Informes de la Construcción
Vol. 69, 545, e176
enero-marzo 2017
ISSN-L: 0020-0883
doi: http://dx.doi.org/10.3989/ic.15.065

Design proposal for ultimate shear strength of tapered steel plate girders

Propuesta de cálculo de la resistencia a cortante de vigas armadas de acero de canto variable

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ABSTRACT

Numerous experimental and numerical studies on prismatic plate girders subjected to shear can be found in the literature. However, the real structures are frequently designed as non-uniform structural elements. The main objective of the research is the development of a new proposal for the calculation of the ultimate shear resistance of tapered steel plate girders taking into account the specific behaviour of such members. A new mechanical model is presented in the paper and it is used to show the differences between the behaviour of uniform and tapered web panels subjected to shear. EN 1993-1-5 design specifications for the determination of the shear strength for rectangular plates are improved in order to assess the shear strength of tapered elements. Numerical studies carried out on tapered steel plate girders subjected to shear lead to confirm the suitability of the mechanical model and the proposed design expression.

Keywords: tapered steel plate girders; Resal effect; mechanical model; ultimate shear resistance; EN 1993-1-5 rules; design proposal.

RESUMEN

En la literatura pueden encontrarse numerosos estudios experimentales y numéricos sobre vigas armadas de acero de canto constante sometidas a cortante. Sin embargo, muchas estructuras de acero se proyectan, frecuentemente, con elementos de canto variable. El objetivo del artículo es ofrecer una formulación para el cálculo de la resistencia última a cortante de paneles de alma de canto variable, basada en un nuevo modelo mecánico, que es utilizado para mostrar las diferencias de comportamiento entre paneles de alma de canto constante y variable, sometidos a cortante. Las reglas de cálculo de EN 1993-1-5 para determinar la resistencia a cortante de paneles de alma de canto constante se mejoran para poder evaluar la resistencia a cortante de paneles de canto variable. Los numerosos estudios numéricos llevados a cabo sobre vigas armadas de acero de canto variable confirman la idoneidad del modelo mecánico y de la nueva expresión propuesta para el cálculo.

Palabras clave: *vigas armadas de acero de canto variable; efecto Resal; modelo mecánico; resistencia última a cortante; EN 1993-1-5; expresión de cálculo.*

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Cómo citar este artículo/*Citation:* Bedynek, A., Real, E., Mirambell, E. (2016). Design proposal for ultimate shear strength of tapered steel plate girders. *Informes de la Construcción*, 69(545): e176, doi: http://dx.doi.org/10.3989/ic.15.065.

Recibido/*Received*: 02/05/2015
Aceptado/*Accepted*: 05/10/2016
Publicado on-line/*Published on-line*: 28/03/2017

1. INTRODUCTION

1.1. Previous studies

The behaviour of rectangular steel plates subjected to shear load was deeply studied in the last century. In consequence, several theories to predict the ultimate shear capacity of such members were developed. Some of them were taken as a reference and evolved in time and other ones were implemented in design codes. The most important methods to be mentioned are: Basler's model (1), Chern and Ostapenko (2), the Rotated Stress Field Model developed by Höglund (3), (4) and the Tension Field Model developed in Cardiff and Prague by Porter *et al.* (5) and Rockey and Škaloud (6).

Almost all ultimate-shear-strength models for tapered plate girders proposed in literature are derived from the previous presented models for rectangular plates. Several models for tapered girders were developed by: Falby and Lee (7), Davies and Mandal (8), Takeda and Mikami (9), Roberts and Newmark (10), Zárate and Mirambell (11) and Shanmugam and Min (12). Recently, some other numerical studies have been published by Abu-Hamd M. and Abu-Hamd I. (13). In most cases, the models are based on an assumption of a simply supported rectangular plate and do not consider either the actual boundary conditions existing in the flange–web–, stiffener–web– junctions or the geometry of the tapered steel plate girder. In the last years, some researchers, among others Lee *et al.* (14), Mirambell and Zárate (15) and Estrada *et al.* (16) have proved the importance of these effects. The importance of the so-called "Resal effect" which appears in tapered plate girders was also explained in the other work of Zárate (17).

On the other hand, the current European design norm EN 1993-1-5 (18) states that the design rules for the assessment of the ultimate shear resistance of prismatic members may be applied to non-rectangular plate girders if the angle of the inclined flange does not exceed 10°. For larger angles, panels may be assessed assuming it to be a rectangular panel based on the larger of depths of the panel. Previous research conducted by Bedynek *et al.* (19) demonstrated that for some geometry of such panels, the ultimate shear strength results might be significantly overestimated. It happens because of the specific behaviour of tapered panels and the Resals' phenomenon.

In this research an attempt of an extension of the existing design rules included in EN 1993-1-5 for non-prismatic panels were done. Various geometrical parameters such as: the aspect ratio of the panel, the slope of the inclined flange and the web and flange slenderness were taken into account. Moreover, a new mechanical model for assessing the influence of the Resal effect was developed and included in the final version of the proposal for calculating the ultimate shear resistance of tapered steel plate girders subjected to shear and shear-bending interaction.

1.2. Ultimate shear strength for plate girders

Different theories have been developed to analyse the behaviour of steel plated girders subjected to shear, and some of them are used nowadays in design codes. The Rotated Stress Field Method (4) is the one implemented in EN 1993-1-5 (18). This method is based on the assumption that the web panel is under a pure shear stress state caused by a shear force in the slender web before buckling occurs (principal stresses are equal and opposite). Once buckling occurs, compression stress is close to the critical shear buckling stress and therefore, the increase of load is resisted by an increase of the principal tensile stress, until the moment that the yield criterion is fulfilled in the web. The inclination of the principal stresses varies as the load increases. The contribution of the flanges in the resistant mechanism is included considering that the tension band anchors in the flanges until a four hinge mechanism leads to the collapse of the girder (see Figure 1). A more detailed explanation of the failure mechanism in plated girders subjected to shear can be found among others in the works of Johansson *et al.* (20) or Maquoi and Skaloud (21).

2. RESAL EFFECT

2.1. General

The results obtained by Bedynek *et al.* (19) show that tapered plate girders can be classified into four typologies. The dif-

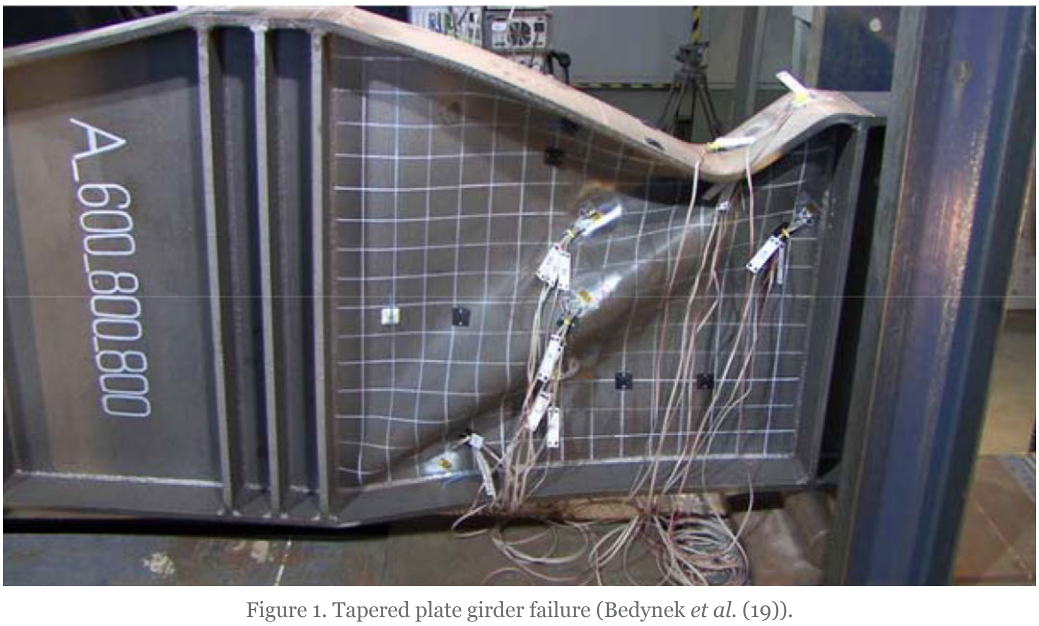

Figure 1. Tapered plate girder failure (Bedynek *et al.* (19)).

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ference among them consists in the direction of the diagonal tension field, which can develop on the shortest or the largest diagonal of the web panel and in the stress state in the inclined flange (tension or compression). These typologies are presented in Figure 2.

For the girders belonging to typologies I and III the diagonal tension field develops on the shortest diagonal of the web panel and the inclined flange is under compression or tension, respectively. For typologies II and IV the diagonal tension field appears on the longest diagonal of the web and the inclined flange is under tension or compression, respectively. Different behaviour observed within each typology of tapered panels also influences their ultimate shear resistance.

Moreover, research conducted by Zárate and Mirambell (11) confirmed that in tapered plate girders an additional vertical load derived from the axial force in the inclined flange appears and its influence on the ultimate shear resistance of the whole web panel should be taken into account. This phenomenon is known as Resal effect (Samartín (22), Florida Department of Transportation (23)) and may cause an increase or decrease of the ultimate shear resistance in tapered plate girders. Its positive or negative influence is strongly correlated with the typology of the web-panel. Graphical illustration of the Resal effect in all considered typologies is given in Figure 3.

For those cases where the moment of inertia of the cross-section increases with the increase of internal forces (typolo-

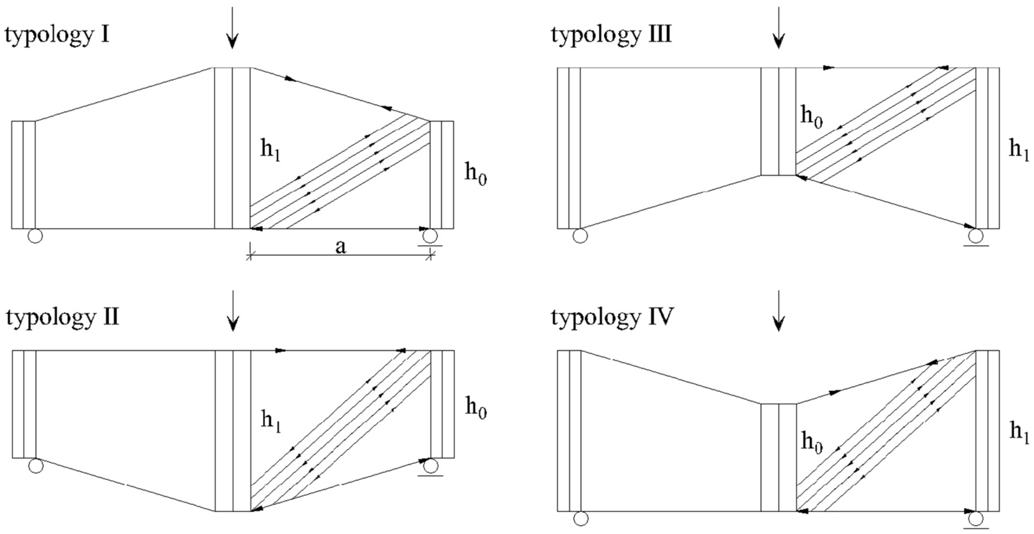

Figure 2. Four typologies of tapered plate girders.

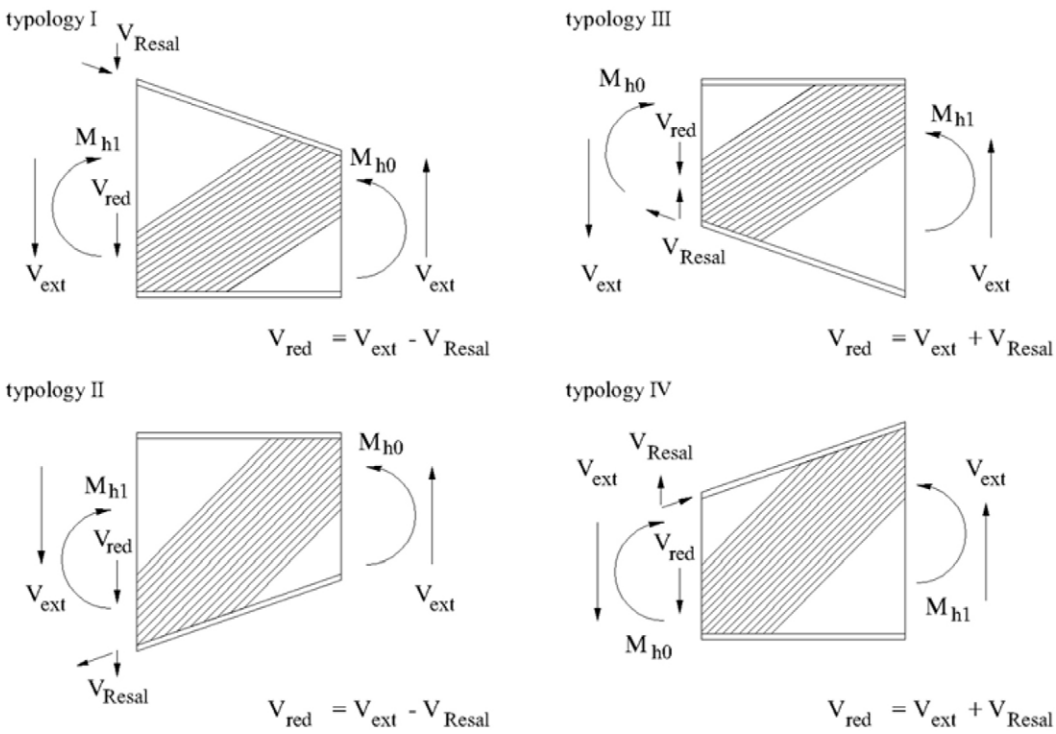

Figure 3. Positive and negative influence of the Resal effect on V_u.

gies I and II), the vertical component of the Resal effect acts against shear force and reduces the shear force design value; in other words, consideration of the vertical component of the Resal effect increases the ultimate shear resistance of tapered girders (positive influence). For typologies III and IV the opposite situation is observed. Due to their smaller bearing capacity, these two typologies are not as common as typologies I and II, but sometimes might be required in a case of non-standard structural solutions where the geometry plays an important role.

2.2. Theoretical mechanical model

Since there is still observed a shortage in bibliography focused on the Resal effect in tapered steel plate girders, additional studies are needed to better understand its actual role.

The study on the influence of the Resal effect starts from defining a simplified two-dimensional mechanical model with simplified boundary conditions in some characteristic nodes (Figure 4).

The main objective of this section is to find a direct correlation between Resal force and the shear load acting on the girder. The model represents a rigid frame consisting of flanges and transversal stiffeners only. It is assumed that all the internal forces are transmitted only by surrounding rods of the frame (excluding contribution from the web). In fact, this simplification is used in order to find an approximated value of the Resal force, not its exact value. The boundary conditions reproduce the real ones: simply supported on the one end without bending moment along the shortest depth and with maximum constant bending moment along the rod representing the central cross-section (the largest depth for typologies I and II). Similar idea of simplification of tapered plate girder by rod structure (in that case by truss) was proposed by Davies and Mandal (8).

Solving the hyperstatic frame with two unknowns X_1 and X_2, it is possible to find a contribution of the internal forces from the flanges and from the transversal stiffeners. Consequently, the axial force *N* (Figure 4) in the inclined flange will be known. The Resal force is obtained directly by projecting the axial force *N* on the vertical axis.

Full procedure of determining the Resal force will be explained for the typology I, however, only the most important steps will be pointed out. Other typologies were analysed in the same way and their final expressions for calculating the Resal force will be presented.

Using the principle of virtual works, two unknowns X_1 and X_2 can be found and expressed by Eqs. 1 and 2:

$$X_1 = \frac{V_{ext}}{\left(1+\frac{1}{\cos(\phi)}\right)} \quad [1]$$

$$X_2 = \frac{a V_{ext}}{\left(1+\frac{1}{\cos(\phi)}\right)} \quad [2]$$

where a is the frame length and ϕ is the angle of the inclined flange (Figure 4).

From the equilibrium of upper right hinge, the axial force *N* in the inclined flange can be found (Eq. 3):

$$N = \frac{V_{ext}}{\left(1+\frac{1}{\cos(\phi)}\right)}\sin(\phi) \quad [3]$$

Next, the Resal force was calculated as the vertical component of axial force *N* and for all typologies is given by Eq. 4:

$$V_{Resal} = \frac{V_{ext}}{\left(1+\frac{1}{\cos(\phi)}\right)}\sin^2(\phi) \quad [4]$$

As it was mentioned before, the Resal effect for tapered plate girders from typologies I and II is favourable. Thus, reduced shear force V_{red} should be calculated according to Eq. 5:

$$V_{red} = V_{ext} - V_{Resal} \quad [5]$$

For typologies III and IV the Resal effect has a negative influence and V_{red} is given by Eq. 6:

$$V_{red} = V_{ext} + V_{Resal} \quad [6]$$

3. NUMERICAL STUDY

3.1. General

Parametric studies were conducted on 85 tapered panels belonging to four typologies. The girders varied by geometric parameters such as the aspect ratio $\alpha = a/h_1$, the slope of the inclined flange ϕ and the web thickness t_w. The range of dimensions for the studied models is presented in Table 1. Nomenclature used in girder's name is the following: 600_800_800_4_180_15 means: smallest depth h_o = 600 mm, largest depth h_1 = 800 mm,

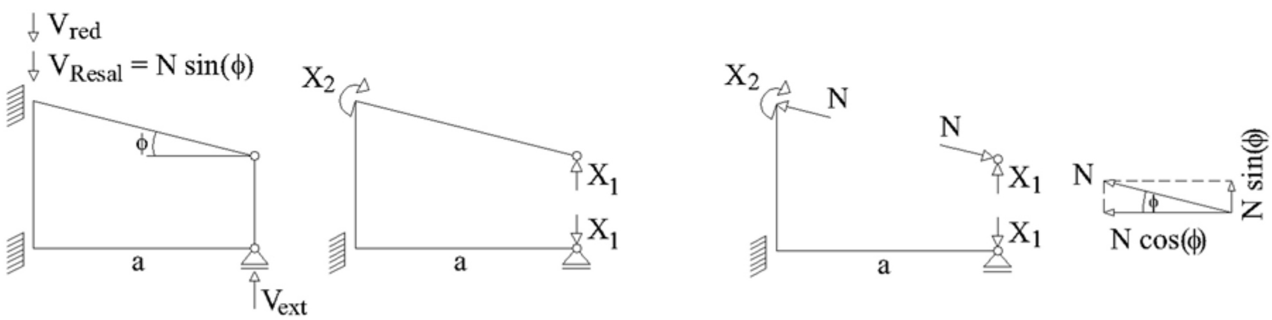

Figure 4. Internal forces in the structural model.

Table 1. Range of dimensions for the studied models.

Variable	Symbol	Unit	Range
web thickness	t_w	[mm]	2 to 8
smaller depth	h_0	[mm]	300 to 1235
larger depth	h_1	[mm]	700 to 3000
panel length	a	[mm]	800 to 8220
aspect ratio	α	-	1 to 4
tangent	tan(ϕ)	-	0.0 to 0.5
angle	ϕ	[°]	0 to 26.6

panel length a = 800 mm, web thickness t_w = 4 mm, flange width b_f = 180 mm and flange thickness t_f = 15 mm (see Figure 1). Due to the large amount of analysed cases, only selected results will be presented.

3.2. Numerical model

In the initial part of the numerical study several nonlinear analyses were performed on the tapered plate girders with use of quad-dominant 4-node shell element S4R5. This finite element, with 5 degrees of freedom (in each node) and linear shape functions, is especially suitable for modelling shell surfaces where large rotations and displacements are expected. A convergence analysis allowed setting the mesh density with a mesh size of 20 mm for all modelled cases. The nonlinear analyses were conducted with the Modified Riks algorithm implemented in ABAQUS (24). Due to the simple shape of the modelled tapered girders, the mesh used in the numerical model could be generated automatically.

All numerical simulations were done using the same bilinear material model with kinematic hardening: steel S275 with yield stress f_y = 275 MPa, Young's modulus E = 210 GPa and Poisson ratio ν = 0.3.

The beams are modelled with a full 3D model for tapered plate girders to reflect the actual boundary conditions between web and flange panels in the continuous span of the girder. The numerical model is thought to be rigid end-post. The modelled girder is simply-supported on two edges and its boundary conditions are presented in Figure 5 and Table 2.

Two different analyses have been considered in this study. First an eigen-value analysis to obtain the critical shear load as the first eigen-value and to obtain the deformed shape for the critical shear load as the first eigen-vector. The second analysis is a geometric and material nonlinear analysis to reproduce the postbuckling behaviour of tapered plate girders up to failure. Geometric imperfections should be introduced in the model in order to develop the second order effects in the web panel. In this case, the deformation corresponding to the first eigen-mode was used.

The numerical model validation based on four experimental tests was presented in Bedynek *et al.* (19). In the same paper the results from the analysis on the structural imperfections in tapered steel plate girders were shown. The studies proved that this kind of structures is not susceptible to the residual stresses and their consideration may decrease the ultimate shear resistance not more than 2 %. For this reason only the geometrical imperfections derived from the deformed shape of the web-panel corresponding to the 1st positive eigen-value were used in the analysis. Also the magnitude of the geometric imperfections was scaled according to EN 1993-1-5, Annex C.5 (18) what in practice means that the maximum amplitude was on the level of the 80% of the geometric fabrication tolerances.

Table 2. Boundary conditions.

	u_x	u_y	u_z	θ_x	θ_y	θ_z
L	0	1	1	1	1	0
R	1	1	1	1	1	0

* 0 – free movement; 1 – restraint.

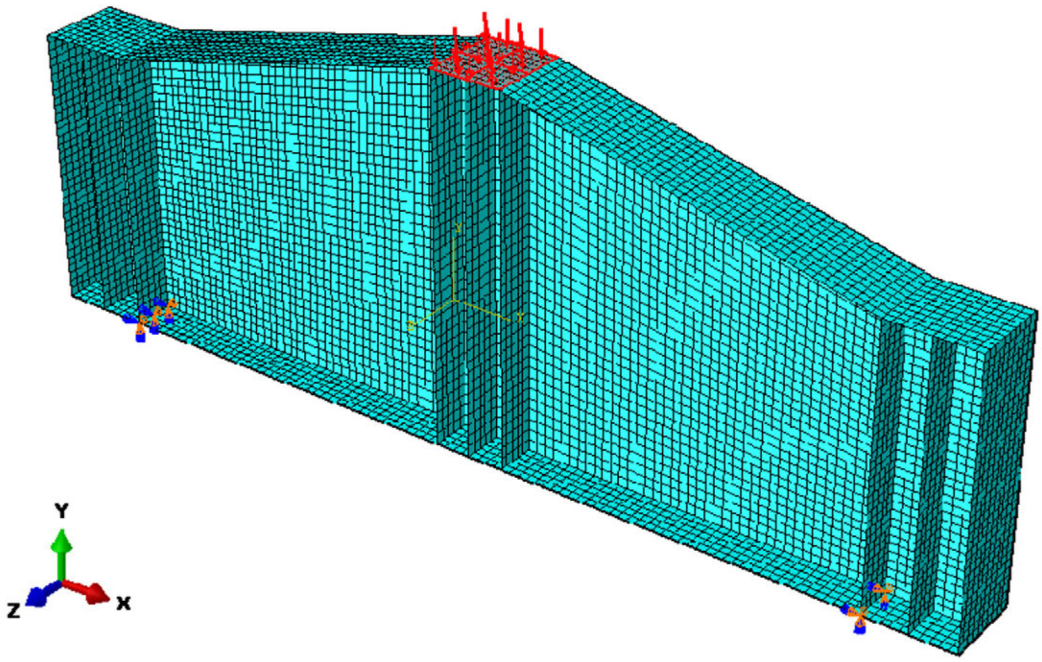

Figure 5. Numerical model.

4. ULTIMATE SHEAR RESISTANCE ACCORDING TO EN 1993-1-5

4.1. Verification of existing design rules provided by EN 1993-1-5

EN 1993-1-5 suggests calculating the ultimate shear resistance of tapered panels treating them as rectangular ones with their largest depth h_1. Unfortunately, for some particular geometries of non-rectangular plates this approach leads to significant overestimation of the results what means that the obtained values of the ultimate shear resistances do not fulfil the safety requirements. It happens especially for the typologies III and IV with larger slopes of the inclined flange. This situation can be partially caused by Resal effect, an additional vertical force derived from the axial force that appears in the inclined flange, which for typologies III and IV is unfavorable and should be taken into account. On the other hand, results obtained for typologies I and II are a bit conservative for some cases. Nevertheless, in general this approach gives a good estimation of the ultimate shear resistance of tapered panels with less than 18 % difference between the analytical (EN 1993-1-5) and numerical solution (see Table 3).

In order to illustrate the differences when ultimate shear resistance of tapered panels is calculated using EN 1993-1-5 simplification for tapered panels, results for some of the studied girders are presented in Table 3. In the column titled "Diff.$_{(1)}$", differences between the numerical (FEM) and analytical (EN 1993-1-5) values of the ultimate shear resistance V_u are shown. The aspect ratio of the girders "A" and "B" is $\alpha = 1$ and $\alpha = 2$, respectively.

New approach which gave better assessment of the ultimate shear resistance for typologies III and IV was presented by Bedynek *et al.* (19). The ultimate shear resistance V_u calculated as for a rectangular plate was multiplied by a reduction factor h_0/h_1. Although that proposal is easy to apply and gives quite good agreement between the numerical (FEM) and analytical (EN) values of V_u (Table 3, last column "Diff.$_{(2)}$"), does not justify a new contribution from the web and flanges neither taking into account the Resal effect.

4.2. Proposal for the design shear resistance in tapered steel plates based on EN 1993-1-5

A new proposal for determining the design shear resistance for tapered steel plate girders based on the existing design rules in EN 1993-1-5 (18) and considering the Resal effect obtained by the simplified model explained above is presented here.

The new proposal is valid for slender, rigid end-posted girders, whose slenderness parameter $\bar{\lambda}_w > 1.8$. Moreover, in this part of the research according to the assumption made for the studied cases, the shear-bending interaction may be neglected. EN 1993-1-5 states that for the cases where the design bending moment M_{Ed} is smaller than the resistant moment of the cross-section consisting of the effective area of the flanges $M_{f,Rd}$, the shear-bending interaction can be omitted.

According to EN 1993-1-5 (18) the design shear resistance $V_{b,Rd}$ for plate girders should be taken as

$$V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd} \qquad [7]$$

where contribution from the web is

$$V_{bw,Rd} = \frac{\chi_w f_{yw} h_w t_w}{\sqrt{3}\, \gamma_{M1}} \qquad [8]$$

being χ_w the shear buckling reduction factor, f_{yw} the yield stress of the web, h_w the depth of the web panel, t_w the web thickness and γ_{M1} the partial safety factor.

Table 3. Critical shear buckling load and ultimate shear resistance for the analysed prototypes. Numerical (FEM) and theoretical results (EN 1993-1-5).

		girder	α	ϕ[º]	tan(ϕ)	FEM V_{cr} [kN]	FEM V_u [kN]	EN V_u [kN]	Diff.$_{(1)}$ [%]	Proposal Bedynek *et al.* (2013) V_u [kN]	Diff.$_{(2)}$ [%]
typology I	A	480_800_800_4_180_15	1.0	21.8	0.40	285.0	364.8	310.5	14.9	310.5	14.9
		600_800_800_4_180_15	1.0	14.0	0.25	238.0	366.9	310.5	15.4	310.5	15.4
		680_800_800_4_180_15	1.0	8.5	0.15	215.8	365.6	310.5	15.1	310.5	15.1
	B	600_1200_2400_4_250_25	2.0	14.0	0.25	177.9	368.4	305.4	17.1	305.4	17.1
		850_1200_2400_4_250_25	2.0	8.3	0.15	130.3	367.9	305.4	17.0	305.4	17.0
typology II	A	480_800_800_4_180_15	1.0	21.8	0.40	256.3	335.4	310.5	7.4	310.5	7.4
		600_800_800_4_180_15	1.0	14.0	0.25	217.3	351.3	310.5	11.6	310.5	11.6
		680_800_800_4_180_15	1.0	8.5	0.15	202.5	354.8	310.5	12.5	310.5	12.5
	B	600_1200_2400_4_250_25	2.0	14.0	0.25	180.0	355.2	305.4	14.0	305.4	14.0
		850_1200_2400_4_250_25	2.0	8.3	0.15	130.5	361.7	305.4	15.6	305.4	15.6
typology III	A	480_800_800_4_180_15	1.0	21.8	0.40	186.6	221.0	310.5	-40.5	186.3	15.7
		600_800_800_4_180_15	1.0	14.0	0.25	188.0	274.9	310.5	-13.0	232.9	15.3
		680_800_800_4_180_15	1.0	8.5	0.15	189.3	310.4	310.5	0.0	263.9	15.0
	B	600_1200_2400_4_250_25	2.0	14.0	0.25	95.4	186.9	305.4	-63.4	152.7	18.3
		850_1200_2400_4_250_25	2.0	8.3	0.15	95.5	263.5	305.4	-15.9	216.3	17.9
typology IV	A	480_800_800_4_180_15	1.0	21.8	0.40	161.6	202.7	310.5	-53.2	186.3	8.1
		600_800_800_4_180_15	1.0	14.0	0.25	168.7	263.3	310.5	-17.9	263.9	-0.2
		680_800_800_4_180_15	1.0	8.5	0.15	176.4	300.5	310.5	-3.3	232.9	22.5
	B	600_1200_2400_4_250_25	2.0	14.0	0.25	94.6	182.1	305.4	-67.7	152.7	16.1
		850_1200_2400_4_250_25	2.0	8.3	0.15	95.1	259.0	305.4	-17.9	216.3	16.5

The contribution from the flanges is given by:

$$V_{bf,Rd} = \frac{b_f t_f^2 f_{yf}}{c\,\gamma_{M1}} \left(1 - \left(\frac{M_{Ed}}{M_{f,Rd}} \right)^2 \right) \qquad [9]$$

where b_f is the flange width, t_f is the flange thickness and f_{yf} is the yield stress of the flanges.

In the last expression *c* is the distance between the plastic hinges in both flanges (Eq. 10).

$$c = a \left(0.25 + \frac{1.6\, b_f t_f^2 f_{yf}}{t_w h_w^2 f_{yw}} \right) \qquad [10]$$

M_{Ed} is the design bending moment calculated at mid-span cross-section $M_{Ed} = a\ V_{Ed}$, and $M_{f,Rd}$ is the moment of resistance of the cross-section consisting of the effective area of the flanges only.

Eqs. 11 and 12 show how to take into account the Resal effect when calculating the ultimate shear resistance of tapered plate girders. For typologies I and II the Resal effect is favourable and reduces the design value of the shear force V_{Ed}. Thus, the design shear resistance should be verified according to Eq. 11:

$$V_{Ed} - V_{Resal} < V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd} \qquad [11]$$

For typologies III and IV its influence is unfavourable increasing the external shear load (Eq. 12):

$$V_{Ed} + V_{Resal} < V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd} \qquad [12]$$

4.2.1. Contribution from the flanges

For tapered plate girders the depth h_w used to calculate the distance *c* in Eq. 10 should be taken as the depth corresponding to the central cross-section when the design bending moment M_{Ed} is calculated. Consequently, for tapered plate girders from typologies I and II, the largest depth $h_w = h_1$ should be used and for typologies III and IV the smallest one $h_w = h_o$ (see Figure 2).

4.2.2. Contribution from the web

For typologies I and II, the best assessment of their ultimate shear capacity is obtained when in Eq. 8 for the contribution from the web, the depth h_w is substituted by the largest one $h_w = h_1$ (Eq. 13). The contribution from the web for typologies III and IV should be calculated using the smallest depth $h_w = h_o$ (Eq. 14).

$$V_{bw,Rd} = \frac{\chi_w f_{yw} h_1 t_w}{\sqrt{3}} \text{ for typologies I and II} \qquad [13]$$

$$V_{bw,Rd} = \frac{\chi_w f_{yw} h_o t_w}{\sqrt{3}} \text{ for typologies I and II} \qquad [14]$$

From the comparison of the results given in Table 4 is easy to observe that ultimate shear resistance between various typologies of tapered plate girders may vary significantly. The ultimate shear strength V_u of girders belonging to typologies I and II is always higher than for the same girders from typologies III and IV. This difference can be caused by two important factors which are: the Resal effect, whose influence in case of typologies I and II is favourable and for typologies III and IV unfavourable, as well as the shear capacity of the web $V_{bw,Rd}$ (Table 4).

The web contribution $V_{bw,Rd}$ for tapered plate girders has been evaluated by analysing the shear buckling reduction factor χ_w from new 85 numerical simulations. The numerical values of χ_w have been obtained from Eqs. 11 and 12, subtracting from the numerical value of the ultimate shear resistance the contribution from the flanges (Eq. 9) obtained

Table 4. Ultimate shear resistance for the analysed prototypes subjected to pure shear.
Numerical results (FEM), results from the 1st proposal (Bedynek *et al.* (2013)) and results from the new proposal.

		girder	V_u (FEM) [kN]	Proposal Bedynek *et al.* (2013) V_u [kN]	Diff.(2) [%]	$V_{bw,Rd}$ [kN]	$V_{bf,Rd}$ [kN]	V_{Resal} [kN]	New proposal V_u [kN]	Diff.(3) [%]
typology I	A	480_800_800_4_180_15	364.8	310.5	14.9	268.4	42.0	20.6	331.0	9.2
		600_800_800_4_180_15	366.9	310.5	15.4	268.4	42.0	9.0	319.4	12.9
		680_800_800_4_180_15	365.6	310.5	15.1	268.4	42.0	3.4	313.8	14.2
	B	600_1200_2400_4_250_25	368.4	305.4	17.1	251.8	53.6	8.8	314.2	14.7
		850_1200_2400_4_250_25	367.9	305.4	17.0	251.8	53.6	3.2	308.6	16.1
typology II	A	480_800_800_4_180_15	335.4	310.5	7.4	268.4	42.0	20.6	331.0	1.3
		600_800_800_4_180_15	351.3	310.5	11.6	268.4	42.0	9.0	319.4	9.1
		680_800_800_4_180_15	354.8	310.5	12.5	268.4	42.0	3.4	313.8	11.5
	B	600_1200_2400_4_250_25	355.2	305.4	14.0	251.8	53.6	8.8	314.2	11.5
		850_1200_2400_4_250_25	361.7	305.4	15.6	251.8	53.6	3.2	308.6	14.7
typology III	A	480_800_800_4_180_15	221.0	186.3	15.7	177.5	39.3	14.4	202.4	8.4
		600_800_800_4_180_15	274.9	232.9	15.3	221.9	37.5	7.5	251.9	8.4
		680_800_800_4_180_15	310.4	263.9	15.0	251.5	38.9	3.2	287.2	7.5
	B	600_1200_2400_4_250_25	186.9	152.7	18.3	138.8	35.8	5.1	169.5	9.3
		850_1200_2400_4_250_25	263.5	216.3	17.9	196.6	45.3	2.5	239.4	9.2
typology IV	A	480_800_800_4_180_15	202.7	186.3	8.1	177.5	34.2	14.1	197.6	2.5
		600_800_800_4_180_15	263.3	263.9	-0.2	221.9	37.5	7.5	251.9	4.4
		680_800_800_4_180_15	300.5	232.9	22.5	251.5	38.9	3.2	287.2	4.4
	B	600_1200_2400_4_250_25	182.1	152.7	16.1	138.8	35.8	5.1	169.5	6.9
		850_1200_2400_4_250_25	259.0	216.3	16.5	196.6	45.3	2.5	239.4	7.6

according to EN 1993-1-5 (18). Thus, the expression for χ_w for typologies I and II stays unchanged (Eq. 15) and for typologies III and IV is given by Eq. 16.

$$\chi_w = \frac{1.37}{\left(0.7 + \bar{\lambda}_w\right)} \qquad [15]$$

Here, it is important to point out that for all typologies of tapered plate girders, the slenderness parameter $\bar{\lambda}_w$, is always calculated as a simply supported rectangular panel with its largest depth, regardless of the typology.

Numerical results for the shear buckling reduction factor are presented in Figure 6a-b and compared with the actual proposal in EN 1993-1-5 (18).

As it can be observed in Figure 6a, the expression for calculating the reduction factor χ_w (Eq. 15) proposed in EN 1993-1-5 (18) gives good approximation for end-posted tapered girders for typologies I and II whose slenderness parameter $\bar{\lambda}_w \geq 1.8$.

On the other hand, from the results shown in Figure 6b, it is possible to find a better adjustment of the shear buckling reduction factor χ_w for tapered plate girders belonging to typologies III and IV. From the numerical analyses of above 40 end-posted tapered plate girders with the slenderness parameter $\bar{\lambda}_w \geq 1.8$ belonging to typologies III and IV, it is concluded that better approximation (Figure 6b) may be achieved by using the modified value of the shear buckling reduction factor χ_w given by Eq. 16:

$$\chi_w = \frac{1.51}{\left(0.7 + \bar{\lambda}_w\right)} \qquad [16]$$

5. ANALYSIS OF THE RESULTS

5.1. Shear

Based on the same examples of the girders presented in Table 3, the results for the ultimate shear resistance V_u obtained according to three different methods are compared in Table 4. The values shown in the first column $V_{u\ (FEM)}$ come from the numerical analysis and they are compared with the proposal by Bedynek *et al.* (19), in column "Diff.$_{(2)}$". The following columns represent the contribution from the web $V_{bw,Rd}$, the contribution from the flanges $V_{bf,Rd}$ and the Resal effect V_{Resal} calculated according to the proposal. The final values of the ultimate shear strength can be found in the column called V_u (new proposal) and in the last one "Diff.$_{(3)}$" these values are compared with the numerical ones. The data from Table 4 refers only to several examples of the studied girders.

The results for the 85 girders obtained according to three methods: EN 1993-1-5, the proposal by Bedynek *et al.* (19)

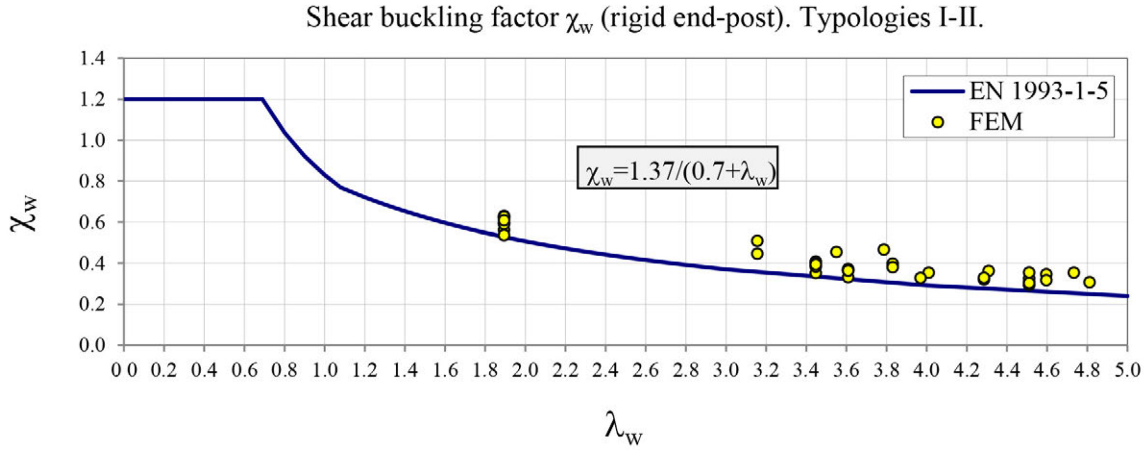

Figure 6a. The factor χ_w for end-posted, slender, tapered panels for typologies I-II.

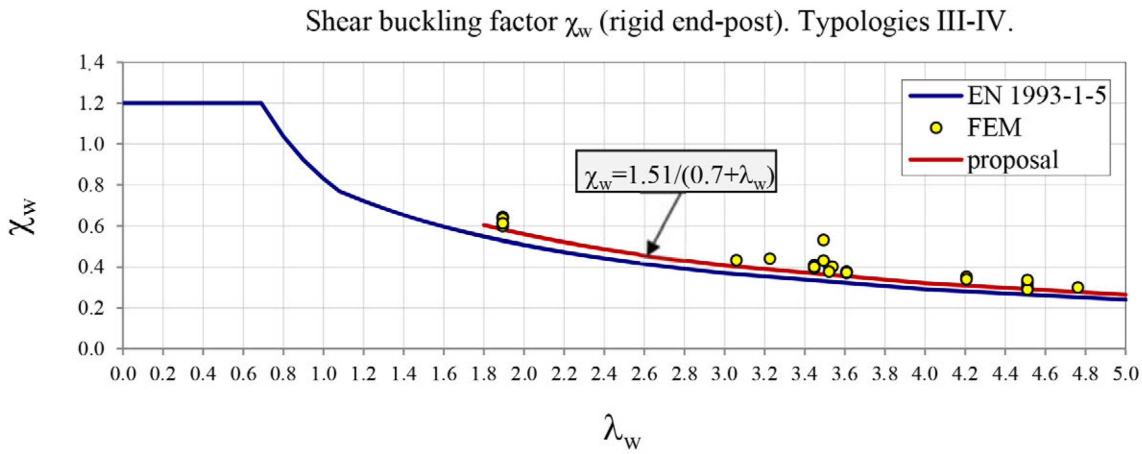

Figure 6b. The factor χ_w for end-posted, slender, tapered panels for typologies III-IV (a new proposal).

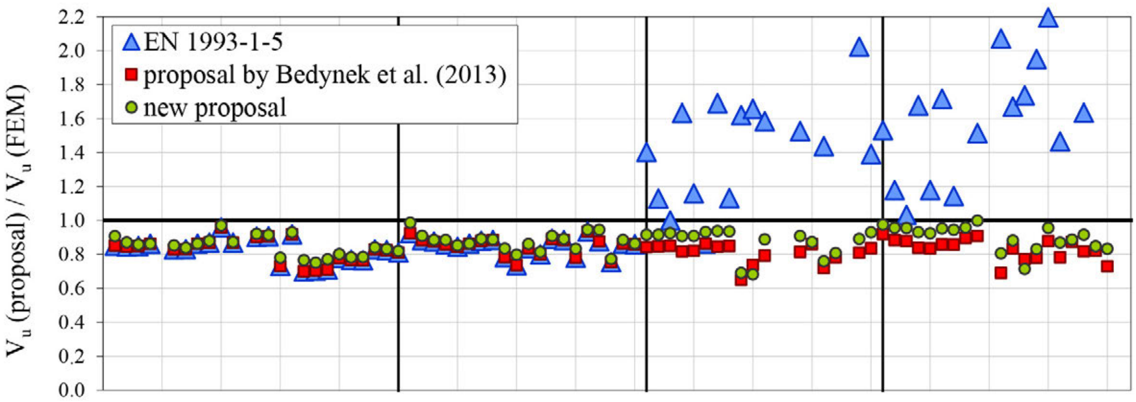

Figure 7. Comparison of the results obtained according to EN 1993-1-5, the proposal by Bedynek *et al.* (19) and the new proposal.

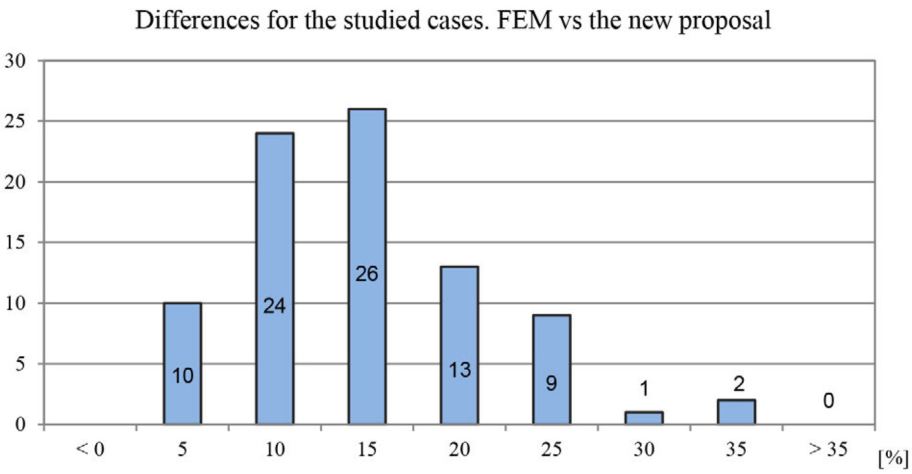

Figure 8. Statistic distribution of the differences between the numerical results and those obtained according to the new proposal.

and the new one are presented in Figure 7 and compared with the results derived from the numerical simulations. Significant improvement caused by applying the new proposal is especially noticeable for typologies III and IV. As it can be observed all values of the ultimate shear strength obtained according to the new proposal are on the safe side and show very good agreement with the numerical tests.

In Figure 8 a statistic distribution of the difference between the numerical results and those obtained according to the new proposal are shown. Almost for all cases these differences does not exceed 25 %. Only for 3 from 85 studied girders (3.5 % of all tests) the difference is greater than 25 %, but always less than 35 %. For 72 % of studied girders the differences are smaller than 15 % what confirms very good accuracy of the new proposal.

The verification of the new proposal was also conducted using the experimental data from three of the four tests under pure shear load presented in Bedynek *et al.* (19). The results are presented in Table 5, and the differences oscillate between 12.5-16.5 %.

5.2. Shear-bending interaction

5.2.1. EN 1993-1-5

For the cases where the design bending moment M_{Ed} is higher than the moment of resistance of the cross-section consisting of the effective area of the flanges $M_{f,Rd}$ and less than the plastic moment of the resistance of the cross-section $M_{pl,Rd}$ ($M_{f,Rd} < M_{Ed} < M_{pl,Rd}$), the shear-bending interaction should be considered. According to EN 1993-1-5, if $V_{Ed} > 0.5\, V_{bw,Rd}$ the combined effects of bending and shear in the web of a I girder should satisfy Eq. 17.

$$\frac{M_{Ed}}{M_{pl,Rd}} + \left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right)\left(\frac{2V_{Ed}}{V_{bw,Rd}} - 1\right)^2 \leq 1 \qquad [17]$$

Table 5. Ultimate shear resistance for the analysed prototypes. Tests results and the new proposal.

	girder*	V_u (test) [kN]	$V_{bw,Rd}$ [kN]	$V_{bf,Rd}$ [kN]	V_{Resal} [kN]	New proposal V_u [kN]	Diff.(4) [%]
typ. I	600_800_800_3.9_180_15	392.0	271.4	46.9	9.2	327.5	16.5
	500_800_1200_3.9_180_15	320.5	244.9	27.7	7.9	280.5	12.5
	480_800_800_3.9_180_15	388.2	256.9	42.5	19.9	339.4	12.6

* all tested girders were made of steel S275.

Some numerical analyses considering shear-bending interaction in tapered steel plate girders for the four different typologies studied in this research have been conducted and the results are presented in the next section.

5.2.2. Comparison of results

In order to check the validity of the new proposal for tapered plate girders under shear-bending interaction, twelve new cases, three for each typology were examined. It is necessary to mention that the situation where influence of bending moment on the ultimate shear is significant and may lead to its reduction is not very common. It happens due the fact that design bending moment M_{Ed} derives from the vertical reaction in the support (ultimate shear force) and in order to achieve its significant level, the span length of the tapered girders should be long enough and the flexural resistance of the flanges should be relatively low, which is not usually recommended from the design point of view.

In this section, the geometries of the studied girders were designed in order to fulfil condition about the design bending moment at the mid-span cross-section (calculated as the distance *a* multiplied by V_u) which should be greater than the moment of resistance of the cross-section consisting of the effective area of the flanges, $M_{Ed} > M_{f,Rd}$ and $V_{cr} < V_u$.

The values of $V_{u\,red}$ (Table 6) were calculated with the same reduction factors as it has been explained before: $\chi_w = \dfrac{1.37}{(0.7+\bar{\lambda}_w)}$ for typologies I and II, and $\chi_w = \dfrac{1.51}{(0.7+\bar{\lambda}_w)}$ for typologies III and IV. Also the influence of the Resal effect V_{Resal} was considered.

In the first three columns of Table 6 a comparison of the results obtained numerically $V_{u\,(FEM)}$ and those calculated according to unmodified method proposed in EN 1993-1-5 ($V_{u\,(EN)}$) are shown. Next, some extra data such as: the bending resistance of the flanges $M_{f,Rd}$, the design bending moment M_{Ed}, the contributions from the web $V_{bw,Rd}$ and flanges $V_{bf,Rd}$ and the Resal force V_{Resal} were calculated according to the new proposal and are presented in the following columns. Finally, in the last two columns the reduced ultimate shear strengths $V_{u,red}$ are compared with their corresponding numerical values $V_{u\,(FEM)}$.

Relevant improvement for typologies III and IV is observed when the contribution from the web is calculated according to Eq. 14 and when the Resal effect is taken into account. For typologies I and II this improvement is not so significant. Consideration of the Resal effect brings the values of $V_{u,\,red}$ closer comparing to the numerical solution for all examined cases.

In general, it can be observed that results obtained according to the new proposal give a very good agreement with the numerical ones, and only for typology III, the values of the ultimate shear resistance are slightly overestimated (not greater than 2 %). When calculating $V_{u,red}$, the partial safety factor, included in design codes, was not taken into account, so its application may bring unsafe values of $V_{u,red}$ on the safe side.

In Figure 9 a graphical comparison of the obtained results is shown. As it was mentioned before for typologies I and II, the benefits from the application of the new proposal are observed in all cases, however the improvement is rather small (up to 2 %).

On the other hand, for typologies III and IV the use of the new proposal improves the results and gives a good assessment for the ultimate shear resistance.

6. CONCLUSIONS

In this paper the structural response of tapered steel plate girders subjected to shear and shear-bending interaction has been studied. Numerical research has considered four different typologies of such girders which have been studied separately.

A new proposal for the assessment of the ultimate shear resistance of tapered steel plate girders on the existing design rules in EN 1993-1-5 and considering the Resal shear value obtained by a simplified model has been presented.

A simplified model to obtain the value of the shear load produced by the Resal effect has been presented, then the shear force acting in the web can be reduced in typologies I and II or increased in typologies III and IV when verifying the shear resistance of the web.

Table 6. Ultimate shear resistance for the analysed prototypes. Shear-bending interaction.

typology	girder	V_u (FEM) [kN]	V_u (EN) [kN]	Diff.(1) [%]	New proposal $M_{f,Rd}$ [kNm]	M_{Ed} [kNm]	$V_{bw,Rd}$ [kN]	$V_{bf,Rd}$ [kN]	V_{Resal} [kN]	$V_{u\,red}$ [kN]	Diff.(3) [%]
I	600_1100_2500_5_180_15	336.5	327.6	2.6	707.5	819.1	332.0	0.0	6.2	333.9	0.8
I	600_1000_4000_5_250_20	332.0	312.1	6.0	1198.5	1248.4	313.8	0.0	1.5	313.7	5.5
I	800_1200_6000_6.5_400_25	545.0	511.9	6.1	2878.8	3071.7	516.8	0.0	1.1	513.1	5.9
II	600_1100_2500_5_180_15	336.6	327.6	2.7	707.5	819.1	332.0	0.0	6.2	333.9	0.8
II	600_1000_4000_5_250_20	347.2	312.1	10.1	1198.5	1248.4	313.8	0.0	1.5	313.7	9.7
II	800_1200_6000_6.5_400_25	549.5	511.9	6.8	2878.8	3071.7	516.8	0.0	1.1	513.1	6.6
III	1100_600_2500_5_180_15	190.5	327.6	-72.0	390.2	493.7	199.6	0.0	3.8	193.7	-1.7
III	1000_600_4000_5_250_20	206.0	312.1	-51.5	728.5	823.1	207.5	0.0	1.0	204.7	0.6
III	1200_800_6000_6.5_400_25	366.8	511.9	-39.6	1938.8	2249.1	379.8	0.0	0.8	374.0	-2.0
IV	1100_600_2500_5_180_15	219.0	327.6	-49.6	390.2	493.7	199.6	0.0	3.8	193.7	11.5
IV	1000_600_4000_5_250_20	236.2	312.1	-32.1	728.5	823.1	207.5	0.0	1.0	204.7	13.3
IV	1200_800_6000_6.5_400_25	426.5	511.9	-20.0	1938.8	2249.1	379.8	0.0	0.8	374.0	12.3

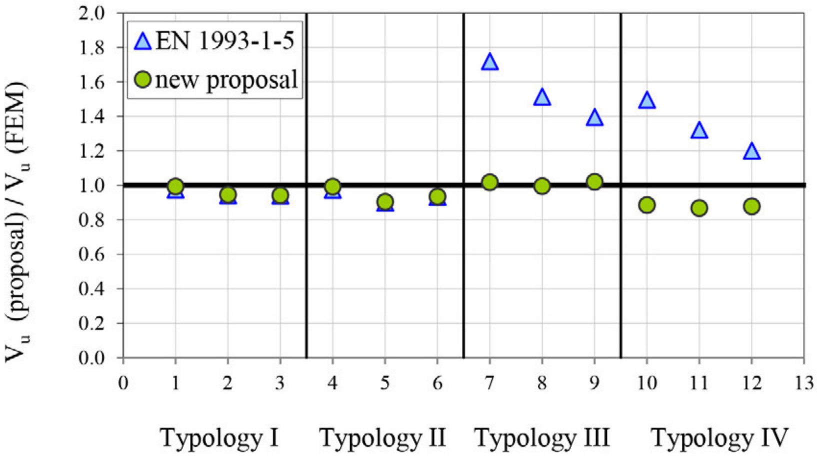

Figure 9. Comparison of the results obtained according to EN 1993-1-5 and the new proposal.

Based on the results obtained from 85 numerical simulations (approx. 20 for each typology) some modifications in expressions proposed by EN 1993-1-5 for rectangular plate girders were made for their application to tapered plate girders. First, the value of the depth used when determining the contribution of the web and used to calculate the distance *c* between two plastic hinges in the flange has been defined as $h_w = h_1$ for typologies I and II or $h_w = h_o$ for typologies III and IV.

Moreover, a new adjustment of the reduction factor χ_w has been presented for typologies III and IV.

A very good agreement between the results obtained according to the new proposal $V_{u\,(proposal)}$ and numerically $V_{u\,(FEM)}$ is observed. In the case where the shear-bending interaction is not considered, the differences between the values of the ultimate shear strength $V_{u\,(FEM)}$ and $V_{u\,(proposal)}$ do not exceed 33 %, whereas in most cases these differences are less than 15 %.

The same design proposal was applied for tapered steel plate girders subjected to shear-bending interaction. Due to presence of the significant bending moments at the mid-span cross-section, an additional reduction of the ultimate shear resistance had to be done. Validation of the new proposal was carried out for 12 girders, 3 from each typology. Also here, a good agreement between the results obtained numerically and these calculated according to the proposal was observed. A significant improvement of the results was especially visible for typologies III and IV.

Presented design proposal not only provides a satisfactory tool for the assessment of the ultimate shear resistance of tapered steel plate girders but also reveals its physical interpretation and respects individual contributions of all parts. Therefore, with use of the proposed method all components of the ultimate shear strength of the whole girder such as: the contribution from the web and flanges or Resal force can be found. This improvement was possible especially thanks to division of tapered plate girders into four different typologies which required an individual analysis according to their geometry.

It is important to point out that the main objective of this proposal is not thought to substitute or change the existing rules EN 1993-1-5 for rectangular plate girders, but only to give a simplified algorithm to assess an approximated value of the ultimate shear resistance of tapered members with considerable slope of the inclined flange. It is strongly recommended to treat the proposal as a reference and for the cases where the high precision is required an additional numerical study of specific girder is needed.

ACKNOWLEDGEMENTS

The authors wish to express their gratitude to Spanish Ministry of Science and Innovation for the financial support provided as a part of the Research Project BIA2008-01897, and for a doctoral scholarship for PhD student Agnieszka Bedynek. Additionally, the authors would like to thank Universitat Politècnica de Catalunya for the grant awarded in 2009 to the first author.

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