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# FORMATIVE ASSESSMENT OF ACADEMIC WRITING WITH ENGLISH LEARNERS IN AN ALGEBRA CLASS 

by

## Robert Anderson

A capstone submitted in partial fulfillment of the requirements for the degree of Masters of Arts in Education.

Hamline University
St. Paul, Minnesota

May 2016

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To my wife who took on a little more to give me the time I needed finish while caring for our newborn son. To Arthur, my son, who gave me a new perspective on students, and to my students who push me to learn something new each day.
"These brave claims about the theoretical possibility of expressing complex ideas in any language are not wishful thinking."

- Guy Deutscher, Through the Language Glass


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## CHAPTER ONE: Introduction

My research question grew out of how I see my role changing as an English Learner teacher. Although I would not list language and math as my favorite subjects as a learner, I am drawn to how language works within the subject of mathematics and the role that language plays in student understanding. I also wanted to pair this perspective with our school's initiative of increased use of formative assessment, assessment that gives teachers feedback on how to adjust their instruction to better support all learners. My research question comes at the intersection of these three topics: mathematics, language, and formative assessment. My research question is: how does a writing-based formative assessment as a part of explicit mathematics language instruction affect students' understanding of mathematical concepts?

This first chapter will trace my journey in my understanding of what educational equity is and how it became part of who I am as a teacher. It will also reveal how my perspective developed on the importance of language in learning and how language and content work together to build understanding of ideas and the ability to express those ideas.

## CHAPTER ONE

The cold January wind confronted us as we turned north off of Lake Street and headed for Andersen Elementary, a school in the inner city of a large Midwestern metropolitan area. While many of my college classmates were spending their January Term learning Spanish in Latin America or exploring history and politics in Europe, we
were pushing on against the cutting Minnesota wind that, for college students at 7 am , was strongly suggesting that we should head back to bed.

But we weren't thinking about the cold, the snow, or our friends on the beach. We were anxiously anticipating our first day as classroom assistants at Andersen Elementary. My previous experiences in school as a student shaped my perspective up to this point, but this new experience would change the way I saw schools and students. School had always been a comfortable place for me. I had grown up in schools that consisted of almost entirely white (89.8\% White), English-speaking students (98.4\% Non-EL); students that were just like me (Minnesota Department of Education). After studying the inequities that exist in different schools, reading books like Jonathan Kozol's Savage Inequalities, and talking with classmates who had more diverse educational experiences than me, I was nervous about this new setting. We would be working with groups of students who were much less white ( $95.7 \%$ Non-white) and spoke much less English (72\% EL) than the schools that I attended (Minnesota Department of Education).

The two weeks were challenging. We felt exhausted, like the feeling I now have the first day of school each year after summer break. Except this time we were repeating that first day for two weeks straight. The early mornings, cold January weather, the mid school year grind, and classrooms full of energetic elementary students with diverse needs and broad ranges of abilities, caused us to question whether we saw a future in teaching or not, but by the end we had learned so much that this experience confirmed our initial desires to be teachers. I experienced cognitive dissonance as I tried to understand the differences among the schools that I grew up in and the school we were
working in. I learned the importance of stepping outside my comfort zone. I learned that equity work is hard work but that it is the work we need to focus on.

I signed up for the class because I was considering becoming a teacher and thought that this would be a good experience to see if that is truly what I wanted to do. But it turned out to be much more influential. It not only helped me make up my mind about teaching, but it kicked off my journey in working towards understanding and creating educational equity for English Learners (ELs).

This was my first experience working with students that did not speak English as their first language. I was unaware of their struggle and what it meant to be an English Learner. I remember there were times when it was frustrating when working with students at Andersen Elementary who struggled with math. I always experienced success in math. It didn't cross my mind at the time that what may have been holding these students back was not that they didn't "get it," but that there was a break down in the communication of the ideas. The way we have always taught math works better for some students than it does for others, and for me, a native English speaker, the communication of mathematics concepts always worked well for me. For example, over the years in math class I learned that "times," "multiply," and "product" all signify the function of multiplication. A student who is just learning English might be confused when similar terms are interchanged. Each subsequent field experience during my licensure program was spent working more and more with ELs as I became more interested in how to best help these students learn.

Before this experience I pictured myself as a social studies teacher leading students to fall in love with the content that I always found so intriguing. But the more
experiences that I had with ELs, the more I found myself intrigued by how students learn, not what they learn. The what is important, but the how, the journey, is ultimately the focus of learning and growing. So with each new experience with ELs, I saw the how as the focus of education. And with ELs, language is the key.

The content was not ignored or treated as secondary to how that content was taught, but my role as a teacher was evolving. My focus became how to combine content and language in the classroom to support language diverse students. I was learning that language instruction supports the understanding of the content and the content gives language a context. Through professional development I learned more and more about the role and importance of academic language, the discourse, syntax, and vocabulary, of different content areas. I saw academic language as the key for ELs' academic success while also recognizing that language support is beneficial for not just ELs but also for students from non-white and lower socioeconomic backgrounds.

With a push from another English Language teacher and support from our administration we organized a co-taught Language Arts 11 class. I worked closely with a language arts teacher to develop a curriculum where we taught the Language Arts content with a focus on supporting language development. We provided individualized writing support and differentiated grammar instruction for students within their larger writing assignments. For example, while conferencing with a student who was having trouble writing in a consistent tense, we organized grammar activities to support the student's understanding of the topic. We taught reading comprehension strategies and the language that accompanies these strategies like how to formulate deep questions, how to express different types of connections, and how to use different types of writing structures to
respond to different types of literature. Beforehand, texts like The Great Gatsby and Macbeth were often considered too difficult for ELs, but our comprehensive language support provided ELs access to the same content as every other student.

Building on the success of the co-taught Language Arts class, I organized a cotaught Economics class for seniors. Here, where teachers often would have made modifications that may have lowered expectations, I worked with the economics teacher to build a more supportive curriculum for ELs. Our language support included instruction on reading strategies, note taking, and vocabulary. It also included the explicit modeling of and the expectation that students would use academic language to express their ideas. We built in cause and effect language and structures for students to use when accurately describing supply and demand shifters, and we supported students in organizing their academic writing when they wrote about how the principles of economic thinking explain economic enigmas. We brought together language and content so students could understand and express the complex ideas we learned.

Given the amount of reading and writing students are expected to do, language is often understood to be an important aspect of language arts and social studies. However, language is not always considered as essential as it is in language arts and social studies. This is sometimes used to describe math since we predominantly think of math as numbers and formulas. While there are certainly some language-reduced operations in a math class, mathematics has its own language. Whether the language of mathematics is expressed in mathematical expressions or with word problems, students need language to understand a teacher's explanation as well as to be able to express their own thinking.

While supporting ELs with their mathematics homework at the high school level, I often find that students struggle with expressing their ideas. If a student is confused $\mathrm{s} /$ he often struggles to express what exactly $\mathrm{s} / \mathrm{he}$ is confused with. Or a student may know how to complete a mathematical operation and can demonstrate their understanding by "finding x," but when I ask the student to explain how, the student cannot find the words. These dilemmas are not unique to mathematics. ELs often have similar struggles in other content areas; it is clear that many of these struggles are due to a lack of academic language.

In an effort to support academic language development in mathematics I will be co-teaching an Algebra II class during the 2015-2016 school year. We will incorporate explicit academic language instruction into the existing Algebra II curriculum. We are going to focus on implementing writing strategies where students use target vocabulary and sentence structures to explain, in writing and orally, how they solved certain math problems.

Our major initiative next year in our school is implementing formative assessments in the classroom. We are going to pair our work with academic language writing strategies with the formative assessment initiative so that we are collecting information on the impact of our instruction on student understanding. The formative assessment will assess students' ability to express their understanding of the mathematical processes we are studying using target vocabulary and language structures. We want students to understand the mathematics, and we also want students to be able to express their ideas like mathematicians.

This research is important for several different reasons. First, the explicit academic language strategies aim to support ELs' understanding of challenging mathematical concepts. ELs, as a group, are less proficient on the Minnesota Comprehensive Assessments (MCAs) and the math portion of the ACT. In Minnesota, $71.4 \%$ of all students are proficient on the MCA math test but only $47.6 \%$ of English learners are proficient (Minnesota Department of Education). Academic language development in mathematics will support a deeper understanding of mathematical concepts. With a stronger understanding, students will be better prepared for future education and/or careers. Closing the gap in language is an important step in closing the gap in achievement.

Secondly, the formative assessment piece will provide valuable information to my co-teacher and me regarding how effective our strategies are. By tracking student performance and taking close notes, we will be able to reflect and then make betterinformed decisions in our classroom. This will help us be more effective in organizing and delivering instruction that promotes student success.

Finally, implementing this project in a math class provides a space to collaborate more closely with the mathematics department in the hopes of implementing academic language supports more widely across the department and the building. I already have close collaborative relationships with the language arts and social studies departments where we are able to discuss the role of academic language and provide appropriate language support for ELs. In fact, because of our collaboration, some teachers are now able to provide effective language support on their own. I hope to build close collaborative relationships with math teachers. In doing so I hope to support math
teachers in understanding the role of academic language and how to support ELs, and all students, in developing academic language.

## Summary

This chapter covered my journey in working with ELs and my development as a teacher for ELs. The next chapter will review the research and literature connected to the topics of my research question. Chapter Three will outline the research proposal and explain why I chose to organize the research in this manner. Chapter Four explores the results of the study. Chapter Five will analyze the results and draw some conclusions.

## CHAPTER TWO: Literature Review

## Introduction

While supporting $11^{\text {th }}$ and $12^{\text {th }}$ grade English learners with their math homework, I felt like we were always a step behind. We struggled to keep up with understanding enough mathematics, and we were just getting by with each assignment, quiz, or test. I did not feel that I was supporting students in reaching a deeper understanding, nor did I feel like I was supporting students' language development. I needed to come up with a more systematic approach to supporting students' language development. As an English language teacher, I think that an important aspect in supporting student success in mathematics is by teaching the language of mathematics along with the content. My research question is: how does a writing-based formative assessment as a part of explicit mathematics language instruction affect students' understanding of mathematical concepts? I will use two sub questions to explore my research question. The first is: do higher levels of academic language, as measured by a writing rubric, on written formative assessment correlate with English language learners' assessment results in an Algebra class? The second is: what impact does an academic writing activity have on student understanding in mathematics? In order to better understand these questions this literature review will synthesize research on English learners, the role of academic language in content learning, especially in mathematics classes, and the importance of formative assessment in monitoring progress and adjusting instruction for English learners.

## English Learners

The terms English learner (EL) or English language learner (ELL) refer to a student who speaks a language other than Standard American English at home and needs academic English support at school (Minnesota Department of Education, 2015). Students are identified as Limited English Proficient (LEP) for state accountability systems (Minnesota Department of Education, 2015). The term English as a second language (ESL) refers to the program and the teaching license a teacher holds (Minnesota Department of Education, 2015). This term has become less popular as a label for classes and students since a student may be learning English as a third or fourth language. A teacher may also be referred to as an English language (EL) teacher, and the program may go by the same name. Currently, the most common term used to describe students learning English is English Learners (ELs). I will use ELs to describe the students that I am working with in this study since this is what our state and district use.

English learners represent a fast growing section of the United States' population. In 2005, 12.4 percent of the United States' population, 35 million people, was immigrants. In 2008, 20 percent of young people in the United States had immigrant parents. By 2040, it is projected that 33 percent of children will be from an immigrant family (Suárez-Orozco, Suárez-Orozco, \& Todorova, 2008). In 2009, 11 percent of all students in the United States were classified as English learners, and 20 percent of all students were classified as English learners at some point in their education (Callahan, 2013). Current and former English learners are a growing segment of the school population that has language needs that must be addressed while supporting all students.

Students are identified as English learners if there is evidence of another language used at home and if the academic language assessment shows that students qualify for academic language support. If there is presence of a language other than English at home, students are screened using an academic language assessment to see whether they qualify for language support services. Students receive academic language support appropriate to their needs and take an academic language assessment every year to track growth and to see if they meet the exit requirements. The ACCESS assessment is an annual assessment of ELs' academic English that measures students' listening, speaking, reading, and writing in English. Upon meeting the exit requirements, where students have demonstrated that they have acquired academic language comparable to native English-speaking peers, students are no longer consider English learners. For example, in our district students exit the English language program when they score a five or higher on the annual ACCESS language assessment. At this level students have developed academic language that is approaching the language of English-proficient peers (WIDA Consortium, 2012). While students are approaching proficiency levels of their peers, they may need continued support to develop the academic language that is comparable to their peers. Despite no longer being labeled as an EL, these students may still face a language gap compared to native English speaking peers.

Along with immigrant families there are also language gaps across socioeconomic status. Studies show that the vocabulary of 4-year-olds from low-income families is about one-third of the size of children from middle-class families; this makes it more difficult for these children to comprehend reading or participate in activities that rely on vocabulary (Darling-Hammond, 2010). Schleppegrell (2012), referring to the language
students encounter at school, states, "children do not all come to school prepared in the same ways to engage in these new contexts and registrars" (p. 411). Our schools have students with major language gaps. These students may be immigrants first learning English, may come from immigrant families who do not have a strong grasp of English, may live in poverty, or may be faced with several of these challenges compounding their struggles.

This language gap leads to a potential achievement gap. Johnson (2009) makes this connection: "Gaps in language lead to larger gaps in literacy and learning, and gaps in literacy and learning lead to gaps in achievement" (p.3). The gaps in achievement may lead to different outcomes and different opportunities for these groups of students. According to Callahan's 2013 study, the dropout rate for all students during the 20102011 school year was 14.4 percent. The dropout rate for English learners was 24.8 percent, and the rate for socioeconomically disadvantaged students was 17.6 percent. This disparity has implications for individuals since a high school dropout will earn about $\$ 200,000$ less than a high school graduate and nearly $\$ 1$ million less than a college graduate over his/her lifetime (DoSomething.org). Gaps in language may lead to missed opportunities for individual students and their families. Acquiring the language one needs to be successful in school provides students with the tools they need to graduate high school and pursue more promising opportunities in the future.

Not only do missed educational opportunities have an impact on individuals, but the population growth of immigrant families combined with the low graduation rates could have an impact in shaping our country's civic and economic future (Callahan, 2013). Callahan (2013) states that democratic society depends on informed citizens to
make decisions that are in the best interest of the community, but high school dropouts are less likely to vote. High school dropouts are also more likely to receive social services (Levin, Belfield, Muenning, \& Rouse, 2007 as cited by Callahan, 2013). Having a higher percentage of dropouts in a growing section of the population could complicate the economic and civic stability of our country.

## Academic Language: The Language of School

Acquiring the language necessary for full participation in school is essential to a student's immediate and long-term success. Much research has focused on this type of language, which is referred to as academic language. Teachers may be surprised when a student is able to hold a colloquial conversation proficiently in English but struggles with reading and writing in class. A teacher may reason, if the student is a proficient English speaker, this student must have enough English to understand the content of this class. However, holding a conversation and understanding increasingly complex content require two very different types of English proficiency.

Jim Cummins' research led to the distinction between basic interpersonal communicative skills (BICS) and cognitive academic language proficiency (CALP). Cummins found that it took children about two years to develop BICS, the type of language necessary for conversation, but took five to seven years to acquire CALP, the academic language that a student would need to read a textbook or comprehend a lecture (Himmele \& Himmele, 2009). A student may seem proficient in a conversation, but may struggle with the academic language demands of the classroom. As students advance through their educational experience and the content becomes more complex, the
language used to express more complex ideas also becomes more challenging and it may be more difficult for students to keep up.

There are many different facets to academic language. Himmele \& Himmele recognize the complexity of academic language, but offer their most concise summary: "Academic language is the language of books" (2009, P. 21). Jeff Zwiers (2004) describes academic language as the "linguistic glue that holds tasks, texts, and tests of school together" (p. 60). Academic language is the language, words and phrases, that describe content-area knowledge, express abstract concepts and thinking processes, and create organization and clarity in content discourse (Zwiers, 2004). Johnson (2009) identifies six areas of academic language: the language of the standards, the language of the curriculum, the language of instruction, the language of assessment, the language of textbooks, and the language of cognitive actions. A student faces a wide range of language through his/her day, and since this type of language is usually learned in the classroom, it is essential that teachers teach this language (Himmele \& Himmele, 2009).

Johnson (2009) presents an example of the challenges students face when they do not have a strong grasp of academic language. The example asks the reader to answer two questions and rate his/her level of confidence. The first presents the challenge without knowledge of Spanish academic language.

Solamente queda un pedazo en que se puede construer, y el cine ocupara todo eso completament. En ese frase, la palabra pedazo significa
a. mucho de algo
b. un groupo complete
c. una seccion de tierra
d. la resulta de un chance

Answer: $\qquad$ Confidence: $\qquad$ \%

The second question provides some of the academic language in English.
Only remains un pedazo en question se puede construer, y el theater occupied all eso space. En ese sentence, the word pedazo means
a. great amount
b. complete group
c. section of land
d. result of chance

Answer: $\qquad$ Confidence: $\qquad$ \% (p. 6-7)

By simulating the perspective of an English learner, one can see the importance of acquiring academic language and the impact it has on confidence and understanding content.

## The Language of Mathematics

Each content area has its own vocabulary, syntax, and discourse. Mathematics is sometimes considered less demanding in terms of language because there is less reading and writing than classes like language arts or social studies. Cavanagh (2005) shares that because of the foundation in numbers, math has been considered a universal language, but poses as many challenges for English learners as subjects that include more reading. Barrow (2014) builds on this idea: "math is not a universal language, and educators need to be attentive to the nuances and complexity of the English language" (p.35).

However, Kenney (2005) compares the language of mathematics to a foreign language for many students since the language of mathematics is learned mostly at school and does not "originate as a spoken language" (p. 3). Previous research reveals the specific language demands of mathematics.

Mathematics has its own language. Halliday (1978) identified and defined the challenges of language within the "mathematical register." While mathematics draws on the use of everyday language, it also uses language in new ways. Halliday (1978) defined a register as "a set of meanings that is appropriate to a particular function of language" along with the words and structures that express those meanings, and the "mathematical register" as the meanings that go with the language of mathematics (p. 195).

Moschkovich defines the language of mathematics as not just as a vocabulary list unique to the subject, but "the communicative competence necessary and sufficient for competent participation in mathematical discourse practices" (2012, p. 17). Mathematics is a complex subject, and the language necessary to fully participate in mathematics classes is also complex.

The mathematics register contains a variety of features. Kenney (2005) identifies four major actions of math verbs as a foundation for understanding the processes in mathematics: modeling and formulating, transferring and manipulating, inferring, and communicating. However, these actions are not applied evenly or in the same order and require different types of proficiencies (Kenney, 2005). Mathematics has symbolic notation accompanied by oral and written language and supplemented by graphs and other visual displays (Schleppegrell, 2007). Within those types of communication there is also technical vocabulary, dense noun phrases, being and having verbs, conjunctions
with technical meanings, and implicit logical relationships (Schleppegrell, 2007). For example, students must know terms that are unfamiliar in everyday conversation like "exponent" and "quotient" or words that have double meanings like "table" and what a question is asking when it asks the student to find the "difference" between two numbers (Cavanagh, 2005). Students also must be able use this vocabulary in meaningful patterns (Schleppegrell, 2007).

Understanding the meaning of mathematics vocabulary is not enough, students need the to be able to comprehend and understand the language that connects the vocabulary. Consider the grammatical structure of a long noun phrase in mathematics: the volume of a rectangular prism with sides 8,10 , and 12 cm . This noun phrase contains a classifying adjective (rectangular) before the noun and qualifiers ( 8,10 , and 12 cm ) after the noun (Schleppegrell, 2007, p.143). Deconstructing those dense noun phrases is difficult for English learners.

Schleppegrell (2007) presents an example from O'Halloran's (2003) analysis of how translating a math problem to a written or spoken form requires deep understanding of math and its grammatical structures. The problem is represented symbolically as:

$$
a^{2}+(a+2)^{2}=340
$$

When written or spoken, it can be represented as:
The sum of the squares of two consecutive positive integers is 340 (p.196).
The grammatical challenge here is that the written form presents processes as nouns. Multiplication, division, addition, and subtraction are processes, but are often written or verbally described as things (Schleppegrell, 2007). This use of language is
different from the everyday language students use and is different from the academic language they may be learning in other content classes.

The different types of communication a teacher uses in the classroom also complicate the language of mathematics. Schleppegrell (2007) highlights four different types of talk that a mathematics teacher may use as researched by Setati (2005), Moschkovich (2002), and Gee (1999): procedural, conceptual, contextual, and regulatory. Procedural talk lays out the steps to solving a problem while conceptual talk reasons why certain procedures are used. Contextual talk is used to bring in background information when working with word problems, and regulatory talk is used for classroom management (Schleppegrell, 2007). Kenney (2005) states that students often lack fluency with mathematics because they learned mathematical processes out of context and therefore must work harder to decode the language and context while also working through processes that they do not understand very well. As shown by the research above, the language of mathematics presents a complex challenge for English learners trying to learn the content and the language. Not only must students understand a new language, but they must also recognize the different ways in which that new language is used.

## Building Academic Language in Mathematics

A growing body of research is revealing the best practices to support students learning academic language and mathematics. According to Moschkovich (2012), "instruction should provide opportunities for students to actively use mathematical language to communicate about and negotiate meaning for mathematical situations" (p. 19). Previous research discusses a range of approaches for supporting the development
of academic language across modalities as well as general practices for engaging English learners.

Moschkovich (2012) outlines five recommendations to connect mathematical content to language: 1) focus on students' mathematical reasoning, not accuracy with language, 2) shift to a focus on mathematical discourse practices, 3) recognize and support students to engage with the complexity of language in math classrooms, 4) treat everyday language and experiences as resources, and 5) uncover the mathematics in what students say and do. The attitude underlying these recommendations is that teachers need to use the language students have and build more complex mathematical language from that foundation.

## The Four Modalities in Language Acquisition

Reading, writing, listening, and speaking present different challenges for students learning academic English. Previous research identifies the challenges English learners face and best practices for supporting student development of the four modalities.

## Reading

Beal, Adams, \& Cohen (2010) found that overall math performance correlates with English proficiency. In their study, they found that reading skills were related to math performance, but measures of English conversational proficiency were not. If a student struggles with reading, $\mathrm{s} / \mathrm{he}$ faces challenges in reading in math class as well. Just because the amount of texts that students are reading are shorter does not mean that the texts are easier to comprehend.

In order to successfully read in mathematics class, a student needs to know the appropriate background information, and readers must read to understand the author's
intention (Fuentes, 1998). Along with a lower reading proficiency, an English language learner may also lack the necessary background information needed to understand a mathematical word problem and may need support in developing appropriate background knowledge. In order to be a successful reader in mathematics, students must apply metacognitive behaviors like setting a purpose and activating prior knowledge before reading, recognizing text structures, using fix-up strategies, and monitoring comprehension while reading, and reflecting and summarizing after reading (Fuentes, 1998). Fuentes (1998) also describes a strategy of including an equation under the written word problem so that students can see the pattern of how word problems are represented symbolically. Barwell (2003) treats mathematical word problems as a unique genre that consists of a three-part structure including a set-up, items of information, and a question. An abnormal use of tense is also a common factor (Barwell, 2003). Most of texts that students read are organized into patterns, and recognizing these patterns is important for student comprehension (Johnson, 2010). According to Schleppegrell (2007), it may be important to unpack the dense noun phrases in word problems and make what was implicit in the word problem more explicit for students. It is important for teachers to support students, especially English learners, in understanding the organizational features of mathematics word problems and in applying appropriate active reading strategies with attention to the different grammatical structures or mathematics texts.

## Listening and Speaking

Interaction is important for developing academic language. Unfortunately there are few opportunities in the school day for students to interact as many teachers dominate
most of the talk in the classroom (Echevarria, Vogt, \& Short, 2008). When students are asked to respond it is often for simple recall, which limits students' opportunities to produce more complex language (Echevarria et al., 2008). According to Echevarria et al. (2008), well-organized, meaningful interaction in the classroom increases brain stimulation, motivation, and attention; it also reduces the risk students feel in participating and provides more processing time. While it may be tempting for teachers to dominate classroom talk, English learners need the chance to use English the most. The opportunity to discuss ideas and information promotes language growth. Zwiers (2014) builds on this idea: "language is meant to bridge information gaps, to communicate ideas and information to others who don't already know them-to be used to get things done. Students' language doesn't need to be perfect or even correct, but it needs to communicate."

Zwiers (2014) offers a number of resources like math conversation posters, opinion formation cards, and a math paired conversation protocol for students to have meaningful interaction where students practice speaking and listening in mathematics class and "use the facts, grammar, and vocabulary in connected sentences to clarify, fortify, and negotiate complex ideas." Wilson Vazquez (2014) used a screencasting app as a tool for students to narrate how they solved math problems and found a positive relationship between students' use of more complex language and increased understanding of math content.

Students need the opportunity to use language with one another and to see it modeled by the teacher. Schleppegrell (2007) states that while students need to discuss ideas and support conclusions with each other, talking with each other alone will not lead
to the development of the mathematics register. Adams (2003, as cited by Schleppegrell, 2007) suggests that teachers provide explicit support in helping students recognize and use the technical language of mathematics by making connections between everyday language and the language of mathematics and evaluating students' abilities to use more technical language. Explicit instruction and assessment of the language of mathematics supports students' language development.

## Writing

Writing is another means of output for students to construct their understanding, communicate with others, or demonstrate what they have learned. The National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics (1989) encouraged teachers to build more writing experiences into the teaching of mathematics with the understanding that writing provides the opportunity to practice mathematical communication that supports the development of deeper understanding. Despite this recommendation, in a study on teacher beliefs and practices regarding writing in the mathematics classroom, Quinn \& Wilson (1997) found that high school teachers are changing their beliefs about how mathematics should be taught but are not changing how they teach. Bossé \& Faulconer (2008) state that students learn mathematics more effectively and at a deeper level when writing and reading are directed at learning mathematics.

Academic writing across content areas provides the opportunity to synthesize learning and "is the hallmark of rigorous learning" (Johnson, 2009, p. 101). Academic writing requires more developed language than conversational exchanges and writers do not have immediate access to verbal feedback or clarifying questions (Johnson, 2009). In
the mathematics classroom writing may be constructing and analyzing data tables, creating story problems, explaining mathematical processes, applying methods for writing mathematical notation, or summarizing mathematical concepts (Johnson, 2009). A teacher, as Johnson (2009) suggests, may incorporate writing strategies like writealouds, sentence combining, or collaborative writing to promote the development of academic writing. According to Rothstein \& Rothstein (2007), a teacher may include more writing in order to: gain insights into students' mathematical thinking, identify misunderstandings, assess habits and attitudes, and/or evaluate their own teaching.

As outlined in research, the mathematics classroom offers opportunities to include writing within the curriculum. Previous studies connect the type of writing students produce and writing strategies with engagement, motivation, and deeper content understanding. Factual and technical writing is more important than descriptive writing and students can write about procedures, explanations of findings, and arguments about theorems (Schleppegrell, 2007). In a case study of a class, Badii (2006) found that journal writing in a high school mathematics class increased student motivation and led to higher test scores. Langeness (2011) found that using student-authored word problems and drawing pictures to represent word problems created more engagement and deeper understanding and that the practice of orally explaining steps to solve a problem was helpful for all students.

Other studies recognize the importance of modeling the strategies that are implemented in the classroom. Ediger's (2006) review of writing strategies in the mathematics classroom found that writing in mathematics needs to be modeled in contextual situations where students are learning to write and writing to learn.

Fortescue's (1994) study showed that students writing improved after the teacher modeled the activity where students described a math activity and students read their procedures to each other. The study found that $70 \%$ of students reported that this activity helped them better understand mathematics. Modeling the writing while implementing strategies with technical writing in mathematics can increase engagement, build writing skills, and deepen students' understanding of math.

Kenney (2005) shares that using writing strategies in mathematics class can promote small group interaction and provide additional follow-up opportunities for more writing. Writing forces students to learn the material and should be used at the beginning of a lesson and then used as a tool to help students explain and refine their thinking throughout (Kenney, 2005). Journals, creating similar problems, directed expository writing, and structured writing guides for problem solving are examples of ways a mathematics teacher can include writing in the classroom that encourage students to share a wide range of responses (Kenney, 2005). Rothstein, Rothstein \& Lauber's (2006) Planning Wheel organizes 10 strategies for writing in mathematics that focus on building terminology and vocabulary, planning and organizing, integrating other subjects, and writing creatively. Using a variety of writing in the classroom allows teachers to see both how and why students are learning the content, an important part of the assessment process (Kenney, 2005).

## Assessment of English Learners

Many times when one thinks about assessment in the classroom, usually summative assessments come to mind. Summative assessments are used to evaluate student learning at the end of a learning unit in reference to a standard or benchmark and
may include a final exam, project, or paper (Eberly Center). Much of what we know regarding assessing EL students comes from summative assessments (Alvarez, Ananda, Walqui, Sato, \& Rabinowitz, 2014). Research shows that English learners perform lower than native English speakers on large-scale, summative assessments (Abedi \& Levine, 2013).

One line of research on ELs and assessment focuses on the impact of language within large-scale assessments. Sato, Rabinowitz, Gallagher, \& Huang (2010) sought to study how accommodations make high-stakes assessments more accessible to English language learners by studying the effects of linguistic modifications on the assessment so that students' mathematical understanding is assessed, not their language ability. The researchers found that the linguistically modified test items measured math understanding more reliably than the original test items and recognized a need for more analysis of test items (Sato et al, 2010). Linguistic demands are one factor that limit ELs' success on large-scale assessments.

There are limitations with large-scale assessments that are often used to analyze English learners' academic performance. Durán (2008) notes that these large-scale assessments often measure more than the intended skill because of the wide variety of linguistic and cultural backgrounds of ELs. For example, a test item that refers to a situation in which an EL has little or no experience is also assessing that student's background knowledge. Secondly, large-scale assessments only sample a limited amount of what students know and can do (Durán, 2008). Also, real-world learning environments are too complex to be represented by assessments that present skills and knowledge in an isolated manner (Durán, 2008). So while large-scale, summative assessment data
provides important information, there are limitations in how summative assessment data can be used for supporting the content and language learning of English learners.

## Formative Assessment

Summative assessments are assessments of the learning that has already happened, but formative assessments are assessments used for future learning. Formative assessments are tools that provide ongoing feedback to teachers to improve their teaching and to learners to improve their learning and may include a drawing to represent a topic or a short summary of what was learned after a lecture (Eberly Center). Alvarez et al (2014) define formative assessment as " a continuous cycle that entails gathering evidence of and judging student learning; providing feedback to students about their learning; and using assessment data to adjust subsequent instruction as needed (p. 2). Formative assessment promotes learning, seeks evidence of learning in multiple ways, monitors learning, provides useful feedback, and helps students become autonomous learners (Alvarez et al., 2014). Alvarez et al (2014) argue that formative assessment may be more important for English learners than other students because of the continuous cycle of gauging understanding and providing feedback for students to learn.

It is necessary to consider the role of academic language and second language acquisition in designing formative assessments that seek to measure students' content understanding and language development. When using formative assessment with a lens on English learners, it is important for teachers to understand language demands within tasks, how language is used across disciplines, and how to scaffold language development (Alvarez et al, 2014). In a study on the use of technology to promote the development of academic English speaking skills, Wilson Vazquez (2014) reported that a
classroom teacher used screencasts recorded by students as a formative assessment to plan and differentiated future math lessons for students. Other types of formative assessments could be transformed to assess students' understanding of mathematical concepts and development of academic language skills. Meskill (2010) describes the use of moment-by-moment formative assessments like instructional conversations where the teacher guides students towards the target academic language and focuses on specific forms of language. Kenney (2005) explains that students' written responses reveal what students are thinking and provide insight into how a teacher should approach further learning. Language learning formative assessments should be authentic communication with the complexities of communication including context, production, interactivity, and adaptivity (Meskill, 2010).

## Research Question

My research question comes at the intersection of the topics reviewed above: English learners, academic language and writing in mathematics, and formative assessment. My research question is: how does a writing-based formative assessment as a part of explicit mathematics language instruction affect students' understanding of mathematical concepts?

## Summary

English learners face the challenge of learning new content, a new language, and new content in a new language. While mathematics is sometimes regarded as a content area with fewer language demands, research clearly dispels that myth. Best practice for instruction for ELs in mathematics includes attention to all four language modalities,
reading, writing, listening, and speaking, with meaningful interaction that builds on students' language levels in order to push them to use more complex academic language.

This chapter reviewed the literature connected to my research question. Chapter Three will outline my research proposal and the different types of research methods that will be used to investigate my research question.

## CHAPTER THREE: Methodology

## Research Question

This study was designed to explore the relationship between writing strategies, formative assessment, and student performance in an Algebra II class with English learners. My research question was: how does a writing-based formative assessment as a part of explicit mathematics language instruction affect students' understanding of mathematical concepts? To investigate this question I also explored these questions: do higher levels of academic language, as measured by a writing rubric, on written formative assessment correlate with English learners' assessment results in an Algebra class? And what impact does an academic writing activity have on student understanding in mathematics?

This chapter provides an overview of the research methods that I used to investigate these questions and why I chose to design my research in this manner. This chapter begins with an explanation of mixed methods and provides some important background information about the school where the study took place and about the students that participated in the study. Next, I will discuss how I collected data and describe the tools I used. The chapter concludes with a description of how the data will be analyzed.

## Research Paradigm

In order to explore my research question I used quantitative and qualitative data in a mixed methods paradigm. Both types of research offer different advantages. Quantitative research provides means of testing a theory by analyzing the relationship among variables (Creswell, 2009). I incorporated quantitative measures in order to
investigate the correlation between written formative assessments and student performance on quizzes and tests in an Algebra II class. Qualitative research involves collecting data in the participants' setting where the researcher analyzes and interprets the data to connect to larger themes (Creswell, 2009). I included qualitative measures in my research design because the context in which the research takes place is important. Qualitative measures provide a way to incorporate and better understand the numerous factors that impact learning in the classroom. While the quantitative measures were ultimately what we used to measure student progress, the qualitative measures provide the context of the narrative for each student in the case study.

## Research Strategy

I used a concurrent embedded strategy framework as part of a case study for my research design. In this approach, quantitative and qualitative methods are used to collect data during the same data collection period with one method, quantitative or qualitative, as the primary method that guides the research and the other as a secondary, supportive role (Creswell, 2009). In my study I used the quantitative research to track student progress, and the qualitative measures to provide the supporting narrative. The embedded approach is the best design for my research because the concurrent embedded approach is useful in using qualitative data to describe aspects from a quantitative study that cannot be quantified (Morse, 1991 as cited by Creswell, 2009). There are many different variables in the classroom; therefore, using qualitative research approaches helped provide valuable information that was not evident in the quantitative research approaches. Creswell (2009) states that a concurrent embedded approach provides a means for using quantitative and qualitative data to show two different pictures to
provide a more comprehensive description. A classroom is a complex environment. Ignoring the value of qualitative data in this type of study would have been irresponsible; therefore, the qualitative data provided a structure of a case study to better understand the quantitative measurements.

The quantitative portion of the study will follow a pre-experimental case study design with a group of five students. I identified a baseline of student performance in an algebra class and then exposed students to an intervention.

## Setting and Participants

The study took place in a large suburban high school in the upper Midwest with an enrollment of 2312 students. The school was predominantly white ( $82 \%$ ) with a small population of English learners (1\%). The study took place in an Algebra II classroom where I co-taught the class with a mathematics teacher and in an academic support class for English language learners.

The Algebra II class had 33 total students, five of which were English learners. The mathematics teacher and I taught the class together taking turns leading the class and providing more individualized support while the other taught. We worked on incorporating explicit academic language instruction around the language of mathematics while also teaching the content. Our support for content and language growth included a number of different supports based on the needs of the students. One support was the writing practice that is outlined in this study.

There were a total seven students in the academic support class. Four of these students were also in the Algebra II class. The academic support class was organized to support students' academic language development in all of their classes. Because this
group of students had diverse backgrounds and needs, much of the class consisted of individualized instruction and one-on-one support for reading, writing, listening, and speaking.

Below is a description of each participant in the case study. Pseudonyms were used to protect the identities of the students in the case study.

Billy was a Hmong student who mostly spoke English but also spoke some Hmong at home. He was an 18-year-old senior in high school. He was born in the United States, and was considered a long-term English Language Learner. Billy has been in the same school district his entire school career. His academic speaking and listening skills were stronger than his academic reading and writing skills. He earned a C in his first semester of Algebra I and a D - in the second semester two years prior to taking Algebra II. Billy had a 1.67 cumulative GPA. Billy's overall language skills were classified as emerging; he had general and some specific language of content areas and was able to use some expanded sentences in written paragraphs. Billy was in the cotaught Algebra II class and the academic support class.

Scott was a Spanish-speaking Latino that moved to the United States in the summer of 2015. He was an 18-year-old junior in high school. He started the year with some basic English language skills like being able to greet others and the ability to read numbers. Scott attended school in Central America until moving to the United States. Scott was in the Algebra II class and the academic support class.

William was a Spanish-speaking Latino. He moved to the United States from Central America in August 2015. William was a 17-year-old junior in high school. He only attended school through the 8th grade in Central America due to dangerous
conditions in his community. At the beginning of the 2015-2016 school year William did not know any English. William was in the Algebra II class and the academic support class.

Don was a Spanish-speaking Latino from Central America who moved to the United States in the summer of 2014. He was an 18-year-old junior. He completed 10th grade in the same high school the year before. Don had a 3.67 cumulative GPA. Don's academic English language abilities were considered beginning; he had some general language related to content areas and was able to respond with phrases or short sentences. Don was in the Algebra II class and the academic support class.

Kelly was a Spanish-speaking junior from South America. She lived in the United States for the last two years. Kelly had a grade point average of 3.88. Her academic language abilities were expanding; she had some specific and technical language of content areas and was able to use a variety of sentence lengths of varying complexity. Kelly was only in the Algebra II class. She did not want to take the academic support class so that she could take other electives. Since she was not in the academic support class, she completed the activities from the academic support class on her own.

## Tools

WIDA's writing rubric (Appendix B) was used to evaluate students' writing on a scale from 1-6 across the categories of linguistic complexity, vocabulary usage, and language control. A score of 1 describes writing that contains single words or set phrases of the highest frequency vocabulary that is generally comprehensible when copied from a source. Higher scores on the rubric indicate greater linguistic complexity, more
developed vocabulary, and a stronger control of language up to a score of 6 , which indicates native-like English ability.

The quizzes and tests that are used in this Algebra II class are standard across all of the Algebra II classes in the school. The mathematics teachers collaborated and designed these assessment tools together and aligned them with state standards and district curriculum maps (Appendix G).

The written formative assessments (Appendix D) consisted of a math problem from the unit of study that was already solved and students were asked to describe in writing how the problem was solved. Students completed this assessment without any target language identified on the assessment, but they were free to draw on other resources like class notes while working on this assessment.

The guided academic writing activities (Appendix E) were assignments that students initially completed in their academic language support class, a separate period during the day where students work on the language and academic skills they need to be successful in all of their content classes. After students completed this writing activity several times as a group, we implemented it in the algebra class and students completed it independently. With this assignment, students used key content vocabulary and other academic language like sentence frames and signal words to construct a paragraph describing a mathematical process they studied in Algebra II class.

The interview (Appendix C) was a short list of questions that students completed after a quiz or test. It was a self-assessment used to collect qualitative information regarding how confident they felt on the test, how much they prepared, and what students found challenging about that assessment.

## Methods

The research question that I addressed was: how does a writing-based formative assessment as a part of explicit mathematics language instruction affect students' understanding of mathematical concepts? I investigated this question through two sub questions: 1) do higher levels of academic language, as measured by a writing rubric, on written formative assessments correlate with English language learners' assessment results in an Algebra class? And 2) what impact does an academic writing activity have on student understanding in mathematics?

The case study included a series of individual and guided writing activities followed by teacher made, common assessments. In September students took a basic algebra skills review test (Chapter 0 Test) to establish a baseline of student performance.

Students were then introduced to the formative writing activity before taking the next assessment in class. On this formative assessment students were given a math problem from the current chapter that was already solved and were asked to write a paragraph to describe the steps on how the problem was solved (Appendix D). The formative assessment was scored using a writing rubric from WIDA (Appendix B), an organization that promotes academic language development. After the formative assessment, students took a teacher created test (Chapter 1) in algebra class.

During the second month of the class (Chapter 2), students completed another formative writing assignment (Appendix D), and then completed a guided writing assignment together in the academic support class. As a class, we discussed the problem and developed sentences together using target vocabulary and language frames. Students then took the Chapter Two test (Sample test in Appendix G) and completed a short
teacher created, survey that asked: 1) What did you do to prepare for this quiz/test? 2) What grade do you expect to earn on this assessment? Why? 3) What was challenging about this assessment? (Appendix C).

Following the test I recorded student data. I recorded each student's score and calculated the percentage. I also identified which questions on the test were related to the questions that students wrote about in the formative and guided writing activities. I recorded how students did on the questions on the test that were related to the writing activities and how students did on the questions that were not directly related to the questions used in the writing prompts.

The third unit, at the end of the second month of class, followed the same process as Unit Two. After a couple classes of instruction, students participated in a formative writing activity in the Algebra II class where they explained a problem that was already solved. The next week students completed the guided writing activity together during the academic support class. Students took the Chapter 3.1-3.3 Quiz and I recorded the students overall test score and how they did on the questions that were related and unrelated to the questions on the formative and guided writing tasks. The second half of Unit Three followed a similar pattern. After two or three days of learning new content, students completed a formative writing task in the Algebra II class. A couple of days later students completed the guided writing activity together with teacher guidance during the academic support class. At the end of the second month, students took the Chapter Three test and completed the post assessment survey. After the test, I recorded students’ scores on the test and how each student performed on individual questions related to and unrelated to the writing activities.

The first half of Chapter Four at the beginning of the third month followed the same pattern as Chapters Two and Three, but during the second half of Chapter Four we started doing the guided writing activity as an exit ticket with all of the algebra students. The guided writing activity consisted of a problem that was already solved was projected on the board along with target vocabulary and target sentence frames. All students were instructed to write a paragraph describing the posted solved problem during the last 5 minutes of class using the model or their own language. Like the other assessments, I recorded the number of questions students got correct on the test that were similar to the questions posed during the writing activities.

The remainder of the study followed a formative writing activity, test, guided writing activity, test pattern. After the first few days of instruction in a new unit, students would participate in a formative writing activity as an exit ticket during the algebra class. A problem that was already solved was posted on the board and students were instructed to write a paragraph explaining how the problem was solved. The next week students would complete the guided writing activity as an exit ticket at the end of class. With this activity, a problem that was already solved was posted on the board at the front of the class along with a list of key vocabulary and sentence frames that students could use to write their paragraph. After the guided writing activity, students would take a teachercreated common assessment. Following the assessment, students completed the survey, and I recorded student performance on the assessment and on the questions related to the writing activity and questions unrelated to the writing activities. Because of the pacing of Units Seven and Ten during the sixth month of the study, students completed only one
writing activity before each assessment instead of doing a formative writing and guided writing activity before each assessment.

I kept a journal to record student observations throughout the study to provide context and a narrative to help me better understand the effect of a guided academic writing intervention on English language learners' performance on algebra assessments.

## Data Analysis

The quantitative data, written formative assessment and quiz scores, were weighted more heavily than the qualitative data, interviews and observations. The quantitative data was analyzed to look for patterns between students' formative and guided writing and assessment (quiz and test) scores. The qualitative data was coded and then integrated into the quantitative data by uncovering students' feelings and opinions during the data collection period to create a more holistic picture of student performance.

## Ethics

This study used the following measures to protect participants' identity:

1. Families were notified of the study in English and their home language and signed permission for their student to participate in the study. Students could withdraw from the study at any time without any penalty.
2. The school district reviewed the procedure and data collection methods and gave permission for the study to proceed.
3. The human subjects review board at Hamline University reviewed the procedure and data collection process and granted permission.
4. The data collection and study uses pseudonyms to protect the identity of the students.
5. Data was kept in a password-protected Google Drive account.

## Summary

Chapter Three described the research methods for this study and provided information about the students participating in this study. It described the process and the tools that will be used and explained how the data will be analyzed once it is collected. Chapter Four will explore the results of the study.

## CHAPTER FOUR: Results

## Introduction

This case study took place in an Algebra II and in an Academic Support Class for English learners at a large suburban high school from September 2015 through February 2016. The goal of the study was to explore the relationship between writing strategies, formative assessment, and performance of ELs on tests in an Algebra II class. My research question was: how does a writing-based formative assessment as a part of explicit mathematics language instruction affect students' understanding of mathematical concepts? This chapter presents the results of students' writing scores and assessment scores.

## Results

The quantitative data in the study included the students' formative and guided writing activities scored using the WIDA writing rubric, their overall test results, and their results on the test questions that were similar to the questions in the formative and guided writing activities.

The five students participated in two different writing activities over the course of the study. During an in-class formative assessment, students wrote a paragraph to describe how a problem was solved. Following the formative writing activity students took a quiz later in the week. The next week students wrote a paragraph describing how a problem was solved as part of the guided writing activity. With this activity, target vocabulary and language frames were posted along with the problem. Following the guided writing activity students took a test.

## Writing Levels and Assessment Performance

The WIDA Writing Rubric (Appendix B) measures writing in three areas: linguistic complexity, vocabulary usage, and language control. Each area is scored on a scale of zero to six. Zero indicates that a student did not write anything, and a score of six indicates that the student is reaching native-like English language writing ability.

Linguistic complexity measures how much a student wrote, the variety of the sentences used, and how well the piece is organized. The first example below is a sample from one of Scott's guided writing activities where he scored low on the rubric in the area of linguistic complexity:

Firs, subtitute the poins into the equation. then subtract -3 and -3 that equals -6. Next subtract 1 and 3, that equals -2. Finally divide -6 and -2 the final answer is 3 .

The next example shows the first part of the guided writing activity from Kelly where she scored higher in the area of linguistic complexity:

First thing to do is to know the slope formula, that is $y_{2}-y_{1} / x_{2}-x_{1}=$ slope. The equation for this formula is -3-3/1-3 that equals $-6 /-2$. You can get to know the formula better by plotting what equals what...

Kelly scored higher in the area of linguistic complexity because her writing included a wider variety of sentence structures and included sentences that varied in length.

The second area of the rubric, vocabulary usage, measures the types of vocabulary words that students use in their writing. A student that scores low on the scale uses only the highest frequency vocabulary from school and may lack the vocabulary specific to the
content area. A student that scores higher in the vocabulary usage area uses technical vocabulary from the content area and consistently uses the right word in the right place.

The third area of the rubric is language control. Language control measures how comprehensible the writing piece is. A writing sample that scores low on the rubric is generally comprehensible. The writing may be copied or adapted from a model and comprehensibility may be impeded by errors. A writing sample that scores on the rubric is comparable to English proficient peers. Below is one of William's writing samples that scores low on the rubric in the area of language control:
$y \leq 3 x-2$ First -2 second point sets for to remove the line Leter draw the lene
The sample below is Billy's writing sample from the same activity:
You will have to start at -2 on the graph then up 3 and automatly right $1 . \leq$ its a solid line and shade under.

Billy scored higher than William's in the area of language control because it is more comprehensible.

Table 1 shows average writing scores through the course of the study. The average writing score is the average of all three areas of the writing rubric for each writing activity.

| Table 1 |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Average writing scores |  |  |  |  |
|  | Linguisitic <br> Complexity | Vocabulary <br> Student | 4.2 | Language <br> Control |
| Kelly | 2.5 | 4.3 | Average |  |
| Don | 2.7 | 3 | 4.4 | 4.3 |
| Billy | 1.6 | 2.7 | 3.2 | 2.9 |
| Scott | 1.3 | 1.6 | 2.8 | 2.7 |
| William | 1.5 | 1.6 | 1.6 |  |

Table 1 shows the range of writing abilities of the students in the case study. The writing scores include all writing activities that students completed independently.

Figure 1 graphs students' writing scores over the course of the study and provides a more detailed picture of their writing trends.


Over the course of the study students' writing scores generally improved. There are some fluctuations from score to score, but the general trend over the course of the study shows that students' writing scores were higher at the end of the study than at the beginning.

The central question of this case study was whether there was a connection between more developed academic writing skills and performance on classroom assessments (Appendix H, Table 2). The two students with the highest average writing score also had the average test scores. Scott and William had relatively similar writing scores and relatively similar test scores. Billy had the third highest average writing score but the lowest average test score. Students that had higher average test scores also had more assessments where they scored above the class median.

In order to analyze my research question of how a writing activity affects student understanding of mathematics content, it was important to break down how students did on test questions that were related to the writing prompts and how they did on test questions that were not related to the writing prompts (Appendix H, Table 3). Two students, Kelly and Billy, scored higher on the test questions that were related to the writing activities. The other three students scored lower on the questions related to the writing activities, but their percentages in the two categories, related and unrelated, are within two percentage points. There was not a clear pattern in the results of the questions that were related or unrelated to the writing prompts to determine how the writing activities impacted test scores.

## Formative and Guided Writing Activities

While it was important to examine the relationship between test questions that were related or unrelated, it was also important to consider the relationship between questions that were related to the different types of writing activities. The formative writing activities asked students to describe a math problem that was already solved but did not provide any language support. The guided writing activities provided some target
vocabulary and language frames as supports for students to use as they wrote. For all of the students except Billy, the average writing score on the guided writing activities was higher than the average score on the formative writing activities (Appendix H, Table 4). Students generally scored higher on writing activities when they had language support.

Each writing activity was based on a problem similar to questions on the test. So to investigate the connection between the writing activities and student understanding of the content, I recorded students' performance on the test questions that were directly related to the problems in the writing activities (Appendix H, Table 5). Four of the five students scored higher on the test questions that were related to the formative writing activities than they did on the test questions related to the guided writing activities. Billy was the only student that scored higher on the test questions related to the guided writing activities.

## Analysis

Results of the case study do not present a simple answer for my research question, but field notes and student background information provide some context for understanding some of the themes within the data. These themes include the amount of time each student has studied in the United States, how they used language supports while writing, and considerations of the different types of writing tasks.

Generally, the average writing scores and average test scores reflected the amount of time each student has attended school in the United States. Kelly has lived in the United States for just over two years and has attended school in the same district during that time. Don has lived in the United States for just over a year and a half and has attended school in the same district during that time. Scott and William arrived in the

United States in the summer of 2015 and have attended school since the beginning of the school year. For these four students, Tables 1 and 2 suggest a connection between time in the country, writing scores, and test scores, but while there may be a connection, it is not clear whether one score is dependent on the other.

Along with time in the country, past educational experiences may also explain some of the differences among this group of students. Kelly and Don had the highest average test scores. Kelly and Don had math classes in their home country before moving to the United States and were in the prerequisite math class in the same district the previous year. Within the group of students who have been in school in the United States for less than one year, Scott had the most math classes in his previous educational experiences. Scott attended school in Central America for 11 years and took math classes with similar content before moving to the United States this fall. William attended school for nine years and had over a year gap before resuming school in the United States. Scott shared that his classes in Central America covered some of the similar topics as the Algebra II class while William reported that math class was very different in his home country. Although they lived in the United States for similar amounts of time, the previous experiences in math could account for the differences between those two students in their math test scores.

The exception to this trend is Billy. Billy is a second-generation immigrant, has lived in the United States all of his life, and has attended school in the same district since kindergarten. His writing scores fall below two students who have been in the United States for much less time than he has and he recorded the lowest average test scores. Given Billy's educational background, Billy's test scores also do not fit the trend of the
rest of the group. Despite having the third highest writing score, Billy had the lowest average test score. Billy attended schools in the same district and went through a similar math sequence as other non-ELs in the class. In one survey after Billy did well on a test, he reported that the content was familiar from a previous class. However, it is unclear why some content that was covered in previous classes was familiar while other content was not. Overall, Billy's past educational experiences are much different from the other four students in the case study.

Billy's results highlight the differences in the different types of language covered in the literature review: Basic Interpersonal Communication Skills (BICS) and Cognitive Academic Language Proficiency (CALP). According to Cummins, while it may only take two years for students to acquire the social language needed for communication, BICS, it takes five to seven to acquire academic language, CALP (Himmele \& Himmele, 2009). Billy has developed the BICS to communicate in English with teachers and classmates, but the results of his writing activities suggest that he still needs support in developing his academic language proficiency.

All of the students, except Billy, scored higher on the guided writing activities than they did on the formative writing activities. This is not surprising since the guided activities highlighted target vocabulary and language frames that students could use. While most students scored higher on the guided writing tasks, that did not necessarily transfer to improved performance on questions on the tests that were related to the guided writing tasks. All students, except Billy, scored higher on the test questions that related to the formative writing activities than on the questions related to the guided writing activities.

A true connection between the different writing scores and test scores is difficult to discern. A first look at the results suggests that the formative writing activity resulted in higher test scores since students scored higher on the test questions related to the formative writing activities than the questions related to the guided writing activities. However, the formative writing activities and the guided writing activities covered different types of questions, and the guided writing activities came later in each unit when the mathematical content was usually more complex. The differences in the types of math questions may account for the fact that students scored higher on the test questions relating to the formative writing activities than the questions related to the guided writing activities.

While the formative and guided writing activities do not allow for a direct comparison, the two writing activities provide insight into how students understand mathematical processes and how they engage in the writing process. An important theme in the results is the differences in how students participated in the writing activities and how that participation changed over the course of the study. Students used the vocabulary and language frames in the guided writing activities to varying degrees. Don frequently used the vocabulary and language frames provided on the guided writing assignments and would also use resources like his notes and notes that were written on the board. As the school year went on, William tried to use more of the guided writing features whereas earlier in the year he would rely on using a translator or asking a classmate for help. In many of the guided writing activities there was little evidence that Billy used the vocabulary and language provided. Kelly frequently used the language
supports as a model to create well-developed paragraphs that drew on some of the language provided but also used much of her own language within her writing.

Kelly and Don seemed more familiar with how to use the language supports. It often took the group of newcomers (Scott and William) considerably more time to complete the writing activities. While Kelly, Don, and Billy finished their writing activities in the class time provided, Scott and William often needed additional time during the academic support class to finish their writing. It would be expected for a student will less developed academic writing skills to take a longer time to produce language, but even with additional supports like a translator or key vocabulary and language frames highlighted it would sometimes take William an additional 20 minutes to produce basic writing even when using a translator. However, this changed over the course of the study as students became more familiar with the writing process. As the study progressed, William took less time to complete the writing tasks, and instead of relying on a classmate for help, he would make use of his resources like class notes and a translator more independently. At the beginning of the year, Scott and William did not write anything on the first formative writing activity. For the next few writing activities, they would write in Spanish. Then Scott and William would use a translator to help write in English. By the end of the study both still used a translator, especially during the formative writing activities, but they were completing the writing activities more efficiently and independently while also making use of language and vocabulary from the guided writing activities. One of the highlights of the case study was towards the end of January when everyone finished the guided writing activity in class.

The differences in students' language production and their struggles demonstrate the challenges of the language of mathematics. The complex features of the language of mathematics covered in the literature review came to light over the course of the study. Moschkovich identifies the language of mathematics as more than a vocabulary list unique to the subject; defining it as "the communicative competence necessary" for participation (2012, p. 17). This includes symbolic notation, written language accompanied with graphs and visuals, technical vocabulary, and dense noun phrases (Schleppegrell, 2007). All of these aspects of the language of mathematics presented challenges for students while they were writing. While the activities used mathematics problems that were already solved and also provided some language support, describing these processes required students to use highly developed language. While Kelly demonstrated the English language skills to be able to describe these complex processes, the complexity of the language required to accurately describe the mathematical processes were often beyond the English academic language levels of Scott and William which is possibly why Scott and William seemed frustrated early in the year and often relied on a Spanish-English translator to produce language.

The strongest theme running through the results is how the data reflects different groups of English learners. While individual ELs vary, they can be organized into different groups.

Students can be grouped by the length of time within the country, their language abilities, and past educational experiences. Kelly and Don had similar patterns of writing and testing scores. Both of these students are still relatively new to the United States
since this is only their second full year in school in the United States. While relatively new to the United States, they have developed intermediate English language skills.

While Scott and William have been in the United States a similar amount of time, they have different educational background experiences. They are both newcomers, but William has had an interrupted education since he was not in school for a couple years before coming to the United States. William fits the definition of a Student with Limited or Interrupted Formal Education (SLIFE). A SLIFE student is defined as a student who comes from a home where English is not the primary language, has two years less school than EL peers, is two years below the expected grade level in reading and math, and may be preliterate in their primary language (Minnesota Statutes, 2015). A student that fits this definition has different needs than a newcomer who has had more formal, uninterrupted education. A newcomer who attended school consistently in her/his country before coming to school is more likely to have more developed literacy skills in her/his first language than a student that was not in school consistently. A SLIFE student may be limited in developing academic English if academic language in her/his first language is underdeveloped.

Billy fits into a different category of EL students. Given that he has been in an EL program from all of his educational experience he is considered a Long Term English Learner (LTEL). LTELs, defined as English learners that have been enrolled in school in the United States for 6 years or more and are not making progress towards language proficiency, often have more developed social language skills but may have less developed literacy skills in their first language (Long-term English learners, 2012). Less developed academic language skills in the student's home language may impact the
student's development of English academic language, and a student may have academic language gaps in both languages. While Billy was able to sustain a variety of social conversations, his performance on the writing activities and on tests suggests that he may fit the definition of an LTEL.

These groups of students: intermediate ELs (Kelly and Don), newcomers (Scott and William), SLIFE (William), and LTEL (Billy) have different needs because of their language levels and their past educational experiences. The results of this case study have provided some quantitative and qualitative data on the differences among these subcategories of ELs.

## Summary

Chapter Four reviewed the results of the data that was collected throughout this case study and then incorporated journal notes to analyze the themes to provide a more complete narrative. The theme that came out of the data was the relationship between the time in the country, past educational experiences, and performance in writing and on tests. The data and notes also described differences among student participation in the writing activities and how these differences fit into different subcategories of EL students.

Chapter Five will reflect on what I have learned throughout the case study. It will consider implications and limitations of the data while also making recommendations for future research. Chapter Five will also include a plan for how this information will be shared, and I will reflect on how this case study fits in with the larger picture of my learning as a teacher.

## CHAPTER FIVE: Conclusions

## Introduction

Chapter Five is a reflection on what I have learned throughout this case study as I investigated my research question: how does a writing-based formative assessment as a part of explicit mathematics language instruction affect students' understanding of mathematical concepts? Chapter Five will discuss the implications and the limitations of my case study. It will also recommend areas of future research related and present a plan for communicating my results. Finally, this concluding chapter will describe the capstone's place in my journey as an educator.

## Discussion

As I reflect on the research question that I set out to investigate while considering all of the work that goes into trying to answer a question like this in a complex setting like a classroom, I found that there is not a simple answer to be drawn from the results. While the quantitative data collected for this case study has some limitations, when combined with the qualitative data it builds context around writing in the mathematics classroom from which we can learn. The narrative of the experience of the students in this case study created new learning regarding language development for English learners in a content area that is often times regarded as less demanding in terms of language.

The analysis of the results suggest a connection between overall writing scores and test scores, but this also may be attributed to the amount of time a student has attended school in the United States and their previous educational experiences. A student that has attended school in the United States longer has had more exposure to and direct instruction in English. For example, understanding the directions in class gives
students who have been in the United States for a longer period of time an advantage. Writing, the focus of this study, was studied in the most detail, but the other three language modalities (reading, listening, and speaking) also play a role in how students comprehend the content in the classroom, and it was difficult to isolate the effect of the writing activities.

The case study highlights the different needs of different types of English learners. ELs vary not just in their level of language development but also in their first language literacy and past educational experiences. Intermediate level ELs with first language literacy require different support than newcomers. Newcomers vary depending on their first language literacy and with how many years they have been in school and whether that experience was interrupted or not. Long term English learners may have different needs depending on their strengths and weaknesses and these are likely going to be much different than the needs of newcomers.

The students in the case study did not necessarily perform any better on the test questions related to the writing activities than on test questions not related to the writing activities (Appendix H, Table 3), and there does not appear to be a clear connection between the more supported writing activity and improved performance on test questions (Appendix H, Table 5). However, from the data collected it appears that the guided writing activities scored higher on the writing rubric than the formative writing activities, and perhaps most importantly, students' writing scores improved over the course of the study (Figure 1).

Until I was teaching mathematics everyday I did not realize the large role that language played in a mathematics class. As an EL teacher, I knew that language was
important, but I could not have stated its level of importance for student understanding, especially for English learners. A math teacher shared with me once in reference to an EL student in his mathematics class, "It's math, so there is less language." It appeared that this math teacher may have identified with the attitude described by Cavanagh (2005) in the literature review that mathematics has been considered a universal language because of its foundation in numbers. At that time I did not have the knowledge to refute that belief. However, this case study gives me a counterpoint to the less language in mathematics argument. As covered in the literature review, Barrow (2014) states, "math is not a universal language, and educators need to be attentive to the nuances and complexity of the English language" (p. 35). There are instances where a teacher asks the class to compute and share one number as a single answer, but understanding the complex processes in mathematics requires a specific vocabulary and an understanding of how the language of mathematics fits together to understand teacher's directions, read word problems, and describe the processes.

In this case study students not only had to navigate the language demands, but they also had to understand challenging mathematical concepts while dealing with cultural inequities. Each of these factors (language demands, content, and cultural inequities) occur a spectrum from less to more challenging. Across the language demands spectrum, some units of study required little reading while others had more language demands with more challenging vocabulary or more reading. Regarding content, certain units had mathematical processes that followed easier patterns while others were more challenging. This connects to Schleppegrell's (2007) presentation of

O'Halloran's (2003) analysis of how translating a math problem to written form requires deep understanding of math and its grammatical structures.

Each unit also presented cultural challenges where some units did not include any cultural references, but others had many unfamiliar examples in word problems. For instance, the Chapter Ten test referenced 13 different examples that may or may not be familiar to students. Snowfall measurements, GPA, and a fundraising raffle is likely to be familiar to many students in Minnesota, but these examples added another layer of difficulty for students who are new to the country. Not only did the students in this case study need to learn the content, they also had to navigate varying language demands and possibly confusing cultural references. In the midst of all of these considerations, we were trying to study the impact of writing activities.

One of the advantages of using the writing activities as a formative assessment in the classroom was that it helped us better understand where students needed support. We used formative assessment as Alvarez et al (2014) defined it as a cycle of gathering evidence, assessing learning, providing feedback, and adjusting instruction (p. 2). We used a number of different strategies to support students in understanding the content, the language, and the culture within the constraints of the class. The writing activities were just one piece of what was happening and was what was most closely studied, but it provided a means to analyze the supports for diverse ELs in a mainstream classroom.

## Implications

Co-teaching a mathematics class was an experience unlike my co-teaching experiences in language arts and social studies. The literature review cited a 1997 study that found teachers were not changing how they taught even though their beliefs about
how mathematics should be taught was changing (Quinn \& Wilson). When I came across this study from almost 20 years ago I thought that it might be outdated; it also begged the question why are teachers not changing the way they teach if their beliefs are changing? I now have the perspective of a mathematics teacher, and I feel like this finding, from well before my teaching career began, at least partly describes my experience.

Because of the amount of content that we were required to cover in the co-taught Algebra II class, I felt limited in what we could do. The pacing needed to cover the required content, common assignments, and common assessments across all Algebra II classes created an inexorable pattern in the classroom: warm-up, notes, practice, assessment, and repeat. We were able to do some things differently with two teachers to better to support students, but I still found myself thinking that I was not doing the best I could. I felt that we were almost always moving too fast and we lacked variation in how we helped students engage with the content. Many times students seemed like passive receptors than active learners.

While we may not be able to change the amount of material we are required to cover, although that does not mean that we should not do what we can to push for structural changes, close analysis from this case study provides some insight into opportunities where we can better engage students. Future instruction needs to address equity considerations across language and cultural diversity, include more opportunities to communicate ideas in multiple ways, and include differentiated writing activities.

This case study only analyzed two writing strategies. For the sake of the study, students participated in the same activity. No matter what their language level was,
students did the same assignment. Students were able to draw on different resources to help them with the assignments like class notes, dictionaries, or translators, but because of the different needs of the students and their different language levels, I think that students would have benefitted from more differentiation within the assignments. At times, especially early in the study, the newcomers seemed confused and frustrated with the assignment. It is possible that this was because the process was unfamiliar, but it is also possible that the activities were well beyond their language levels. For the more advanced students like Kelly and Don, the writing activities were at their level or provided enough scaffolding to support them in the activity. However, this was not the case for William and Scott, and more basic writing activities early in the year may have provided the scaffolding for them to develop their academic writing skills at a more appropriate pace.

As identified in the literature review, Mathematics instruction needs to give students opportunities to use language to communicate and negotiate meaning (Moschkovich, 2012). We did this to a degree by focusing on writing in the classroom. We implemented some of the suggestions of best practice covered in the literature review and did what the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics (1989) encouraged teachers to do and built more writing experiences into our curriculum. We emphasized factual and technical writing as suggested by Schleppegrell (2007). We also provided some guiding structures for students as they wrote. A more comprehensive approach would include more attention to all of the language modalities. It would benefit students to write in different ways with classmates and to use academic language in more often in structured
discussions. Writing is only one part of providing students the opportunities to negotiate meaning and build deeper understanding in any content area.

There are also implications regarding equity for the diverse learners in our classroom. One example is the choice of examples used in instruction and assessment. Examples that are unfamiliar for some students create an advantage for other students. If examples are unfamiliar, a teacher is no longer only assessing a student's understanding of the content but also their understanding of the cultural reference.

A teacher also must consider how $\mathrm{s} /$ he teaches the content and the opportunities that are provided for students to interact with the content. As a native English speaker who always did well in mathematics classes, I was always comfortable with the standard approach of notes, practice, assess, and repeat. I need to consider how my advantages impact how I choose to teach students who have different perspectives and needs than I did.

These implications are not limited to students learning English. There were many during the formative writing activities where ELs and non-ELs would share something along the lines of, "I know how to do it, but I can't explain it." Students expressed this directly at times or it was evident in their inability to fully explain the math problems that were already solved. All students vary in their level of academic language development, so it is important that the strategies that are used to support academic language development for ELs are also implemented to support academic language development for all students.

## Limitations

The quantitative data collected has limitations in its ability to present a clear answer to the research question of whether the writing strategies affected students' test scores. However, considerations of the limitations of the data provide important insight into the complexity surrounding teaching, and it requires me to dig deeper into the data and the context to try to understand the situation in a more complete way. The data in this case study is limited because it does not present a complete picture of a complex situation, there was not a control group, and the possibility of teacher bias.

While the data provides a general sense of students' writing ability, it does not show the entire picture. Each writing activity was summarized by a rubric. The student's writing ability and/or the student's understanding of the content could have impacted a student's score on the writing activities. For example, although the math problems used in the writing activities were already solved in order to measure student writing, it may be harder for a student to describe a process if the student does not fully understand the process. On the other hand, the student may understand the process, but struggle with producing the language to accurately describe it. I was trying to address both of these considerations by having the math problem solved and by providing some target vocabulary and language frames, but as discussed previously, the possible lack of coordination between the writing task and any given student's language level limits the attempts to control the tasks.

The quantitative data is limited because of the small sample size. Only 5 students participated in this case study. This is a too small of a number to draw any concrete conclusions from the data alone. A more robust study that included more ELs would
provide more data to analyze any connection between a writing activity and content assessments.

The data is limited because there is not a control group. The test questions that are related to writing tasks and the test questions that are not related to the writing tasks are different, as are the test questions related to formative writing activities and the questions related to the guided writing tasks. Using the formative writing activity and the guided writing activity could have interfered with one another if both writing activities were used to write about the same activity. Ultimately we do not know what students would have scored on any given assessment if they had not done the writing activities. It is possible that scores may have been lower or higher. As a review activity and formative assessment I do not think that it detracted from students' understanding; however, there is always the opportunity cost of what we could have done with that time instead of writing.

The order of the class and activities may also have affected the scores. The formative writing activities came before the guided writing activity in every unit of study. We used the formative writing activity to get a better idea of students' understanding across the class and often adjusted our instruction the next day based on the results of the formative writing tasks. In the class the more complex mathematical processes occurred later in each unit. Therefore, when students participated in the formative writing activities they wrote about less complex mathematical processes, and when they participated in the guided writing activities they wrote about more complex tasks. The complexity of the test questions could be reflected in the results. We could have changed the order of the writing activities but this would have complicated how we used the formative writing activity to plan class. Alvarez et al (2014) states that formative
assessment is used to seek evidence of learning, monitor learning, and provide feedback. The formative writing activities were used to get a sense of where students were at and what type of language support might be needed for when students participated in the guided writing activities.

A student's performance for any given question on a test could be impacted by other factors including how the question was written, how much a student studied, or student attendance. While class assignments generally mirrored how test questions were written, there was some variation in how it was written or what example was used. Students also reported different amounts of studying prior to assessments. Some only completed the review activities while others put additional time in outside of school to study for tests. There is a lot of variation in why students scored what they did on any given test (language demands, mathematical complexity, cultural considerations, time spent studying, absences, and more). Any of these factors could impact a student's overall test score or a student's score on any given test question.

Teacher bias also needs consideration. While the tests were standard across all Algebra II classes, my co-teacher and I were the ones who selected a problem for each writing activity. Our selection of which problem to use for the writing prompts depended on: what processes we thought would provide opportunities for more writing, the pacing of the class, and when we had time to plan the activity together. There is also the possibility of bias in the scoring of the writing assessments. I used a common rubric to assess and score all of the writing prompts. I did this throughout the year and then reviewed all of the scores at the end of the study to double check for consistency. However, since I scored all of the writing activities myself, there is a possibility of bias
within my scoring. Future research should consider my limitations in designing studies to understand the connection between academic language development and performance on content assessments.

## Recommendations for Further Research

As I attempted to answer my research question about how writing activities affect test scores in mathematics, I found myself asking more and more questions. It was the new questions that I had not originally considered that made this exploration more engaging and helped me better understand the complexity of the classroom.

Future research could explore any number of writing strategies to discover which strategies show the most promise for ELs. This research could also investigate whether different writing activities work better for different populations of ELs (newcomers, SLIFE, LTELs, advanced). Additional research could shed new light on how to scaffold and differentiate the writing process for students with different academic writing abilities. In differentiating writing activities and providing writing activities that are at more appropriate levels for newcomers it would be interesting to investigate how to structure writing activities so that, while appropriate for students with less developed writing abilities, the writing activities do not lose any of the complexity of the content. For example, how does a teacher create a writing task that supports students writing about a complex process if they students do not yet have enough developed language to describe the more complex process?

This case study only includes analysis of the writing and performance of English learners. Further research could investigate how writing strategies designed for ELs affect other groups of students and compare academic language development for the different
groups. A good question usually leads to more than answers. As teachers if we expect students to answer the questions that we pose, we need to investigate and answer challenging questions as well. It is in modeling this learning that sustains us.

## Presentation of Results

This case study will be published online on Hamline University's Digital Commons. I will also share individual results with my students. As I meet with each student at the end of the quarter regarding their progress, I will share my findings about their progress. My co-teacher and I will also use the results as we plan to teach a similar course next year. We will use what we have learned about students writing to incorporate more diverse writing assignments that are more appropriately leveled for students at different levels of academic language development.

The results of this study can be used to reach beyond my classroom. I plan to share the findings of this study with other EL teachers in my district as we continue to develop strategies to support ELs in more mainstream classrooms. I also plan to share my findings with the math department in my building and in our district in order to empower classroom teachers to support academic language development in their classrooms for all of our students.

There is also the potential to reach a wider audience by presenting at a conference. I plan to present the findings of this case study in a presentation at the Minnesota Council of Teachers of Mathematics 2017 Spring Conference to reach more teachers of mathematics and the MinneTESOL 2016 Fall Conference to share my findings with more EL teachers.

## Conclusion

Chapter Five discussed my new understandings from the case study as well as its implications and limitations. Working towards educational equity is often like a cold, windy walk on a dark January morning in Minnesota. It can feel tiring, lonely, and at times, futile. Walking with the wind is easy, and so is going with the status quo for those of us who are already adequately equipped. However, it is the walk into the headwind, pushing back against the status quo that creates changes in the lives of others. Teaching the content and language of mathematics presents challenges with pacing and differentiation, but using content as a context for language development and language as a support for understanding content we aim to build deeper understanding with students who have language needs that have been overlooked.

As my case study concluded, the spring was around the corner and the days were beginning to get longer and lighter just as the case study helped me see the depth of language in a new content area and presented more questions.

## REFERENCES

Abedi, J., \& Levine, H. G. (2013). Fairness in assessment of English learners.
Leadership, 42(3), 26-38. Retrieved from
http://ezproxy.hamline.edu:2052/login.aspx?direct=true\&db=keh\&AN=87860324
$\& s i t e=$ ehost-live
Alvarez, L., Ananda, S., Walqui, A., Sato, E. \& Rabinowitz, S. (2014). Focusing formative assessment on the needs of English language learners. Retrieved from http://www.wested.org/resources/focusing-formative-assessment-on-the-needs-of-english-language-learners/

Barrow, M. A. (2014). Even math requires learning academic language. Phi Delta Kappan, 95(6), 35-38. doi:10.1177/003172171409500608

Badii, F. (2006). Promoting interaction and engagement through journals in a high school mathematics classroom: A lesson from the field. Retrieved from http://hamline.summon.serialssolutions.com/

Barwell, R. (2003). Patterns of attention in the interaction of a primary school mathematics student with english as an additional language. Educational Studies in Mathematics, 53(1), 35-59. doi:10.1023/A:1024659518797

Beal, C. R., Adams, N. M., \& Cohen, P. R. (2010). Reading proficiency and mathematics problem solving by high school English language learners. Urban Education, 45(1), 58-74. doi:10.1177/0042085909352143

Bossé, M. J., \& Faulconer, J. (2008). Learning and assessing mathematics through reading and writing. School Science and Mathematics, 108(1), 8-19. doi:10.1111/j.1949-8594.2008.tb17935.x

Callahan, R. (2013). The English learner dropout dilemma: Multiple risks and multiple resources. ( No. 19). Santa Barbara, CA: California Dropout Research Project. Retrieved from http://www.cdrp.ucsb.edu/pubs_reports.htm

Cavanagh, S. (2005). Math: The not-so-universal language. Education Week, 24 (42), 1. Retrieved from http://www.edweek.org/ew/articles/2005/07/13/42math.h24.html Creswell, J. W. (2009). Research design: Qualitative, quantitative, and mixed methods approaches (3rd ed.). Thousand Oaks, CA: Sage Publications.

Darling-Hammond, L. (2010). The flat world and education: How America's commitment to equity will determine our future. New York: Teachers College Press.

DoSomething.org. 11 facts about high school dropout rates. Retrieved from https://www.dosomething.org/facts/11-facts-about-high-school-dropout-rates

Durán, R. (2008). Assessing English-language learners' achievement. Review of Research in Education, 32, 292-327.

Eberly Center. Formative vs summative assessment. Retrieved from https://www.cmu.edu/teaching/assessment/basics/formative-summative.html

Echevarria, J., Vogt, M., \& Short, D. (2008). Making content comprehensible for English learners: The SIOP model (3rd ed.). Boston: Pearson/Allyn and Bacon.

Ediger, M. (2006). Writing in the Mathematics Curriculum. Journal Of Instructional Psychology, 33(2), 120-123.

Fortescue, C. M. (1994). Using oral and written language to increase understanding of math concepts. Language Arts, 71(8), 576-580.

Fuentes, P. (1998). Reading comprehension in mathematics. The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 72(2), 81-88. doi:10.1080/00098659809599602

Gee, J. P. (1999). An introduction to discourse analysis: Theory and method. London and New York: Routledge.

Halliday, M. A. K. (1978). Language as a social semiotic: The social interpretation of language and meaning. Baltimore: University Park Press.

Himmele, P. \& Himmele, W. (2009). The language-rich classroom. Alexandria, VA: ASCD.

Johnson, E. R. (2009). Academic language! academic literacy: A guide for K-12 educators. Thousand Oaks, Calif: Corwin.

Kenney, J. M., \& Association for Supervision and Curriculum, D. (2005). Literacy Strategies for Improving Mathematics Instruction. Alexandria, Va: Assoc. for Supervision and Curriculum Development.

Langeness, J. (2011). Methods to improve student ability in solving math word problems. Retrieved from http://hamline.summon.serialssolutions.com/

Levin, H. M., Belfield, C. R. , Muennig, P., \& Rouse, C. E. (2007). The Costs and Benefits of an Excellent Education for all of America's Children. Center for Benefit-Cost Studies in Education (CBCSE): Teachers College Press, New York.

Long-term English learners, Education Code § AB-2193 (2011-2012).

Meskill, C. (2010). Moment-by-moment formative assessment of second language development. In H.L. Andrade \& C. J. Cizek (Eds.), Handbook of formative assessment (pp. 198-211). New York: Routledge.

Minnesota Department of Education. (2015). English Learners. Retrieved from http://www.education.state.mn.us/MDE/JustParent/EngLearn/index.html

Minnesota Department of Education. (2015). Minnesota Report Card. Retrieved from http://rc.education.state.mn.us/\#

Minnesota Department of Education. (2015). Title III. Retrieved from http://www.education.state.mn.us/MDE/SchSup/ESEA/TitleIII/index.html

Minnesota Statutes 2015, section 124D.59, subdivision 2a ; MINN. STAT. 124.59D (2015) ; MINN. STAT. ANN. 124.59 (2015).

Morse, J. M. (1991, March/April). Approaches to qualitative-quantitative methodological triangulation. Nursing Research, 40(1), 120-123.

Moschkovich, J. N. (2002). A situated and sociocultural perspective on bilingual mathematics learners. Mathematical Thinking and Learning, 4(2\&3), 189-212.

Moschkovich, J. (2012). Mathematics, the common core, and language: Recommendations for mathematics instruction for ELs aligned with the common core. Retrieved from http://ell.stanford.edu/papers/practice

National Council of Teachers of Mathematics (NCTM). 1989. Curriculum and Evaluation Standards for School Mathematics. Reston, Va.: NCTM.

O'Halloran, K. L. (2003). Educational implications of mathematics as a mutisemiotic discourse. In M. Anderson, A. Saenz-Ludlow, S. Zellweger, \& V. V. Cifarelli (Eds.), Educational perspectives on mathematics as semiosis: From thinking to
interpreting to knowing (pp. 185-214). Brooklyn, NY, and Ottawa, Ontario: Legas.

Quinn, R. J., \& Wilson, M. M. (1997). Writing in the mathematics classroom: Teacher beliefs and practices. Clearing House, 71(1), 14.

Rothstein, A., \& Rothstein, E. (2007). Writing and mathematics: An exponential combination. Principal Leadership, 7(5), 21.

Rothstein, A., Rothstein, E., \& Lauber, G. (2006). Write for mathematics. Thousand Oaks, CA: Corwin.

Sato, E., Rabinowitz, S., Gallagher, C., \& Huang, C. (2010). Accommodations for English language learner students: The effect of linguistic modification on math test item sets (NCEE Report 2009-4079). Washington, DC: National Center for Education Evaluation and Regional Assistance.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. Reading \& Writing Quarterly, 23(2), 139-159. doi:10.1080/10573560601158461

Schleppegrell, M. J. (2012). Academic language in teaching and learning. The
Elementary School Journal, 112(3), 409-418. doi:10.1086/663297
Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. Journal for Research in Mathematics Education, 36(5), 447-466.

Suárez-Orozco, C., Suárez-Orozco, M. M., \& Todorova, I., (2008). Learning a new land: Immigrant students in American society (1st ed.). Palo Alto, Calif; Cambridge, Mass: Belknap Press of Harvard University Press.

WIDA Consortium. (2012). Performance definitions for the levels of English language proficiency in grades K-12. In The English language learner can do booklet (4). Retrieved from: https://www.wida.us/downloadLibrary.aspx

WIDA Consortium. (2013). Writing rubric of the WIDA Consortium, Grades 1-12. In English Language Proficiency Standards and Resource Guide, 2007 Edition, Seventh Printing (RG-56). Retrieved from: https://www.wida.us/downloadLibrary.aspx

Wilson Vazquez, A. (2014). Math talk: Screencasting and English learners. Retrieved from http://hamline.summon.serialssolutions.com/

Zwiers, J. (2004). The third language of academic English. Educational Leadership, 62(4), 60-63

Zwiers, J. (2014). Academic language \& literacy. Retrieved from http://www.jeffzwiers.org/

## APPENDIX A

Can-Do Descriptors of WIDA Levels

Can Do Descriptors: Grade Level Cluster 9-12
For the given level of English language proficiency and with visual, graphic, or interactive support through Level 4, English language learners can process or produce the language needed to:

|  | Level 1 Entering | Level 2 Beginning | Level 3 Developing | Level 4 Expanding | Level 5 Bridging |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | - Point to or show basic parts, components, features, characteristics, and properties of objects, organisms, or persons named orally <br> - Match everyday oral information to pictures, diagrams, or photographs <br> - Group visuals by common traits named orally (e.g., "These are polygons.") <br> - Identify resources, places, products, figures from oral statements, and visuals | - Match or classify oral descriptions to real-life experiences or visuallyrepresented, content-related examples <br> - Sort oral language statements according to time frames <br> - Sequence visuals according to oral directions | - Evaluate information in social and academic conversations <br> - Distinguish main ideas from supporting points in oral, content-related discourse <br> - Use learning strategies described orally <br> - Categorize content-based examples described orally | - Distinguish between multiple meanings of oral words or phrases in social and academic contexts <br> - Analyze content-related tasks or assignments based on oral discourse <br> - Categorize examples of genres read aloud <br> - Compare traits based on visuals and oral descriptions using specific and some technical language | - Interpret cause and effect scenarios from oral discourse <br> - Make inferences from oral discourse containing satire, sarcasm, or humor <br> - Identify and react to subtle differences in speech and register (e.g., hyperbole, satire, comedy) <br> - Evaluate intent of speech and act accordingly |  |
|  | - Answer yes/no or choice questions within context of lessons or personal experiences <br> - Provide identifying information about self <br> - Name everyday objects and pre-taught vocabulary <br> - Repeat words, short phrases, memorized chunks of language | - Describe persons, places, events, or objects <br> - Ask WH-questions to clarify meaning <br> - Give features of contentbased material (e.g., time periods) <br> - Characterize issues, situations, regions shown in illustrations | - Suggest ways to resolve issues or pose solutions <br> - Compare/contrast features, traits, characteristics using general and some specific language <br> - Sequence processes, cycles, procedures, or events <br> - Conduct interviews or gather information through oral interaction <br> - Estimate, make predictions or pose hypotheses from models | - Take a stance and use evidence to defend it <br> - Explain content-related issues and concepts <br> - Compare and contrast points of view <br> - Analyze and share pros and cons of choices <br> - Use and respond to gossip, slang, and idiomatic expressions <br> - Use speaking strategies (e.g., circumlocution) | - Give multimedia oral presentations on grade-level material <br> - Engage in debates on content-related issues using technical language <br> - Explain metacognitive strategies for solving problems (e.g., "Tell me how you know it.") <br> - Negotiate meaning in pairs or group discussions | 를 |

The Can Do Descriptors work in conjunction with the WIDA Performance Definitions of the English language proficiency standards. The Performance Definitions use three criteria (1. linguistic complexity; 2. vocabulary usage; and 3. language control) to describe the increasing quality and quantity of students' language processing and use across the levels of language proficiency.

WIDA Consortium. (2012). Performance definitions for the levels of English language
proficiency in grades K-12. In The English language learner can do booklet (4).
Retreived from: https://www.wida.us/downloadLibrary.aspx

Can Do Descriptors: Grade Level Cluster 9-12
For the given level of English language proficiency and with visual, graphic, or interactive support through Level 4, English language learners can process or produce the language needed to:

|  | Level 1 Entering | Level 2 Beginning | Level 3 Developing | Level 4 Expanding | Level 5 Bridging |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | - Match visual representations to words/phrases <br> - Read everyday signs, symbols, schedules, and school-related words/phrases <br> - Respond to WH- questions related to illustrated text <br> - Use references (e.g., picture dictionaries, bilingual glossaries, technology) | - Match data or information with its source or genre (e.g., description of element to its symbol on periodic table) <br> - Classify or organize information presented in visuals or graphs <br> - Follow multi-step instructions supported by visuals or data <br> - Match sentence-level descriptions to visual representations <br> - Compare content-related features in visuals and graphics <br> - Locate main ideas in a series of related sentences | - Apply multiple meanings of words/phrases to social and academic contexts <br> - Identify topic sentences or main ideas and details in paragraphs <br> - Answer questions about explicit information in texts <br> - Differentiate between fact and opinion in text <br> - Order paragraphs or sequence information within paragraphs | - Compare/contrast authors' points of view, characters, information, or events <br> - Interpret visually- or graphically-supported information <br> - Infer meaning from text <br> - Match cause to effect <br> - Evaluate usefulness of data or information supported visually or graphically | - Interpret grade-level literature <br> - Synthesize grade-level expository text <br> - Draw conclusions from different sources of informational text <br> - Infer significance of data or information in grade-level material <br> - Identify evidence of bias and credibility of source |  |
| 年 | - Label content-related diagrams, pictures from word/phrase banks <br> - Provide personal information on forms read orally <br> - Produce short answer responses to oral questions with visual support <br> - Supply missing words in short sentences | - Make content-related lists of words, phrases, or expressions <br> - Take notes using graphic organizers or models <br> - Formulate yes/no, choice and WH- questions from models <br> - Correspond for social purposes (e.g., memos, e-mails, notes) | - Complete reports from templates <br> - Compose short narrative and expository pieces <br> - Outline ideas and details using graphic organizers <br> - Compare and reflect on performance against criteria (e.g., rubrics) | - Summarize content-related notes from lectures or text <br> - Revise work based on narrative or oral feedback <br> - Compose narrative and expository text for a variety of purposes <br> - Justify or defend ideas and opinions <br> - Produce content-related reports | - Produce research reports from multiple sources <br> - Create original pieces that represent the use of a variety of genres and discourses <br> - Critique, peer-edit and make recommendations on others' writing from rubrics <br> - Explain, with details, phenomena, processes, procedures |  |

The Can Do Descriptors work in conjunction with the WIDA Performance Definitions of the English language proficiency standards. The Performance Definitions use three criteria (1. linguistic complexity; 2. vocabulary usage; and 3. language control) to describe the increasing quality and quantity of students' language processing and use across the levels of language proficiency.

WIDA Consortium. (2012). Performance definitions for the levels of English language proficiency in grades K-12. In The English language learner can do booklet (4).

Retreived from: https://www.wida.us/downloadLibrary.aspx

## APPENDIX B

WIDA Writing Rubric (9-12)

| Writing Rubric of the WIDA ${ }^{\text {TM }}$ Consortium* Grades 1-12 |  |  |  |
| :---: | :---: | :---: | :---: |
| Level | Linguistic Complexity | Vocabulary Usage | Language Control |
| $\begin{gathered} 6 \\ \text { Reaching } \end{gathered}$ | A variety of sentence lengths of varying linguistic complexity in a single tightly organized paragraph or in well-organized extended text; tight cohesion and organization. | Consistent use of just the right word in just the right place; precise Vocabulary Usage in general, specific or technical language. | Has reached comparability to that of English proficient peers functioning at the "proficient" level in statewide assessments. |
| $5$ <br> Bridging | A variety of sentence lengths of varying linguistic complexity in a single organized paragraph or in extended text; cohesion and organization. | Usage of technical language related to the content area; evident facility with needed vocabulary. | Approaching comparability to that of English proficient peers; errors don't impede comprehensibility. |
| 4 <br> Expanding | A variety of sentence lengths of varying linguistic complexity; emerging cohesion used to provide detail and clarity. | Usage of specific and some technical language related to the content area; lack of needed vocabulary may be occasionally evident. | Generally comprehensible at all times, errors don't impede the overall meaning; such errors may reflect first language interference. |
| 3 <br> Developing | Simple and expanded sentences that show emerging complexity used to provide detail. | Usage of general and some specific language related to the content area; lack of needed vocabulary may be evident. | Generally comprehensible when writing in sentences; comprehensibility may from time to time be impeded by errors when attempting to produce more complex text. |
| 2 <br> Emerging | Phrases and short sentences; varying amount of text may be copied or adapted; some attempt at organization may be evidenced. | Usage of general language related to the content area; lack of vocabulary may be evident. | Generally comprehensible when text is adapted from model or source text, or when original text is limited to simple text; comprehensibility may be often impeded by errors. |
| 1 <br> Entering | Single words, set phrases or chunks of simple language; varying amounts of text may be copied or adapted; adapted text contains original language. | Usage of highest frequency vocabulary from school setting and content areas. | Generally comprehensible when text is copied or adapted from model or source text; comprehensibility may be significantly impeded in original text. |

* Note: Raters at MetriTech. Inc. use this rubric to rate responses on the ACCESS for ELLs test. Teachers and test

WIDA Consortium. (2013). Writing rubric of the WIDA Consortium, Grades 1-12. In
English Language Proficiency Standards and Resource Guide, 2007 Edition,
Seventh Printing (RG-56). Retrieved from:
https://www.wida.us/downloadLibrary.aspx

## APPENDIX C

Interview Questions

Below are the questions that students will respond to after completing a quiz or test in Algebra II:

1) What did you do to prepare for this assessment?
2) What grade do you expect to earn on this assessment? Why?
3) What was challenging about this assessment?

## APPENDIX D

Formative Writing Activity Samples

## Chapter Two Formative Writing



Chapter Three Formative Writing

## Exit Slip:

Explain how I could use elimination to solve this problem. You don't actually have to solve it


Chapter Five Formative Writing


Chapter Ten Formative Writing

## Exit Slip:

Explain how the expected value was found:

| Payoff | $\$ 3$ | $\$ 5$ | $\$ 1$ | $-\$ 3$ | $-\$ 2$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Probability | 0.10 | 0.12 | 0.5 | 0.1 | 0.18 |

$.3+.6+.5+-.3+-.36$
$=0.74$

## APPENDIX E

Guided Writing Activity Samples

## Chapter Three Guided Writing



Chapter Four Guided Writing


## Chapter Five Guided Writing

## Exit Slip:

Explain how the following exponential equation was solved:

## Original Equation: <br> $$
5.2^{x}-4=33
$$

Language:

$$
5.2^{x}=37
$$

- First, ... Next, ... After that,... Finally, ...
- Rewrite using $\qquad$ -

$$
\log _{5.2} 37=x
$$

- Rewrite so $\qquad$ -.
-Rewrite so

$$
\frac{\log 37}{\log 5.2}=x
$$

Vocabulary:
Exponential Form, Log Form, Change of base Property, Add, subtract, multiply, divide

```
    2.19 = x
```

Chapter Ten Guided Writing


APPENDIX F
Student Writing Samples

## Kelly Chapter One Formative Writing

increase by $3 \%$ each year. Write a recursive formula for calculating his salary.
$U_{0}=42,000$
$U_{n}=U_{n-1} * 1.03$
$42000,43260,445578,45894.534, \ldots$
Explain how the problem was solved.


## Kelly Chapter Two Guided Writing

A student solved the problem below. Chapter 2 Guided Writing

Here are students' scores from a history test: 26, 68:72, Med
Are there any outliers?

| $M_{\text {in }}: 26$ | $I Q R=Q_{3}-Q_{1}$ |
| :--- | :--- |
| $Q_{1}: 70$ | $I Q R=85-70$ |
| $M_{\text {ed }}: 76$ | $I Q R=15$ |
| $Q_{3}: 85$ |  |
| $M_{a x}: 97$ |  |

Outlier Fences
7
26 is an outlier
(1)

Min: 26
$I Q R=Q_{3}-Q_{1}$
$I Q R=85-70$
$I Q R=15$

Using the vocabulary and language in the box, explain how to solve the problem.

| Key Vocabulary | Supporting Language |
| :--- | :--- |
| 5 number summary <br> minimum <br> quartile 1 <br> median <br> quartile 3 <br> maximum <br> interquartile range <br> formula <br> outlier fences | First, next, after that, then, finally |

first yow have to organize the number or you can put the secpence in th Calculator $B$ it will gill yaw the med, Q1, Q3, max, min B X (mean) after having those 5 number Summary. Take the solve the I QR by taking the Q3-Q that would be $85-70=15$. Having the IDe now the oullie is (am be solved that means $Q_{1}-(1.5 * I Q R) B(13+11.5 * 1 Q R)$ that means $\rightarrow 0-(15 * 15) \quad 85+15 \times 15)=475 \$ 1075$ Because 26 is less than 47.5 that means is an outlier

## Kelly Chapter Three Formative Writing



## Kelly Chapter Three Guided Writing



## Kelly Chapter Seven Guided Writing



## Don Chapter One Formative Writing

Jeff has received a job that the starting pay in $\$ 42,000$ for the first year and his salary will increase by $3 \%$ each year. Write a recursive formula for calculating his salary.

$$
\begin{aligned}
& U_{0}=42,000 \\
& U_{n}=U_{n \cdot 1} * 1.03
\end{aligned}
$$

## Explain how the problem was solved.


answer is


## Don Chapter Two Formative Writing

Algebra II 2.1 Formative Writing
Below is a five number summary from a data set..
$9,10,13,15,20,21,25,26,26,30,45$

## Minimum: 9

Quartile 1: 13
Median: 21
Quartile 3: 26
Maximum: 45

## Explain how a student found the five number summary.

$M_{i}$. is the number mare smallest.
Qi: 15 the mideam of 9 and 21 of the data.
Med. is the middle number.
Q3. Is the midean of the second term and the last number M. is the number note bigger

Don Chapter Four Guided Writing


Don Chapter Five Guided
First Rewrite in log form.
next Rewrite using the change of mise, property A ter that Calculate
fondly check with calculator

Billy Chapter One Formative Writing

Jeff has received a job that the starting pay in $\$ 42,000$ for the first year and his salary will increase by $3 \%$ each year. Write a recursive formula for calculating his salary.

$$
\begin{aligned}
& U_{0}=42,000 \\
& U_{n}=U_{n-1} *+3
\end{aligned}
$$

## Explain how the problem was solved.



Billy Chapter Two Formative Writing

Algebra II 2.1 Formative Writing
Below is a five number summary from a data set..

$$
9,10,13,15,20,21,25,26,26,30,45
$$

Minimum: 9
Quartile 1: 13
Median: 21
Quartile 3: 26
Maximum: 45

Explain how a student found the five number summary.


Q1 is between the middle of mini \& median. And
the $Q_{3}$ is between median i Max.

## Billy Chapter Three Guided Writing



$$
\begin{aligned}
& \text { Af First Sub the points into the eqaution. then Subtract } \\
& -3 \text { and }-3 \text { the at equal }-6 \text { Next Sub land }-3 \text { equal } \\
& -2 \text {. then divid }-6 \xi-2=3
\end{aligned}
$$

Billy Chapter Five Guided Writing


Billy Chapter Six Formative Writing
you will nave to start at -2 on the graph then up 3 and automatly Right 1 . (6) its a solid and Shade under.

Billy Chapter Seven Guided Writing
first you find the polynomail, then After you rewrite with an $=0$ then you Save using T chert (both sides)

Billy Chapter Ten Guided Writing


## Scott Chapter One Formative Writing

$$
\text { Jeff has received a job that the starting pay in } \$ 42,000 \text { for the first year and his salary will }
$$ increase by $3 \%$ each year. Write a recursive formula for calculating his salary.

$$
\begin{aligned}
& U_{0}=42,000 \\
& U_{n}=U_{n-1} * 1.03
\end{aligned}
$$

Explain how the problem was solved.

## Scott Chapter One Formative Writing



## Scott Chapter Two Formative Writing



## Scott Chapter Three Guided Writing

A student solved the problem below.

## Chapter 3.3 Guided Writing

What is the slope of a line that passes through the points $(3,3)$ and $(1,-3)$ ?
$\frac{y_{2}-y_{1}}{x_{2}-x_{1}}-3-3--6 \quad$ slope $=3$ bot

Using the vocabulary and language in the box, explain how to solve the problem.



Scott Chapter Four Guided Writing



The graph is a vertical fefreckon because - outside

The graph $h$ streches $b \times a$
multiple of 2 because

Scott Chapter Five Guided Writing
original Equation $5.2^{t}-4=33 \quad 5^{\text {th }}$ offer first numbers
$5.2 x=37$
are added or subtracted $\log 5.237=x$
are anstributed after $\frac{\log 37}{\log 5.2}=x=2.19$
ind istrubuyes to $\log$ and
then divide then and give to the
result
Scott Chapter Seven Formative Writing


Scott Chapter Ten Guided Writing

$$
12 P_{3}=1320
$$

$$
12 c_{3}=220
$$

First indentify whether the are or not names so you Know i permutation of combination are well and them you can Solve using calcutat

## William Chapter One Formative Writing

Jeff has received a job that the starting pay in $\$ 42,000$ for the first year and his salary will increase by $3 \%$ each year. Write a recursive formula for calculating his salary.
$U_{0}=42,000$

$$
U_{n}=U_{n-1} * 1.03
$$

Explain how the problem was solved.

William Chapter Three Formative Writing


William Chapter Four Guided Writing


William Chapter Five Formative Writing


William Chapter Six Formative Writing


William Chapter Ten Guided Writing
the first is factorial permutationy combination permutaxion because possible aptionsore there The.secandivg order matters because bossible groups of these can be picked fer the infield?

## APPENDIX G

Sample Test/Quiz

## Algebra 2 Quiz 3.1-3.3 Name

1. Consider the sequence
$U_{0}=-2$
$U_{n}=U_{n-1}+3$
a) Graph the first four terms of the sequence
b) What is the $y$-intercept of the line that contains these points?
c) What is the slope of the line that contains these points?
2. Write a recursive formula for a sequence whose points lie on the line $y=15-8 x$.
3. What is the slope of the line that contains the points $(-3,1)$ and $(2,5)$ ?
4. What is the slope of the line $4 x+8 y=5$ ?
5. The graph to the right gives the horsepower and weight in tons of various cars.
a) Draw (on the graph) the line of best fit.
b) Using your line from (a), predict the weight of a car with 180 horsepower.

| Day \# | 0 | 7 | 21 | 60 | 65 | 82 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Height <br> (inches <br> ) | 7.5 | 9.3 | 13.0 | 23.1 | 24.3 | 28.8 |

6. I planted some raspberry bushes. When the bushes went into the ground, they were 7.5 inches tall. The table below represents heights of one bush over the course of the summer.
a) Plot the points (label your graph) on the axes to the right.
b) Plot a line of best fit on the above data.
c) Which variable is the independent variable?
d) Which variable is the dependent variable?
e) Find the equation of the line of best fit.
f) Use your line to predict the height after 100 days.
7. Convert the linear equation $y=\frac{3}{4}(x-8)+4$ to slope / intercept form.
8. Write the equation of the line that passes through the points $(-4,7)$ and $(3,4)$ in point / slope form.
9. Write the equation of the line parallel to $y=\frac{1}{3} x+4$ that passes through the point $(2,5)$. Your answer should be in point/slope form
10. Write the equation of the line perpendicular to $y=7-5 x$ that passes through the point $(-6,11)$. Your answer should be in point/slope form.
11. $\frac{5 x+2}{7}=\frac{-7 x+1}{4}$

White Bear Lake Area High School Math Department, Algebra II Common Assessment

## APPENDIX H

Data Tables

| Table 1 |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Average writing scores | Linguisitic | Vocabulary | Language |  |
|  | Complexity | Usage | Control | Average |
| Student | 4.2 | 4.3 | 4.4 | 4.3 |
| Kelly | 2.5 | 3 | 3.2 | 2.9 |
| Don | 2.7 | 2.7 | 2.8 | 2.7 |
| Billy | 1.6 | 1.6 | 1.6 | 1.6 |
| Scott | 1.3 | 1.5 | 1.4 | 1.4 |
| William |  |  |  |  |


| Table 2 |  |  |  |
| :--- | :---: | :---: | :---: |
| Writing score, test score, and class comparison   <br> Student Average Writing Score Average Test Score | Scores Above Median <br> Kelly | 4.3 | $84 \%$ |
| Don | 2.9 | $78 \%$ | 9 |
| Billy | 2.7 | $56 \%$ | 4 |
| Scott | 1.6 | $75 \%$ | 1 |
| William | 1.4 | $64 \%$ | 5 |


| Table 3 |  |  |
| :--- | :---: | :---: |
| Scores on test questions related to and unrelated writing activities |  |  |
| Student | $\frac{\text { Related }}{}$ | $\underline{\text { Unrelated }}$ |
| Kelly | $88 \%$ | $83 \%$ |
| Don | $75 \%$ | $77 \%$ |
| Billy | $65 \%$ | $54 \%$ |
| Scott | $74 \%$ | $75 \%$ |
| William | $55 \%$ | $56 \%$ |


| Table 4 <br> Average <br> writing | scores on formative and |  |
| :--- | :---: | :---: |
| Student | Formative Writing |  |$\quad$| writing activities |
| :---: |
| Stuided Writing |


| Table 5 |  |  |
| :--- | :---: | :---: |
| Percent correct of features related to writing activities |  |  |
| Student | Formative Writing | Guided Writing |
| Billy | $54 \%$ | $64 \%$ |
| Scott | $81 \%$ | $62 \%$ |
| William | $52 \%$ | $40 \%$ |
| Kelly | $100 \%$ | $54 \%$ |
| Don | $82 \%$ | $76 \%$ |

