal{K}_{\mathcal{M}}}^T(t)\mathcal{F}_{\mathcal{M}}^T\mathcal{M}^T \end{pmatrix}\right\} \leq 0 \tag{20}$$

$$\begin{pmatrix} \varGamma_{\mathcal{M}}(\mathcal{S},\mathcal{W},\gamma,\delta,\mu)+\varLambda(\nu,\mathcal{W}) & \mathcal{S} & \mathcal{W}^T & \mathcal{S}\mathcal{L}^T & \mathcal{W}^T\mathcal{M}^T & \mathcal{W}^T\mathcal{N}_{\mathcal{M}}^T & 0 \\ \mathcal{S} & -\mathcal{Q}^{-1} & 0 & 0 & 0 & 0 & 0 \\ \mathcal{W} & 0 & -\mathcal{R}^{-1}+\nu\epsilon_{\mathcal{M}}\mathcal{F}_{\mathcal{M}}\mathcal{F}_{\mathcal{M}}^T & 0 & 0 & 0 & 0 \\ \mathcal{L}\mathcal{S} & 0 & 0 & -\gamma I_q & 0 & 0 & 0 \\ \mathcal{M}\mathcal{W} & 0 & 0 & 0 & -\delta I_s & 0 & 0 \\ \mathcal{N}_{\mathcal{M}}\mathcal{W} & 0 & 0 & 0 & 0 & -\mu I_m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\varUpsilon_{\mathcal{M}} \end{pmatrix} \leq 0 \tag{21}$$

Theorem 1 provides a sufficient condition for the existence of a robust non-fragile guaranteed cost controller under multiplicative control gain perturbations. Next, we show the theorem for a design method of a robust non-fragile guaranteed cost controller under additive one.

Theorem 2 Consider the uncertain system (1) with the quadratic cost function (3). There exists the state feedback gain matrix K such that the control law (4) with the additive control gain perturbations of the form (6) is a robust non-fragile guaranteed cost control if there exist $\mathcal{S} > 0, \mathcal{W}, \gamma > 0, \delta > 0, \mu > 0$ and $\nu > 0$ satisfying the LMI condition (23). In (23), $\varUpsilon_{\mathcal{A}}$ is the matrix given by $\varUpsilon_{\mathcal{A}} \stackrel{\triangle}{=} I_s - \nu(I_s + \epsilon_{\mathcal{A}}\mathcal{M}\mathcal{F}_{\mathcal{A}}\mathcal{F}_{\mathcal{A}}^T\mathcal{M}^T)$ and $\varGamma_{\mathcal{A}}(\mathcal{S},\mathcal{W},\gamma,\delta,\mu)$ is the matrix given by (24).

If the solution of the LMI (23) exists, then the state feedback gain matrix $K \in \Re^{m\times n}$ is obtained as

$$K = \mathcal{W}\mathcal{S}^{-1} \tag{25}$$

Proof: The result of **Theorem 2** is derived in a similar way as for **Theorem 1**. □

5. Design of Optimal Robust Non-Fragile Guaranteed Cost Controllers

Since (13) and (23) are LMIs in $\mathcal{S} > 0$, $\mathcal{W}$, $\gamma > 0$, $\delta > 0$, $\mu > 0$ and $\nu > 0$, the matrix inequalities (13) and (23) define a convex solution set of $(\mathcal{S}, \mathcal{W}, \gamma, \delta, \mu, \nu)$. Therefore various efficient convex optimization algorithms can be used to test whether the LMIs are solvable and to generate particular solutions. Moreover, its solutions parametrize the set of guaranteed cost controllers. The parametrized representation can be exploited to design the guaranteed cost controller with some additional requirements. In particular, the optimal robust non-fragile guaranteed cost control which minimizes the upper bound on the quadratic cost function (10) can be determined by solving a certain optimization problem. In this section, we consider the problem of the optimal robust non-fragile guaranteed cost control.

We now consider to design the optimal robust non-fragile guaranteed cost controller. In (10), the upper bound $\mathcal{J}^*$ depends on the initial vector $x(0)$. Thus in order to avoid this dependence, we assume that the initial vector $x(0)$ is zero mean random vector satisfying $E\{x(0)x^T(0)\} = I_n$[6, 17]. In this case, we consider the value of the quadratic cost function as its expectation. Then the upper bound on the quadratic cost function (10) is given as $E\{\mathcal{J}^*\} = Tr\{\mathcal{P}\}$. Therefore we seek to minimize $Tr\{\mathcal{P}\}$ subject to the constraint (13) or (23) (see **Remark 2**). Namely the problem of designing the optimal robust non-fragile guaranteed cost control is reduced to the following constrained optimization problem.

$$\begin{pmatrix} \varGamma_{\mathcal{A}}(\mathcal{S},\mathcal{W},\gamma,\delta,\mu) & \mathcal{S} & \mathcal{W}^T & \mathcal{S}\mathcal{L}^T & \mathcal{W}^T\mathcal{M}^T & \mathcal{S}^T\mathcal{N}_{\mathcal{A}}^T & 0 & \mathcal{E} & \mathcal{S}^T\mathcal{N}_{\mathcal{A}}^T \\ \mathcal{S} & -\mathcal{Q}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathcal{W} & 0 & -\mathcal{R}^{-1}+\nu\epsilon_{\mathcal{A}}\mathcal{F}_{\mathcal{A}}\mathcal{F}_{\mathcal{A}}^T & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathcal{L}\mathcal{S} & 0 & 0 & -\gamma I_q & 0 & 0 & 0 & 0 & 0 \\ \mathcal{M}\mathcal{W} & 0 & 0 & 0 & -\delta I_s & 0 & 0 & 0 & 0 \\ \mathcal{N}_{\mathcal{A}}\mathcal{S} & 0 & 0 & 0 & 0 & -\mu I_m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\varUpsilon_{\mathcal{A}} & 0 & 0 \\ \mathcal{E}^T & 0 & 0 & 0 & 0 & 0 & 0 & -\nu I_r & 0 \\ \mathcal{N}_{\mathcal{A}}\mathcal{S} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\nu I_{l_{\mathcal{A}}} \end{pmatrix} \leq 0 \quad (23)$$

$$\varGamma_{\mathcal{A}}(\mathcal{S},\mathcal{W},\gamma,\delta,\mu) \stackrel{\triangle}{=} H_e\{A\mathcal{S}+B\mathcal{W}\} + \gamma\mathcal{D}\mathcal{D}^T + \delta\mathcal{E}\mathcal{E}^T + \mu\epsilon_{\mathcal{A}}B\mathcal{F}_{\mathcal{A}}\mathcal{F}_{\mathcal{A}}^T B^T - \mathcal{E}\mathcal{E}^T \quad (24)$$

$$\underset{\mathcal{S},\mathcal{W},\gamma,\delta,\mu,\nu}{\text{Minimize}}[Tr\{\mathcal{P}\}] \ \text{subject to}$$
(13) (or (23)) and
$$\mathcal{S}>0, \mathcal{W}, \gamma>0, \delta>0, \mu>0 \ \text{and} \ \nu>0 \quad (26)$$

Note that in the constrained optimization problem (26), the constraint (13) is adopted in the case of the multiplicative control gain perturbations of the form (5) and the constraint (23) is adopted in the case of the additive one of the form (6).

Since (13) and (23) are LMIs in $\mathcal{S}, \mathcal{W}, \gamma, \delta, \mu$ and ν, we now introduce a complementary variable $\mathcal{Z} \in \Re^{n\times n}$ satisfying the following relation.

$$\begin{pmatrix} \mathcal{Z} & I_n \\ I_n & \mathcal{S} \end{pmatrix} \geq 0 \Rightarrow \mathcal{Z} - \mathcal{S}^{-1} \geq 0 \quad (27)$$

Therefore we see from the relation $\mathcal{S}^{-1} = \mathcal{P}$ that the minimization problem of $Tr\{\mathcal{P}\}$ can be transformed into that of $Tr\{\mathcal{Z}\}$. Note that the condition (27) is also the LMI in $\mathcal{Z}$ and $\mathcal{S}$. Consequently, the constrained optimization problem (26) is reduced to the following constrained convex optimization problem.

$$\underset{\mathcal{Z},\mathcal{S},\mathcal{W},\gamma,\delta,\mu,\nu}{\text{Minimize}}[Tr\{\mathcal{Z}\}] \ \text{subject to}$$
(13) (or (23)) and (27) and (28)
$$\mathcal{Z}>0, \mathcal{S}>0, \mathcal{W}, \gamma>0, \delta>0, \mu>0 \ \text{and} \ \nu>0$$

If the optimal solution of the constrained convex optimization problem (28), denoted by $\mathcal{Z}^*, \mathcal{S}^*, \mathcal{W}^*, \gamma^*, \delta^*, \mu^*$ and ν^* is obtained then the optimal robust non-fragile guaranteed cost control law for all unknown matrices $\Delta_{\mathcal{A}}(t)$ and $\Delta_{\mathcal{B}}(t)$ and all control gain perturbations $\Delta_{\mathcal{K}_{\mathcal{M}}}(t)$ or $\Delta_{\mathcal{K}_{\mathcal{A}}}(t)$ is given by (4) with the state feedback gain matrix $K = \mathcal{W}^*(\mathcal{S}^*)^{-1}$.

From the above discussion, the following theorem can be developed.

Theorem 3 Consider the uncertain system (1) with the quadratic cost function (3). There exists an optimal robust non-fragile guaranteed cost control for all uncertainties $\Delta_{\mathcal{A}}(t)$ and $\Delta_{\mathcal{B}}(t)$ and all control gain perturbations $\Delta_{\mathcal{K}_{\mathcal{M}}}(t)$ or $\Delta_{\mathcal{K}_{\mathcal{A}}}(t)$, if there exist the optimal solution which satisfy the following constrained convex optimization problem.

$$\underset{\mathcal{Z},\mathcal{S},\mathcal{W},\gamma,\delta,\mu,\nu}{\text{Minimize}}[Tr\{\mathcal{Z}\}] \ \text{subject to}$$
(13) (or (23)) and (27) and
$$\mathcal{Z}>0, \mathcal{S}>0, \mathcal{W}, \gamma>0, \delta>0, \mu>0 \ \text{and} \ \nu>0$$

If the optimal solution of the constrained convex optimization problem (28) is obtained, then the optimal robust non-fragile guaranteed cost control law for all unknown matrices $\Delta_{\mathcal{A}}(t)$ and $\Delta_{\mathcal{B}}(t)$ and all control gain perturbations $\Delta_{\mathcal{K}_{\mathcal{M}}}(t)$ or $\Delta_{\mathcal{K}_{\mathcal{A}}}(t)$ is given by (4) with the state feedback gain matrix $K = \mathcal{W}^*(\mathcal{S}^*)^{-1}$.

Furthermore, the upper bound on the quadratic cost function is computed as $Tr\{(\mathcal{S}^*)^{-1}\}$.

Remark 1 In this paper, the design problem of the robust non-fragile guaranteed cost controller for the uncertain system (1) has been considered. By the way, It might not be necessary to consider uncertainties in the input matrix, because by introducing additional actuator dynamics and constituting an augmented system, uncertainties in the input matrix are embedded in the system matrix of the augmented system[18]. However, even if such augmented system is constituted, the dimension of the resulting LMI needed to be solved is more larger than one of this paper. Namely, the amount of computation required to solve becomes large for problems with large state dimension. Additionally, the resulting controller in this case is not static feedback controller but dynamic one, i.e. the controller is more complex than the proposed controller. One can see from this fact that the proposed design method is useful.

Remark 2 In the above, the minimization problem of $Tr\{\mathcal{P}\}$ instead of the upper bound $\mathcal{J}^* \stackrel{\triangle}{=} x^T(0)\mathcal{P}x(0)$ is considered. However, the minimization problem of the

$$\frac{d}{dt}x(t) = \begin{pmatrix} -1.0 & 1.0 \\ 0.0 & 2.0 \end{pmatrix} x(t) + \begin{pmatrix} 0.1 \\ 0.1 \end{pmatrix} \Delta_{\mathcal{A}}(t) \begin{pmatrix} 0.25 & 0.05 \end{pmatrix} x(t) + \begin{pmatrix} 0.0 \\ 2.0 \end{pmatrix} u(t) + \begin{pmatrix} 0.050 \\ 0.175 \end{pmatrix} \Delta_{\mathcal{B}}(t)u(t) \quad (31)$$

upper bound $\mathcal{J}^* \stackrel{\triangle}{=} x^T(0)\mathcal{P}x(0)$ can be directly considered. In this case, we introduce a scalar variable σ satisfying the following LMI condition instead of the complementary variable $\mathcal{Z} \in \Re^{n\times n}$ which satisfies the constraint (27).

$$\begin{pmatrix} \sigma & x^T(0) \\ x(0) & \mathcal{S} \end{pmatrix} \geq 0 \Rightarrow \sigma - x^T(0)\mathcal{P}x(0) \geq 0 \quad (29)$$

Therefore in this case, considering the LMI condition (29), the design problem of the optimal guaranteed cost control is reduced to the following constrained convex optimization problem.

$$\begin{aligned} &\underset{\sigma,\mathcal{S},\mathcal{W},\gamma,\delta,\mu,\nu}{\text{Minimize}} \ [Tr\{\sigma\}] \ \text{subject to} \\ &\text{(13) (or (23)) and (29) and} \qquad (30) \\ &\sigma > 0, \mathcal{S} > 0, \mathcal{W}, \gamma > 0, \delta > 0, \mu > 0 \ \text{ and } \ \nu > 0 \end{aligned}$$

If the optimal solution of the constrained convex optimization problem (30), denoted by $\sigma^* > 0$, $\mathcal{S}^* > 0$, $\mathcal{W}^*$, $\gamma^* > 0$, $\delta^* > 0$, $\mu^* > 0$ and $\nu^* > 0$, is obtained then the optimal non-fragile guaranteed cost control law for all unknown matrices $\Delta_{\mathcal{A}}(t) \in \Re^{p\times q}$ and $\Delta_{\mathcal{B}}(t) \in \Re^{r\times s}$ and all control gain perturbations $\Delta_{\mathcal{K}_\mathcal{M}}(t) \in \Re^{k_\mathcal{M}\times l_\mathcal{M}}$ or $\Delta_{\mathcal{K}_\mathcal{A}}(t) \in \Re^{k_\mathcal{A}\times l_\mathcal{A}}$ is given by (4) with the state feedback gain matrix $K = \mathcal{W}^*\left(\mathcal{S}^*\right)^{-1}$. Furthermore, the upper bound on the quadratic cost function is computed as $x^T(0)\left(\mathcal{S}^*\right)^{-1}x(0)$.

6. Illustrative Examples

In order to demonstrate the efficiency of the proposed controller, we have run a simple example.In this example we deal with the multiplicative control gain perturbations of the form (5) and using **Theorem 3**, we consider to design the optimal robust non-fragile guaranteed cost controller. Also, the simulation results are shown for the proposed optimal robust non-fragile guaranteed cost controller, the conventional guaranteed cost controller designed without thinking of control gain perturbations based on the existing results[6),8)] and the standard linear quadratic regulator (LQR) which is designed for the nominal system.

Note that the control problem considered here are not necessary practical. However, the simulation results stated below illustrate the distinct feature of the proposed robust non-fragile controller.

Consider the uncertain linear system (33). In this example, we assume that $k_\mathcal{M} = l_\mathcal{M} = 1$ (i.e. $\Delta_{\mathcal{K}_\mathcal{M}}(t) \in \Re^{1\times 1}$, $\mathcal{F}_\mathcal{M} \in \Re^{1\times 1}$ and $\mathcal{N}_\mathcal{M} \in \Re^{1\times 1}$) and $\mathcal{F}_\mathcal{M}$ and $\mathcal{N}_\mathcal{M}$ in (5) and $\epsilon_\mathcal{M}$ are given as $\mathcal{F}_\mathcal{M} = 1.2$, $\mathcal{N}_\mathcal{M} = 1.0$ and $\epsilon_\mathcal{M} = 0.5$ respectively.

Now we select the weighting matrices $\mathcal{Q} = 1.0I_2$ and $\mathcal{R} = 0.5$. Then by applying **Theorem 3** and solving the constrained optimization problem (28), we obtain the following optimal solution.

$$\begin{aligned} \mathcal{Z}^* &= \begin{pmatrix} 19.76483 & 28.87171 \\ 28.87171 & 184.14328 \end{pmatrix} \\ \mathcal{S}^* &= \begin{pmatrix} 6.56252\times 10^{-3} & -1.02893\times 10^{-3} \\ -1.02893\times 10^{-3} & 7.04381\times 10^{-4} \end{pmatrix} \\ \mathcal{W}^* &= \begin{pmatrix} 9.28699\times 10^{-8} & -2.88031\times 10^{-1} \end{pmatrix} \\ \gamma^* &= 2.27982\times 10^{-2},\ \delta^* = 1.09736 \\ \mu^* &= 2.43430\times 10^{-1},\ \nu^* = 7.40741\times 10^{-1} \end{aligned} \quad (32)$$

Thus, we get the following state feedback gain matrix.

$$K = \begin{pmatrix} -8.31594 & -53.03892 \end{pmatrix} \quad (33)$$

Furthermore, the upper bound on the quadratic cost function $E\{\mathcal{J}^*\} = Tr\left\{\left(\mathcal{S}^*\right)^{-1}\right\}$ can be computed as

$$E\{\mathcal{J}^*\} = 2.03908\times 10^3 \quad (34)$$

On the other hand, selecting the same weighting matrices $\mathcal{Q} = 1.0I_2$ and $\mathcal{R} = 0.5$, the feedback gain matrix for the conventional guaranteed cost control based on the existing results[6),8)], denoted by $K_\mathcal{C}$, has been derived as

$$K_\mathcal{C} = \begin{pmatrix} -7.87642\times 10^{-1} & -7.13502 \end{pmatrix} \quad (35)$$

Also the optimal gain matrix for LQR, denoted by $K_{\mathcal{LQ}}$ has been obtained as

$$K_{\mathcal{LQ}} = \begin{pmatrix} 3.33332\times 10^{-7} & 5.00000 \end{pmatrix} \quad (36)$$

For numerical simulations, the initial value for the uncertain linear system (31) is selected as $x(0) = \begin{pmatrix} 5.0 & -1.0 \end{pmatrix}^T$. Besides, for the uncertain parameters $\Delta_\mathcal{A}(t)$ and $\Delta_\mathcal{B}(t)$ and the control gain perturbations $\Delta_{\mathcal{K}_\mathcal{M}}(t)$, Case 1) and Case 2) in (37) are considered. The results of the simulation of this example are depicted in Fig. 1–8. In these figures, Proposed represents the transient time-response, the manipulated control input and the actual control input generated by the proposed

$$
\begin{aligned}
&\bullet \text{ Case 1) : } \Delta_{\mathcal{A}}(t) = \sin(2.00\pi t), \quad \Delta_{\mathcal{B}}(t) = -\cos(5.0\pi t), \quad \Delta_{\mathcal{K}_{\mathcal{M}}}(t) = -\frac{1}{\sqrt{3}}\left(1-\exp(-5.0t)\cos(10.0\pi t)\right) \\
&\bullet \text{ Case 2) : } \Delta_{\mathcal{A}}(t) = 1.0, \quad \Delta_{\mathcal{B}}(t) = 1.0, \quad \Delta_{\mathcal{K}}(t) = \frac{1}{2\sqrt{2}} \quad \text{for} \quad 0 \le t \le 0.5 \\
&\qquad \Delta_{\mathcal{A}}(t) = -1.0, \quad \Delta_{\mathcal{B}}(t) = -1.0, \quad \Delta_{\mathcal{K}}(t) = -\frac{1}{2\sqrt{2}} \quad \text{for} \quad 0.5 < t \le 1.5 \\
&\qquad \Delta_{\mathcal{A}}(t) = -1.0, \quad \Delta_{\mathcal{B}}(t) = 1.0, \quad \Delta_{\mathcal{K}}(t) = \frac{1}{\sqrt{2}} \quad \text{for} \quad 1.5 < t \le 2.0 \\
&\qquad \Delta_{\mathcal{A}}(t) = 1.0, \quad \Delta_{\mathcal{B}}(t) = -1.0, \quad \Delta_{\mathcal{K}}(t) = -\frac{1}{\sqrt{2}} \quad \text{for} \quad t > 2.0
\end{aligned}
\tag{37}
$$

guaranteed cost controller. Furthermore, Conventional and LQR show the time histories of the state and control input for the conventional guaranteed cost controller and LQR with the optimal gain matrix $K_{\mathcal{LQ}}$ (36) for the nominal system, respectively.

From Fig. 1–4, we find that the proposed guaranteed cost controller (Proposed in figures) and the conventional guaranteed cost controller (Conventional in figures) stabilize the linear dynamical system (31) with parameter uncertainties and control gain perturbations. However, the closed-loop uncertain system for LQR with the optimal gain matrix $K_{\mathcal{LQ}}$ is unstable.

On the other hand, we see from Fig. 5–8 that though the uncertain closed-loop systems for Conventional and LQR are unstable, the proposed controller stabilizes the closed-loop system with plant uncertainties and control gain perturbations. Namely this result shows that although the conventional guaranteed cost controller designed without thinking of control gain perturbations is fragile under the control gain perturbations, the proposed guaranteed cost controller is not fragile.

Therefore the effectiveness of the proposed robust non-fragile guaranteed cost controller is shown.

7. Conclusions

In this paper, a design method of a robust non-fragile guaranteed cost controller for linear continuous-time systems with structured uncertainties which are included in both the system matrix and the input one under control gain perturbations has been presented and the multiplicative control gain perturbations of the form (5) and the additive one of the form (6) have been considered. Additionally, an optimal robust non-fragile guaranteed cost control which minimizes an upper bound on a given quadratic cost function has been discussed. Finally, simple numerical examples are given for illustration of the proposed robust non-fragile guaranteed cost controller and the simulation result has shown that the closed-loop system is well stabilized in spite of plant uncertainties and control gain perturbations.

We have shown that the proposed robust non-fragile guaranteed cost controller can be easily obtained by solving LMI and the design problem of the optimal robust non-fragile guaranteed cost controller has been reduced to a constrained optimization problem. Therefore, the proposed robust non-fragile guaranteed cost controller can be easily obtained by using software such as MATLAB's LMI Control Toolbox and Scilab's LMITOOL.

The future research subjects are extension of the proposed design to such a broad class of systems as large-scale interconnected systems and discrete-time systems and output feedback systems.

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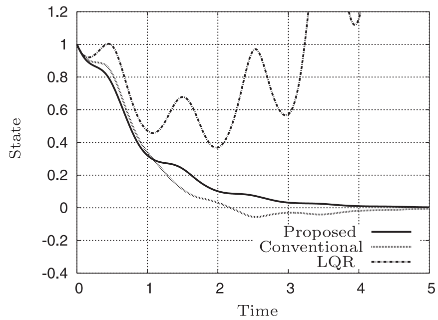

Figure 1: Transient time-response of the state variable $x_1(t)$: Case 1)

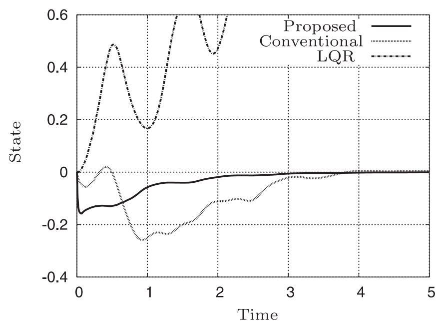

Figure 2: Transient time-response of the state variable $x_2(t)$: Case 1)

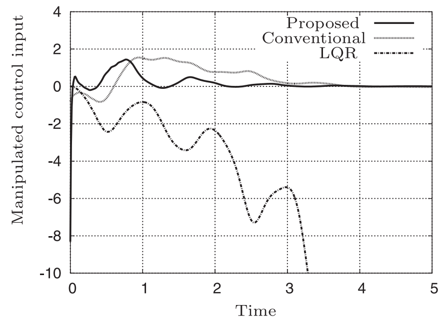

Figure 3: Time histories of the manipulated input $u(t) \triangleq Kx(t)$: Case 1)

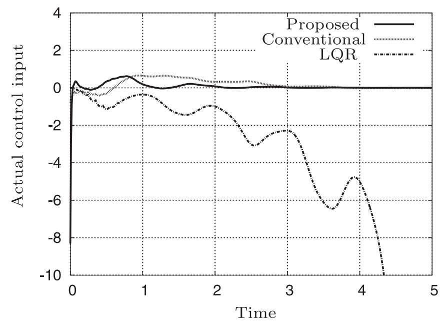

Figure 4: Time histories of the actual control input $u(t) \triangleq K(t)x(t)$: Case 1)

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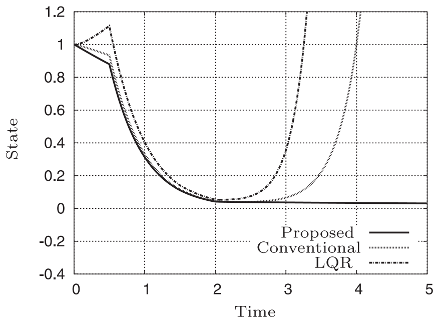

Figure 5: Transient time-response of the state variable $x_1(t)$: Case 2)

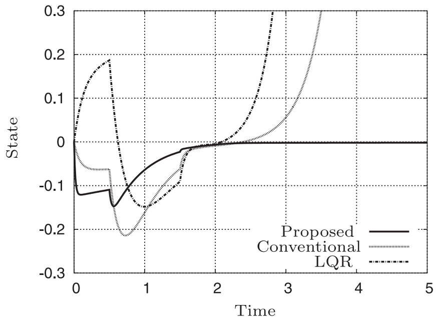

Figure 6: Transient time-response of the state variable $x_2(t)$: Case 2)

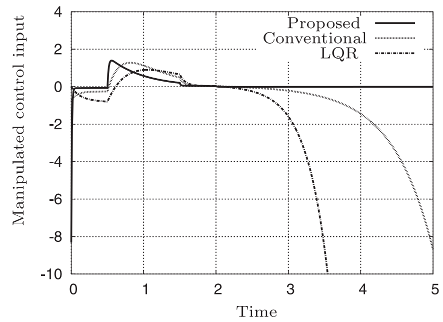

Figure 7: Time histories of the manipulated input $u(t) \triangleq K(t)x(t)$: Case 2)

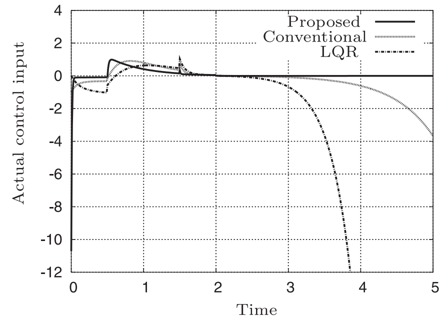

Figure 8: Time histories of the actual control input $u(t) \triangleq K(t)x(t)$: Case 2)