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EFFECTIVENESS OF COGNITIVELY GUIDED INSTRUCTION PRACTICES IN

BELOW GRADE-LEVEL ELEMENTARY STUDENTS

by

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A capstone submitted in partial fulfillment of the
requirements for the degree of Master of Arts in Teaching.

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CHAPTER 1

Introduction

Overview

As a first-year teacher, I was introduced to Cognitively Guided Instruction (CGI) on two separate occasions. My first introduction to this approach, a daylong professional development session held at my school, caught my attention in a powerful way. "You'll have kindergarteners working with fractions by Halloween," the facilitators told us. As my colleagues and I sat in what we expected to be just another training on best teaching practices to get us geared up for the new school year, my excitement began to build. Over the course of the day, we learned about this constructivist approach to teaching math and its benefits. Instead of teaching our students algorithms, procedures, and facts to memorize, we were urged to draw on their natural curiosity and inquisition to build deep conceptual understanding and engage in rich discourse about the mathematical concepts that serve as the foundation to future understanding. I left this session energized, excited, and eager to shift our focus to building this strong conceptual foundation in our elementary students. But many of my colleagues did not share my excitement. They doubted whether our students, many of whom were significantly below grade-level and needed intensive intervention in math, would benefit from this new style. They worried

about giving up the fast-paced approach to teaching math that allowed us to cover nearly two years of content in just ten months. They worried that CGI was not a practical solution for our situation.

I heard a separate group of peers and colleagues express similar reservations when I was introduced to CGI a second time, this time in a graduate course about teaching math to elementary students. Like me, most of my peers taught at schools serving high-needs populations, where many students were below grade-level in math. Though we all agreed that the approaches we were being exposed to were fascinating and powerful, I heard the phrase "I just wish this would work for my kids" more times than I can count. Though my fellow teachers believed in the validity and importance of the approach just like I did, they doubted its effectiveness when working with students who were so far below grade-level.

As I learned more and reflected on the most effective elements of my own math education, I was struck by the power of using a CGI-based instructional approach in the elementary school classroom. I believe deeply in the importance of building strong conceptual understanding and flexible thinking, and agree that the focus in a math classroom should be on using conceptual understanding to shape procedural fluency rather than seeing procedural fluency as the primary goal of instruction. However, I understood my colleagues' hesitation and doubt. As I spent the next few years trying to find the right approach to teach my students—most of who were severely below grade-level when they began second grade—I found myself constantly living in the tension between approaches. I was not confident enough that a CGI-based teaching model would effectively catch my students up to grade-level in the short time I had with them to fully

shift to using CGI as my primary approach to instruction. But, I also was not comfortable focusing only on procedural fluency and prioritizing rapid growth at the expense of deep conceptual understanding. This tension has led me to explore my research question: *What is the effect of Cognitively Guided Instruction practices on the math beliefs and abilities of below grade-level second grade students?*

In my work with over one hundred students who have entered second grade significantly below grade-level, I have noticed that their difficulties with math are often accompanied by a dislike of all math activities and extremely low self-confidence in their own math abilities. Thus, this study will examine the way that Cognitively Guided Instruction practices affect students' math abilities and beliefs, as both are powerful components of students' elementary math experiences that undoubtedly impact them when they leave second grade.

In this chapter, I provide context and background for my research question. I will explain my personal interest in this topic, as well as my motivation for studying it. In doing so, this chapter will examine the impact that this work will have on educators, students and their stakeholders, and educational policy makers. By better understanding the effect of constructivist approaches on the math achievement and beliefs of students who are below grade-level, I hope to be able to improve my instruction for the students who need it the most while working more efficiently and strategically to close the achievement gap that students are already experiencing at such a young age. I also hope to gather data to share with my administration so that our school and others like it can set up all students to be successful mathematicians now and in the future.

Context

In order to understand the significance of this question, it is important to understand the context in which I teach. I spent my first two years of teaching at a charter school in a major metropolitan city in the Upper Midwest whose mission is to close the achievement gap and give all students access to a high quality college-preparatory education. The school serves predominantly low-income students of color, and the majority of students are learning English as a second language. After that, I moved to a smaller city on the East Coast, where I teach at a charter school serving predominantly African American students and families. This school has a similar mission of providing a college preparatory education for all students, regardless of race or economic status.

In both schools, the majority of my students have entered second grade significantly below grade-level in math. Many of them begin school without the prerequisite skills needed to meet kindergarten standards, and though they make progress each year, it is not enough to get them to grade-level. While some students come in needing review of just a few concepts, I have had students begin second grade who cannot count to twenty, and others who are not able to recognize numbers past ten. Often, these low-achieving students have also internalized negative beliefs and attitudes about math that contribute to their difficulty. I have seen students enter second grade already convinced that they will never be good at math, while others have displayed high levels of anxiety and frustration around the subject.

Though students come in with a wide range of abilities, my goal is the same for all of them: that they leave second grade at or above grade-level. Because of the urgency of this task and the extensive amount of content that must be taught in one school year,

teachers at these schools often use an "I Do, We Do, You Do" approach to teaching math. In this approach, teachers decide on a strategy or procedure that students will use to solve a particular type of problem. They introduce the procedure by explicitly modeling it before guiding students through it together. Students are then released to execute the procedure on their own while receiving rapid feedback from the teacher. Though this approach does not prioritize conceptual understanding, it is relied on so commonly because of the speed at which it allows teachers to move through material. While I believe deeply in the importance of building conceptual understanding and engaging students in discourse about mathematical concepts, I have found it hard to completely abandon this explicit approach to teaching math, quite simply because I do not know if I could get through everything I need to teach without it.

Experiencing the Tension

During my first year of teaching, I immediately felt the pressure to cover almost two years of material in just ten short months with my students. I focused on fostering an active discourse environment in which students discussed concepts and solution strategies with each other and assessed students' conceptual understanding in lessons when I had time, but often found myself sacrificing those elements of my instruction because of the urgency I felt to catch my students up to grade level. However, as I learned about CGI and other constructivist approaches to teaching math in professional development sessions and my graduate coursework, I began to wonder if my urgency and speed were actually doing my students a disservice in the long run. By going so quickly and sometimes sacrificing conceptual understanding for procedural fluency, was I setting them up for massive confusion and conceptual breakdown in the future? I thought about

this question as I sat in a graduate class one night during my first year of teaching and learned about the importance of building conceptual understanding from an early age. As we watched five and six year-old children solve complex story problems while teachers listened to, interpreted, and then utilized their thinking to drive instruction, we saw how effectively this type of teaching could be used to build deep conceptual understanding of foundational skills. In addition to building the conceptual understanding that all teachers want their students to have, CGI-based instruction models have been found to teach flexible thinking, problem-solving skills, and build engagement and joy in elementary students (Jacobs & Ambrose, 2008). Like I frequently did when I left this class, I felt a renewed commitment to focusing on encouraging student-led discourse and building conceptual understanding with my students the next day. But when I got to school, I was reminded of the tension that I felt. How could shift my focus to bigger conceptual understandings when several of my students still needed to master adding and subtracting within ten? How could this approach allow me to teach remedial kindergarten skills like identifying numbers while also introducing second grade content like counting coins and making change? As much as I wanted to devote more time to CGI-based instruction and transfer its practices to the rest of my instruction, I struggled to find balance in the tension.

The next year, I became the math planner for the second grade team at my school. As I planned lessons for our students, I attempted to incorporate more principles of CGI into our approach. Instead of deciding on one procedure or strategy that we expected students to use, lessons included discourse about possible strategies and allowed students to come up with their own solutions to new problems. I planned more CGI-based

problem-solving lessons and set out to do them three or four times per week, with ways to differentiate so the work would be accessible to all of our students. Our leadership team was eagerly on board with these adjustments, and we began the year ready to make these constructivist elements a priority. But yet again, tensions arose. Because so many of my students were below grade-level, time was extremely important. We had a tight schedule to keep, which became tighter when we received beginning of the year assessment data and identified the kindergarten and first grade skills that we would need to address before moving on to second grade content. And even though everything was differentiated to meet the needs of our wide range of learners, certain grade-level concepts were still inaccessible to the majority of students. As the year went on, we slowly abandoned some of the CGI-based practices in exchange for more time spent explicitly teaching the more basic skills that our students had not yet mastered, like skip counting, identifying coins, and counting to one hundred. By the spring, I had significantly lessened my focus on using story problem lessons as a way to respond to students' thinking and instead frequently reverted back to the speed and efficiency of teacher-centered direct instruction. We did not always have time to discuss different strategy choices in lessons, and I occasionally taught students procedures and strategies before eliciting their own thinking. But, as we took mid- and end of year-assessments, we were pleased with our results. Students were growing, and they were getting closer to (and some far surpassing) grade-level expectations. But still, I wondered if we were celebrating too early. What were our students not getting from our math instruction that they needed? When would their lack of conceptual understanding become apparent? Though my teaching did not always reflect it, I still believed deeply in the importance of

using CGI and other constructivist approaches to teaching math, and I continued to wonder if these strategies would be effective when working with students who enter second grade significantly behind grade level.

Personal Educational History and Beliefs

In addition to being motivated by questions that have come of my own teaching experiences, my desire to study this topic also comes from my own educational experiences. As a child and adolescent, I had the enormous fortune of having exceptional math teachers who used constructivist teaching practices to prioritize and build conceptual understanding. I was able to build a deep conceptual understanding of mathematical concepts in elementary school and beyond, which cultivated a love of math and the flexibility and confidence to continue learning and solving problems into adulthood. As a teacher, though, I often hear parents express frustration that they were "never good at math," or that they "just never liked math very much as a child." I believe that my early experiences of math were foundational to my enjoyment of the subject, and that all students can have a strong relationship with math if they are taught in a way that builds understanding and allows them to feel successful in different ways from a young age.

As a teacher, I believe that it is my responsibility to create this confidence, joy, and engagement in my students. As a teacher at a school that serves primarily low-income students of color who often already experience the achievement gap that prevents far too many low-income students and students of color from receiving the education that they deserve, this responsibility is even more important. In my experience, charter schools provide many incredible services to their students and work to close the

achievement gap in powerful ways. However, we can always learn from other successful educators and approaches. While I currently have not found a way to consistently integrate constructivist approaches into my math instruction, I believe that their proven success makes researching their effectiveness with students who are below grade-level an urgent priority. Additionally, because these practices draw upon students' life experiences and allow them to be sources of knowledge and information in the classroom, they help provide the culturally responsive education that students of color deserve.

Impact on Stakeholders

The answer to the question, *What is the effect of Cognitively Guided Instruction practices on the math beliefs and abilities of below grade-level second grade students?* has significant importance for educators, students, their families, and educational policy makers alike.

Better understanding the effect that CGI practices have on students who are below grade-level has the potential to fundamentally shift the way that educators and administrators think about teaching math to these students. It will help relieve the tension between the speed and urgency with which I currently teach and the desire to focus on the conceptual understanding and rigorous discourse that I know are so foundational to high math achievement (Carpenter, Fennema, Franke, Levi, & Empson, 2015). In short, this research will help educators identify and use the most effective strategies for teaching elementary students who are below grade-level in math, and will better allow educators to catch these students earlier in their educational careers and send fewer below grade-level students to the next grades.

As it affects educators, this research will also have a powerful impact on students and their families. Research has shown that using constructivist approaches like CGI helps engage students in mathematics and builds feelings of enjoyment and accomplishment as students grapple with and solve complex problems (Jacobs & Ambrose, 2008). The students that I teach are coming into second grade below grade-level, and many have accepted that they simply will never be good at math. After just two years of elementary school they are already missing key understandings, skills, and mindsets, and they deserve better. Better understanding how to best teach them will give them a mathematics education that will set them up for success and confidence in elementary school and beyond.

Finally, policy makers will greatly benefit from better understanding the impact of using constructivist approaches with students who are below grade-level. Having a clearer idea about how to best instruct these students will lead to more effective curricula, teacher training programs, and models of instruction. It will allow students to enter high school and college better prepared in the field of mathematics and with a stronger conceptual foundation than many currently have. And most importantly, it will make education more equitable for all students. All students deserve access to the best teaching approaches, and I hope that this research will help clarify what those approaches are.

Conclusion

While CGI instructional practices have been found to be quite successful at building conceptual understanding, flexible thinking, and engagement in elementary students, these approaches present challenges when students are significantly below grade-level. In this introduction, I have presented my research question, which attempts

to better understand this tension: *What is the effect of Cognitively Guided Instruction practices on the math beliefs and abilities of below grade-level second grade students?* I have described my personal and professional motivation for exploring this question, as well as the beliefs that are driving my research. I have also explained the significance of this research on educators, students, and policy makers. My capstone will involve a study of below grade-level students' achievement after participating in Cognitively Guided Instruction-based classroom practices, as well as surveys about their beliefs and perceptions about math before and after this instruction. In Chapter Two, I will explore literature on Cognitively Guided Instruction and other constructivist approaches to teaching math, their principles and benefits, and the mechanisms by which they work. I will examine currently used approaches to teaching students who are below grade-level, and will look at the specific challenges that teachers face when working with these students.

CHAPTER 2

Literature Review

Introduction

In 2003, 23% of fourth graders and 32% of eighth graders in the United States

performed below grade-level on standardized math assessments (Witzel & Riccomini,

2007). In an effort to correct this massive problem, educators have turned to Cognitively

Guided Instruction (CGI) as a way to improve students' conceptual understanding and

math abilities. Because it allows teachers to guide students to build strong conceptual

understanding of key concepts, many educators believe that CGI can develop stronger

mathematicians and build better enjoyment of math. However, there is a perception that

CGI has been used primarily with students who are performing on grade-level, and less

frequently with students who are not meeting grade-level standards. The research

question, *What is the effect of Cognitively Guided Instruction practices on the math*

beliefs and abilities of below grade-level second grade students?, requires a thorough

understanding of constructivist teaching practices (CGI) and the different components of

students' mathematical experiences that they influence. This literature review focuses on

four important themes that are central to the research question: Cognitively Guided

Instruction (CGI), conceptual understanding in mathematics, mathematics ability, and

mathematics beliefs. In doing so, it will explain CGI and its key components as an instructional approach, and will discuss the major areas that CGI aims to address. Examination of these themes will highlight ways in which CGI positively impacts students' math abilities and beliefs, as well as ways in which it can be supplemented to better meet the needs of all students.

Cognitively Guided Instruction

  Overview. Cognitively Guided Instruction (CGI) is a constructivist approach to teaching math that uses problem solving and rich discourse to build conceptual understanding of fundamental mathematical ideas (Carpenter, Fennema, & Franke, 1996). CGI was initially developed in the late 1980s by Thomas Carpenter and Elizabeth Fennema, who noticed that, instead of entering school with little mathematical understanding as educators thought, students in fact begin elementary school with a "great deal of informal or intuitive knowledge of mathematics" that can serve as the foundation of their mathematical understanding (Carpenter et al., 1999, p. 4). Carpenter and Fennema (1999) argued that, instead of being taught traditional algorithms, formulas, and procedures for fundamental mathematical concepts, students can use this implicit understanding to find multiple solutions to complex problems. As students use their intuitive knowledge of mathematical concepts to solve story problems and are cognitively guided by teachers, they discover "big ideas" and key properties of math, and learn how to think, reason, and dialogue mathematically (Carpenter, Franke, & Levi, 2003). If students' informal understandings are accessed and developed properly in a CGI classroom, children develop key conceptual understandings and grade-level skills without the need for explicit procedural or formulaic instruction.

CGI is rooted in the belief that elementary-aged children are naturally curious, and that students have the capacity to use this curiosity and inquisitiveness to find solutions to complex story problems, even without the traditional, formal instruction that schools often provide (Carpenter et al., 1996; Carpenter et al., 1999). As Carpenter and colleagues (1999) initially studied elementary students' mathematical learning and achievement, they discovered that the ways in which children naturally think and reason about math did not align with the math instruction that they were receiving in school. They found that adults' mathematical reasoning is often entirely different from the way that young children think about math, yet formal math instruction for students was commonly based on teachers' understandings and thinking patterns instead of those of elementary children (Carpenter et al., 2015; 1999). In an attempt to correct this discrepancy, the researchers continued to study children's thinking and created an instructional approach that supports and extends development of their intuitive mathematical thinking and reasoning. They found that, as teachers learned to better understand the ways in which children's mathematical thinking develops, their teaching fundamentally shifted in ways that were reflected in students' learning (Carpenter et al., 2015, p. 200). As Franke and Kazemi (2001) explained, CGI brings together research on how children's mathematical thinking develops and research on teaching to enable teachers to offer the most effective form of instruction. CGI, then, is not a prescription or recipe for instruction, but rather a "philosophy, a way of thinking about the teaching and learning of mathematics" (Franke & Kazemi, 2001, p. 103).

Instructional components. As an instructional practice, CGI focuses on using story problems and rigorous questioning as instructional techniques to extend and

promote deep conceptual understanding in students. In a typical CGI classroom, students are presented rigorous but contextualized story problems. They are given the freedom to solve the problems using whatever strategy they select, and engage in rich discourse with their instructor and their peers about the problem and their strategy choice (Carpenter, et al., 1996).

The use of story problems is central to this approach because story problems allow students to make meaning of mathematical concepts and apply their existing understandings in new situations. Jacobs and Ambrose (2008) examined the ways in which teachers effectively use story problems to guide instruction. They found that using story problems in instruction allows math to be meaningful to students, as they work to solve authentic and relevant problems. When problems are meaningful, students are better able to make sense of the story and apply their existing mathematical understandings to solve. They also found that story problems build engagement and enjoyment in elementary students, allow teachers to identify and address misconceptions as they arise, and encourage students to "construct strategies that make sense to them rather than parrot strategies they do not understand" (Jacobs & Ambrose, 2008, p. 260). As students wrestle with conceptually rigorous problems, they invent and discover different mathematical strategies that provide information about key concepts including addition, subtraction, multiplication, and division. They then have the opportunity to share these strategies with their peers, learn from classmates' strategies, and continue to develop more efficient and sophisticated strategies as they progress (Carpenter et al., 1999).

Though most instruction in CGI occurs around the story problem, teachers who are well versed in CGI can adjust and manipulate story problems to elicit a variety of strategies and conceptual understandings. In their beginning work with teachers implementing CGI-based approaches, Carpenter and colleagues (1999) found that both the structure of the story problem and the magnitude of the numbers involved influence the strategies that students produce and use to solve. Thus, when careful decisions about problem type and magnitude are made, CGI-based approaches can be used to encourage development of various strategies and solutions. In order for this to happen, instructors must have a thorough understanding of the way that students reason mathematically and are likely to solve problems (Carpenter et al., 1999). Thus, strong content knowledge by instructors is a key component of successful implementation of CGI-based approaches.

Benefits of CGI. Since its introduction in educational spheres nearly thirty years ago, researchers have identified several benefits of using CGI as an instructional approach in the elementary school classroom (Carpenter, et al., 1999; Carpenter, et al., 2003; Franke & Kazemi, 2001; Ladson-Billings, 2000; Moscardini, 2014). Among its primary benefits is the fact that CGI-based approaches allow teachers to utilize and extend children's already existing mathematical knowledge to build deep conceptual understanding that is meaningful instead of procedural. As Carpenter and his colleagues (1999) explained, "until recently, we have not clearly recognized how much young children understand about basic number ideas, and instruction in early mathematics too often has not capitalized on their rich store of informal knowledge" (p. xiv). Students enter elementary school with a rich knowledge base about mathematics, and CGI allows teachers to access that prior knowledge and take advantage of preexisting understandings

and curiosities. Building on this knowledge allows students to make connections and come to understandings that educators previously did not expect them to make. In their work integrating arithmetic and algebraic thinking into elementary school math, Carpenter and colleagues (2003) saw "glimpse[s] of the profound mathematical thinking of which ordinary children are able" when they are encouraged to build on their preexisting understandings (p. v). Because it elicits this powerful reasoning, research by the National Council of Teachers of Mathematics (2000) and the National Research Council (2001) has found that math instruction that builds on children's thinking and existing knowledge, like CGI, produces rich instructional environments and leads to gains in student achievement.

Research has also shown that the conceptual understanding that CGI-based instruction produces builds a strong foundation for elementary students to be successful in later grades. Carpenter and colleagues (2003) found that the conceptual understanding that children build in elementary school "provides children with a solid basis for extending their knowledge of arithmetic to learn algebra" (p. xi). CGI-based instruction contributes to this future success in a few ways. Firstly, when students have a solid understanding of fundamental concepts and operations, they are prepared to be successful as they learn more rigorous skills and concepts (Carpenter et al., 1999). Additionally, as Carpenter and colleagues (2003) explained, "students who learn to articulate and justify their own mathematical ideas, reason through their own and others' mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics" (p. 6). The kind of thinking that CGI-

based instruction develops serves students well in later grades, and contributes to greater mathematical success that extends beyond elementary school.

Another benefit that researchers have identified is that CGI encourages the development of deep critical thinking and flexible thinking in students. Using alternative approaches to problem solving and prompting students to come up with their own solutions instead of formally introducing algorithms and procedures builds flexible thinking in students (Jacobs & Ambrose, 2008). In a study on the effectiveness of CGI-based instruction in classrooms in Scotland, Moscardini (2014) found that CGI benefitted students because it gave them opportunities to "lead in their learning as opposed to being the passive recipients of knowledge" (p. 74). As students create their own approaches to solving problems and hear their peers share their own strategies, students expand their thinking and practice thinking critically about problems and solutions. This critical thinking that develops as a result of CGI-based instruction has also been shown to spread to other subject areas, benefitting students holistically (Ladson-Billings, 2000). As Ladson-Billings (2000) shared at the 1999 CGI Institute for Teachers, "the thinking that students develop in a CGI classroom is not likely to be constrained to mathematics" (p. 8). The benefits of CGI benefit students mathematically and beyond.

In addition to benefitting students, implementing CGI-based instruction has also been shown to benefit educators. Research on teachers who implement CGI in their classrooms has found that shifting towards this model of instruction contributes to professional growth in educators. Franke and Kazemi (2001) tracked the development of teachers trained in CGI for four years following the initial professional development. They found that teachers who used CGI-based teaching practices in their classrooms were

better able to analyze and respond to students' thinking, and that many of the teachers they followed experienced significant professional growth (p. 105). Moscardini's (2014) work with teachers in Scotland supports this finding, as teachers were found to gain deeper insight into their students' mathematical understandings after implementing CGI in their classrooms. CGI, then, has been found to benefit both students and teachers alike in many powerful ways.

Criticisms of CGI. While research has illustrated several benefits of using CGI-based instruction in the elementary classroom, criticisms exist as well. Though its supporters do not often consider this to be a criticism, there is widespread agreement among researchers that executing CGI effectively requires what Carpenter and colleagues (2003) referred to as "a complex work of teaching" (p. v). In order to effectively implement CGI-based instruction, teachers must have a strong understanding of children's thinking, be able to detect this thinking quickly, and know how to respond to misconceptions and requests for support in the moment (Jacobs & Philipp, 2010). Because it depends on higher-rigor teaching moves, executing CGI-based instruction well requires more professional development for educators (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). This professional development requires an investment of time and resources, and often includes changes to already existing curricula, which some educators find threatening (Ladson-Billings, 2000).

Another criticism of using CGI-based instruction in elementary classrooms is the lack of predictability that it produces. As Ladson-Billings (2000) discussed, using CGI makes instruction less predictable than it is in traditional approaches (p. 8). Many schools expect daily objectives, reports on weekly mastery of those objectives, and predictable

routines and assessments. CGI-based instruction does not fit well with these expectations, and teachers who use CGI must leave flexibility in their lessons and preparation to allow for students thinking to take the group in a variety of different directions. This unpredictability may be uncomfortable to educators who are used to tight structures and routine, though it is not argued that CGI-based instruction has a negative impact on student learning.

CGI-based instruction focuses on students' mathematical thinking in order to guide their understanding of new concepts, and has been shown to benefit students and educators alike, though it requires more teacher content knowledge and comfort with unpredictability in instruction. These findings suggest that CGI-based instruction can be used to improve all students' math abilities, including those who are below grade-level, when done carefully and by well-trained teachers. By understanding students' thinking through CGI, teachers are able to better support the development of conceptual understanding in students, which will be discussed in greater detail in the following section. This research suggests that the use of CGI-based instruction in my study will positively influence students' mathematical abilities, even though they are below grade-level, as long as it focuses on building conceptual understanding. It also highlights the importance of teachers who work with below grade-level students being extremely knowledgeable about the content they teach and able to understand students' thinking, respond to misconceptions, and guide their students to deep understandings. Thus, it will be important that the CGI-based lessons used in the study include a large focus on conceptual understanding and that I am well-equipped to guide my students to conceptual growth in order to maximize the benefits of CGI-based instruction.

Conceptual Understanding

In order to fully understand CGI as an instructional practice, it is important to know what conceptual mathematical understanding is, why it is a key component of CGI-based instruction, and how it can be developed in the elementary math classroom.

Overview. According to the National Research Council, conceptual understanding and procedural fluency are the two major components of mathematical proficiency (as cited in Kilpatrick, Swafford, & Finnell, 2001). Procedural fluency is defined as "the understanding of the rules and steps to be able to solve a mathematics problem," while conceptual understanding is "an understanding of the underlying relationships for why the procedure works" (Kanive, Nelson, Burns, & Ysseldyke, 2014, p. 83). In other words, procedural fluency is the "how," or the ability to solve a problem. Procedural fluency involves the ability to successfully carry out an algorithm or execute a procedure; a student who can successfully carry out the standard American algorithms for addition and subtraction (what many adults remember as "borrowing" and "carrying") show procedural fluency. However, conceptual understanding is the "why," or the ability to understand why a procedure or algorithm works and what is being done when numbers are manipulated (Kanive et al., 2014). A student who shows strong conceptual understanding would be able to explain with understanding what is happening when they borrow or carry and how the numbers are being manipulated. As Baroody & Benson (2001) explained, "many people view teaching as telling or showing children something they need to know, then having them imitate and practice it" (p. 156). This instructional practice, where students are shown a procedure or skill and then asked to repeatedly practice it until they achieve independence, is a procedural approach to teaching math.

Students learn *how* to do something, but not *why* they are doing it or even what exactly it is that they are doing. However, as Carpenter and colleagues (2003) argued, learning mathematics involves "learning powerful mathematical ideas rather than a collection of disconnected procedures for carrying out calculations" (p. 1). Thus, while procedural fluency is important, it does not independently lead to proficiency; conceptual understanding is also needed to achieve true mathematical proficiency (Carpenter et al., 2003).

Research has shown that conceptual understanding begins to develop even before formal math instruction begins. In fact, Baroody and Benson (2001) found that students begin to develop conceptual understanding as early as the preschool years. Conceptual understanding includes a variety of different understandings and conceptions, including understanding of number (often referred to as "number sense"), the ability to understand different operations and number manipulations, and relational thinking (Baroody & Benson, 2001; Carpenter et al., 1996; Jacobs et al., 2007). Relational thinking, a key component of conceptual understanding, involves "looking at expressions and equations in their entirety, noticing number relations among and within these expressions and equations," and using those relations to solve problems (Jacobs et al., 2007, p. 260). Instead of simply carrying out a procedure or algorithm, relational thinking uses fundamental properties and conceptual understandings to solve algebraic problems (Jacobs et al., 2007). As such, it is a key marker of a student with strong conceptual understanding.

Importance of conceptual understanding. Conceptual understanding has been found to be important in elementary mathematics instruction for a variety of reasons. At

the early stages of math instruction, conceptual understanding of number is the foundation for future mathematical instruction (Baroody & Benson, 2001). In their initial studies on CGI in 1989, Carpenter and colleagues found that students in CGI classes scored just as well as control classes on a test of number skills, even though the CGI classes placed much less emphasis on number skills than the control classes did (Carpenter et al., 1999). Though these students did not practice fact fluency like those in control classes did, they were able to recall number facts just as well as (and in some cases, better than) students in the control class (Carpenter et al., 1999, p. 109). This finding suggests that the conceptual understanding that these students developed in their CGI classes provided them with an important foundation from which they could then perform other key mathematical operations and tasks.

Conceptual understanding in mathematics is also important because it has often been found to be lead to greater overall mathematical achievement. As Fyfe and colleagues (2014) explained, conceptual instruction and understanding "is thought to support key learning processes including knowledge integration and procedure generation" (p. 504). That is, students who receive conceptual instruction and develop strong conceptual understanding are better able to learn new concepts and apply those concepts to solve problems. Conceptual understanding has also been found to aid in problem solving and the generation of accurate problem-solving procedures (Fyfe et al., 2014), another way in which it leads to greater mathematical achievement.

Additionally, building conceptual understanding has been found to be an effective way of helping students who struggle mathematically. Burns (2011) found that while interventions that focus on building fact fluency improve students' performance on a

variety of different math problems, they have more of an impact on students who have basic conceptual understanding of underlying concepts than on students who do not have this conceptual understanding. Further, studies have shown that interventions that focus on building conceptual understanding are more effective than those that address and reinforce procedural fluency. Kanive and colleagues (2014) argued that "interventions that target students' conceptual understanding have been shown to be effective in correcting students' misconceptions of fundamental mathematical principles and in establishing an understanding of underlying mathematics concepts for problem-solving" (p. 83). Because they are able to address misconceptions and build strong conceptual foundations, these interventions are more effective than those that simply target procedural fluency.

Building conceptual understanding. Educators involved in CGI and beyond have spent a considerable amount of time studying how teachers can best promote the development of conceptual understanding in mathematics students. A common strategy for developing conceptual understanding is altering the traditional order of instruction. Traditionally, educators introduce and model a procedure or problem-solving approach, and then students use the procedure to solve problems. However, when the order of instruction is changed, students are allowed to work with problems on their own before being instructed about a particular strategy or procedure (Fyfe et al., 2014). As Fyfe and colleagues (2014) found, "when instruction includes procedures, it may be best to delay instruction to give learners a chance to generate procedures on their own" (p. 503). This switch allows students the opportunity to create their own solution strategies and come to their own understandings about underlying concepts, and opportunity that they do not

always have when their task is to simply regurgitate a procedure that they have been taught. When they create these authentic procedures and understandings, students are able to carry out procedures and solve problems with understanding (Carpenter et al., 1999). This change in the order of instruction can be done in the early stages of math instruction and exploration by creating opportunities for students to explore new problems independent of explicit instruction. Teachers can also create this effect by allowing students to create their own strategies to solve problems while teachers interact and converse with them as they work through solutions (Baroody & Benson, 2001, p. 156). Teachers can also change the order of instruction when introducing basic addition, subtraction, multiplication, and division facts. When this is done, students are able to develop their own derived strategies and can expose what Brickwedde (2012) referred to as "key algebraic properties of number operations" (p. 1), including the distributive, associative, and commutative properties, that can aid future development of more advanced algebraic understandings.

However, despite the success that has been found by changing the order of instruction, Fyfe and colleagues (2014) found that the effectiveness of this approach depends on the type of content that is being taught. They found that children who were asked to solve equality-concept problems, which depend on students understanding that the equals sign indicates equality in quantity on both sides of an equation rather than a "get the answer" sign, were more successful when they had received instruction about the concept prior to being asked to solve problems than when they were allowed time to work independently and develop their own approaches and strategies (Fyfe et al., 2014). This finding suggests that instructors must carefully select the concepts for which they

choose to use this kind of instruction. Simply put, the type of instruction matters. Specifically, Fyfe and colleagues (2014) found that when instruction involves both procedures and concepts (as word problems do), allowing students to work by themselves and develop their own approaches first is a better approach than beginning by offering explicit instruction because it allows students to build on and strengthen their existing understandings. However, when the instructional point is solely conceptual, like it is with equivalency problems, providing conceptual instruction first seems to be beneficial, because it guides students in their problem solving when concepts are unfamiliar (Fyfe et al., 2014). Thus, while allowing students to grapple with problems and discover their own strategies and solutions can be an effective way to build conceptual understanding, teachers must carefully select the problems that they choose for this approach.

Educators can also develop conceptual math understanding through teacher questioning. Research has found that effective teacher questioning elicits and demands a particular kind of dialogue that builds conceptual understanding in students, shifting the balance from more "teacher talk" to more student discourse (Franke et al., 2009). Franke and colleagues (2009) described this effective style of teacher questioning as "a probing sequence of specific questions" (p. 390), which often leads students to complete and accurate explanations of their thinking. They found that teachers who use a rigorous and extensive form of questioning that involves multiple follow-up questions elicit more descriptive and elaborative thinking than teachers who do not use questioning or who ask a few simple questions (Franke et al., 2009). Good questioning pushes students to verbalize their conceptual understanding and expand their thinking. As Franke and colleagues (2009) described, "beyond providing answers, students must describe how

they solve problems and why they propose certain strategies and approaches" (p. 381). In order to be most effective, students also "must be precise and explicit in their talk, especially providing enough detail and making referents clear so that the teacher and fellow classmates can understand their ideas" (Franke et al. 2009, p. 381).

This form of teacher questioning has been found to benefit all students in the class and "lead to increased student mathematical knowledge and understanding" (Franke et al., 2009, p. 381), because it allows teachers to better understand students' thinking, students to solidify and correct their own thinking, and lets other students connect what is being discussed with their own understandings. This discourse can serve to either strengthen existing understandings or correct existing misunderstandings (Franke et al., 2009). It also provides opportunities for students to extend their ongoing understandings to build towards more complex concepts and understandings (Jacobs et al., 2007). In addition to helping teachers monitor their students' thinking, it encourages students to help each other build more comprehensive mathematical understandings by sharing ideas and strategies. And, the actual act of talking about math has also been found to help students deepen their conceptual understanding (Franke et al., 2009).

In addition to using questioning to further develop the understanding of new concepts, discussing misconceptions has been found to build conceptual understanding in elementary students. Fyfe and colleagues (2014) found that the "activation of misconceptions" (p. 515), in which teachers recognize a misconception in a student's work and engage in dialogue about it, is an important component of strengthening conceptual understanding. When a teacher notices a misconception and asks the child to verbalize it, the misconception can be understood and corrected at the conceptual level.

In conjunction with strong teacher questioning, activating and correcting misconceptions can develop strong conceptual understanding in elementary students (Franke et al. 2009). Building strong conceptual understanding in elementary mathematics students leads to higher levels of achievement in elementary school and beyond. This section of literature suggests that CGI can be effective in improving even low-achieving students' mathematical achievement because of its focus on building conceptual understanding. My research will attempt to build conceptual understanding and confidence in order to lead below grade-level students to higher achievement. Higher achievement is measured in students' mathematical abilities, a theme which will be discussed in greater detail in the following section.

Math Abilities

In order to study how CGI affects students' math abilities, it is important to fully understand the different components of students' mathematical abilities, how are developed, and the factors that lead to low mathematical ability in elementary aged children.

Overview. Mathematical ability is an overall measure of students' procedural fluency and conceptual understanding (Kilpatrick et al., 2001). Students can be identified has having high, average, or low math ability, and those with low math ability can be students with learning disabilities or those who simply struggle in math for other reasons. These students, who struggle with math ability for reasons other than a learning disability, are referred to as low-achieving or low-attaining mathematics students (Moscardini, 2010). Math ability has been a growing concern in the United States in recent years, and it is not just students with learning disabilities who are struggling

(Witzel & Riccomini, 2007). The 2000 National Assessment of Educational Progress (NAEP) found that only 2% of students in the United States were able to attain advanced levels of math achievement by the twelfth grade (Witzel & Riccomini, 2007). Additionally, large numbers of students in the United States are continuing to score below basic levels of proficiency in math. In 2003, 23% of fourth graders and 32% of eighth graders were below proficiency in the United States (Witzel & Riccomini, 2007). Though these numbers are staggering, as Pool and colleagues (2012) found, it is important to identify and address the needs of these low-attaining students as early as possible in the elementary years, because "students who fail to develop proficiency and automaticity and computational skills…and problem solving in the primary grades are more likely to experience difficulties in math curriculum later (p. 211). Though many students struggle with math ability in the United States, bringing these students to proficiency is a growing priority in math instruction (Pool, Carter, Johnson, & Carter, 2012).

  Challenges. A variety of factors have been found to prevent students from developing high levels of mathematical achievement in elementary school. Research has shown that one of the most limiting factors in the development of students' math ability is their language abilities and reading skills. While many students with low reading ability also struggle with low math ability, Vista (2013) found that reading comprehension ability, or a student's ability to understand what they are reading, mediates the relationship between reading ability and math growth. Similarly, Vilenius-Tuohimma and colleagues (2008) found that, even when gender and parental education are controlled for, there is a strong relationship between students' reading and problem

solving abilities. This finding suggests that a major factor in elementary students' ability to perform at grade-level in math is their ability to read with understanding, with students who struggle to read having a much more difficult time achieving high levels of math ability (Vista, 2013).

Similarly, students' language capabilities have been found to powerfully affect their math performance. In a test of the relationship between working memory and math skills, Wilson and Swanson (2001) found that verbal skills significantly predict students' math ability. Students with better verbal skills were more likely to have higher math achievement, and students with lower verbal skills struggled more (Wilson & Swanson, 2001). This is especially important for students who are learning English as a second language. For these students, language difficulties have been found to hinder math performance (Orosco, 2014). As Orosco (2014) found, learning math in a new language is more "arduous" because of a variety of factors (p. 45). Students who are learning English for the first time have limited vocabulary development and do not have prior math content knowledge in English. They also struggle to solve word problems and comprehend what is being asked of them in a new language (Orosco, 2014). These challenges often lead to low math performance in students who are learning English as a second language (Orosco, 2014).

Another limiting factor in the development of students' mathematical ability is the lack of particular foundational skills that many elementary students have. Research has found that elementary students are often limited in their ability to develop proficiency in math by their understanding of number concepts (Kamii & Rummelsburg, 2008). In her work with first graders at a Title-I school, Kamii and Rummelsburg (2008) found that

many first grade students had little or no understanding of number concepts, which are foundational to mathematical success in elementary school and beyond. Using simple assessments, she found that students were not able to conserve number and did not have what she refers to as "a strong cognitive foundation for number" (Kamii & Rummelsburg, 2008, p. 389).

Additionally, research has found that many elementary students are held back in their math achievement by a lack of visual-spatial skills. In their test of the relationship between working memory and math skills, Wilson and Swanson (2001) found that visual-spatial measures in working memory tests significantly predicted students' math ability. Students with greater performance on visual-spatial measures were more likely to show high math achievement, while lower performance on visual-spatial tasks was correlated with low math achievement (Wilson & Swanson, 2001). More specifically, van Garderen (2006) found significant positive correlations between spatial-visualization measures and a student's ability to solve word problems. Students with lower spatial-visual skills were found to use less sophisticated types of imagery when solving problems, which likely limits performance by interfering with students' ability to understand, represent, and solve word problems. This finding suggests that the lack of visual-spatial and visualization skills limits students' math achievement by impeding their ability to mentally represent and understand and subsequently solve math problems.

In addition to being held back by specific skills, students' math abilities can be limited by their negative beliefs about math. This is especially true for low-performing students, who are more likely to hold negative attitudes towards math (Phillips, Leonard, Horton, Wright, & Stafford, 2003). As Phillips and colleagues (2003) explained, "low-

attaining students begin to develop strong negative attitudes towards school and mathematics" (p. 107). Additionally, math anxiety has been found to have a powerful effect on students' achievement (Jameson, 2014; Ramirez et al., 2016). These negative attitudes contribute to lower math achievement in a variety of ways, which will be discussed in much greater detail in the last section of this chapter.

Finally, low-attaining students' mathematical abilities are also frequently affected by behavioral challenges and social skills. Pool and colleagues (2012) found that difficulties with motivation, attention, and self-regulation might play an important role in students' academic achievement. Interventions that addressed motivation, behavior, and self-regulation in struggling students were found to be successful at improving third graders' math performance (Pool et al., 2012). This finding suggests that social and behavioral challenges are another factor that can limit the development of mathematical ability in elementary students.

Strategies to improve math ability. Though there are many factors contributing to low math achievement in elementary students, research has identified a variety of instructional tools and strategies that can be used to improve students' math abilities. Specifically, research shows that interventions and targeted instructional approaches can be effective ways to improve students' mathematical performance (Moscardini, 2010). An effective way to improve low-attaining students' math abilities is to strengthen their foundational skills (Kamii & Rummelsburg, 2008). After discovering that many first graders' math achievement was held back by their lack of number concept, Kamii and Rummelsburg (2008) found that physical knowledge activities could be used to build number concept and strengthen mathematical ability in students. Physical knowledge

activities are "those in which children act on objects physically and mentally to produce a desired effect" (Kamii & Rummelsburg, 2008, p. 390). Kamii and Rummelsburg (2008) referred to these activities as "constructivist activities," which included physical manipulation of number to build number sense in students. Students who engaged in these constructivist activities scored significantly higher than those who did not on an end of year assessment (Kamii & Rummelsburg, 2008). They "quickly strengthened their foundation for number concepts" and built a "good cognitive foundation" for learning arithmetic later in the year (Kamii & Rummelsburg, 2008, p. 394). Research has also found that students' foundational skills can be improved by teaching strategy instruction for approaching word problems (Orosco, Swanson, O'Connor, & Lussier, 2011). This strategy instruction can involve practicing the academic language that is required to solve word problems and teaching students how to apply their contextual experiences to solve new problems (Orosco et al., 2011).

The use of CGI and similar problem solving approaches has also been identified as a way to improve students' mathematical abilities. Historically, educators have thought that direct instruction is the most effective way to teach struggling students and students with learning disabilities, but more recent studies have shown that low-achieving students and those with learning disabilities can benefit from CGI (Moscardini, 2010). As Moscardini (2010) explained, even lower-attaining students and students with learning disabilities can "invent, transfer, and retain strategies for solving arithmetical problems" when CGI is used as an instructional approach (p. 130). Further research supports this claim, finding that even mathematically low-performing students are able to effectively invent strategies for solving word problems and apply those strategies to accurately to

find solutions when CGI and other constructivist approaches are used, as long as teachers are effectively trained in how to use these approaches and believe in students' abilities to perform (Hankes, 1996; Jacobs et al., 2007; Moscardini, 2010). Though Moscardini (2010) found that teachers initially had concerns that this type of constructivist learning would not work for their low-attaining students with learning disabilities, after using CGI, "no teacher expressed concerns about the suitability of CGI for any of the children they were working with" (p. 134). In fact, teachers reported that many of their students exceeded their expectations in their problem-solving abilities (Moscardini, 2010).

One of the primary benefits of using CGI with low-attaining students is that it allows them to create and use strategies that hold meaning and can be done with understanding, instead of forcing them to use algorithms, which can be harmful when students attempt to use them without understanding (Kamii & Dominick, 1998). Instead of being taught a procedure for adding double-digit numbers, for example, students are presented a problem and given the freedom to solve it in a way that makes sense to them, while their teacher guides their thinking through questioning and discourse. Another benefit of CGI and other instructional techniques that involve problem solving is that they require students to share explanations of their thinking and collaborate with each other, which have been found to predict higher mathematical achievement in elementary students (Webb et al., 2008). When students explain their thinking and work collaboratively, they have opportunities to clarify misconceptions, internalize new understandings, and strengthen the connections between new learning and their previous mathematical understanding (Webb et al., 2008). Additionally, collaboration and having students share explanations of their thinking has been found to help students who are

learning English as a second language participate in math instruction (Maldonado, Turner, Dominquez, & Empson, 2009).

CGI can also help low-attaining students, especially those who are learning English as a second language, because the nature of word problems allows more students to access the problem. Instead of being presented without any context or explanation, word problems can be told as stories, with as much elaboration and detail as is needed for students to understand (Turner, Celedón-Pattichis, Marshall, & Tennison, 2009). This prevents students from being held back by limited language proficiency. Additionally, hearing these problems as stories allows students who are just beginning to learn English to draw on their existing "funds of knowledge" and use that information to access and solve (Turner et al., 2009, p. 30). As they use their background knowledge and experiences to solve, students are also able to conceptualize and represent mathematical relationships in ways that have meaning for them, which builds understanding and leads to higher achievement (Turner et al., 2009).

Teachers can also develop stronger math abilities in students by promoting mathematical discourse in their instruction. Research has shown that there is a positive relationship between teachers who elicit more student thinking in mathematical discourse and student achievement, which suggests that sharing student thinking contributes to higher levels of math ability (Webb et al., 2008). When students share their mathematical thinking, they are encouraged to create multiple strategies and compare strategies with their peers, which provides more opportunities for students to internalize new understandings (Webb et al., 2008). It allows other students to benefit from their peers' thinking, and, even when incorrect ideas or strategies are shared, promotes the evaluation

of concepts and addresses common misconceptions (Webb et al., 2008). When teachers use what they know about students' mathematical thinking to guide discussions and students' learning, achievement improves (Webb et al., 2008).

Though many strategies that can be used for all students have been found to effectively improve the abilities of low-attaining students, research has also found that differentiation for struggling students an important part of improving achievement. Differentiation can take many forms. Christenson and Wager (2012) found that giving all students the same word problem, but allowing children to select numbers that are "just right" for them is an effective way of differentiating for lower-attaining students (p. 196). In this approach, all students solve the same word problem, but are instructed to select numbers that are neither too easy nor too difficult for them to solve. This approach allows students of all math abilities to access the problem and benefit from its instructional value without being held back by the range of numbers it uses (Christenson & Wager, 2012).

In addition to allowing students to use different number ranges when solving problems, teachers can also differentiate their instruction by allowing students to use different strategies (Christenson & Wager, 2012). Instead of prescribing a particular strategy that students must use to solve, this approach involves letting students use the strategy that makes the most sense for them at that point in their development of math ability. While using a number line might be the right strategy for one student, a less sophisticated choice like direct modeling might be the appropriate strategy for another (Christenson & Wager, 2012). Letting students use different strategies allows all students to access the problem and solve it in a way that has meaning and builds understanding (Christenson & Wager, 2012).

Teachers can also make accommodations for students who struggle with language and reading proficiency. Research has shown that language affects not only reading proficiency, but math proficiency as well, and must be addressed in order to boost mathematical achievement (Pace & Ortiz, 2015). Teachers can address language proficiency by integrating reading comprehension practice, vocabulary instruction, and problem solving strategies into math instruction for students who are learning English as a second language and those who struggle with reading proficiency (Orosco et al., 2011; Orosco, 2014; Pace & Ortiz, 2015). Research has also found that teachers can accommodate for students with limited language proficiency by reading word problems aloud multiple times (Christenson & Wager, 2012). These oral readings allow students who struggle with language or reading to understand and access the problem, preventing them from being limited by their language abilities.

In addition to differentiating for lower-attaining students, intensive data-driven interventions are also effective in raising math achievement in elementary students. In an examination of practices that support achievement of low-income students in diverse schools, Brown (2015) found that schools that produce gains in student-achievement heavily rely on data when planning instruction. This data was used to identify and plan interventions for low-attaining students in math and was found to lead to higher academic achievement (Brown, 2015). Interventions can be used for specific foundational skills, like subitizing, which hold students back in their math abilities (Warren, deVries, & Cole, 2009).

An example of this kind of intensive, data-driven instruction is the Math Recovery® program (Math Recovery: US Math Recovery Council, 2016). Math Recovery

works with students in their second year of school who are in the bottom 25% of their class (Phillips et al., 2003). Math Recovery intervention teachers create specialized and individualized intervention plans for each student, which use specific tasks to push struggling students to move toward more sophisticated problem-solving strategies (Phillips et al., 2003). In doing so, this program has been found to significantly improve elementary students' arithmetical strategies, an improvement that was sustained even after students left the program (Phillips et al., 2003).

Many elementary students in the United States struggle to achieve grade-level proficiency in mathematics for a variety of reasons. Though there are many factors limiting the ability of these low-attaining students, research has identified many strategies and approaches that can be used to improve performance, including CGI. This research informs the research question by suggesting that CGI can be an effective strategy for improving below grade-level student's math ability. However, the literature on differentiation and intervention suggests that CGI must be used carefully and in combination with other instructional techniques in order to effectively address all of the factors that limit low-attaining students' achievement (Phillips et al., 2003; Warren et al., 2009). The following section will discuss students' math beliefs and attitudes, which have also been found to powerfully affect student achievement. Understanding the impact of CGI on below grade-level students' experience of math and their math beliefs is crucial in order to meet this study's goal of supporting low-attaining students as they reach proficiency in math.

Math Beliefs

Math beliefs, including math anxiety, math self-concept, and math affect, have been found to powerfully affect student achievement in math (Jameson, 2014; Pinxten, Marsh, De Fraine, Van Den Noortgate, & Van Damme, 2014; Ramirez et al., 2016; Wigfield & Meece, 1988). In order to fully understand how to help low-attaining students improve their math ability, it is important to understand the definition of these beliefs, how they affect student performance, and how they are developed.

Math anxiety. Math anxiety is defined as the feeling of "tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551). Though typically thought of as a concern for older students and adults, neurological research shows that math anxiety is a biological reaction that can be detected in the brain of students as early as age seven (NCTM, 2013). Certain features, including its precision, make math particularly anxiety inducing for some students (Wigfield & Meece, 1988). Two primary components of math anxiety have been identified: negative affective reactions to math and worries about succeeding in math (Wigfield & Meece, 1988). Negative affective reactions include the fear, nervousness, and discomfort that students experience when thinking about math and engaging in mathematical tasks (Wigfield & Meece, 1988). Worries about succeeding in math are more cognitive experiences of concern, and have been found to decrease student enjoyment and performance (Jameson, 2014). Importantly, current research suggests that math anxiety contributes to students' poor math performance in the United States (Jameson, 2014).

Math affect and self-concept. Math affect is a measure of how much students enjoy math (Pinxten et al., 2014). Math self-concept is defined as "students' beliefs in their own domain-specific and/or global academic capabilities" (Pinxten et al., 2014, p. 153). Math low self-concept arises from negative experiences or stereotypes about one's math abilities and, like math anxiety, has been found to have a powerful, negative effect on students' mathematical achievement (Pinxten et al., 2014).

Effect of math beliefs on math ability. Research has shown that math anxiety, affect, and self-concept have a powerful impact on students' math achievement. Math anxiety has been found to decrease students' math achievement in a variety of ways (Jameson, 2014; Ramirez et al., 2016). As Ramirez and colleagues (2016) explained, "children's capability for improving their math skills is contingent on children feeling comfortable with mathematics in general" (p. 95). When children feel anxiety instead of comfort, they often experience "task impairment," which causes students to perform worse on mathematical tasks because of the anxiety they experience (Wigfield & Meece, 1988, p. 214). In a study on the neurological effects of math anxiety, Young and colleagues (2012) found that the part of a student's brain that is responsible for mathematical reasoning is less active when they experience math anxiety. This finding suggests that math anxiety actually inhibits the brain functioning that supports mathematical thinking. Consistent with this finding, Ramirez and colleagues (2016) found that students who experienced higher math anxiety used fewer of the advanced strategies that they had been taught when solving math problems. Thus, it appears that math anxiety limits students' mathematical achievement by interfering with their

mathematical reasoning and ability to use the strategies that they have been taught to solve mathematical tasks.

Math anxiety has also been found to negatively impact achievement by affecting students' motivation and effort (Wigfield & Meece, 1988). Wigfield and Meece (1988) found that students who experience high levels of math anxiety may not put as much effort into math, which negatively affects math achievement over time. Additionally, these students report that they value math less than students who do not experience math anxiety, which further leads to lower expenditure of effort and lower performance.

Although math anxiety has been found to have negative effects on development of math ability in elementary students, math self-concept and affect have a strong, positive effect on math achievement in elementary school (Pinxten et al., 2014). Students who are more confident in their mathematical abilities and who enjoy math more have higher performance in elementary school, an effect that can be leveraged to boost student achievement, especially when working with students who are performing below grade-level.

Improving math beliefs. Because of their powerful impact on student achievement, it is important to understand how math anxiety, affect, and self-concept can be improved in elementary students. One important finding is that math anxiety relates to low self-confidence in one's math ability (Jameson, 2014; Stuart, 2000). This finding suggests that, in order to relieve students of their math anxiety, teachers must work to build their self-concept (Stuart, 2000). Stuart (2000) suggested several ways in which teachers can do this. Teachers can build students' self-concept and minimize their anxiety by letting students share different strategies for solving various problems. This teaches

students that there is not one right way to solve math problems and begins to decrease the

anxiety to solve things in a particular way (Stuart, 2000). Teachers can also emphasize

the importance of mistakes and help students realize that they engage in math as part of

their daily lives (Stuart, 2000). In doing so, teachers can build positive self-concept in

students and begin to reduce the math anxiety that negatively impacts their math

achievement.

As they attempt to minimize students' experiences of math anxiety, teachers

should also work to improve students' math affect. Math affect (the extent to which they

enjoy math) has been found to positively relate to math self-concept, which decreases

anxiety and improves performance (Pinxten et al., 2014). This relationship suggests that

encouraging students to participate in math activities that they enjoy from a young age

can build high self-concept and achievement in mathematics. Pinxten and colleagues

(2014) refer to this instructional move as "making math more attractive" (p. 170). When

math is more attractive, or enjoyable, students are likely to have higher self-concept and

lower anxiety, which leads to the development of higher math ability (Pinxten et al.,

2014).

Research also suggests that teachers can improve students' math enjoyment by

allowing them to solve problems in ways that make sense to them, instead of requiring

students to use specific and prescribed strategies or algorithms to solve problems

(Buschman, 2003). As Buschman (2003) explained, "when children are given the

opportunity to solve problems 'their way,' they take great pride and pleasure in

developing their own strategies, instead of simply practicing strategies that adults have

shown them" (p. 540). Letting students solve problems in a variety of ways allows them

the satisfaction of completing a challenging but engaging task, which has been found to lead to greater enjoyment of problem solving (Buschman, 2003). Additionally, allowing students this flexibility helps them turn their fear of mistakes into an appreciation of mistakes. Buschman (2003) found that "young children want to learn from their mistakes, and their enjoyment of problem solving increases when children know that mistakes will be used as stepping stones to new learning" (p. 540). When mistakes no longer represent a failure to repeat a specific strategy, they can be used as a tool to improve and deepen understanding. As students learn to value mistakes and use their own strategies to solve problems, their math affect and self-concept improve, leading to higher achievement and proficiency.

As noted, math anxiety, affect, and self-concept can powerfully influence elementary students' math abilities. Math anxiety has been found to impede student achievement, while self-concept and affect lead to higher performance and proficiency. When working to improve performance in below grade-level students, math beliefs must be considered and addressed. More research needs to be done to measure the impact of CGI on students' math beliefs, and determine if CGI is an effective way to raise student achievement by improving math anxiety, affect, and self-concept. My research will attempt to address this question in order to better understand the effect of CGI on low-attaining students' math performance.

Conclusion

This chapter has focused on several themes that inform the research question, *What is the effect of Cognitively Guided Instruction practices on the math beliefs and abilities of below grade-level second grade students?* It has explored the themes of

Cognitively Guided Instruction (CGI), conceptual understanding, math abilities, and math beliefs in order to better understand the effect of CGI on students' math experience. The body of literature suggests that, by focusing on how children think mathematically and encouraging them to use this thinking to invent and share strategies for solving complex problems, teachers can use CGI to build strong conceptual understanding in even low-attaining students. Though students struggle to develop proficient math ability in elementary school for a variety of reasons, the literature indicates that CGI, when used in combination with other specific skill-building interventions and scaffolding practices, can be used to effectively improve math ability in students who are below grade-level. Additionally, the research suggests that, while math anxiety and low math self-concept can impede students' performance, there are ways in which educators can work with students to limit the experience of these negative emotions and improve students' enjoyment of math, leading to higher achievement. These strategies should be integrated with CGI and other skill-building and scaffolding practices to maximize students' math experiences. However, more research is needed to identify exactly how CGI should be used in combination with other instructional practices to positively affect students' math beliefs and abilities. The next chapter will provide an overview of this study's methodology, which will address this need by further examining the ways in which CGI affects math ability in below grade-level students and the ways in which it alters their beliefs and feelings about math.

CHAPTER 3

Methods

Research Question

What is the effect of Cognitively Guided Instruction practices on the math beliefs and abilities of below grade-level second grade students?

Overview

In order to know how teachers can continue to meet the needs of elementary students who are performing below grade-level in mathematics, this study examines the effect a Cognitively Guided Instruction (CGI)-based intervention model has on these students' math abilities. Additionally, since research has shown that students' math beliefs, including their self-concept (self-confidence), math affect (the degree to which they enjoy math) and math anxiety, can powerfully affect their math proficiency, this study will also examine the effect that CGI-based instructional practices have on these measures (Jameson 2014; Pinxten, Marsh, DeFraine, Van Den Noortgate, & Van Damme, 2014; Ramirez, Chang, Maloney, Levine, & Beilock, 2016; Young, Wu, & Menon 2012). By understanding how CGI-based practices affect students' math beliefs and abilities, as well as any limitations with this instructional approach, this research will

allow educators to best support elementary students who are below grade-level in math and encourage their development as early as possible.

In Chapter 2, I described the importance of building strong conceptual understanding in elementary mathematics students, and explored how CGI research and developmental frameworks been used to develop this understanding. I also examined components of math proficiency and the challenges that students who are below grade-level often face, as well as the importance of math beliefs, including self-concept, affect, and anxiety. This chapter explains the methods used in this research to examine the effect of CGI-based practices on below grade-level students' math beliefs and abilities. It describes the setting in which this study took place, the participants, and the methodology and measures that I used to answer the question: *What is the effect of Cognitively Guided Instruction practices on the math beliefs and abilities of below grade-level second grade students?*

Research Paradigm and Method

This study used a mixed methods paradigm. According to Creswell (2014), mixed methods research "involves the collection of both qualitative (open-ended) and quantitative (closed-ended) data in response to research questions or hypotheses" (p. 217). As Creswell explained, mixed methods designs are chosen because of their "strength of drawing on both qualitative and quantitative research and minimizing the limitations of both approaches" (p. 218). Though the mixed methods paradigm is relatively new, it has been extensively used in the field of education and allowed for a more thorough examination of the ways in which CGI-based practices affects students' math beliefs and proficiency than a strictly quantitative or qualitative study would have.

Many researchers have examined the effect of constructivist approaches to teaching math on elementary students' math abilities (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Franke & Kazemi, 2001; Hankes, 1996; Moscardini, 2010) and the development of math beliefs and their effect on math proficiency (Ayodele, 2011; Jameson, 2014; NCTM, 2013; Pinxten, Marsh, De Fraine, Van Den Noortgate, & Van Damme, 2014; Ramirez, Chang, Maloney, Levine, & Beilock, 2016; Wigfield & Meece, 1988). However, few studies have looked at the ways in which CGI-based instructional practices impact both students' beliefs and their abilities. In order to best understand this effect, a mixed methods approach was needed. Researchers have used quantitative measures to examine the ways in which constructivist instructional approaches, including CGI, affect the development of elementary students' math proficiency (Fyfe, DeCaro, & Rittle-Johnson, 2014; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Moscardini, 2010). Additionally, studies have used quantitative data to study the importance of various math beliefs on students' math proficiency (Ayodele, 2011; Pinxten, Marsh, De Fraine, Van Den Noortgate, & Van Damme, 2014; Ramirez, Chang, Maloney, Levine, & Beilock, 2016; Wigfield & Meece, 1988). Researchers have also powerfully used qualitative data to more deeply understand how CGI practices affect the development of students' conceptual understanding and leads to greater math proficiency (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke, & Levi, 2003; Franke et al., 2009; Moscardini, 2010). This study combined these quantitative and qualitative analyses to provide a more complete picture of the impact of CGI-based practices on below grade-level elementary students' mathematical development.

This study collected quantitative data through the use of pre- and post-assessments of content proficiency and surveys about students' math self-concept, math anxiety, and math affect. Additionally, it collected qualitative data through the collection of student work samples, field notes, and other public class artifacts (including public sharing notes and communally created explanations of work). Qualitative data was also gathered through the use of audio recordings of conversations with select participants at various points throughout the study.

In order to best support students who are below grade-level in math, teachers must develop their proficiency of grade-level content while also building enjoyment and engagement and limiting feelings of anxiety (Jameson 2014; Pinxten, Marsh, DeFraine, Van Den Noortgate, & Van Damme, 2014; Ramirez, Chang, Maloney, Levine, & Beilock, 2016; Young, Wu, & Menon 2012). This mixed methods design allowed me to measure students' proficiency as they engaged in CGI-inspired intervention model while also understanding the ways in which this instruction impacted their math beliefs in order to better understand how teachers can best support elementary students who struggle to perform at grade-level in math.

Setting

This study took place at an urban public school in the Northeast region of the United States. The school serves 445 students from Kindergarten through 6th grade. During the 2016-2017 school year, 98% of students were African American or Latino/a, and 2% were Caucasian. 81% of students were eligible for Free- and Reduced-Price Lunch, and 6% of students were learning English as a second language and identified as having limited English proficiency. At the beginning of the school year,

32% of second graders scored below grade-level in math proficiency according to the nationally normed NWEA Measures of Academic Progress (MAP) math assessment. In the most recent school year, 62% of third and fourth grade students scored in the "proficient" or "above proficient" category in mathematics on the state test.

Participants

Participants in this study were identified because their scores on the NWEA MAP mathematics assessment indicate that they were performing significantly below grade-level in math at the end of first grade. Based on the school's grouping procedures, in which students are grouped for math instruction based on their ability levels in order to provide low-performing students with interventions as soon as possible, the ten lowest-performing (based on NWEA MAP mathematics assessment scores) of these second grade students were invited to participate] in this study. These ten students and their families were informed of the study and invited to participate at the beginning of the year. Parents and legal guardians of these students were informed of the purpose, methodology, and potential risks of the study according to Hamline University's Human Subject Committee regulations, and were given the option to consent to participation with or without audio recording of their students' conversations during instruction. Parents and guardians who agreed to allow their student to participate returned a signed consent form and were assured that they could ask their student to be removed from the study at any time. Of the ten students who were eligible to participate, eight consented to take part in the study with permission to take audio recordings of conversations, nine consented to take part without audio recording, and eight participated through completion.

Of the students who participated in the study, 6 entered second grade scoring more than one year below grade-level. The remaining 2 participants scored more than two years below grade-level. Because of the school's procedures for grouping students, none of the participants in this study were students with identified learning disabilities, and no participants were identified as having limited English proficiency.

Procedures

Grouping. Participants in this study formed a homogenous ability-based group for math instruction, based on the school's procedures for grouping students for math instruction. The decision to use homogenous ability groups and work with these ten students in the same group was based solely on the school's policy, not on recommendations from CGI research. Grouping students this way allowed for instruction and pacing that targeted their specific needs. Additionally, since many students were significantly below grade-level at the beginning of the school year, these groupings allowed students to work with the specific skills that they needed in order to reach grade-level proficiency.

Within this group of nine students, five participants were randomly selected at the beginning of the study to serve as specific case studies to provide additional qualitative data about the development of strategy use and conceptual understanding. These five students, whose parents or legal guardians all consented to the use of audio recordings of their work in addition to general participation in the study, had additional work samples collected throughout the study, and their interactions with the researcher during work time were audio-recorded three times each over the course of the study.

Instruction. Students in this study received four weeks of CGI-based math instruction as their primary mathematical instruction, in direct alignment to the instruction that their peers in different instructional groups received. However, because these students made up the lowest-performing group of students in the grade, the number range and pacing of instruction was often adjusted relative to that of their peers to meet their needs. The four-week unit of instruction used in this study focused on working with word problems that required addition and subtraction to solve and included a variety of problem types (see Appendix A for a calendar of instructional tasks). This sequence was chosen because it is a foundational unit of study about addition and subtraction that reviews and introduces key concepts and skills around place value, Base 10, adding, and subtracting, which must be developed in order for students who are below grade-level to meet grade-level standards.

Instructional framework. Students participated in 45 minutes of math instruction each day on Mondays through Thursdays. The 4-week timeframe was chosen because that is the length of the first unit of 2nd grade, which is a foundational unit of study about addition and subtraction that reviews and introduces key concepts and skills around place value, Base 10, adding, and subtracting.

Daily instruction followed the same format each day and included a warm-up, work time, and public sharing. Each day started with a 15-minute warm-up, in which students practiced with tens frames, identified how much they needed to "get to 10" from different numbers, and broke numbers down to add numbers by getting to 10. After the warm-up, the remaining 30 minutes of instruction were spent solving and sharing. Each day, students solved rich story problems presented in a familiar context. Students began

by reading the problem together and discussing it in context to make sure they understood

the action or context of the problem before solving. Reading the story together and

discussing it in context also allowed students to access the academic language of the

problem and build their visualization skills. After discussing the problem, students were

given 15 minutes of work time to solve the problem using whatever strategy they wanted.

Students were always given access to a variety of manipulatives (Unfix Cubes, counters,

Base 10 blocks, and plentiful scratch paper) and were allowed to select and use

whichever they chose with no teacher intervention. At the beginning of the study,

participants were given the choice of two number pairs and asked to select a pair to solve

in the story. For the first several days, the researcher guided participants in this choice to

make sure that they were selecting a number range that was in their zone of proximal

development. As the study progressed, more number pairs were given and students were

allowed to self-select the pair that challenged them while still allowing them to solve the

problem. While students worked, the teacher circulated and took field notes, recorded

conversations as scheduled, and used open-ended questioning to both understand and

cognitively guide students' thinking and solving (see Appendix B for the template used to

record field notes and the open-ended questions used when checking in with students

during work time). For the first three days of the study, participants worked

independently to establish routines and procedures. For the remainder of the study, they

were allowed to work collaboratively and talk through the problem with students near

them. However, all participants were required to record their solution strategy and answer

on their own paper, even if they worked collaboratively. At the end of work time, the

lesson culminated with 15 minutes of public sharing. Students were selected to share

based on their solution and strategy choice and were given the chance to explain their

thinking to their peers. Sharers were instructed to walk the group through the strategy

they used and, if they used any manipulatives, were asked to show in numbers what they

did with manipulatives. While students shared, the teacher recorded their work publically

for all members of the group to see and reference later. After instruction was completed

and students were released to their next class, the researcher returned to her field notes to

record statements from public sharing and to expand on any particularly relevant

statements or observations from the lesson.

Data Tools

In order to measure changes in students' math beliefs and abilities from the

beginning to the end of the study, and in order to examine shifts in participants'

conceptual understanding and strategy use, a series of measurement tools and instruments

were used to collect data during the course of the study.

Math abilities assessment. Students completed a pretest, which included a series

of tasks conducted in a one-on-one interview designed to assess a variety of skills and

understandings, to measure their math abilities before the study began (see Appendix C

for specific tasks and assessment forms). They completed an identical posttest again after

the conclusion of the four-week unit in order to measure the impact of CGI-based

practices on their math abilities. The pre- and posttests were completed individually and

privately per the directions for each task and assessed several components of students'

math abilities. The pre- and posttest assessed students' proficiency in a variety of grade-

level skills upon which the unit of instruction focused. Additionally, because number

sense, visual-spatial skills, and number concepts have been found to contribute to low-

performing students' difficulties with math, these skills were assessed as well (Kamii & Rummelsburg, 2008; van Garderen, 2006; Wilson & Swanson 2001).

Fluency. The first component assessed on the pre- and posttest was fluency. Students' addition fact fluency was assessed using an abbreviated version of Kamii's Basic Fact Assessment, which methodically asks students to solve an ordered sequence of addition facts, beginning with doubles before moving into adding one, two, three, four, and finally five to an initial number quantity (Kamii, 1985). Students were given six facts to solve orally and their responses were recorded on the assessment sheet. In addition to recording students' answers, the researcher noted whether students answered in two seconds or fewer, which indicates automaticity. If students did not respond with automaticity, the researcher noted the time that it took them to respond and any strategy that they used to solve. If students used their fingers, the researcher noted if they used their fingers to direct model both quantities or if they used them to count on (and if so, which quantity they began with).

Subitizing. Subitizing has been found to be a foundational mathematical skill that contributes to the development of other important skills and abilities (Clements, 1999). Participants' ability to subitize, or immediately recognize and identify the number of objects shown, was measured on the pre- and posttest with a series of six cards. Students were shown three five-wise tens frames and three non-standard dice subitizing cards. Their ability to immediately (within two seconds) identify the number of dots shown on each card without counting was recorded, along with their response when asked how many dots they saw.

Number sense. Students' number sense was measured with a number comparison task. Students were shown a series of five pairs of cards and asked to identify which of the two numbers was greater. The researcher recorded their response on the assessment sheet and noted any additional comments or solution strategies.

Base 10. Students' ability to work within the Base 10 system was assessed with a "Get to 10" task. In this task, students were shown a series of cards with numbers on them and were asked either "how many to get to ten?" or "how many to get to the next ten." Students were shown four number cards that had ten as the following decade, and three number cards that required them to work with higher decades. Students' responses were recorded on the assessment sheet, as well as the time it took them to respond and any strategies or calculation methods that they used to produce an answer.

Story problem strategy use. Students' abilities to accurately solve word problems involving addition and subtraction, as well as their strategy use, number range, and comfort with different problem types were assessed using a variation of Brickwedde's (2005) Early Base Ten Assessment. The researcher read each problem aloud to participants, as indicated on the assessment sheet, and recorded the students' answer and the solution strategy that they used. The researcher also adjusted the number range of the problem, if needed, in response to the student's performance. The researcher noted whether the student was able to accurately solve the problem and at what stage of base ten development the child was at for each problem. After completing the Early Base Ten Assessment, the researcher completed the Individual Student Profile for each participant to identify participants' choice of strategy use for each problem type.

Math beliefs inventory. In addition to math proficiency, students' math beliefs were measured before and after they participated in a CGI-inspired intervention model. Before the study began, students completed a math beliefs inventory, which measured their math self-concept, math affect, and math anxiety (see Appendix D). The questions on this inventory were based on Ayodele's (2011) questionnaire on self-concept of mathematics and adjusted for use with elementary students. Students completed the inventory privately and in writing, and were informed before beginning the survey that their responses would be kept confidential and were being collected so the researcher could know how to best help them enjoy math instruction. Students completed the same inventory again after the four-week instructional period had concluded, with their responses again recorded privately in writing.

Teacher observation and record-keeping tools. Throughout the study, the researcher collected data on the development of students' solution strategies and on changes in students' conceptual understanding of key concepts. Field notes, work samples, and audio recordings were collected throughout the course of the study to provide data on these developments. Though field notes, work samples, and audio recordings were collected using students' names, any identifying information was removed for research purposes, and a pseudonym was used when discussing individual participants in the research.

Field notes. During every instructional block, the researcher recorded field notes about participants' solution strategies, answers to open-ended questions, explanations, performance during warm-ups, and other general notes that offered information about their conceptual understanding and development of strategy use (see Appendix B for the

template used for recording field notes). In addition to recording observations and notes during instruction, the researcher also recorded more thorough notations after the lesson concluded, expanding on important statements by students or noted shifts in conceptual understanding or strategy use. Field notes were recorded for all participants on every day of the study, and were used both to collect data and to determine what problem type and number range the group was ready for in upcoming instruction.

Work samples and other artifacts. In addition to the field notes collected, work samples and other public artifacts were gathered throughout the course of the study to collect more data on students' strategy use and development of conceptual understanding. Work samples from all participants were collected three times during the survey: during the first week, at the end of the second week, and during the fourth week. Additionally, individual work samples from the five participants who were randomly selected as case studies were collected every day. Other public artifacts, including communally created documents from public sharing (which included students' demonstrations of their solution strategies as well as written statements from their explanations) were collected when the artifact recorded the emergence of a new solution strategy or evidenced a shift in conceptual understanding among participants. These work samples and artifacts were collected to provide qualitative data on changes in students' use of solution strategies and conceptual understandings of key ideas and concepts throughout the course of the study.

Audio recordings. To supplement the collection of work samples and public artifacts, conversations with the five students who were selected as case studies were audio recorded and subsequently transcribed three times each during the study. These five students were recorded during their interactions with the researcher during work

time, as they responded to open-ended questions and explained their strategy use and understanding of the problem. Transcripts of these audio recordings were used to provide qualitative data on the development of participants' conceptual understanding of key ideas and concepts over the course of the study.

Data Analysis

Results of students' performance on the pre-and posttests and their responses on the math abilities inventory before and after the study took place will be quantitatively analyzed to initially examine the effect of CGI practices on students' math beliefs and abilities. These instruments will be used to determine whether participants' performance on grade-level content and levels of math anxiety and enjoyment change after participating in four weeks of CGI-based instruction. Students' proficiency before and after their experience engaging in the CGI-inspired intervention model, as measured by their performance on the pre- and posttests, will be compared to analyze whether CGI-based practices effectively improve proficiency in students who are below grade-level. Students' responses on the math beliefs inventory before and after the four-week unit will be compared to analyze whether students' math beliefs, enjoyment, and anxiety change after participating in CGI-based instruction.

In addition to the quantitative analyses described above, qualitative analyses will be performed on the field notes, work samples, and transcripts of audio recordings collected throughout the four-week unit. These work samples, notes, and transcripts will be analyzed to track changes in students' solution strategy use throughout the course of the study, as well as shifts in their conceptual understanding of key concepts. Specifically, these artifacts will be used to determine how students' solution strategies

changed during the unit (both individually for each participant selected as a case study and for the group as a whole) and if students' ability to utilize solution strategies to accurately solve story problems improved throughout the unit. The qualitative analysis of these artifacts will focus on answering the following research sub-questions:

1. Does the intervention model based on CGI practices improve students' ability to invent or incorporate new solution strategies that they can apply with understanding to new problems?

2. Does the intervention model based on CGI practices improve students' ability to explain their mathematical reasoning and problem-solving approach?

3. Does the intervention model based on CGI practices improve students' conceptual understanding of the processes carried out when solving story problems involving addition and subtraction?

4. How do students' math beliefs and attitudes impact their achievement?

Finally, these artifacts will be examined for any trends or challenges that appear, which, in combination with the quantitative data collected, will demonstrate the effect of CGI-based instruction on students who are below grade-level.

Conclusion

Through a mixed methods analysis of students' performance on grade-level tasks, responses to questions regarding their beliefs about math, and work samples and other artifacts, this study examines the research question: *What is the effect of Cognitively Guided Instruction practices on the math beliefs and abilities of below grade-level second grade students?* Students' math proficiency and measures of math self-concept, affect, and anxiety before and after participating in a CGI-based instruction model will be

compared in order to better understand how CGI practices affect students who are below grade-level. Work samples, public artifacts, and transcripts of recorded conversations during work time will also be analyzed to further inform the ways in which CGI-based instruction affects students' solution strategies, conceptual understandings, and attitudes about math during the four-week instructional unit. In Chapter Four, I will discuss the results of this study, as well as the ways in which students' thinking and problem-solving abilities were affected by their participation in a CGI-inspired intervention model. These results will allow elementary teachers to better support below grade-level students by examining the strengths of the CGI research base to inform an instructional approach and revealing areas in which it can be supplemented to encourage grade-level proficiency in all students.

CHAPTER 4

Results

Introduction

This study asked, *What is the effect of Cognitively Guided Instruction practices on the math beliefs and abilities of below grade-level second grade students?* In order to answer that question, this chapter will discuss and analyze the results of the math abilities assessments and math beliefs inventories that were taken at the beginning and end of the study. Additionally, data from field notes, work samples and other artifacts, and audio recordings of conversations between participants and the researcher that were collected over the course of the study will be analyzed.

This study was carried out over a four-week period in September of 2016. At the beginning of the school year, the parents and guardians of the 10 lowest-performing mathematics students in second grade were invited to participate in this study. Of those 10 families informed about the study, 9 parents and guardians consented for their child to participate. 1 student received consent to participate but moved away 4 days after data collection began and is thus not included in the data analysis. Participants' scores on the NWEA-MAP mathematics assessment from the end of first grade were used to determine eligibility for this study and students' level of math proficiency. Of the 8 participants, 6

entered second grade scoring more than one year below grade-level. The remaining 2

participants scored more than two years below grade-level (see Table 1 for detailed

scores). These participants received four weeks of Cognitively Guided Instruction-based

math instruction. This daily instruction included a warm-up, work time in which the

researcher conferenced with students to guide their learning, and public sharing of

solution strategies.

Table 1

NWEA-MAP Mathematics Scores

Participant	NWEA-MAP Mathematics Score*	Grade-level performance
1	160	> 1 year below grade level
2	158	> 1 year below grade level
3	156	> 1 year below grade level
4	139	> 2 years below grade level
5	151	> 1 year below grade level
6	138	> 2 years below grade level
7	150	> 1 year below grade level
8	161	> 1 year below grade level

*The beginning of year 2nd grade score is 177. A score of 162 indicates that a student is performing at the level of a beginning-of-year 1st grader. Participants who scored below 162 were determined to be performing more than one year below grade-level. A score of 140 indicates that a student is performing at the level of a beginning-of-year kindergartener. Participants who scored below 140 were determined to be performing more than two years below grade level.

Before and after the study, participants completed a math abilities assessment and

math beliefs inventory. These measures were used to assess changes in students' math

abilities and beliefs. Additionally, the researcher collected field notes and student work

samples during the study. Conversations between the researcher and participants were

audio-recorded three times during the study for each of the five students who were

selected as case studies. This chapter contains a quantitative analysis of participants'

scores on the math abilities assessment and math beliefs inventory, as well as a

qualitative analysis of all other data collected. This mixed-methods paradigm will be used to answer the following research sub-questions:

1. Does the intervention model based on CGI practices improve students' ability to invent or incorporate new solution strategies that they can apply with understanding to new problems?

2. Does the intervention model based on CGI practices improve students' ability to explain their mathematical reasoning and problem-solving approach?

3. Does the intervention model based on CGI practices improve students' conceptual understanding of the processes carried out when solving story problems involving addition and subtraction?

4. How do students' math beliefs and attitudes impact their achievement?

In addition to answering these questions, this chapter will discuss the trends that emerged in order to evaluate the effectiveness of CGI-based instruction for students who are below grade-level.

Instructional Timeline

In the week before the study began, participants completed a pretest in a one-on-one interview with the researcher. The pretest included a series of tasks designed to measure participants' math abilities and key understandings (see Appendix C for specific tasks and assessment forms). Participants also completed the math beliefs inventory to measure their math beliefs and attitudes (see Appendix D). During the study, students participated in 45 minutes of math instruction each day on Mondays through Thursdays. Each instructional session began with a 15-minute warm-up. Warm-up tasks included working with tens frames, "get to ten" tasks, and using tens to add friendly numbers. The

remaining 30 minutes of instruction were used for solving and sharing about the day's story problem. Students read the problem together and discussed it in context before solving it using any strategy they chose. Students always had access to manipulatives (Unfix Cubes, counters, Base 10 blocks, and scratch paper) and were free to use whichever they chose. While they worked, the researcher circulated and conferred with participants. Each day closed with 15 minutes of public sharing while the researcher recorded the sharer's work for all participants to see. At the conclusion of the study, participants completed a posttest (which was identical to the pretest) in order to measure changes in their math abilities during the study. They also completed the math beliefs inventory at the conclusion of the study to measure any changes in their math beliefs over the course of the study.

Math Abilities Assessment

The math abilities assessment measures a variety of math skills and abilities that are important for grade-level proficiency. All participants completed the math abilities assessment before the study began and again after it was completed in order to measure changes in their math abilities from the beginning to the end of the study. The assessment was administered privately and individually by the researcher and measured fluency, subitizing, number sense, Base 10 understanding, and story problem strategy use. Participants' performance on the pretest was compared to performance on the posttest to analyze the impact of the constructivist approach to instruction on these below grade-level second graders' key mathematical abilities.

Fluency. The fluency portion of the math abilities assessment consisted of 6 math facts that students were asked to solve (see Appendix C for assessment forms). In this

task, participants were shown one fact card at a time and were asked to solve the fact as quickly as they could. The researcher recorded their response and noted how long it took them to solve. If the participant solved the fact within 2 seconds, they were determined to have recalled it automatically (Kamii & Rummelsburg, 2008). If the participant did not solve automatically, the researcher also recorded the way in which they used their fingers or other tools to solve.

During the study, participants did not participate in explicit fact instruction. All fact practice occurred in the context of warm-up activities or daily problem solving. Any strategies that emerged were discovered by participants and were only shared publically when they were used as part of a solution strategy.

The results of the fluency portion of the assessment indicate that participants' accuracy and automaticity improved during the four weeks of the study (see Table 2 for individual participants' scores on the pretests and posttest). All 8 participants increased the number of facts that they answered correctly from the pretest to the posttest, and all but one participant increased the number of facts that they were able to automatically recall. Among all 8 participants, there was an average increase of nearly 1 fact (0.9) answered correctly from the pretest to the posttest, indicating that participants improved their fluency during the course of the study. Additionally, there was a larger average increase of exactly 3 facts recalled automatically from the pretest to the posttest. These results show that participants in the constructivist-based teaching model experienced an increase in both fact fluency and automaticity during the four weeks of the study. This finding is important because it suggests that students who are below grade-level can improve these important skills without being explicitly taught addition facts. Though

participants in this study were not directly taught strategies for adding and subtracting

and did not spend class time memorizing facts, they experienced growth in this important

area, a finding consistent with previous research (Carpenter et al., 2015).

Table 2

Fluency Scores on Pre- and Posttest

Participant	Pretest Correct	Posttest Correct	Pretest Automatic	Posttest Automatic
1	5	6	1	5
2	5	6	0	4
3	6	6	1	6
4	4	5	0	4
5	5	6	4	6
6	3	5	0	2
7	4	5	2	2
8	6	6	3	6
Average	4.75	5.63	1.38	4.38

In addition to the increase in fluency and automaticity that was found from the

pretest to the posttest, another important trend emerged regarding participants' strategies

for solving facts that were not automatically recalled. On the pretest, 3 participants were

unable to count on to add when solving a fact that was not automatically recalled. Instead

of holding one number mentally and representing the other with their fingers, these 3

participants relied on Direct Modeling and showed both numbers on their fingers before

counting each finger. This, of course, led to confusion when the sum of the fact exceeded

ten, and represents a less sophisticated and more inefficient way of adding. On the

posttest, however, all 3 of these participants were able to count on by holding one number

in their head and using their fingers to count on by the other. This shift indicates that

these students were able to adopt more sophisticated and efficient strategies of adding

over the course of the study. In fact, 2 of these participants were able to solve 3+6 by

holding 6, the larger number, mentally and counting on by 3, the smaller number. In addition to adopting a yet more sophisticated and efficient way of solving, this finding indicates that these participants developed an understanding of the commutative property of addition during the study. Thus, these participants in the study showed increases in accuracy and sophistication while also exhibiting evidence of an important conceptual mathematical understanding.

Though the constructivist approach to teaching math that was utilized in this study did not involve explicit teaching or practice of addition and subtraction fact fluency, participants showed an increase in accuracy and sophistication, a finding consistent with previous research (Carpenter et al., 2015). This finding suggests that second grade students who are performing below grade-level were able to improve this crucial skill without spending time receiving direct instruction. Though it is likely that participants' scores increased over the four weeks of the study in part because they had returned from summer vacation, where many students do not practice math, to daily exposure and practice, it important to note that these students who are performing significantly below grade-level were able to experience this increase in both accuracy and sophistication without any explicit fact instruction. This finding supports the notion that, even without direct instruction, students who engage in constructivist forms of mathematical instruction can indeed improve their basic foundational skills.

Subitizing. Subitizing, the skill of automatically recognizing a quantity on sight without counting, has been shown to be a foundational skill that contributes to the development of other important math skills and abilities (Clements, 1999). Because foundational skills like subitizing are so important to moving low-performing

mathematics students closer to grade-level, the math abilities assessment measured its changes in participants' subitizing abilities over the course of the four-week study. On this portion of the assessment, participants were shown a series of six dot cards for two seconds each. After seeing each card, the participant was asked to identify the number of dots that they saw. The researcher recorded their responses and noted any strategies that the participant used to answer (see Appendix C for assessment forms and recording tools).

As Table 3 shows, the pretest revealed that all eight participants had great difficulty subitizing. At the beginning of the study, only two participants were able to accurately recognize the number of dots on any of the six cards, and of those two participants, only one actually subtilized (the other was able to quickly count, pointing her finger at each dot before the two seconds expired and the card was taken away). The remaining six participants immediately attempted to count the number of dots on each card in a similar manner on the pretest, though they were unable to count quickly and accurately enough to correctly identify the number on each card. Notably, these six participants were not even able to recognize the tens frame with three dots, nor were they able to accurately count those dots within the two seconds that they were given too examine the card. These results indicate that participants had great difficulty subitizing at the beginning of the study, a finding that was not surprising given their below grade-level proficiency.

Table 3

Subitizing Scores on Pre- and Posttest

Participant	Pretest Correct	Posttest Correct	Pretest Automatic	Posttest Automatic
1	3	6	3	5
2	0	3	0	3
3	0	2	0	2
4	0	1	0	1
5	0	4	0	3
6	0	0	0	0
7	0	2	0	2
8	1	3	0	3
Average	0.50	2.63	0.38	2.38

Participants showed some slight improvement in subitizing over the course of the study, though most participants still struggled to subitize numbers larger than five and only one participant was able to accurately subitize all cards on the posttest (see Table 3 for detailed results on the pretest and posttest). The posttest showed an average growth of about 2 cards accurately identified and 2 cards automatically recognized, suggesting that improvement in accuracy was due to students being able to subitize automatically, not being able to count more quickly or efficiently. However, participants were by no means performed well on the subitizing task after completion of the study. Only one participant was able to accurately identify all 6 cards on the posttest, while exactly half of the participants were able to accurately identify fewer than three cards (including one participant who was not able to identify any). It is possible that, being such a discrete and specific task, subitizing is a skill that must be taught more directly. Though students worked with tens frames in the warm ups to their daily instruction, they always had an opportunity to count and were not forced to subitize. Given participants' low performance on this task after four weeks of instruction, these results might suggest that a

constructivist approach to teaching math to students who are significantly below grade-level should include some direct practice of this important skill.

Number sense. Participants' number sense was measured by assessing their ability to compare two numbers. In this task, participants were shown a series of five cards, each of which were labeled with two numbers. Participants were asked to identify either which number was less or which number was greater. The researcher recorded their response, which was used to indicate the participant's ability to recognize and compare the value of numbers.

On the pretest, participants accurately compared 3.25 of the 5 number cards (see Appendix C for assessment forms). They showed very slight improvement on the posttest, where the average score was 4 out of 5 cards compared correctly (see Table 4 for each participant's individual scores). However, only three of the eight participants were able to accurately answer all 5 comparisons on the posttest, indicating that most participants still lacked some number sense at the conclusion of the study.

Table 4

Number Sense Scores on Pre- and Posttest

Participant	Pretest Correct	Posttest Correct
1	4	5
2	3	5
3	4	4
4	2	2
5	2	4
6	3	4
7	3	3
8	5	5
Average	3.25	4.00

Interestingly, the most commonly missed comparison both before and after the study was the comparison of 400 and 40. Six participants missed this comparison on the

pretest and four missed it on the posttest. Of these ten total errors, eight times participants responded that that the two numbers were the same. This belief that 400 and 40 are the same number suggests that participants' number sense and place value understanding are still underdeveloped, as they did not recognize that adding a zero changed the value of the number. However, the comparison between 10 and 1, which is similar in that the only difference between the two numbers is a zero, was only missed twice on the pretest and once on the posttest. This discrepancy might suggest that students are comfortable enough with place value in single- and double-digit numbers to recognize that adding a zero changes the number but that their place value understanding does not yet extend to the hundreds. If so, this comfort could have grown out of the constructivist approach to math that encouraged students to utilize place value when solving problems. Or, it could be explained by students' familiarity and recognition of the both ten and one as numbers, which might lead them to be able to more accurately compare them than two larger numbers that are less familiar, like 40 and 400. Since participants in this study did not work with numbers into the hundreds, this task would need to be assessed later to fully determine the effectiveness of this model on improving students' number sense.

Base ten understanding—landmarks to 10. Participants' Base 10 understanding was assessed using a "get to ten" task. In this task, students were shown a series of seven cards, each with a number printed on it. After being shown the card, the student was asked how much it would take to get to the next ten. The researcher recorded their response, and noted whether they knew automatically or had to calculate their answer.

Overall, participants were highly accurate on this task both before and after the study. On the pretest, participants correctly answered 5.88 out of 7 questions. The

average rose to 6.63 on the posttest, showing slight improvement on an already high-scoring task (see Table 5 for more detailed results). However, an important trend emerged regarding participants' automaticity when getting to the next ten. Before the study began, only three participants were able to answer any questions without calculating in some way, and the average number of questions answered automatically was only 0.5. Notes recorded during the pretest explain that most students used their fingers to determine how much was needed to get to the next ten. On the posttest, though, the average number of questions answered automatically rose to 3.75. Further, all participants were able to answer at least one question automatically on the posttest, with one participant recognizing six of the seven cards automatically and two participants recognizing five of the seven automatically. Thus, there was a noticeable increase in participants' level of automaticity from the beginning of the study to the end. This increase shows that participants' Base 10 understanding increased over the course of the study, as the ability to immediately recognize how much is needed to get to a new ten evidences much stronger Base 10 understanding than the earlier reliance on counting that participants exhibited on the pretest. This finding suggests that the constructivist intervention and its focus on conceptual understanding and strategy use contributed to increases in participants' Base 10 understanding and automaticity, an extremely important component of math proficiency (Carpenter et al., 1993).

Not surprisingly, nearly all of the participants who made significant improvements in fact automaticity also showed considerable improvement in Base 10 automaticity. For example, Participant 1 answered 4 more facts fluently on the posttest than on the pretest, and also automatically identified 4 more "get to ten" cards on the

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posttest than on the pretest. Participant 3 answered 5 more facts fluently on the posttest than on the pretest, and automatically identified 4 more "get to ten" cards at the end of the study than at the beginning. This correlation makes sense because of the connection between fluency and the ability to recognize and utilize sums of ten. Additionally, there was a moderate connection between participants' improvement in fluency and Base 10 recognition and their ability to subitize. Though participants struggled with subitizing in general, those who showed greater subitizing ability on the posttest also showed considerable improvement in fluency and Base 10 recognition. Future research should examine the relationship between fluency and Base 10 recognition and subitizing in order to better understand how to help low-achieving students improve these important skills.

Table 5

Base 10 Scores on Pre- and Posttest

Participant	Pretest Correct	Posttest Correct	Pretest Automatic	Posttest Automatic
1	7	7	1	5
2	6	7	0	4
3	6	7	0	4
4	4	5	0	2
5	7	7	0	3
6	3	6	0	1
7	7	7	1	6
8	7	7	2	5
Average	5.88	6.63	0.50	3.75

Base ten understanding—reconfiguring & decomposition of number.

Participants' strategy selection was measured using Brickwedde's (2005) Early Base Ten Assessment (see Appendix C). In this task, the researcher read a series of four story problems aloud to the participant. Participants were asked to solve a Join, Result Unknown problem, a Separate, Result Unknown problem, a Join, Change Unknown

problem, and a Compare, Difference Unknown problem. Participants were given blank

paper and a pencil as well as Unifix cubes and were allowed to solve however they chose.

As they solved, the researcher recorded their solution strategy and answer and adjusted

the numbers if students were not able to access the problem. For each problem type, the

evolution of participants' strategy choice and accuracy was examined to determine the

impact that the intervention model had on students' math abilities (see Table 6 for results

broken down by problem type).

Table 6

Strategy Use on Pre- and Posttest

Problem Type	Correct Pretest	Correct Posttest	DM* Pretest	DM* Postets	CS* Pretest	CS* Posttest	FS* Pretest	FS* Posttest
JRU	7	7	8	3	0	5	0	0
SRU	7	8	8	5	0	2	0	1
JCU	7	8	8	2	0	4	0	2
CDU	5	7	8	6	0	1	0	1

*DM= Direct Modeling
  CS=Counting Strategy
  FS=Flexible Strategy

The first question on this portion of the assessment was a Join, Result Unknown

problem. On the pretest, 7 participants were able to accurately solve this problem (the

remaining participant made a counting error though his representation matched the

problem and could have led to an accurate solution). On the posttest, the same 7

participants solved the problem accurately. Additionally, 2 participants on the pretest

were unable to access the prescribed numbers and worked with adjusted numbers, and 1

participant worked with adjusted numbers on the posttest. Though there was no change in

participants' level of accuracy from the beginning of the study to the end, a notable trend

emerged regarding strategy use. On the pretest, all 8 participants used Direct Modeling to

solve the problem, building both numbers with cubes and then counting them all. On the posttest, 5 participants used a counting strategy instead of Direct Modeling, and 2 of the participants who used Direct Modeling counted on from the larger number instead of counting all of the cubes. This shift in strategy use indicates that nearly all of the participants adopted more efficient and sophisticated solution strategies for addition over the course of the study.

Next, participants solved a Separate, Result Unknown problem. On the pretest, 7 participants accurately solved, while all 8 participants were able to solve accurately on the posttest. 1 participant needed adjusted numbers on the pretest and all participants used the prescribed numbers on the posttest. Like the Join, Result Unknown problem, all 8 participants relied on Direct Modeling to solve before the study began. On the posttest, 5 participants used Direct Modeling, 2 used a counting strategy, and 1 used a flexible strategy (relying on place value to solve). Importantly, only 1 participant showed Base 10 understanding in their Direct Modeling on the pretest by using sticks of ten to represent and solve rather than building the number as a collection of ones, while 4 of the 5 Direct Modelers showed Base 10 understanding on the posttest. So, while participants relied more on Direct Modeling subtracting than they did to add, their strategy increased almost uniformly in both sophistication and efficiency over the course of the study.

The third question on this portion of the assessment was a Join, Change Unknown problem. Seven participants were able to accurately solve this problem on the pretest, and all 8 participants solved accurately on the posttest. One participant used adjusted numbers on the pretest and all participants used the prescribed numbers on the posttest. 7 participants used Direct Modeling to solve on the pretest, though there was more variety

in strategy use at the end of the study. On the posttest, only 2 participants used Direct Modeling, while 4 counted on and 2 used a flexible strategy, indicating another shift towards more sophisticated strategy use over the course of the study.

The final question on this task was a Compare, Difference Unknown problem. This was the only problem on the Early Base Ten Assessment that participants largely struggled with before the study began. On the pretest, 6 participants were unable to access the problem with its prescribed numbers and worked within an adjusted range. Even with those numbers adjusted, only 5 participants accurately solved the problem on the pretest. On the posttest, 1 participant needed adjusted numbers, and 7 participants accurately solved. Though more participants had access to the problem and accurately solved on the posttest than on the pretest, most participants still relied on Direct Modeling to solve this problem at the conclusion of the study (see Table 6 for complete dispersal of strategy use). It is likely that, since this problem was harder for students to access and understand, they relied a less sophisticated solution strategy than they used to subtract in the Separate, Result Unknown problem.

The results of the Early Base Ten Assessment show that, while participants' ability to accurately solve these four problem types was high on both the pre- and posttest, participants did improve their ability to use more efficient and sophisticated strategies when adding and subtracting. This finding suggests that participants' daily work with story problems and their exposure to different solution strategies during daily discourse and sharing led to improvement in this important area over the course of the study. The data indicates, then, that students who are below grade-level are able to

understand and internalize new and increasingly complex solution strategies when they are exposed to them during peer sharing.

Summary. Comparing participants' performance on the math abilities assessment before and after they received 4 weeks of a CGI-based intervention suggests that this constructivist approach to teaching math had a positive effect on a specific set of students' math abilities. In particular, participants made notable improvements towards proficiency in fact fluency and automaticity, Base 10 understanding, and strategy sophistication. Of course, a key difference between this constructivist approach and a more traditional instructional style is that these skills were not directly or explicitly taught during the study. Participants did not spend time memorizing addition or subtraction facts, nor did they see the instructor model specific strategies for problem solving that they were expected to repeat. Because participants did not receive direct instruction but were cognitively guided in their learning over the course of the study, these findings suggest that a Cognitively Guided Instruction-based teaching model can effectively improve these particular foundational skills in below grade-level students.

Data from the math abilities assessment also indicates that there were key areas in which participants made little improvement or did not near proficiency by the end of the study. At the end of the study, participants still largely struggled with the important and foundational task of subitizing and scored very low in this area on the posttest. It is possible that subitizing is such a specific task that it must be taught directly, with a teacher modeling the skill and then offering repeated and isolated practice. In this study, participants received no direct subitizing instruction and most made little or no improvement in the skill, supporting the idea that subitizing must be taught in a more

explicit manner. It is also possible, though, that the constructivist approach provided opportunities to guide participants to improve their subitizing abilities that the instructor missed. Though participants did not work with tens frames or dice during the study, the instructor could have led students to subitize by more actively discouraging them from routinely counting cubes. Instead, if participants had ben asked to quickly identify how many cubes they saw, perhaps their subitizing skills would have improved more. Thus, it is hard to distinguish whether the lack of improvement was due to the model itself or the researcher's implementation of the model.

Math Beliefs Inventory

The math beliefs inventory is a twelve-question survey designed to measure students' math self-concept (students' beliefs in their own math abilities), affect (the extent to which students enjoy math), and anxiety (Ayodele, 2011; see Appendix D). Participants completed the survey privately before the study began and again once it was over. Students circled one of five faces to indicate their level of agreement with each statement (the researcher explained the response that each face represented for each question to ensure that there was no confusion). The face that represented complete agreement with the statement was given 5 points, the face that represented complete disagreement was given 1 point, and points in between decreased in that order. Each survey was then scored and participants' scores for each category were added together. Higher scores in each category indicated that the participant had higher levels of the particular trait.

Both before and after the study, students reported very high levels of math self-concept and affect, as well as low levels of math anxiety (see Table 7 for results). No

meaningful changes in student's math self-concept, affect, or anxiety were evident from the pre- to the posttest. However, two interesting trends emerged that were captured in the researcher's field notes throughout the study.

Table 7

Math Beliefs on Pre- and Posttest

Category	Points Possible	Pretest Average	Posttest Average
Self-Concept	20	18.0	18.25
Affect	20	18.05	17.95
Anxiety	20	2.15	1.90

First, there was a large discrepancy between participants' reported feelings of self-concept and their behavior at the beginning of the study. Though participants' responses to the survey showed extremely high levels of self-concept, low levels of confidence and efficacy were actually observed during the first two weeks of the study. At the beginning of the study, students exhibited many negative reactions to feedback during individual work time. They often responded negatively when the researcher asked them guiding questions and seemed to immediately assume that discussion with a teacher meant that they had made a mistake. At the beginning of the study, participants also relied heavily on teacher affirmation when solving and were often hesitant continue working without being told that they were solving the problem correctly. On the third day of the study, I recorded in my field notes that 7 of the 8 participants had raised their hands in the first two minutes of work time to ask if their representation was correct, a behavior that contradicts the self-confidence that participants reported feeling on the math beliefs inventory. In response to one of these inquiries, I asked one particular participant to explain to me how they were solving. Instead of simply engaging in the

discussion, the participant showed her lack of confidence and immediately assumed that she had made a mistake. Her response was reflective of many experiences in the first week when she grunted and said "Ahh, come on! I can never do it right!" Thus, while participants responded with high levels of self-concept on the pretest, their behaviors during work time actually indicated that their self-confidence was low and that they relied on frequent assurance from the teacher that they were on the right track (and became easily frustrated when they did not receive it). As the study progressed, these reactions decreased and participants exhibited higher levels of self-confidence. In the fourth week of the study, only one student raised his hand in search of teacher approval for his solution, and students responded much more neutrally to teacher questioning. While no visible increase in participants' reports of self-concept were observed on the math beliefs inventory (though the report of self-concept remained high on the posttest), the researchers' field notes do indicate that participants' exhibited more self-confidence and less reliance on teacher approval as the study progressed.

A somewhat contradictory trend emerged around participants' judgment of their own capabilities when they had to select the number range that they would use for the daily problems. Each day, participants were given a variety of number pairs that they could select to use in the problem. This practice was designed to allow all students to work with the same problem using a number range that they could access. Higher achieving students who needed to be challenged could select larger numbers and lower-achieving students could select smaller numbers that made the problem accessible to them. However, although participants did not exhibit the high levels of self-concept that they reported on the survey when engaging in discussion, their confidence was readily

apparent as many of these low-performing students selected number ranges far beyond their current capacity. This overconfidence was widespread during the first several days of the study, as 6 of the 8 participants had to be urged during the first two days of the study to start with numbers that were more accessible to them. When asked why they chose those numbers, one participant told the researcher that "these numbers are more fun cause they're bigger," and another explained "I like the harder ones." These responses suggest that participants knew that the numbers they were selecting were larger and thus more rigorous, but did not understand that they were selecting numbers that were beyond their current range. This finding was unexpected, and perhaps suggests that another factor holding these low-performing students back is their inability to accurately select tasks that are appropriate for their current level of ability. If students are working on developmentally appropriate problem types but are consistently selecting number ranges that make problems inaccessible to them, they are likely experiencing a lack of success while also not getting the practice that they need with foundational skills. This finding answers sub-question 4 by suggesting that, while high levels of math self-concept have been shown to positively impact math proficiency (Pinxten et al., 2014), overconfidence might actually be detrimental to student performance.

Observations and Field Notes

The quantitative data gathered in the math abilities assessment and the math beliefs inventory offers important insight into how the study's instructional approach affected students' math beliefs and abilities. In order to better understand the specific ways in which students' abilities were affected, field notes, work samples and other public artifacts, and conversations between the researcher and participants were also

collected and analyzed over the course of the study. Three major trends emerged from these documents that will be discussed below.

   Access. As the study progressed, participants' access to story problems, or their ability to understand the action of the problem and work with the numbers given, steadily increased. During the study, field notes and work samples indicate that participants became better able to retell the story, answer comprehension questions about the story, and represent accurately. At the beginning of the study, many students struggled to understand the action of the problem and were unable to represent the problem accurately. Participants' solutions often represented a different action than what was present in the problem and frequently led them to select the wrong operation to solve. Data from the first two weeks of the study suggests that these errors were caused by participants' inability to even understand or explain the story. A field note from the first week read, "[I am] worried about kids even *accessing* the problem. Today it took 4 tries for one of the higher students in the study group to even retell a simple JCU accurately. Not a language or memory issue, he was inverting the action. Follow-up comprehension [questions] were all over the place." Sometimes, selecting the wrong operation to solve indicates that a child is actually a flexible thinker. For example, if a child attempts to find the difference between two numbers by adding or counting up from the smaller number, it suggests that they have a strong conceptual understanding of the relationship between addition and subtraction and are able to use a flexible strategy to solve. However, participants at the beginning of the study were simply recalling the action of the story incorrectly. In one instance, a student retold that the actor in the story problem lost 12 Pokemon cards when in fact the problem stated that the actor got 12 Pokemon cards for

his birthday. If this student was not able to accurately describe the story, it is unlikely that he was able to accurately represent or solve it. This concern was raised several more times during the beginning of the study, and notes stating that access was an issue appear 11 times in the first two weeks.

Transcripts of early conversations between the researcher and participants further illustrate this early lack of access. During the first week of the study, participants were asked to solve a Join, Result Unknown problem (Mikayla has 7 books in her book baggie. Then she gets 8 more books from the library to put in her book baggie. How many books are in her book baggie now?) The following conversation between the researcher and a participant occurred within the first several minutes of work time.

Researcher: Let's think about this problem. What is happening?

Participant: So, I know Mikayla goes to the library.

R: What happens at the library?

P: She gets some books.

R: How many books does she get from the library?

P: 7?

R: Let's check. Show me where you see that in the problem.

P: [points to the number 7]

R: Let's go back and read the story again.

[Participant fluently reads story aloud; only mistake is reading "book bag" instead of "book baggie" the first time it appears in the story]

R: So how many books does she get from the library?

P: 7.

> R: Right here it says that she gets 8 more books from the library. So how many books does she get from the library?
>
> P: 8.

In this instance, the participant was able to read the problem fluently, but either did not understand the action of the story or could not remember key information for long enough to retell it. She also struggled to effectively go back into the story to find the important information. This lack of access clearly prevented the student from accurately solving and likely has held her back from reaching grade-level proficiency.

This early data shows that, despite introducing problems as actionable stories and including familiar contexts that students could relate to, participants initially struggled to access story problems. This finding is not surprising given their low levels of math proficiency. However, towards the end of the second week of the study, access began to improve. Field notes from the beginning of the third week reflect this shift, noting that for the first time, all selected participants were able to accurately retell the story on their first try. Only 4 notes about story problem access appear in the last two weeks of the study, a decrease of more than half from the 11 notes recorded in the first two weeks. Additionally, three participants who had initially struggled heavily to access problems began to do so with much less difficulty and coaching. In the final week of the study, two of these participants were able to access every story problem. This change shows that, as the study went on, more participants were able to access problems and solve in ways that matched the action of the story. This increase in access is crucial in moving low-performing students closer to grade-level.

Strategy sophistication. Field notes and work samples collected over the course of the study also show consistent movement towards more sophisticated and efficient solution strategies. Throughout the study, three common strategy types emerged and made up the large majority of solutions utilized. These strategies included Direct Modeling with no Base 10 evidence, Counting, and Direct Modeling with Base 10 evidence (see Figures 1, 2, and 3 for examples). Each week, the researcher recorded participants' solution for one problem in order to capture trends and compare strategy choice from week to week. Table 8 shows the frequency of these strategies as the study progressed. Consistent with findings from the Early Base Ten Assessment, this data shows steady movement each week away from the less sophisticated strategy of Direct Modeling with no Base 10 evidence towards the more sophisticated and efficient strategies of Direct Modeling with Base 10 evidence and Counting. Looking at individual participants' work samples across the four-week period reveals that this willingness to try new strategies happened at the aggregate and individual level. All participants, at some point in the study, tried each of these three strategies at least one time. Additionally, even though the numbers got larger in the later weeks, participants used more complex strategies. Though these work samples show that participants were not always able to successfully use new strategies accurately on their first try, the data indicates that they were willing to try new strategies after being exposed to them in the public share. This finding answers sub-question 1 and supports the idea that, even without direct instruction, below grade-level students were able to internalize and utilize new and increasingly complex solution strategies after four weeks of CGI-based math instruction.

Figure 1. Direct Modeling with no Base 10 evidence.
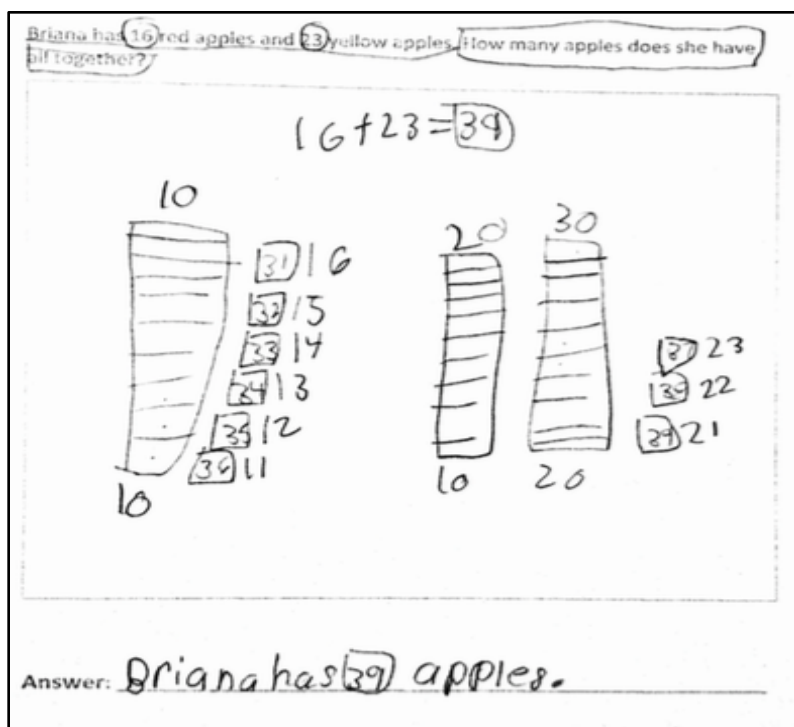

Figure 2. Counting.

Figure 3. Direct Modeling with Base 10 evidence.

Table 8

Frequency of Strategy Use

	Direct Modeling Without Base 10	Direct Modeling With Base 10	Counting
Week 1	7	1	0
Week 2	2	4	1
Week 3	2	2	3
Week 4	1	4	3

Research sub-question 2 examined whether participants' abilities to explain their mathematical thinking improved during the study. Though students were able to incorporate new strategies over the course of the study, there is not sufficient evidence to suggest that participants improved their abilities to explain their mathematical reasoning. With a few exceptions, much of the individual conferring that took place during the study focused on helping participants articulate and explain their mathematical thinking.

During the second week of the study, 16 of 33 field notes recorded include a comment that the student needed heavy scaffolding to accurately explain how they solved. In 9 of these cases, the student solved accurately but could not initially explain how they solved. In the remaining 7 cases, the student did not solve accurately and was unable to explain their solution. These numbers only decreased slightly throughout the study. 13 of 31 conferencing notes from Week 3 included similar notes, and 11 of 29 notes in Week 4 mentioned that students still relied on scaffolding to explain their thinking.

Transcripts of individual conferences between the researcher and participants show that, even though students were able to understand new strategies well enough to use them accurately, they struggled to articulate how they applied them in their own work. In particular, students often needed support using place value language when describing their representations. In one example during the third week of the study, a student was able to Direct Model with Base 10 understanding but could not explain why she used three sticks of ten and five individual cubes to represent the number 35. When prompted, she was able to accurately build other two-digit numbers using tens and ones, suggesting that she understood place value well enough to use it but lacked the language and deep conceptual understanding to articulate her thinking. Participants' difficulty explaining their mathematical reasoning could be caused by a variety of factors. Firstly, it is possible that four weeks was not enough time to develop these skills in students. Because participants were significantly below grade-level, it is possible that much of the mathematical discourse that they had experienced up to this point was inaccessible to them, leading to large deficits in academic language and articulation skills. It is also likely that participants' difficulties articulating their thinking have contributed to their

low math achievement up to this point. In hindsight, the study did not include a heavy emphasis on developing participants' academic language, and future work with below grade-level math students should include more explicit instruction around academic language and explaining one's thinking.

Additionally, it is possible that this difficulty actually reveals that students did not develop strong enough conceptual understanding to be able to explain their thinking. Perhaps they began to develop this understanding, to the extent that they were able to incorporate new strategies into their own work, but did not build a deep enough conceptual foundation to truly articulate their reasoning. Finally, it is possible that students did develop strong conceptual understandings during the study, but that the academic language needed to articulate these new understandings takes longer to develop than the understandings themselves. Perhaps participants simply need more time to be able to explain their thinking than they do to incorporate new conceptual understandings into their solution strategies.

Accuracy. Data from field notes and work samples highlights an overall trend towards higher levels of accuracy throughout the course of the study. Work samples from all participants were collected on the 3rd, 7th, and 15th day of the study. These work samples show steady improvement in participants' abilities to accurately solve story problems, even as the number range increased. On the 3rd day of the study, participants solved a Join, Result Unknown problem with a number range within 20. Only 3 participants were able to accurately solve, while only 4 representations matched the story. On the 7th day, participants solved a Part, Part, Whole-Part Unknown problem with a number range within 50. 4 participants accurately solved, while 2 made representation

errors and 2 made counting errors on accurate representations. Finally, participants

solved a Part, Part Whole-Whole Unknown problem on the 15th day of the study. 7

participants solved this problem accurately, while the remaining participant accurately

represented but made a counting error when solving.

These findings reinforce data from the Early Base Ten Assessment and indicate

that participants' solutions to story problems became increasingly accurate as the study

progressed. Interestingly, participants' accuracy improved in two different areas that

often plague low-performing math students. In addition to improving in counting and

calculation, these work samples also show that students' representations were more

sophisticated and accurate at the end of the study than they were at the beginning. This

improvement addresses research sub-question 3, which examines whether the

intervention model based on CGI practices improved students' conceptual understanding

of the processes carried out when solving story problems involving addition and

subtraction. In order to accurately represent and solve using increasingly sophisticated

strategies, students must conceptually understand the processes of addition and

subtraction, which suggest that their conceptual understanding improved throughout the

course of the study.

This development of conceptual understanding was also evident in conversations

between the researcher and participants during work time. In one recorded conversation,

a participant explained why she counted on from the larger number when adding even

though it was not listed first in the problem. She identified the commutative property of

addition, saying, "I did that because when you add it doesn't really matter which one you

do first. Like I can do this one first or the other one first and I'm still gonna get the right

answer because when I add the two addings [addends] the order doesn't really matter." Another participant explained why he counted up to find the difference between two numbers instead of subtracting, saying "You can subtract and it will give the difference, but I like to add more so I count instead. I'm still right cause I'm finding their difference." With this explanation, the participant showed his understanding of the relationship between addition and subtraction. These conversations show that this constructivist approach to teaching math was effectively able to introduce key mathematical concepts to these below grade-level students, even without direct instruction. Though not all participants verbalized these concepts, the understandings that they represent were seen in nearly all participants over the course of the study.

Conclusion

This chapter has examined data from the math abilities assessment, math beliefs inventory, and qualitative analyses of field notes, work samples and other artifacts, and audio recordings of conversations between participants and the researcher. The data presented in this chapter suggests that participants improved in specific math abilities over the course of the study, including fluency and automaticity, Base 10 understanding, and solution accuracy and sophistication. There is also data to support that the intervention led to increases in students' conceptual understanding of addition and subtraction. These findings indicate that a CGI-based intervention can be an effective approach for improving below grade-level students' foundational and grade-level skills and understandings. However, other key skills like subitizing and explaining one's mathematical thinking did not meaningfully improve over the course of the study, perhaps suggesting that some skills should be more explicitly introduced or addressed in

the context of a constructivist approach. Finally, though the data showed a discrepancy between participants' reported and observed math beliefs, there was a shift towards higher levels of math affect and self-confidence over the course of the study. In Chapter 5, I will discuss the implications of these findings, as well as limitations and next steps from this study that must be considered as educators work to best support below grade-level students.

CHAPTER 5

Conclusion

Introduction

This study asked, *What is the effect of Cognitively Guided Instruction practices on the math beliefs and abilities of below grade-level second grade students?* In order to answer this question, Chapter 4 presented and analyzed the results of the math abilities assessments and math beliefs inventories that participants completed at the beginning and end of the study. It also examined data from field notes, work samples and other artifacts, and audio recordings of conversations between participants and the researcher that were collected throughout the study. This chapter will further discuss these findings and their importance in elementary mathematics education, as well as their limitations and possibilities for future research.

Findings

Students who participated in this Cognitively Guided Instruction-based intervention made significant improvements in fluency, Base 10 recognition, and Early Base 10 understanding. Participants' accuracy and automaticity on the fluency task improved from the pretest to the posttest, indicating that their recall of addition fact improved throughout the course of the study. This result is aligned to Carpenter and

colleagues' finding that, despite not emphasizing instruction of number facts as much as control classrooms, CGI classrooms do not show lower achievement on number facts assessments. In fact, evidence suggests that they show better number recall than control classes (Carpenter et al., 2015). Additionally, by learning these number facts through problem solving, students are allowed to "build upon an understanding of properties of operations and number sense" (Carpenter et al., 2015, p. 5), an opportunity that they would not have if they had simply learned the facts through rote memorization and practice. Even though participants were significantly below grade-level and have struggled with fluency since kindergarten, they made important growth throughout the four weeks of the study.

Additionally, students' performance on two separate tasks shows that their Base 10 understanding improved throughout the course of the study. Participants improved their accuracy and automaticity on the "get to 10" task and also improved their ability to use more efficient and sophisticated solution strategies on the Early Base 10 Assessment as the study progressed. This movement from less sophisticated solutions to more efficient and sophisticated strategies is in line with previous findings that even low-achieving students can "invent, transfer, and retain strategies for solving arithmetical problems" when they participate in CGI-based instruction (Moscardini, 2010, p. 130).

Students' ability to move between strategies and adopt new and more efficient ways of solving during the study suggests that participants developed conceptual understanding of addition and subtraction instead of simply memorizing and regurgitating procedural understandings. This demonstrated growth in conceptual understanding during the CGI-based intervention is consistent with research that suggests that conceptual

understanding can be developed through the use of teacher questioning (Franke et al., 2009) and by letting students develop their own problem solving approaches instead of providing explicit instruction (Fyfe et al., 2014). It is also similar to previous studies that show that CGI-based instructional models allow students to develop deep conceptual understanding, which leads to gains in student achievement (Fyfe et al., 2014; National Council of Teachers of Mathematics, 2000; National Research Council, 2001). Additionally, this shift in strategy use during the study shows evidence of the flexible thinking that CGI-based instruction encourages (Jacobs & Ambrose, 2008). Participants were not bound to one particular way of solving, but were able to flexibly incorporate new solution strategies into their understanding and use them when it was appropriate. This flexibility is an important component of future mathematical success and will be foundational to participants' continued growth.

Though some skills improved during the CGI-based intervention, participants still had difficulty with important skills at the end of the study. Though participants showed some modest improvement in number sense, only 3 of the 8 participants were able to accurately answer all five of the comparison questions on the posttest. Students struggled the most with the comparison between a two-digit number and a three-digit number. Because the study only lasted for four weeks and the participants were significantly below second grade-level when they entered second grade, they did not do any work with three-digit numbers over the course of the study. Research from similar student-centered approaches towards learning suggests that interventions focused on building students' number sense are most productive when they include physical manipulation of the numbers being assessed (Kamii & Rummelsburg, 2008), so it is likely that participants'

scores would have improved if they had worked with three-digit numbers during the intervention.

Similarly, participants showed little improvement in their ability to subitize. Research has shown that subitizing, the ability to automatically recognize a quantity on sight without counting by ones, is a foundational skill that contributes to the development of other math abilities (Clements, 1999). Since participants were significantly below grade-level at the beginning of the study, it is not surprising that they had trouble subitizing when the study began. In fact, their difficulty subitizing has probably contributed to their low levels of proficiency (Clements, 1999). However, despite seeing growth in other important skills during the study, participants improved very little on the subitizing task.

During the subitizing task, many participants attempted to count the number of objects on each card. Some were able to count quickly enough to accurately identify the quantity shown, while others were not. Although most research has found that subitizing precedes counting and is necessary for counting to develop, other researchers have argued that subitizing is actually a form of rapid counting, and that it develops as a shortcut to counting (Clements, 1999). This data seems to contradict the theory that subitizing is a necessary prerequisite for counting, as participants were able to count but not automatically recognize quantities. However, participants had very little practice subitizing during the study. They practiced identifying number cards during warm-ups, but were always given enough time to count the number of objects. It is possible that giving them this time built the habit of counting and that they never developed their subitizing skills because they simply didn't have to. Thus, it is possible that subitizing

requires more practice and direct instruction, and that participants did not improve because they did not have this practice. Clements (1999) supports this idea, saying that "conceptual subitizing must be learned and therefore be fostered" in classroom activities and instruction (p. 402). It is also possible, though, that participants do actually have the ability to subitize but did not demonstrate that ability because they were used to counting when they saw number cards and simply responded out of habit. This could support the idea that subitizing is a shortcut to counting; perhaps students will later subitize when counting becomes too tedious and inefficient.

It is also possible that there is a link between students' language capacity and ability to subitize. Research has found significant positive correlations between spatial-visualization measures and a student's ability to understand, access, and solve word problems (van Garderen, 2006). Students who have a hard time visualizing and spatially representing problems are limited in their ability to develop the language they need to truly understand and access story problems. It is possible that participants struggled to subitize because their limited language abilities have prevented them from developing the visual-spatial skills that they need in order to subitize. It is also possible that their difficulties subitizing are indicative of limited spatial-visualization skills, which contributed to their initial difficulty accessing and solving word problems.

Additionally, research has found that rectangular patterns are easier for children to subitize (Clements, 1999). Participants did show improvement on the rectangular tens frames but struggled more on linear and "domino" arrangements. This is consistent with findings that students as old as college-age struggle more with these complex patterns (Clements, 1999). It is also possible, then, that participants did improve their ability to

subitize less complex arrangements, but that the assessment did not capture that improvement because it did not contain many of the rectangular patterns.

Finally, it is also possible that the four-week study was simply too short for participants to develop sophisticated subitizing strategies. As second graders, these students have been engaged in rigorous math instruction for at least two years, and it is perhaps unreasonable to assume that an additional four weeks is all they needed to mature significantly in this skill. Regardless of why participants showed such little improvement on the subitizing task, their difficulty with this important skill is likely holding them back from developing further sophisticated problem solving strategies, and their dependence on counting by ones will limit their capacity to work efficiently with multi-digit numbers. More research is needed to examine why they are struggling and how they can best be supported.

Participants also showed little improvement in their ability to explain their reasoning during the study. They struggled to use academic language to describe their solution strategies and articulate their thinking as they solved, even after the study concluded. Though research suggests that CGI-based interventions help students develop the skills to explain their thinking (Carpenter et al., 2015), I should have supported students more in their academic language development during the study. During the study, I focused more on strategy use and sophistication than on articulating this thinking. Looking back, I should have included more explicit instruction about academic language development to support students as they explained their thinking.

It is also possible that academic language develops slower than conceptual understanding. Students showed improvements in conceptual understanding during the

study, but not in academic language usage. Once students understand something themselves, they might simply need more time to develop the language they need to articulate their thinking to others. Four weeks is a very short window for significant academic language growth to occur. Future research should continue to examine the development of academic language in CGI-based interventions, and future interventions should include more emphasis on developing these skills in participants.

Finally, participants reported very high levels of math affect (the extent to which they enjoy math) and self-concept (their beliefs in their own math abilities), and low levels of math anxiety at both the beginning and end of the study. There was an initial discrepancy, though, between participants' self-reports and observations of their beliefs. At the beginning of the study, the researcher observed low levels of self-concept and high levels of anxiety. Because research has found that low math self-concept has powerful negative effects on achievement (Pinxten et al., 2014), it makes sense that low-performing students showed lower levels of math self-concept. However, students did not recognize their anxiety or low self-concept, which had negative effects on their achievement. Despite exhibiting low self-concept, students reported and felt overconfident in their abilities. Because anxiety has been found to interfere with mathematical reasoning and strategy use and thus have harmful effects on math achievement (Wigfield & Meece, 1988; Young, Wu, & Menon, 2012), it would make sense that confidence and a lack of anxiety would allow students to experience higher achievement. However, their overconfidence led students to misjudge their abilities and select tasks that were not beneficial for them. At the beginning of the study, students tried to use number ranges that were far too challenging for them, which prevented them from

engaging in meaningful practice. While I expected higher levels of confidence to lead to greater academic success, it actually caused students to struggle and miss valuable opportunities for practice. Future research should continue to examine the impact of overconfidence or misjudging ability in low-performing mathematics students.

Limitations

This study has several limitations that should be addressed in future research. The first of these limitations are its sample size and length. The study only included 8 participants and lasted for 4 weeks, which limits the amount of data that was able to be collected and minimizes the ability to see the impact of the intervention. 4 weeks is a relatively short amount of time to see important improvements, especially with low-achieving students. A longer study would have better allowed me to investigate the impact of the intervention, particularly in the areas of academic language development and subitizing skills. 8 students is also a very small sample size, and a larger sample would have allowed for discovery of larger trends. Additionally, none of the participants were learning English as a second language or had any identified learning disabilities. Future research should examine the impact of CGI-based interventions on these particular populations.

Additionally, this study is limited in its ability to truly draw conclusions about the efficacy of a CGI-based approach to math instruction. Because I implemented it, this study tested my ability to effectively carry out a CGI-based intervention, and not the effectiveness of the approach itself. Many of the primary CGI researchers argue that the success of CGI-based programs is heavily reliant on the instructor's professional development and decision-making (Carpenter et al., 2015; Jacobs, Franke, Carpenter,

Levi, & Battey, 2007). Because of this, any shortcomings of the intervention are likely due to my particular implementation of the model, and are not reflective of the approach itself.

Implications

Despite its limitations, the data collected in this study suggests that there is significant value in using a CGI-based instructional approach with below grade-level students. At the beginning of the study, I wondered if a CGI-based intervention would be effective for some of the lowest performing math students. This study shows that, even in a relatively short period of time, below grade-level students derived numerous important benefits from the intervention. They improved their fluency and Base 10 understanding and developed conceptual understanding and flexible thinking skills that will be important for their future mathematical growth.

These findings also show that particular foundational skills like fluency can be improved in low-achieving students without devoting valuable instructional time to their teaching. In this intervention, students spent their time engaged in rich problem solving exercises and engaging in discourse with each other, instead of doing lower-rigor activities like flashcards or fact practice. However, their fluency improved in the short four-week window of the study. This finding shows that instructional time can be preserved and this important skill can be remediated without sacrificing the grade-level content that students need to build the foundation of their mathematical understanding.

Finally, the high levels of math affect and self-concept that participants reported show that these struggling students have not given up on themselves as mathematicians. They still find math to be enjoyable. I had expected that the lowest-performing students

would have internalized their difficulties in math and would be frustrated by their lack of success. However, this data shows that it is possible for low-achieving students to enjoy math and remain confident in their own mathematical abilities, and future research should continue to examine ways in which this enjoyment can be fostered and preserved.

Future Research

In order to more fully understand the implications of this research and maximize the benefits of a CGI-based instructional approach, future research is needed. Specifically, future research should examine more carefully the development of subitizing in below grade-level students. This data raised questions about how subitizing develops in low-achieving students and its relationship to other foundational skills. Research should continue to examine how subitizing can best be supported in below grade-level students, as well as the impact that it has on the development of other important skills like fluency and strategy sophistication.

Future research should also continue to assess the impact of CGI-based interventions using a longitudinal model. In order to truly understand the impact that this approach has on low-performing students, their progress over longer periods of time must be tracked. While this study suggests that there are several short-term benefits of a CGI-based model, more time is needed to determine whether it can provide the long-term support that these students need to become proficient in math.

Conclusions

In the United States, large groups of elementary students are performing below grade-level in math (Witzel & Riccomini, 2007). This study suggests that CGI-based intervention models have important benefits for these below grade-level students. In

addition to improving specific skills like fluency and Base 10 understanding, this model builds conceptual understanding and flexible thinking, which will be crucial for students' future mathematical success. Instead of explicitly teaching isolated skills and procedural understanding, educators must continue to challenge these struggling students and provide them with opportunities to solve rigorous problems and be led by their own thinking. In doing so, we will build the foundational skills that these students need to be successful academically and as mathematical thinkers in the world.

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APPENDIX A: CALENDAR OF INSTRUCTIONAL TASKS

Week 1	Join-Result Unknown (within 20) Separate-Change Unknown (within 20)	Join-Change Unknown (within 20) PPW-Whole Unknown (within 20)	Join-Result Unknown (within 20) Separate-Result Unknown (within 20)	Separate-Result Unknown (within 20) Join-Result Unknown (within 20)
Week 2	Separate-Change Unknown (within 20) Compare-Difference Unknown (within 20)	Separate-Result Unknown (within 20) Compare-Difference Unknown (within 20)	PPW-Whole Unknown (within 50) Compare-Difference Unknown (within 20)	PPW-Whole Unknown (within 50) Separate-Result Unknown (within 20)
Week 3	Compare-Difference Unknown (within 20) Join-Result Unknown (within 20)	Measurement Division (groups of 10) Compare-Difference Unknown (within 20)	Multiplication (groups of 10) Join-Change Unknown (within 20)	Join-Result Unknown (within 50) Separate-Result Unknown (within 50)
Week 4	Separate-Result Unknown (within 50) Join-Result Unknown (within 50)	Join-Change Unknown (within 50) PPW-Whole Unknown (within 50)	PPW-Whole Unknown (within 50) Separate-Result Unknown (within 50)	Compare-Difference Unknown (within 50) Measurement Division (groups of 10)

*Since instructional decisions in a CGI-based intervention model are often based on the analysis of students' thinking in previous days, adaptations may be made and problem types may occasionally be addressed in a different order than listed here.

Adapted from:

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction.* Portsmouth, NH: Heinemann.

APPENDIX B: FIELD NOTES TEMPLATE

Date: _____

Problem:

Student	Strategy	Solution	Notes

Open-ended Questions Used to Assess Development of Students' Mathematical Thinking

1. How are you solving this problem? Why are you solving that way?

2. Can you tell me what you are doing to solve this problem? Why does that work?

3. Can you tell me what you did? How do you know you can do that?

4. How did you know to do that? Why did it work?

5. Can you tell me how you figured that out?

6. Tell me more about what you are doing. Why did you choose to do it that way?

Questions adapted from:

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction.* Portsmouth, NH: Heinemann.

Carpenter, T. P., Franke, M. L. & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school.* Portsmouth, NH: Heineman.

APPENDIX C: MATH ABILITIES ASSESSMENT

Name: _____   Date: _____

Part A. Fact Fluency

Teacher will show students each fact card, in order that they appear in the table. Students are asked to answer each addition fact as quickly as they can. Teacher will record students' response, if they solved automatically (within 2 seconds), and how they solved if not automatic (counting or direct modeling with fingers).

Fact	Response	Automatic?	Notes
2 + 2			
6 + 6			
5 + 1			
4 + 3			
3 + 6			
6 + 5			

Adapted from:

Kamii, C. (1985). *Young children reinvent arithmetic: Implications of Piaget's theory.* New York, NY: Teachers College Press.

$$\begin{array}{r} 2 \\ +\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ +\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ +\ 3 \\ \hline \end{array}$$

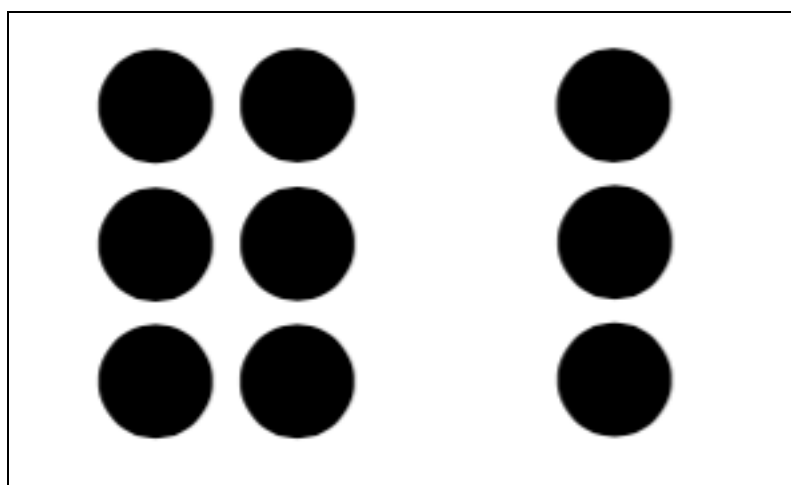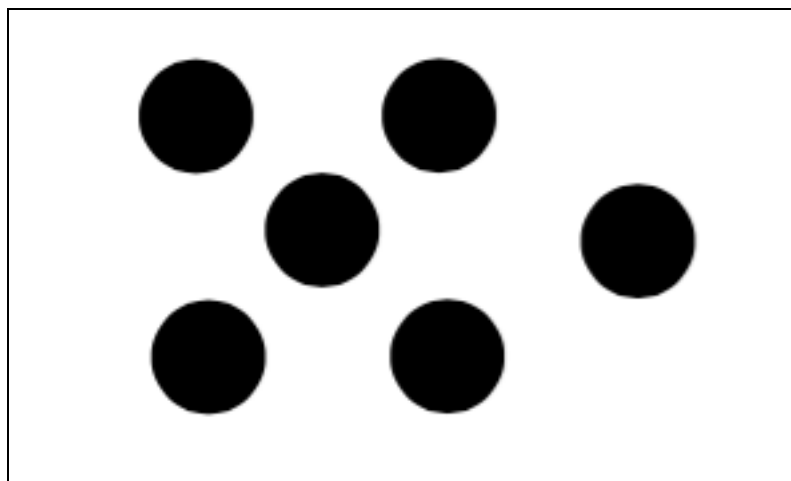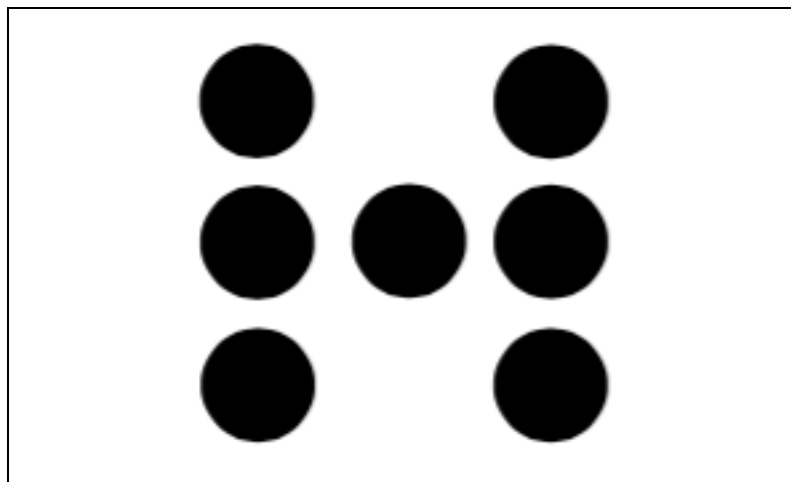$$\begin{array}{r} 3 \\ +\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +\ 5 \\ \hline \end{array}$$

Part B. Subitizing.

Teacher will shuffle cards and show them to students for 2 seconds. Students are asked to name the number of dots they see on the card. Teacher will record students' answer for each card, as well as any notes about their response.

Card	Response	Automatic?	Notes
5—tens frame			
8—tens frame			
3—tens frame			
7—die			
6—die			
9—die			
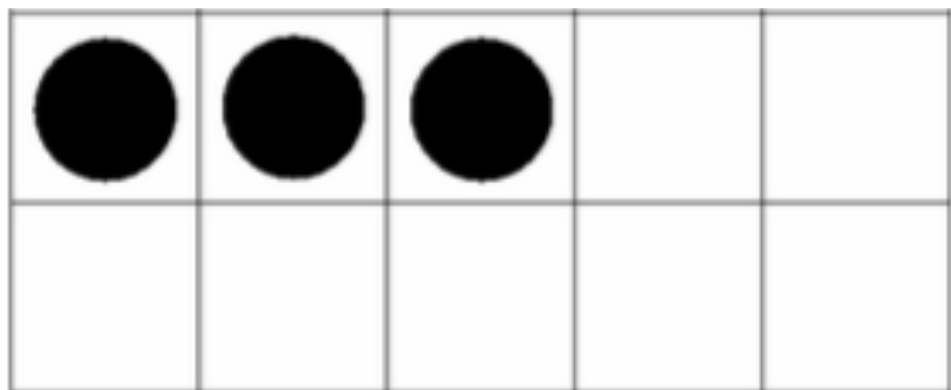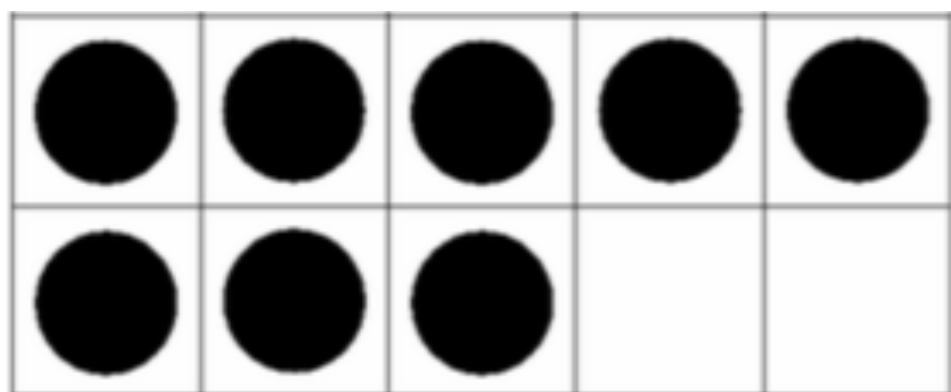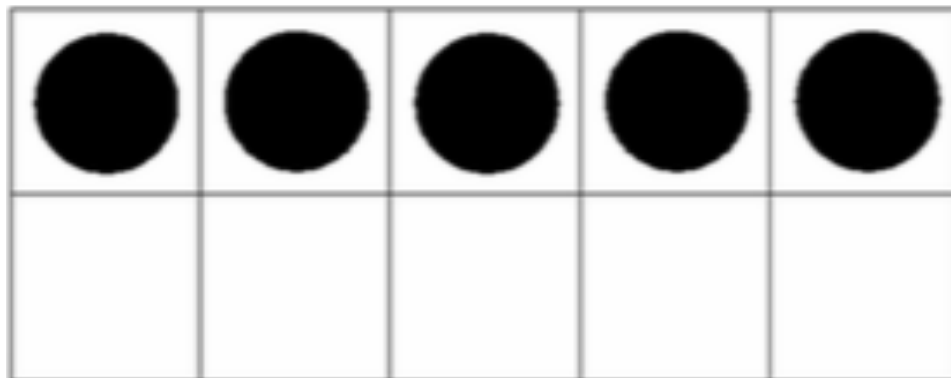
Part C. Comparing Numbers.

Teacher will show students cards one at a time. Each card has two numbers printed on it. Teacher will ask "Which number is greater?" or "Which number is less?" and record students' response.

Numbers	Question	Response
65/56	Which number is less?	
23/26	Which number is greater?	
400/40	Which number is greater?	
10/1	Which number is less?	
6/16	Which number is greater?	

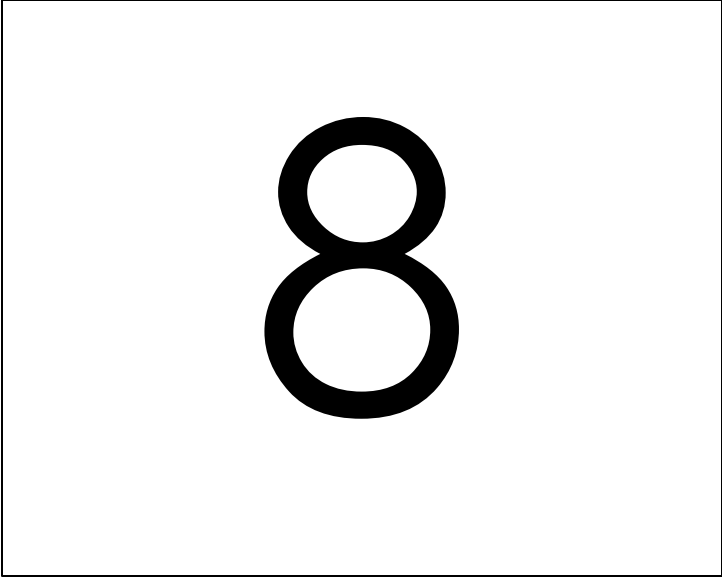| 65	56
23	26
400	40
10	01
6	16

Part D. Base 10.

Teacher will show student each card, one at a time. The teacher will ask the student how much is needed to get to the next ten

Card	Response—Auto or Calc?
8 (get to 10)	
6 (get to 10)	
3 (get to 10)	
1 (get to 10)	
45 (get to 50)	
27 (get to 30)	
38 (get to 40)	

8

6

3

1

45

27

38

Part E. Problem Solving (Early Base Ten Assessment).

Teacher will read each of the following story problems aloud to students and record their solution strategy and answer on the recording sheet. Students will have access to manipulatives (Unifix cubes and Base Ten blocks) and scratch paper to use while solving. After completing the assessment, teacher will fill out the Individual Student Profile for each student.

Join, Result Unknown:
You have 20 cookies on a plate. I give you three more cookies to put on the plate. How many cookies do you have on your plate now?

Separate, Result Unknown:
There are 17 cookies on the plate. Seven of the cookies get eaten. How many cookies are still on the plate?

Join, Change Unknown:
You have 30 cookies already made for a party with friends. How many more cookies do you need to make to have 37 cookies for the party?

Compare, Difference Unknown:
You have 10 cookies on your plate. I have 14 cookies on my plate. How many more cookies do I have than you?

Created by:

Brickwedde, J. (2005). Early base ten assessment: Second edition [Assessment instrument]. Saint Paul, MN; Hamline University.

Early Base Ten Assessment

Directions: Present the following story problems verbally to the child. Record the answer and solution strategy used. The strategy used will expose what stage of base ten development the child is at. The four levels typically witnessed are direct modeling, counting on or counting to (calculating levels), flexible strategies and abstract number strategies (automatic levels). Below is a chart to create a profile of the child based upon the responses to each problem. If a child responds at the automatic abstract number level, there are suggested follow up questions presented. If numbers or the context need to be adjusted, the object is to use these problem types with number choices that draw attention to the explicit base ten relations that exist in composing and decomposing a multidigit number. If the child does not respond to the problem, explicitly direct the child to using cubes or pictures to solve the problem. The assessment administrator should follow the lead of the child to see how far and to what number range his or her base ten understanding might extend before a calculating strategy needs to be utilized.

Join, Result Unknown	Separate, Result Unknown	Join, Change Unknown	Compare, Difference Unknown
You have 20 cookies on a plate. I give you three more cookies to put on the plate. How many cookies do you have on your plate now?	*There are 17 cookies on the plate. Seven of the cookies get eaten. How many cookies are still on the plate?*	*You have 30 cookies already made for a party with friends. How many more cookies do you need to make to have 37 cookies for the party?*	*You have 10 cookies on your plate. I have 14 cookies on my plate. How many more cookies do I have than you?*
Objective: To assess if the child understands how a number is composed in terms of its place value components. To assess a child's intuitive or explicit understanding of 0 + a = a.	**Objective:** To assess if the child understands how a number can be decomposed into its place value components. To assess a child's intuitive or explicit understanding of a − a = 0.	**Objective:** Similar to the Join Result Unknown question but from a different structure.	**Objective:** To assess if a child can compare two numbers with the ones place fixed and tens place is unequal.
Notes:	**Notes:**	**Notes:**	**Notes:**
If the child responds at the *automatic level,* verbally ask, "What if you had 60 cookies and I gave you 8 more?" "What if you had 130 and I gave you 4 more?" "What if it was 105 cookies and I gave you 20 more?"	If the child responds at the *automatic level*, ask "What if it were 74 and 4 were eaten? 124 and 20 were eaten?"	If the child responds at the *automatic level*, ask "What if you had 90 cookies, how many more to have 98 cookies? What if 54 cookies, how many to have 64 cookies?"	If the child does not have an easy time with solving these numbers, try asking the problem with 14 and 24. The teen numbers can be harder for some children. If at the *automatic level*, ask using the numbers 23 & 43 (spreading the distance by greater increments of ten); 136, 236

Individual Student Profile – Early Base Ten Assessment

	JRU (20, 3)	SRU (17,7)	JCU (30,37)	CDU (14,24)
	Adjusted numbers?	Adjusted numbers?	Adjusted numbers?	Adjusted numbers?
Direct Modeling (Calculating Level)				
Counting Strategies (Calculating Level)				███
				███
Flexible, Derived or Abstract Strategies (Automatic Level)				

Created by:

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APPENDIX D: MATH BELIEFS INVENTORY

1. Math is easy for me

2. I can solve any math problem if I put my mind to it

3. Solving math problems makes me feel good

4. Math makes me feel sad

5. I am good at math

6. I can get a good grade in math if I try really hard

7. I am not smart enough to be good at math

8. I know a lot about math right now

9. I want to learn more math

10.   Math makes me feel nervous

11.   I am scared to make a mistake in math

12.   I love solving math problems

Adapted from:

Ayodele, O. J. (2011). Self-concept and performance in secondary school students in
    mathematics. *Journal of Educational and Developmental Psychology, 1*(11),
    176-183.