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# Flow in Porous Media and Environmental Impact

Noujoud M Jawhar

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United Arab Emirates University  
Deanship of Graduate Studies  
M.Sc. Program in Environmental Sciences

# **FLOW IN POROUS MEDIA AND ENVIRONMENTAL IMPACT**

By

**Noujoud M. Jawhar**

A thesis

Submitted to

The United Arab Emirates University

In partial fulfillment of the requirements

for the degree of M.Sc. in Environmental Sciences

**2008**

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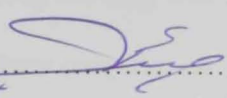
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### **SUPERVISED BY**

<b>Dr. F. Allan</b>	<b>Dr. M. Anwar</b>	<b>Dr. M. Syam</b>
<b>Professor</b>	<b>Professor</b>	<b>Professor</b>
<b>Department of Mathematics</b>	<b>Department of Mathematics</b>	<b>Head Department of Mathematical Sciences</b>
<b>Faculty of Science</b>	<b>Acting Dean, College of Science</b>	<b>Faculty of Science</b>
<b>UAE University</b>	<b>UAE University</b>	<b>UAE University</b>

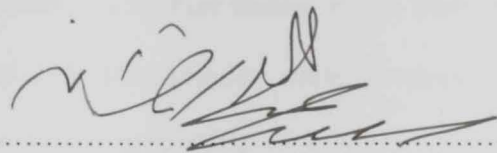
The Thesis of **Noujoud Mohamed Jawhar** for the Degree of Master of Science in Environmental is approved.



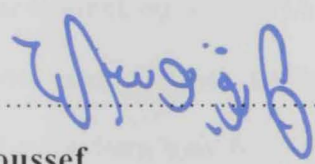
Examining Committee Member, **Dr. Fathi M. Allan**



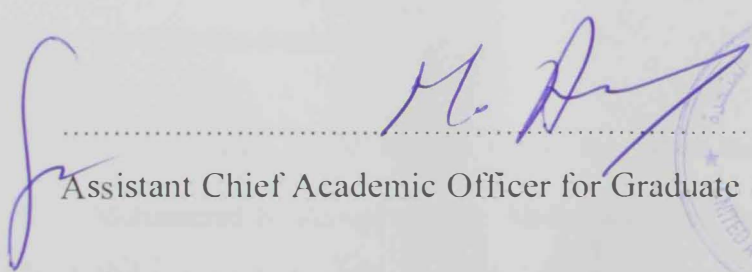
Examining Committee Member, **Dr. Ali Mohammed Sayfy**



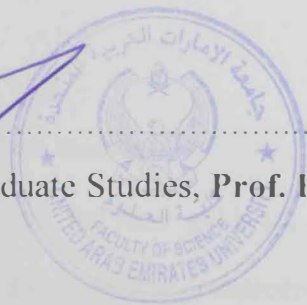
Examining Committee Member, **Dr. Saud Khashan**



Program Director, **Dr. Tarek Youssef**



Assistant Chief Academic Officer for Graduate Studies, **Prof. Ben Bennani**



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## ABSTRACT

Recently, a great deal of interest has been focused on the investigation of transport phenomenon in disordered systems. In particular, fluid flow through porous media has attracted much attention due to its importance in several technological processes such as filtration, catalysis, chromatography, spread of hazardous wastes, petroleum exploration, and recovery. Furthermore, flow through porous media is an important environmental problem that has environmental implications in several areas such as the study of pollution, fate of contaminants, contaminations issues related to agriculture, civil constructions, coastal management, and many more.

In this thesis, the fluid flow through multi-layers porous media is investigated. A mathematical model for the flow velocities is set to describe the flow through these different layers, together with initial and boundary conditions. More attention is made to the velocity profiles at the interface. The model is then solved with two different methods, the shooting method and the finite difference method. We consider a finite width three porous layer problem, where the layers have different permeability values which introduce a discontinuity in the permeability at the interface region. At the interface the continuity of the velocity and shear stress are imposed. A comparison between the nonlinear shooting method and the finite difference approach is then made and it shows that the shooting method is more accurate and more efficient.

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# 1. 0 INTRODUCTION

## 1.1 Porous Medium

A porous medium is a material that consists of a solid matrix with an interconnected void. The interconnectedness of the pores allows the flow of one or more fluids through the material. In the simplest situation which is a single phase flow, the void is saturated by a single fluid. In two phase flow, a liquid and a gas share the void space. Fluid flow in a porous medium resembles that of pouring a cup of water for example over soil and letting the water flow into the soil due to the gravitational forces [1].

A porous material or structure must pass at least one of the following two tests in order to be qualified as a porous medium. The first test, it must contain spaces, called pores or voids, free of solids imbedded in the solid or semisolid matrix. The pores usually contain some fluid, such as water, oil, or a mixture of different fluids. The second test is that it must be permeable to a variety of fluids. That is fluids should be able to penetrate through one face of the material and emerge on the other side. To distinguish between a porous solid and just any solid is a straight-forward process since the infiltration of viscous flow is a predetermined condition for the material to qualify as a porous medium.

Porous materials are encountered literally everywhere in everyday life, in technology, and in nature. Many natural substances such as rocks, and biological tissues such as lungs and bones, and man made materials such as cements, foams and ceramics can be considered as porous media. More examples of porous material include soil which is capable of performing its function of sustaining plant life only

because it can hold water in its pore spaces. Building materials such as bricks, concrete, limestone, and sandstone are examples of porous materials and they are considered to be better thermal insulators because of their porous nature [2].

The concept of porous media is used in many areas of applied science and engineering. It is used in mechanics such as geo-mechanics, soil and rock mechanics, in engineering such as petroleum and construction engineering, in geosciences such as hydrogeology and geophysics, in biology and biophysics, in material sciences, and many more fields of science. However, the most important areas of technology that depend significantly on the properties of porous media are hydrology, which relates to water movement in earth and sand structures, petroleum engineering which is mainly concerned with petroleum, and natural gas production, exploration, well drilling, and logging [1]. Additionally, the flow of blood and other body fluids, and electro-osmosis are few examples where porous media plays a critical role in medicine and biological engineering. Fluid flow through porous media has emerged as a separate field of study because it is a subject of most common interest [3].

The porosity  $\phi$  and the permeability  $k$  are two important quantities that describe the properties of a porous medium. The porosity of a porous medium is defined as:

$$\phi = \frac{\text{pore volume}}{\text{matrix volume}} , \quad (1.0)$$

where the pore volume denotes the total volume of the pore space in the matrix and the matrix volume is the total volume of the matrix including the pore space. Thus, porosity is greater than or equal zero and less than or equal 1. Porosity equals to zero

when the pore volume equals to zero, i.e., there is no flow. Porosity equals to 1 when the pore volume is the same as the matrix volume [4].

The permeability  $k$  describes the ability of the fluid to flow through the porous medium. It is often called the absolute permeability, and it is a quantity that depends only on the geometry of the medium. There has been much effort to establish relations between the permeability and the porosity. However, a general formula seems to be impossible to find, and the permeability is found to be proportional to  $\phi^m$ , where  $m$  is in the range of 3 to 6 depending on the geometry of the medium [3,4].

The fluid flowing in the pore space is characterized by the dynamical viscosity  $\mu$ . The viscosity indicates the resistance in the fluid due to its deformations. At the microscopic level there are friction forces in the fluid caused by the interchange of momentum in collisions between the molecules. Thus, the viscosity of the fluid is set by the strength of the friction forces [3].

Fluid flow through a porous medium is often given by the Darcy equation. Consider a porous medium of absolute permeability  $k$  in a homogeneous gravitation field where one fluid of viscosity  $\mu$  is injected through the medium by applying a pressure gradient  $\nabla P$  across the matrix. Then the flow rate  $U$  of the fluid through the medium is given by Darcy's equation:

$$\vec{U} = -\frac{k}{\mu} (\nabla P - \rho \vec{g}), \quad (1.1)$$

Where  $g$  denotes the acceleration due to the gravitational forces and  $\rho$  is the density of the fluid [4].

## 1.2 Porosity

From a hydrologic point of view, the fundamental interests in a porous medium are its ability to hold and transmit water. Porosity is the most important term among many other terms that relate to the water holding potential of a medium. The porosity  $\phi$  of a porous medium is defined as the fraction of the total volume of the medium that is occupied by void space such as in equation (1.0).

In defining  $\phi$  in this way, all the void space is assumed to be connected. If some of the pore space in a medium are disconnected from the remainder, then effective porosity is introduced which is defined as the ratio of connected void to total volume. Depending on the type of the porous medium, the porosity may vary from near zero to almost unity. For example, certain types of volcanic rocks have very low porosities, while fibrous filters and insulators are highly porous substances [5]. There are many types of void space, but it is important to distinguish between two kinds of pore space. The first one forms a continuous phase within the porous medium, called interconnected or effective pore space. The other one consists of isolated or non interconnected pores dispersed over the medium. Moreover, pore space has a direct effect upon productive value of soils because of its influence upon water holding capacity and upon the movement of air, water, and roots through the soil. For example, when the pore space of a productive soil is reduced 10 percent, movement of air, water, and roots is greatly restricted and growth is very seriously impeded [1].

### 1.2.1 Factors Affecting Porosity

There are many factors that affect porosity, and there has been a lot of effort done to determine approximate limits of porosities values. The factors governing the magnitude of porosity are the following.



**A. Uniformity of grain size:** uniformity or sorting is the gradation of grains.

If small particles of silt or clay are mixed with larger sand grains, the effective porosity will be considerably reduced. Sorting depends on at least four major factors which are size range of the material, type of deposition, current characteristics, and the duration of the sedimentary process [5].

**B. Degree of cementation:** The highly cemented sandstones have low porosities, whereas the soft, unconsolidated rocks have high porosities. The cementation process is very essential because filling void space with mineral material will reduce porosity.

**C. Amount of compaction:** During and after deposition, compaction tends to close voids and squeeze fluid out to bring the mineral particles closer together, especially the finer grained sedimentary rocks. Generally porosity is lower in deeper, older rocks, but exceptions to this basic trend are common [5].

### 1.2.2 Classification of porosity

Additional terms should be introduced regarding terminology with respect to the porosity namely the primary and the secondary forms. The primary porosity refers to the original porosity of the medium upon deposition. Secondary porosity refers to that portion of the total porosity resulting from processes such as dissolution. For example, limestone has a very low primary porosity upon deposition. If raised above sea level, dissolution processes can lead to the formation of caves with very large secondary porosity. A similar interpretation applies to the distinction between solid and fractured rock. The rock matrix has very low porosity. The presence of fractures increases the secondary porosity over the primary porosity. The secondary porosity of a medium is not always greater than its primary porosity, for example, a sand deposit that may become cemented over time to form sandstone. The chemically precipitated cementing agents occupy part of the pore space, therefore reducing the overall porosity [1, 4].

During sedimentation, some of the pore spaces initially developed became isolated from the other pore spaces by various processes such as cementation and compaction. Thus, many of the pores will be interconnected, whereas others will be completely isolated. This leads to two distinct categories of porosity, namely total or absolute and effective. Absolute porosity is the ratio of the total void space in the sample to the bulk volume of that sample regardless of whether or not those void spaces are interconnected. A rock may have considerable absolute porosity and yet have no fluid conductivity for lack of pore interconnection. Examples of this are lava and pumice stone.

Effective porosity is the ratio of the interconnected pore volume to the bulk volume. This porosity is an indication of the ability of a rock to conduct fluids, but it

should not be used as a measure of the fluid conductivity of a rock. It is affected by a number of factors including the type, content and hydration of the clays present in the rock, the heterogeneity of grain sizes, the packing and cementation of the grains, and any weathering and leaching that may have affected the rock. Experimental techniques for measuring porosity must take these facts into consideration [1, 3, 5].

For natural media,  $\phi$  does not normally exceed 0.6. For beds of solid spheres of uniform diameter,  $\phi$  varies between 0.2545 and 0.4764. For man made materials such as metallic foams  $\phi$  can approach the value 1. Table III illustrates the different values of porosity and other properties of common porous materials [4].

### 1.3 DARCY'S LAW

The science of groundwater flow originates from about 1856, in which year the city engineer of Dijon, Henry Darcy, published the results of the investigations that he had carried out for the design of a water supply system based on subsurface water carried to the valley in which Dijon is located, by permeable layers of soil, and supplied by rainfall on the surroundings. Since that time the basic law of groundwater movement carries his name, but the presentation has been developed, and the law has been generalized in several ways [6]. Henry Darcy's investigations revealed proportionality between flow rate and the applied pressure difference. In modern notation this is expressed by equation (1.1), where  $\nabla P$  is the pressure gradient in the flow direction, and  $\mu$  is the viscosity of the fluid. The coefficient  $k$  is the permeability, and it is independent of the nature of the fluid but it depends on the geometry of the medium,  $U$  is the flow rate,  $g$  denotes the acceleration due to the gravitational forces, and  $\rho$  is the density of the fluid.

It is useful to consider as a point of reference, the hydrostatics in a porous medium, the pores of which are completely filled with a fluid of density  $\rho$ . If the pressure in the fluid is denoted by  $p$ , the principles of hydrostatics teach that in the absence of flow the pressure increases with depth, and the local pressure gradient is equal to  $\rho g$ , where  $g$  is the gravity acceleration. Thus, with the positive  $z$ -axis pointing upwards, if there is no flow we have:

$$\frac{\partial p}{\partial x} = 0, \quad (1.2)$$

$$\frac{\partial p}{\partial y} = 0, \quad (1.3)$$

$$\frac{\partial p}{\partial z} + \rho g = 0, \quad (1.4)$$

These equations express equilibrium of the pore fluid. They are independent of the actual pore geometry, provided that all the pores are interconnected [1].

In the case when the pore fluid moves with respect to the solid matrix a frictional resistance is generated, due to the viscosity of the fluid and the small dimensions of the pores. The essence of Darcy's experiments result is that for relatively slow movements the frictional resistance is proportional to the flow rate. If inertia effects are disregarded, and if the porous medium is isotropic, that is the geometry of the pore space is independent of the direction of flow, the equations of equilibrium can be written as:

$$\frac{\partial p}{\partial x} + \frac{\mu}{k} q_x = 0, \quad (1.5)$$

$$\frac{\partial p}{\partial y} + \frac{\mu}{k} q_y = 0, \quad (1.6)$$

$$\frac{\partial p}{\partial z} + \frac{\mu}{k} q_z = 0 , \quad (1.7)$$

Where  $\mu$  is the viscosity of the fluid, and  $k$  indicates the permeability of the porous medium. The quantities  $q_x$ ,  $q_y$ , and  $q_z$ , are the three components of the specific discharge vector, where specific discharge denotes the discharge through a certain area of soil, divided by that area [6].

Darcy's law is a simple mathematical statement which summarizes several familiar properties that groundwater flowing in aquifers demonstrates. Such properties include first if there is no pressure gradient over a distance, no flow occurs. Second, if there is a pressure gradient, flow will occur from high pressure towards low pressure; the greater the pressure gradient the greater the discharge rate. Third, the discharge rate of fluid will often be different through different formation materials, or even through the same material in a different direction, even if the same pressure gradient exists in both cases [3, 5, 6].

### 1.3 PERMEABILITY

It is the term used for the conductivity of the porous medium with respect to permeation by a fluid. It has limited usefulness because its value in the same porous sample may vary with the properties of the permeating fluid and the mechanism of permeation. This quantity is called permeability  $k$ , and its value is uniquely determined by the pore structure. Darcy is the practical unit of permeability [3, 4]. A porous material has  $k$  equals to 1 Darcy if a pressure difference of 1 atmosphere will produce a flow rate of  $1 \text{ cm}^3/\text{sec}$  of a fluid with 1 cP viscosity through a cube having sides 1 cm of length. Thus:

$$1 \text{ Darcy} = \frac{\left( \frac{1 \text{ cm}^3}{\text{sec}} * 1 \text{ cP} \right)}{\left( \frac{1 \text{ atm}}{\text{cm}} * 1 \text{ cm}^3 \right)}, \quad (1.8)$$

One Darcy is a relatively high permeability, and the permeability of most reservoir rock is less than one Darcy.

Measurement of permeability in the case of isotropic media is usually done on linear, mostly cylindrically shaped, core samples. The experiment would be arranged in a way to have either horizontal or vertical flow through the sample. Both liquids and gases have been used to measure permeability. However, liquids sometimes change the pore structure, thus the permeability changes as well, and that is due to the rearrangement of some particles, swelling of certain materials in the pores such as clays, and chemical reactions. In principle, measurement at a single steady flow rate permits calculation of the permeability from Darcy's law, however there is usually considerable experimental error when measuring, that is why it is advised to perform measurements at various low flow rates, plot the low rates versus the pressure drop

and fit a straight line to the data point. According to Darcy's law, this line must pass through the origin. However, the scatter of the data points might sometimes cause the best fitting straight line not to pass through the origin. If the data points cannot be fitted with a straight line, the Darcy's law is not applicable and the system needs to be investigated to find the reasons of the deviation [1, 3, 6].

There are two parameters that describe the permeability, the permeability  $k$ , and the hydraulic conductivity  $K$ , related to each other by the following equation:

$$K = k \left( \frac{\rho g}{\mu} \right), \quad (1.9)$$

Where  $\rho$  is the density of the fluid,  $g$  is the gravity, and  $\mu$  is the fluid's viscosity. The most fundamental property is the permeability  $k$  which only depends upon the properties of the pore space. Because of the factors  $\mu$  and  $\rho$  in this equation, the hydraulic conductivity also depends upon the fluid properties, in particular upon the viscosity. This means that more effort is required to let a more viscous fluid flow through a porous medium. It also means that  $K$  through the viscosity depends upon the temperature. In areas with great fluctuations between the temperatures in summer and winter, seasonal variations in the groundwater discharge may result [5].

One of the most important properties of soils is the velocity of water flow through the pore spaces caused by a given force. A measure of how easily a fluid, for example water, can pass through a porous medium, for example soils. The permeability of soil is defined as the velocity of flow caused by a unit hydraulic gradient. It is not influenced by the hydraulic slope, and this is an important point of difference between permeability and infiltration. Also the term permeability is used for designating flow through soils in any direction. It is influenced most by the physical properties of the soil. In saturated field soils permeability varies between wide limits: from less than 1

foot per year in compact clay soils, up to several thousand feet per year in gravel formations. For unsaturated soils, the moisture content is one of the dominant factors influencing permeability [7, 8, 9].

Furthermore, many physical and chemical characteristics of soils are related to their texture. Fine grained soils have a much larger surface area than coarse grained soils. Thus, their mineral structure is different, resulting in a much greater capacity for sorption of chemicals. On the other hand, well sorted, coarse grained soils have a much smaller sorption capacity and a much larger permeability, for example, cemented sediments include sandstone and shale. Depending on the degree of cementation, sandstone can be very permeable and can serve as an excellent source for water supply. Shale has very low permeability because shale deposits are hydrogeologically significant when they act as confining beds bounding more permeable strata. Carbonate rocks such as limestone have relatively high amounts of void space but have low permeability. Igneous and metamorphic rocks have small amounts of void space about less than 1 percent of the rock mass. In addition, the few present pores are small and not interconnected, resulting in permeabilities that may be regarded as zero for almost all practical problems [5, 10, 11]. However, all rocks such as sedimentary, igneous, and metamorphic can be fractured by earth stresses. Tables I and II illustrate the different values of permeability and hydraulic conductivity of common porous materials. Fractured rock is quite different from unfractured rock and that is because water may move easily through the fractures giving the rock mass a higher permeability although not much capability for storing water within the pore space. In addition, rock that is fractured by earth stresses may develop directional characteristics to its permeability, having a greater potential for allowing flow in certain directions.



**Table I** Values of Hydraulic Conductivity

Material	Hydraulic conductivity K (m/s)
Clay	$10^{-10}$ to $10^{-8}$
Silt	$10^{-8}$ to $10^{-6}$
Sand	$10^{-5}$ to $10^{-3}$
Gravel	$10^{-2}$ to $10^{-1}$

**Table II** Values of Permeability

Material	Permeability k ( $m^2$ )
Clay	$10^{-17}$ to $10^{-15}$
Silt	$10^{-15}$ to $10^{-13}$
Sand	$10^{-12}$ to $10^{-10}$
Gravel	$10^{-9}$ to $10^{-8}$

**Table III** Values of Porosity and Permeability

Material	Porosity	Permeability [ $cm^2$ ]
Brick	0.12-0.34	$4.8*10^{-11}$ - $2.2*10^{-9}$
Cigarette		$1.1.8*10^{-3}$
cigarette filters	0.17-0.49	
Coal	0.02-0.12	
Concrete	0.02-0.07	
copper powder,	0.09-0.34	$3.3*10^{-6}$ - $1.5*10^{-5}$
Fiberglass	0.88-0.93	
granular crushed rock	0.45	
Hair	0.95-0.99	
Leather	0.56-0.59	$9.5*10^{-10}$ - $1.2*10^{-9}$
Limestone	0.04-0.10	$2*10^{-7}$ - $4.5*10^{-10}$
Sand	0.37-0.50	$2*10^{-7}$ - $1.8*10^{-6}$
Sandstone	0.08-0.38	$5*10^{-12}$ - $3*10^{-8}$
silica grains	0.65	
silica powder	0.37-0.49	$1.3*10^{-10}$ - $5.1*10^{-10}$
Soil	0.43-0.54	$2.9*10^{-9}$ - $1.4*10^{-7}$

### **1.4.1 Classification of permeability**

There are two types of permeability, primary permeability, which is also known as the matrix permeability, and secondary permeability. Matrix permeability originated at the time of deposition of sedimentary rocks where secondary permeability resulted from the alteration of the rock matrix by compaction, cementation, fracturing and solution. While compaction and cementation generally reduce the primary permeability, fracturing and solution tend to increase it. For example, in some reservoir rocks, in particular low porosity carbonates, secondary permeability provides the main conduit for fluid migration.

Furthermore, there are many factoring affecting the magnitude of permeability. First factor is the shape and size of sand grains. That is if the rock is composed of large and uniformly rounded grains, its permeability will be considerably high and of the same in both directions horizontally and vertically. Permeability of reservoir rocks is generally lower, especially in the vertical direction, if the sand grains are small and of irregular shape. Cementation is another factor. Both porosity and permeability are influenced by the extent of cementation and the location of the cementing material within the pore space. Fracturing and solution is another factor. Fracturing is not an important cause of secondary permeability in sandstones, except where they are interbedded with shales and limestones [5].

## **1.5 SOIL STRUCTURE**

Soil structure is described in terms of the size and the shape of particles. It is very helpful to realize the differences among soils in order to better understand retention and movement of water, because these phenomena are governed to a large extent by pore size and shape distributions. Large pores can conduct more water,

more rapidly than fine pores. If we rely on the basic relationships between pore-size distributions, flow rates, and suctions, hydraulic properties of soils can be calculated based on observed physical properties of soils. However, measuring soil particle size and structure is much harder than experimental measurements of water content and water movements in soils. Soil is the loose surface of the earth consisting of solid particles, water and air. The solid particles are usually less than 2 millimeters in diameter, and are categorized based on their diameter size. We have sand which has a diameter of 0.05 to 2 millimeters, silt from 0.002 to 0.05 millimeters, and clay less than 0.002 millimeters. As a result, the texture of the soil depends on the relative proportions of sand, silt and clay, and can be defined as a coarse or fine soil. For example, sandy loam is a coarse soil, whereas clay loam is a fine soil [12].

## 1.6 WATER MOVEMENT

In order for water to move from one point to another, two conditions must be met. First, there must be a difference in hydraulic head between the two points, that is  $\Delta H$  which is the difference or change in total water potential between points in the soil. Second, the soil between these two points must be permeable enough to allow the movement of water [12, 2]. Hydraulic conductivity ( $K$ ) is a measure of the ability of a soil to transmit water. The larger the  $K$  of a soil, the greater will be the movement of water through it for any given hydraulic gradient. Darcy's Law for liquid movement in porous media states that the rate of water flow ( $q$ ) through a given soil segment is equal to the hydraulic conductivity of that soil multiplied by the hydraulic gradient that exists in that soil. Therefore, Darcy's Law is written mathematically as follows:

$$q = -K \frac{\Delta H}{L}, \quad (1.10)$$

where  $q$  is the flux, or flow rate in centimeters per hour or day;  $K$  is the hydraulic conductivity in centimeters per hour or day;  $\Delta H$  is the hydraulic head difference between two points in centimeters; and  $L$  is the distance between the two points in centimeters [13].

A soil has a maximum  $K$  value when it is saturated ( $K_{\text{sat}}$ ).  $K$  values are characteristically different for different soils, depending upon soil structure and pore-size distribution. To illustrate this, we use the diagram in Figure 1, in which soils with different pore size distributions are represented by sets of capillaries of varying diameter. The sand contains relatively large pores, but the pores in the clay are finer. At saturation, all pores are filled with water. Large pores conduct much more water than fine pores. When the pore radius is twice as large, for example, about sixteen times of more water can be conducted. It is clear in Figure 1 there are much longer arrows from the larger pores than from the smaller ones [1, 12].

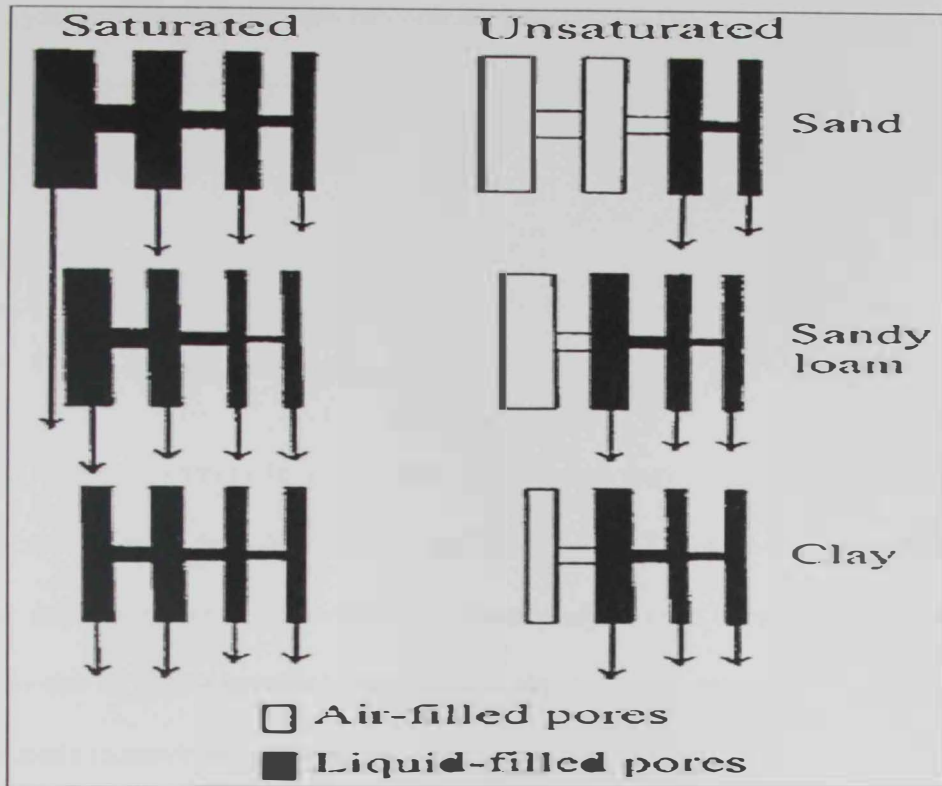


Figure 1: Water Flow Difference [12]

In addition, we notice that the sand is more permeable than the clay at saturation, but the opposite is true when the soils are unsaturated. The large pores, which resulted in a high hydraulic conductivity for the sand at saturation, become filled with air as the soil becomes unsaturated. More water-filled fine pores remain in the clay.

Moreover, the direction (upward, downward, or lateral) and magnitude of water flow in soils depends on the direction and magnitude of hydraulic head gradient and the degree of water saturation of the soil. As a result, we see that there can be no flux of water ( $q$ ) in soil without both a hydraulic gradient  $\frac{\Delta H}{L}$  and hydraulic conductivity ( $K$ ). A soil having a very high hydraulic conductivity will experience little water movement if there is a very low hydraulic gradient. On the other hand, a

high hydraulic gradient between two points in the soil will not cause water flow if  $K$  is essentially zero due to the occurrence of impermeable soil between the two points [13].

### 1.6.1 Water Movement through Soil

Water movement in soil is quite simple and easy to understand, but at the same time it can be complex to be comprehended. It is based on the concept that an object that can move freely, will move impulsively from a higher potential energy state to one of lower potential energy, so is the case with water. A unit volume of water tends to move from an area of higher potential energy to one of lower potential energy [14].

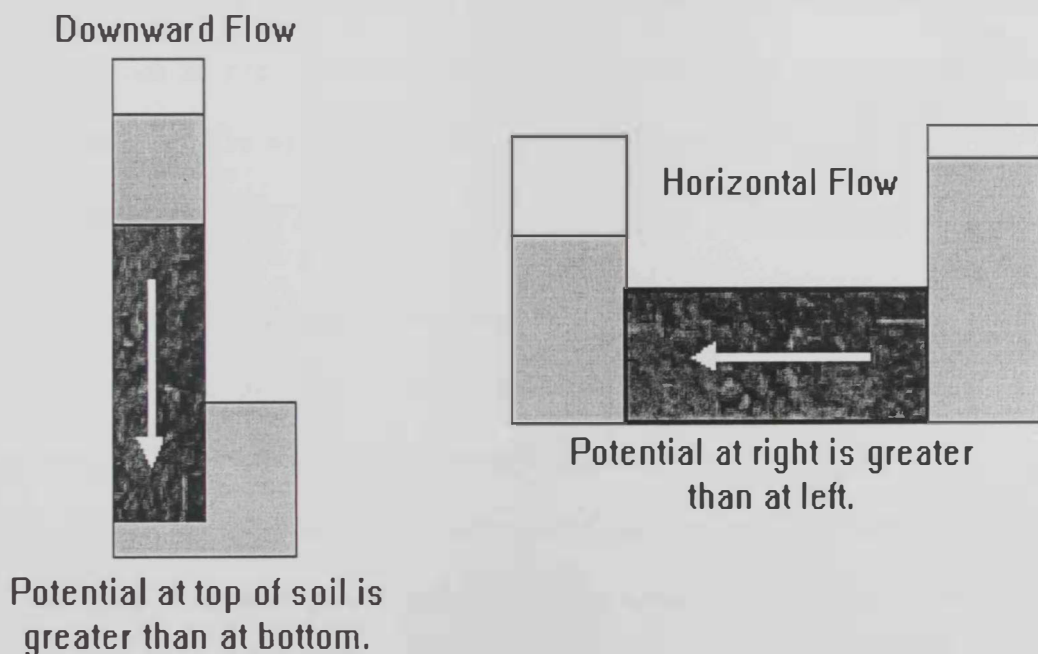


Figure 2: Downward Flow vs. Horizontal Flow

After applying water to the ground's surface, it either evaporates into the atmosphere, or percolates into the soil. The way in which water moves through soil is dependent primarily on the properties of the soil, the interaction between water and the soil, the soil moisture gradient, and the changes in soil properties with depth. The size, numbers and continuity of soil pore spaces affect the rate of water movement and the distance that water can move. The spaces between solid particles are pore spaces, which contain water and air. Even though a fine soil has smaller soil pores than a coarse soil, it has a much greater number of pore spaces than a coarse soil. Therefore, water tends to move more slowly through a fine soil such as clay loam than through a coarse soil such as sand [14]. As water moves into the pore spaces, it displaces the air while filling the pores with water. If the pore spaces are blocked by entrapped air bubbles, the continuity of pore spaces to conduct water is broken. Thus, water movement is reduced through these discontinuous pores. Water movement through soil pores is further influenced by the interaction of water with the solid soil particles, which are composed mainly of mineral materials and a small percentage of organic materials. The water moving through the soil pore spaces is similar to water moving through a capillary tube.

### **1.6.2 How Texture Affects Water Movement through Soil**

Water generally flows downward to deeper depths and from wetter areas to drier ones. This movement of water through soil occurs due to two forces; first the downward pull of gravity and second the forces of attraction between water molecules and soil particles. Just as gravity pulls all objects toward the center of the earth, it pulls water molecules downward through the soil. In sandy soils, this is the primary cause of water draining downward through soil to groundwater. In clay soils, forces of attraction between soil and water molecules also play a key role in determining

movement of soil water. Just as soil types vary in texture and structure, they also vary in their ability to conduct and hold water. Thus, soil pore size is a significant factor in how water moves through soil. In general, water moves through large pores, such as in sandy soils, more quickly than through smaller pores, such as in silty soil, or through the much smaller pores found in clay soil [15].

### **1.6.3 Infiltration**

It is the movement of water into soil from the surface. Water infiltration is largely governed by the surface properties of the soil. A soil high in organic matter, having good structure, and has medium to coarse texture will usually have a rapid infiltration rate. Many other factors, such as water movement within soil, roughness of the ground surface, vegetative cover, and slope, will also affect infiltration [16]. Water already in the soil profile must move downward before more water can enter at the surface. Permeability or hydraulic conductivity is a term that describes the ease with which water moves within a soil. Permeable soils conduct water readily through their mass. Other soils may conduct water slowly or have restricting layers or horizons which limit or prevent downward movement of water. Soils or soil layers which do not conduct water, at all, are termed impermeable. Permeability, like infiltration, is largely determined by texture, structure, and organic matter content [14].



## **1.7 SATURATED VS. UNSATURATED FLOW**

### **1.7.1 Unsaturated water flow**

When the soil pore spaces are partially filled with water, the soil is unsaturated. Unsaturated water flow is slow, and occurs mainly by adjusting the thickness of water films, or capillary water that surrounds the soil particles. Water in unsaturated soil tends to have little movement. The soil moisture gradient, which is the difference in the water content from one soil zone to another, is the driving force for unsaturated water flow. Water flows from pores in wetter soil zones to pores in drier soil zones. Water under unsaturated conditions can move downward, horizontally or upward, depending on the position of the drier soil zone in relation to the wetter soil zone [17].

### **1.7.2 Saturated Water Flow**

Soil is considered saturated when all pore spaces are filled with water. Saturated water flow is rather rapid, and occurs mainly by draining the gravitational water occupying the pore spaces between the soil particles. The size of the soil pores is the main influence on saturated water flow. Like a capillary tube, the soil pores should be continuously connected to each other in order to form a conduit for water to move through the soil. Coarse soil has larger pores, enabling it to conduct saturated water flow faster than fine-textured soil. Gravitational forces are the main driving force for saturated water flow. The direction of saturated water flow is usually downward. Horizontal or lateral water flow in soil under saturated conditions occurs slowly because the force of gravity doesn't assist horizontal water flow. Upward

saturated water flow is very limited because the force of gravity holds back the upward flow [17, 18].

## 2.0 OBJECTIVES AND LITERATURE REVIEW

Many environmental and engineering problems can be characterized by a fluid flow through porous channels with different permeability. Examples of these flows are the oil flow through ground layer and the flow of underground water.

The main objective of this work is to study the fluid mechanics of multi layer flows (More than two layers). The problem of the two layers flows was investigated by several authors [20, 21]. Beavers and Joseph [20] considered the interface region between a porous media and a fluid layer. They presented a description of the velocity gradient using an empirical data that depends on the velocity in the fluid layer and the porous region.

Later, Vafai and Thiyagaraja, [22] showed that the idea of Beavers and Joseph is true only for the linear regime. While Vafai and Kim [23] studied the fluid mechanics of the interface region between a porous medium and a fluid layer, and derived an exact solution for the velocity. Allan and Hamdan [24] studied the fluid mechanics of the interface region between two porous media, and an exact solution was also obtained.

Solbakken, and Andersson [25] studied the lubricated plane channel by means of direct numerical simulations. Their results indicate that the velocity at the interface region of the lubrication plane plays a very significant role in the development of turbulent flow.

Lemos [26] considered a channel partially filled with a porous layer through which an incompressible fluid flows in turbulent regime. At the interface, a jump conditions are assumed and numerical simulation were used to investigate the velocity field. The results obtained indicated that the fluid flow depends heavily on the velocity distribution at the interface.

Specifically, the objectives of this work are:

1. Developing the mathematical model that describes the flow through each region.
2. Specifying the initial and boundary conditions for each region.
3. Developing the interface conditions for each region.
4. Solving the governing equations either numerically or analytically.
5. Study the effect of the two parameters, the Reynolds number and the Darcy number on the velocity at the interface region.

### 3.0 MATERIALS AND METHODS

The methodology of the suggested work is consistent with the research goals. It is divided into the following series of work:

- **Literature Review**

The literature review associated with the physical and mathematical formulation of the problem was reviewed, and the previous work was also presented. A detailed application of the problem was also investigated.

- **Problem Formulation**

The developed mathematical models of the problem were implemented using Mathematica. In addition, a suitable boundary and initial conditions were chosen so that the problem is well posed and exact solution is therefore possible.

- **Solution Techniques**

The method of solution of the problem under consideration was presented. And as expected an exact solution for the system of equations describing the problem was obtained.

- **Computer Program**

Mathematica was used to write the required programs. Two Mathematica programs were developed to solve the non linear system of equations where the unknowns were the velocities at the interface region. A comparison between the results of the two programs was later made to find which method is more efficient.

## 4.0 MATHEMATICAL MODELLING OF FLUID FLOW IN POROUS MEDIA

### 4.1 Simple Mathematical Model

#### 4.1.1 Fluid Flow through a channel of finite depth

A simple mathematical model describing the fluid flow through channels which has finite depth is presented along with suitable boundary conditions. An exact solution is obtained for various settings of the flow; including the existence of two layers of flows with different velocities, and the existence of porous media in one of the sides of the channel [27]. In this model, the attention was devoted to the discussion of several cases where exact solutions can be obtained.

First of all, flow through a channel of finite depth is investigated where parallel flow occurs. A flow is parallel if only one velocity component is different from zero, that is all fluid particles are moving in one direction. For example if the velocity components are  $u$ ,  $v$  and  $w$  and if the components  $v$  and  $w$  are zero everywhere, it follows at once from the equation of continuity that  $\frac{\partial u}{\partial x} \equiv 0$ , which means that the component  $u$  cannot depend on  $x$ . Thus, for parallel flow we have:

$$u = u(y,t) ; v \equiv 0 ; w \equiv 0 , \quad (4.0)$$

Furthermore, from the Navier Stockes the pressure for the y and z directions of the pressure p, we have

$$\frac{\partial p}{\partial y} \equiv 0, \quad (4.1)$$

$$\frac{\partial p}{\partial z} \equiv 0, \quad (4.2)$$

In this case, the pressure depends on x only. Add to that, all the convective terms vanish in the equation for the x direction, so we have:

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial^2 y} + \frac{\partial^2 u}{\partial^2 z} \right), \quad (4.3)$$

And this is a linear differential equation for u(y, z, t) which a simple solution can be obtained for the case of steady flow in a channel with two parallel flats walls. By making the distance between the walls to be 2H, the former equation becomes:

$$\frac{dp}{dx} = \mu \frac{d^2 u}{d^2 y}, \quad (4.4)$$

With boundary condition  $u=0$  for  $y=+H$  or  $-H$ .

Since  $\frac{\partial p}{\partial z}=0$ , the pressure gradient in the direction of flow is constant, as seen from equation (4.4). Thus, if  $\left(\frac{1}{\mu}\right)\frac{dp}{dx} = \alpha$ , which is a constant, the solution for Eqn. (4.4) together with the above boundary conditions will be:

$$u = \left(-\frac{1}{2}\right)\alpha(H^2 - y^2), \quad (4.5)$$

Figure 3 shows the velocity profiles for the flow through a channel for various values of  $\alpha$ .



Another simple solution can be obtained to equation (4.4) known as Couette flow between two parallel flat walls where one of the them is at rest and the other is moving in its own plane with a velocity  $U$  while having the following boundary conditions:

$$U(0)=0, \text{ and } u(H)=U,$$

We obtain the solution which is shown in figure 3;

$$u = \left(\frac{y}{H}\right)U - \left(\frac{H^2}{2}\right) \text{Re}c \left(\frac{y}{H}\right)\left(1 - \frac{y}{H}\right), \quad (4.6)$$

#### 4.1.2 Fluid flow through a two layer channel

In addition, if an imaginary interface is placed at the line  $y=0$ , and parallel to the flow field then one has to worry about the interface velocity. In this case, additional boundary conditions are needed. Assume that two fluids with different velocities are flowing at the two layers of the channel as shown in Figure 4. Assume further that the fluid flow in the two regions are governed by:

$$\frac{d^2 u_i}{d^2 y} = \text{Re}c_i, \quad \text{for } i=1, 2, \quad (4.7)$$

Then the additional boundary condition for the velocities at the interface will be:

$$u_1(0)=u_2(0)=u_{\text{int}}$$

which means that the velocity is continuous. In this case the exact solution is given by:

$$u_1(y) = u_{\text{int}} + \frac{\text{Re}c_1 y}{2} - u_i y - \frac{\text{Re}c_1 y^2}{2}, \quad (4.8)$$

And

$$u_2(y) = u_{int} - \frac{Re c_2 y}{2} + u_1 y - \frac{Re c_2 y^2}{2}, \quad (4.9)$$

To solve for  $u_{int}$ , an additional condition is needed which is given by the smoothness of the velocity:

$$\frac{du_1}{dy}(0) = \frac{du_2}{dy}(0)$$

Using this condition and solving for  $u_{int}$  leads to the following value:

$$u_{int} = \frac{Re c_1 + Re c_2}{4}, \quad (4.10)$$

Figure 5 shows the velocity profiles for different values of  $Re c_1$  and  $Re c_2$ .

#### 4.1.3 Fluid flow through a two layer porous media

Now, consider another case that is similar to the one above where a barrier is placed at the line  $y = 0$  [27]. Let us assume that the upper region is governed by equation (4.4) and the lower region is governed by:

$$Re c_1 + \frac{d^2 U_1}{dy^2} - \frac{U_1}{Da_1} = 0, \quad (4.11)$$

Where quantities have been rendered dimensionless with respect to the characteristic length  $H$ , and the characteristic velocity  $U_\infty$ , using the definitions:

$$U_1 = \frac{u_1}{U_\infty}; \quad y = \frac{Y}{H}; \quad Da_1 = \frac{k}{H^2}; \quad Re = \frac{U_\infty H \rho}{\mu}; \quad \lambda = \frac{\delta F H}{\sqrt{k_1}}, \quad [23]$$

Then the conditions at the interface region will be:

$$\frac{du}{dy}(0) = \frac{dU_1}{dy}(0) \quad (4.12)$$

Solving equations (4.4) and (4.11) subject to (4.12) leads to the following solutions:

$$u(y) = u_{int} + \frac{Recl y}{2} - u_{int} y - \frac{Recl y^2}{2}, \quad (4.13)$$

$$U1(y) = - \frac{e^{-\frac{2}{\sqrt{Da1}} \left( Da1 \left( -1 + e^{\frac{2}{\sqrt{Da1}}} \right) \left( -1 + e^{\frac{y}{\sqrt{Da1}}} \right) Recl - \left( 1 + e^{\frac{1+y}{\sqrt{Da1}}} \right) u_{int} \right)}{-1 + e^{\frac{2}{\sqrt{Da1}}}}$$

Condition (4.12) can now be used to find the value of  $u_{int}$  and it is found to be of the form:

$$u_{int} = \frac{\sqrt{Da1} \left( -1 + e^{\frac{2}{\sqrt{Da1}}} \right) \left( 1 - 2\sqrt{Da1} + e^{\frac{1}{\sqrt{Da1}}} + 2\sqrt{Da1} e^{\frac{1}{\sqrt{Da1}}} \right) Recl}{2 \left( 1 - \sqrt{Da1} + e^{\frac{2}{\sqrt{Da1}}} + \sqrt{Da1} e^{\frac{2}{\sqrt{Da1}}} \right)}$$

Figure 6 shows the velocity profiles for different values of  $Da1$  while  $Recl=10$

Figure 3: Velocity profiles for the flow through a channel for various values of  $Re_1$ . (No interface region).

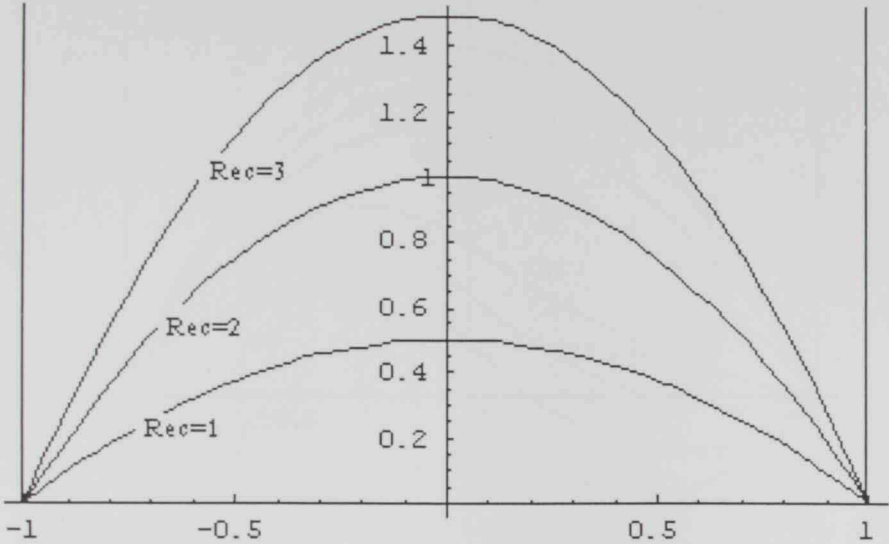


Figure 4: Schematic diagram for the two-layer flow through a channel of depth H.

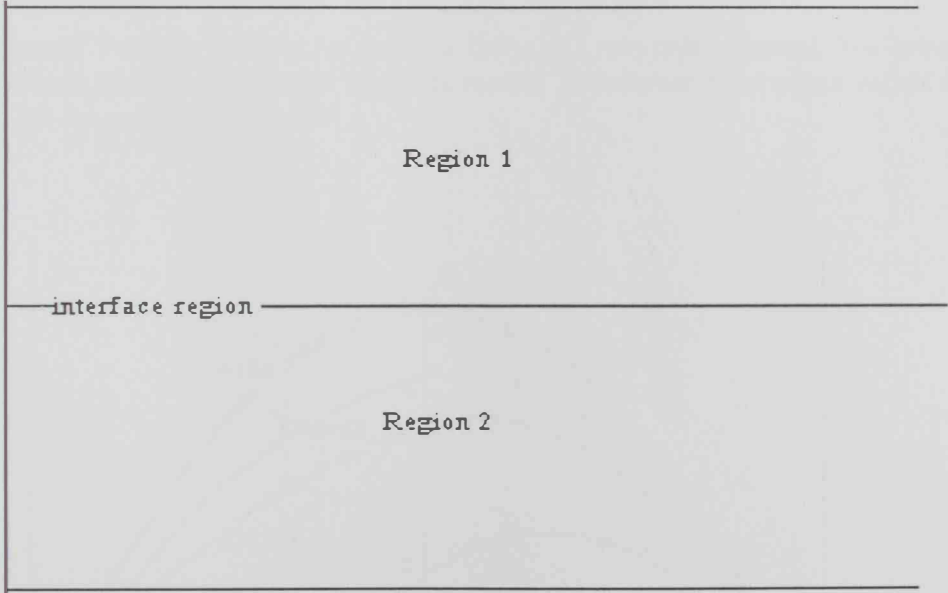


Figure 5: Velocity profiles for the flow through a multi-layer channel for various values of  $Rec_2$  and  $Rec_1$

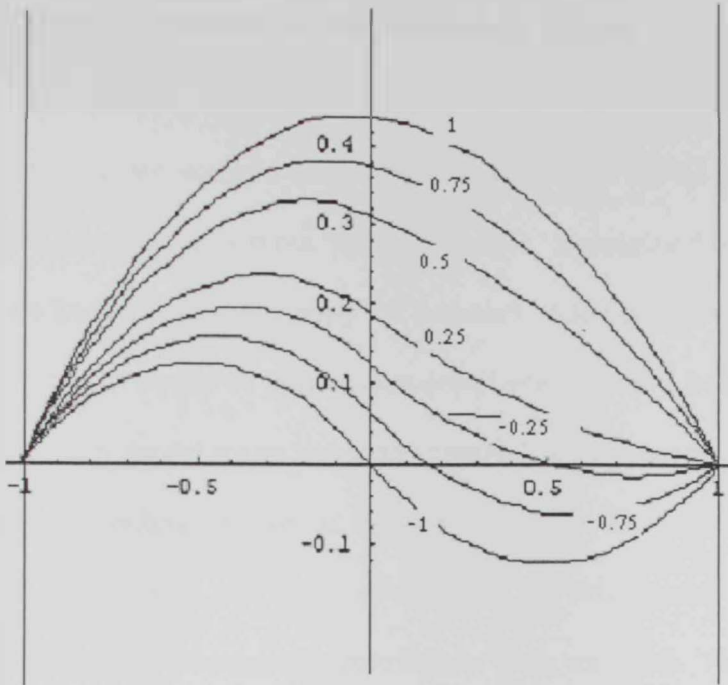
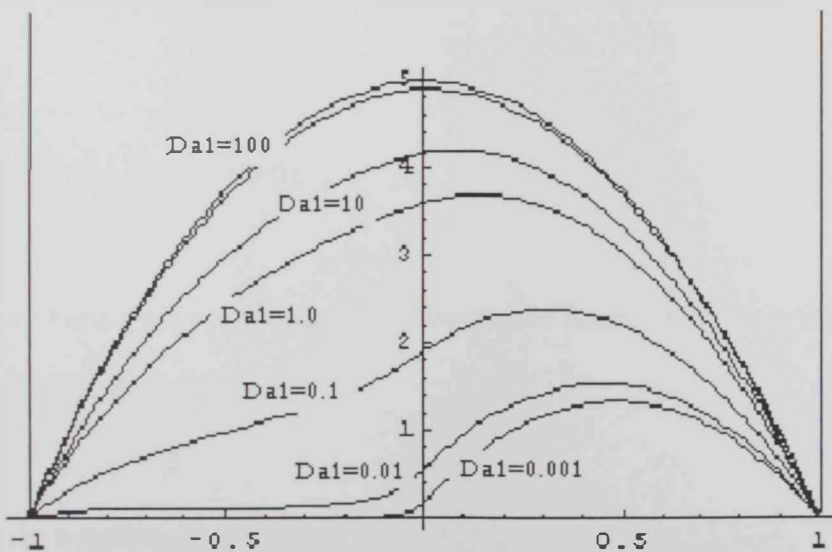


Figure 6: Velocity profiles for the flow through a two-layer channel. The lower layer represents fluid flow through a porous media. Results are for various values of  $Da_1$  and for the value of  $Rec_1=10$



## 5.0 NUMERICAL METHODS

### 5.1 A More Complicated Mathematical Model

A two dimensional shooting method is well known and applied to solve coupled non linear second order boundary value problems. The coupling is manifested by common boundary conditions at the interface. Examples for which the exact solutions are known are used to verify the accuracy validation of the algorithm.

Differential equations play an important role in many fields of science and engineering. They model many important physical phenomena. These differential equations can be ordinary or partial, linear or nonlinear. The exact solution of many differential equations is not easily obtainable. For this reason, researchers have developed numerical schemes to approximate their solutions. The general second order boundary value problem takes the form:

$$y''(x) = f(x, y, y'), \quad a \leq x \leq b, \quad y(a) = \alpha, \quad y(b) = \beta \quad (5.0)$$

where primes denote differentiation with respect to  $x$ .

However, two coupled boundary value problems are more of a concern, more precisely numerically solving the following two problems:

$$y''(x) = f(x, y, y'), \quad a \leq x \leq b, \quad (5.1)$$

and

$$z''(x) = g(x, z, z'), \quad b \leq x \leq c \quad (5.2)$$

Where  $f$  and  $g$  are continuous and differentiable functions, with the following left and right boundary conditions:

$$y(a) = \alpha, \quad z(c) = \beta \quad (5.3)$$

and the interface condition

$$y(b^-) = z(b^+), y'(b^-) = z'(b^+) \quad (5.4)$$

The shooting method solves the two problems separately as initial value problems in their domains with the missing conditions  $y'(a)$  and  $z'(c)$  are set of two parameters  $t$  and  $s$ , ( $y'(a)=t, z'(c)=s$ ). This will result in a system of two nonlinear equations in the two unknowns  $t$  and  $s$ . The two dimensional Newton's method is then used to solve for  $t$  and  $s$ .

The following is a review of the non linear shooting method for second order nonlinear boundary value problems and the deriving of the two dimensional shooting method to solve (5.1) and (5.2) subject to (5.3) and (5.4) is presented.

Consider the nonlinear ODE:

$$y''(x) = f(x, y, y'), \quad a \leq x \leq b \quad (5.5)$$

with boundary conditions

$$y(a) = \alpha, y(b) = \beta, \quad (5.6)$$

The non linear shooting method solves (5.5) as an initial value problem, i.e., solves

$$y''(x) = f(x, y, y'), \quad a \leq x \leq b \quad (5.7)$$

with initial conditions

$$y(a) = \alpha, y'(a) = t, \quad (5.8)$$

where  $t$  is an appropriate parameter so that the solution to (5.7) and (5.8), denoted by  $y(x, t)$ , satisfies the boundary condition (5.6) at  $x=b$ , i.e.,  $y(b, t) = \beta$ . In order for (5.6) to be satisfied, the parameter  $t$  has to be the zero of the function  $y(b, t) - \beta$ . However, the solution  $y(x)$  is not known. In implementation, one solves a sequence of (5.7-5.8) with  $t = t_k$  until the limit of  $y(b, t_k) - \beta$  as  $k$  goes to infinity near zero.

Inspired by the shooting method for second order nonlinear ODEs, we derive a two dimensional shooting method for coupled nonlinear second order ODEs as follows.

Consider the two ODEs:

$$y''(x) = f(x, y, y'), \quad a \leq x \leq b, \quad y(a) = \alpha \quad (5.9)$$

$$z''(x) = g(x, z, z'), \quad b \leq x \leq c, \quad z(c) = \beta \quad (5.10)$$

Where  $f$  and  $g$  are continuous and differentiable functions with respect to their variables.

The interface conditions (coupling conditions) are

$$y(b^-) = z(b^+), \quad y'(b^-) = z'(b^+) \quad (5.11)$$

To solve the above problem, we solve the two initial value problems:

$$y''(x) = f(x, y, y'), \quad a \leq x \leq b, \quad y(a) = \alpha, \quad y'(a) = s \quad (5.12)$$

and

$$z''(x) = g(x, z, z'), \quad b \leq x \leq c, \quad z(c) = \beta, \quad z'(c) = t \quad (5.13)$$

For  $y(x, s)$ ,  $a \leq x \leq b$ , and for  $z(x, t)$ ,  $b \leq x \leq c$ . The parameters  $s$  and  $t$  are to be determined such that (5.11) is satisfied, i.e.,

$$\begin{aligned} y(b^-; s) - z(b^+; t) &= 0 \\ y'(b^-; s) - z'(b^+; t) &= 0 \end{aligned} \quad (5.14)$$

To determine  $s$  and  $t$ , we regard (5.14) as a nonlinear system of two equations of two unknowns, then the two dimensional Newton's method is used [28, 29].

For the following example whose exact solution is known, we apply the algorithm described above to validate its accuracy.



Consider the following two problems:

$$y'' = -y'^2 - \frac{1}{2}y' - y - \frac{x}{4} + \frac{15}{16} + \ln(x), 1 \leq x \leq 2, y(1) = \frac{3}{4}, \quad (5.15)$$

and

$$z'' = z' + 2(z - \ln(x))^3 - \frac{1}{x}, 2 \leq x \leq 3, z(3) = \frac{1}{3} + \ln(3), \quad (5.16)$$

with interface conditions at  $x=2$ ,

$$y(2) = z(2), \text{ and } y'(2) = z'(2), \quad (5.17)$$

It can be verified that the exact solutions to the above problem (5.15) and (5.16) are:

$$y_e(x) = \ln(x) - \frac{1}{4}(x) + 1, \quad (5.18)$$

$$z_e(x) = \frac{1}{x} + \ln(x), \quad (5.19)$$

Applying the algorithm, we solve in order the following initial value problems:

$$y'' = -y'^2 - \frac{1}{2}y' - y - \frac{x}{4} + \frac{15}{16} + \ln(x), 1 \leq x \leq 2, y(1) = \frac{3}{4}, y'(1) = t_k \quad (5.20)$$

And

$$z'' = z' + 2(z - \ln(x))^3 - \frac{1}{x}, 2 \leq x \leq 3, z(3) = \frac{1}{3} + \ln 3, z'(3) = s_k \quad (5.21)$$

Then using the obtained solutions of (5.20),  $y(x; t_k), y'(x; t_k), 1 \leq x \leq 2$ , and of (5.21),

$z(x; s_k), z'(x; s_k), 2 \leq x \leq 3$ , we solve the IVPs:

$$u'' = -2y'u' - \frac{1}{2}u' - u', 1 \leq x \leq 2, u(1) = 0, u'(1) = 1 \quad (5.22)$$

And

$$u'' = v' + 6(z - \ln x)^2 v, 2 \leq x \leq 3, v(3) = 0, v'(3) = 0 \quad (5.23)$$

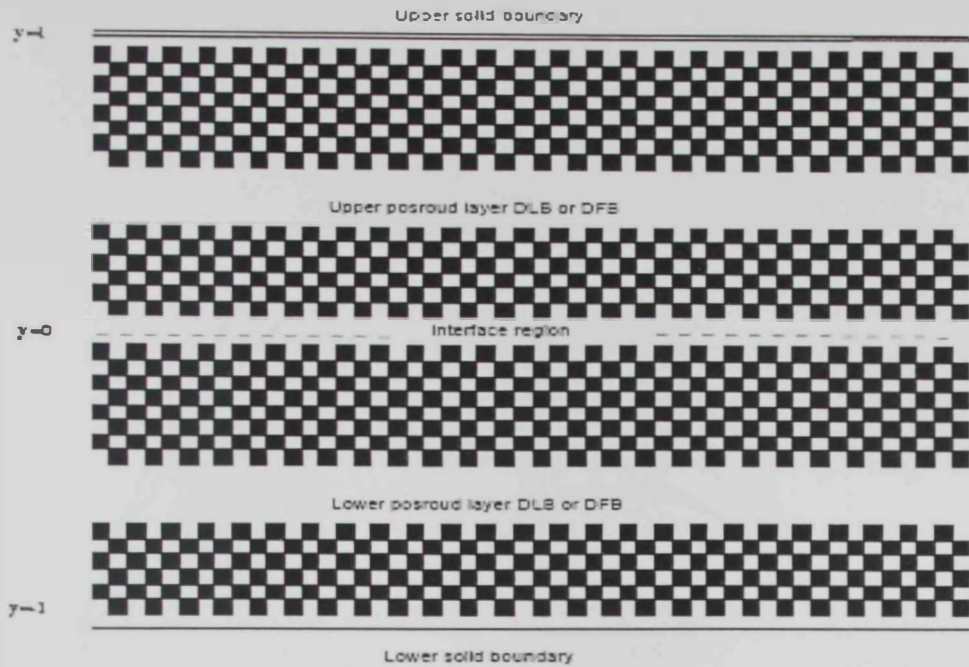
For  $u(x; t_k)$ ,  $u'(x; t_k)$ ,  $v(x; s_k)$ ,  $v'(x; s_k)$ ,

From which  $u(2; t_k)$ ,  $u'(2; t_k)$ ,  $v(2; s_k)$ ,  $v'(2; s_k)$  are used in the calculation of the Jacobian matrix and its inverse to update  $t_k$  and  $s_k$ .

The same algorithm is applied to solve a two layer flow problem. Consider flow through a channel composed of two different porous layers [28].

The upper layer is bounded above by a solid impermeable wall corresponding to  $y = 1$ , and the lower layer is bounded below by a solid impermeable wall corresponding to  $y = -1$ . The  $y = 0$ , corresponds to the interface region. Two combinations of models will be considered when simulating. First, when both layers are modeled by the same model, the Darcy-Lapwood-Forchheimer-Brinkman (DFB) or the Darcy-Lapwood-Brinkman (DLB) model. Second, the two layers are modeled one by the DFB model and the other by the DLB model. For each combination, different values of permeability values are considered.

Figure 7 : Configuration of the two porous layers:



#### Case 1: The DLB-DLB combination

The flow in both layers is governed by the DLB model, where  $Re$  is the Reynold number, and  $C$  is a dimensionless pressure gradient:

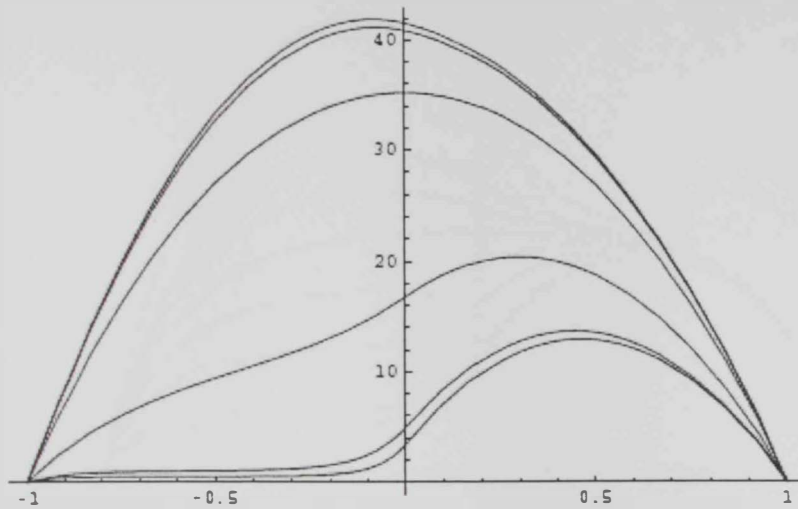
$$u'' = ReC + \frac{u}{k_b}, \quad -1 \leq y \leq 0, \quad y(-1)=0 \quad (5.24)$$

$$u'' = ReC + \frac{u}{k_t}, \quad 0 \leq y \leq 1, \quad y(1)=0 \quad (5.25)$$

With the interface conditions  $u(0^-) = u(0^+)$  and  $u'(0^-) = u'(0^+)$ . The permeabilities are denoted by  $k_b$  and  $k_t$  for the lower and upper layer respectively.

Equations 5.24 and 5.25 are linear, and their exact solutions can be easily found. But we are interested in applying the algorithm discussed above.

Figure 8: Case 1: Flow velocity profiles for both layers, with  $kt = 1$  and  $kb =$  0.005; 0.01; 0.1; 1; 10; 100.  $Re = 10$ ;  $C = -10$ ; Lower graph corresponds to smaller  $kb$ .



#### Case 2: The DLB-DFB combination

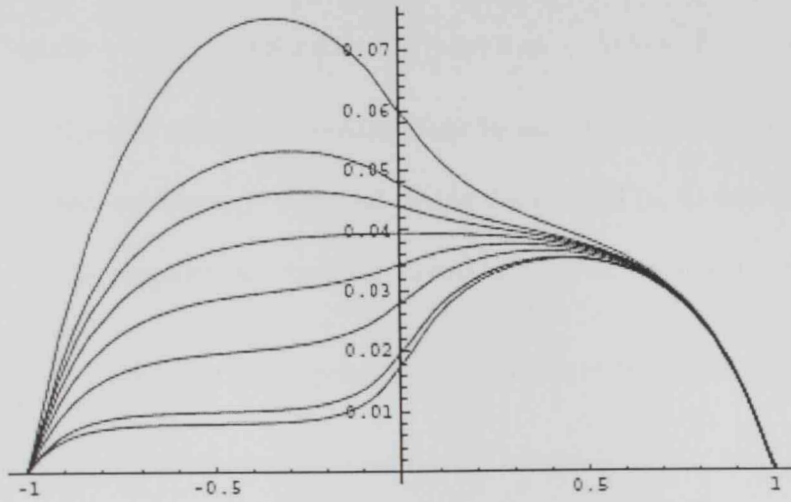
The flow in the top layer is governed by the DLB model, and the lower layer is governed by the DFB, the governing equations are :

$$u'' = Re C + \frac{u}{k_b}, \quad -1 \leq y \leq 0, \quad y(-1) = 0 \quad (5.26)$$

And

$$u'' = Re C + \frac{u}{k_t} + \frac{RC_d}{\sqrt{k_t}} u^2, \quad 0 \leq y \leq 1, \quad y(1) = 0 \quad (5.27)$$

Figure 9: Case 2: Flow velocity profiles for both layers, with  $kt = 0.04$  and  $kb = 0.008; 0.01; 0.02; 0.04; 0.06; 0.05; 0.03; 0.1$ .  $Re = 1$ ;  $C = -1$ ;  $Cd = 0.055$ .



## 5.2 FINITE DIFFERENCE APPROACH

The finite difference method for the linear second order boundary value problem,

$$y'' = p(x)y' + q(x)y + r(x), \quad a \leq x \leq b, \quad y(a) = \alpha, \quad y(b) = \beta, \quad (5.28)$$

requires that difference quotient approximations be used to approximate both  $y'$  and  $y''$ . First, we select an integer  $N > 0$ , and divide the interval  $[a, b]$  into  $(N+1)$  equal subintervals whose endpoints are the mesh points  $x_i = a + ih$ , for  $i=0, 1, \dots, N+1$ ,

where  $h = \frac{b-a}{N+1}$ . Choosing the step size  $h$  in this manner facilitates the application

of a matrix algorithm which solves a linear system involving an  $N \times N$  matrix [29].

At the interior mesh points,  $x_i$ , for  $i=0, 1, \dots, N$ , the differential equation to be approximated is:

$$y''(x_i) = p(x_i)y'(x_i) + q(x_i)y(x_i) + r(x_i), \quad (5.29)$$

Expanding  $y$  in a third Taylor polynomial about  $x_i$  and evaluated at  $x_{i+1}$  and  $x_{i-1}$ , we have, assuming that  $y \in C^4[x_{i-1}, x_{i+1}]$ ,

$$y''(x_i) = \frac{1}{h^2} [y(x_{i+1}) - 2y(x_i) + y(x_{i-1}))] - \frac{h^2}{12} y^{(4)}(\xi_i), \quad (5.30)$$

for some  $\xi_i$  in  $(x_{i-1}, x_{i+1})$ . This is called the centered difference formula for  $y''(x_i)$ .

Then a centered difference formula for  $y'(x_i)$  is obtained in a similar manner resulting in,

$$y'(x_i) = \frac{1}{2h} [y(x_{i+1}) - y(x_{i-1}))] - \frac{h^2}{6} y'''(\eta_i), \quad (5.31)$$

for some  $\eta_i$  in  $(x_{i-1}, x_{i+1})$ .

The use of these formulas in Equation (5.29) results in the equation:

$$\frac{y(x_{i+1}) - 2y(x_i) + y(x_{i-1}))}{h^2} = p(x_i) \left[ \frac{y(x_{i+1}) - y(x_{i-1}))}{2h} \right] + q(x_i)y(x_i) + r(x_i) - \frac{h^2}{12} [2p(x_i)y''''(\eta_i) - y^4(\xi_i)], \quad (5.32)$$

Define  $w_0 = \alpha$ ,  $w_{N+1} = \beta$

And

$$\left( \frac{-w_{i+1} + 2w_i - w_{i-1}}{h^2} \right) + p(x_i) \left( \frac{w_{i+1} - w_{i-1}}{2h} \right) + q(x_i)w_i = -r(x_i), \quad (5.33)$$

Rewrite this to get,

$$-\left(1 + \frac{h}{2} p(x_i)\right)w_{i-1} + (2 + h^2 q(x_i))w_i - \left(1 - \frac{h}{2} p(x_i)\right)w_{i+1} = -h^2 r(x_i), \quad (5.34)$$

And the resulting system of equations is expressed in the tridiagonal  $N \times N$  matrix

$$Aw = b,$$

Which is a nonlinear system that can be solved by any iterative scheme for the

unknowns  $w_i$ .

### 5.3 THE NON LINEAR SHOOTING METHOD

The shooting technique for the nonlinear second order boundary value problem

$$y'' = f(x, y, y'), \quad a \leq x \leq b, \quad y(a) = \alpha, \quad y(b) = \beta. \quad (5.35)$$

is similar to the linear technique, except that the solution to a nonlinear problem can not be expressed as a linear combination of the solutions to the two initial value problems. Instead, we approximate the solution to the boundary value problem by using the solutions to a sequence of initial value problems involving a parameter  $t$ . these problems have the form

$$y'' = f(x, y, y'), \quad a \leq x \leq b, \quad y(a) = \alpha, \quad y'(a) = t, \quad (5.36)$$

We do this by choosing  $t = t_k$ , in a manner to ensure that the limit of  $y(b, t_k)$  as  $k$  goes to infinity equals  $y(b) = \beta$ , where  $y(x, t_k)$  denotes the solution to the initial value problem (5.36) with  $t = t_k$ , and  $y(x)$  denotes the solution to the boundary value problem (5.35).

This technique is called shooting method, by analogy to the procedure of firing objects at a stationary target. We start with a parameter  $t_0$  that determines the initial elevation at which the object is fired from the point  $(a, \alpha)$  and along the curve described by the solution to the initial value problem (5.36).



If  $y(b, t_0)$  is not sufficiently close to  $\beta$ , we correct our approximation by choosing elevations  $t_1$ ,  $t_2$ , and so on, until  $y(b, t_k)$  is sufficiently close to hitting  $\beta$  [29].

We next determine  $t$  with

$$y(b, t) - \beta = 0$$

This is a nonlinear equation and there are many methods available to solve it such as the secant method. We just need to choose initial approximations  $t_0$  and  $t_1$ , and then generate the remaining terms of the sequence by

$$t_k = t_{k-1} - \frac{(y(b, t_{k-1}) - \beta)(t_{k-1} - t_{k-2})}{y(b, t_{k-1}) - y(b, t_{k-2})}, \quad k = 2, 3, \dots$$

## 6.0 Fluid Flow through Multilayer of Finite Depth

In this section, we shall present the mathematical formulations of the models equations governing the three channel problem, where the middle region is of finite width  $H$ , and the upper and lower regions are of finite heights. The mathematical formulations govern the flow of viscous fluid through porous media. The flow through porous media has typically been described by Darcy's law which is only suitable for slow flow through low permeability media. Also, it ignores many physical attributes such as inertial effects and the viscous shear effects. When dealing with viscous fluid flows and high permeability medium, Darcy's law becomes not valid and other flow models which account for these effects have to be adopted. Two popular models are known as Darcy-Lapwood-Brinkman (DLB) and the Darcy-Lapwood-Forchheimer-Brinkman (DFB) models. The DLB model is suitable when viscous shear effects are important and macroscopic inertia is sufficient to describe the flow inertia through the porous material. The DFB model accounts for viscous shear effects, and describes both microscopic and macroscopic inertia in the medium. The flows described by these models are encountered in various natural, physical, biological, and industrial settings.

If we assume that the flow is planar, fully developed and driven by a constant pressure gradient, then, in dimensionless form, the governing equation for the DFB model is

$$\frac{d^2 u}{dy^2} = \text{Re} C + \frac{u}{k} + \frac{\text{Re} C d}{\sqrt{k}} u^2, \quad (6.0)$$

Where  $u(y)$ ,  $-1 \leq y \leq 1$ , is the velocity of the fluid, and the various parameters in (6.0) are defined in terms of physical parameters as follows.

- $Re = \frac{\rho U_{\infty} L}{\mu}$  is the Reynolds number with  $\rho$  is the fluid density,  $U_{\infty}$  is the free stream characteristic velocity,  $\mu$  is the fluid viscosity, and  $L$  is the channel characteristic length.
- $K$  is the permeability of the porous channel.
- $C_d$  is the form drag coefficient.
- $C < 0$  is a dimensionless pressure gradient.

When  $C_d = 0$ , we obtain the DLB model equation

$$\frac{d^2 u}{dy^2} = Re C + \frac{u}{k}, \quad (6.1)$$

In this work, we consider flow through a channel composed of three different porous layers. The upper (lower) layer is bounded above (below) by a solid impermeable wall, corresponding to  $y = -2$  and  $y = 1$ . The  $y = 0$  and  $y = -1$  correspond to the interface region. Since the upper (lower) layer is bounded above (below) by solid impermeable wall, a no slip condition is valid to assume, that is,  $u = 0$  at the solid boundaries  $y = -2$  and  $y = 1$ . At the interface between the three layers along  $y = 0, -1$ , we assume that the velocity and shear stress are continuous. This assumption is realistic and makes it possible to determine the fluid velocity at the interface.

Then we apply the finite difference method to solve the three layer flow problem. The solution is approximated at grid point  $y_i$ , and the solution,  $u(0)$ , and its derivative  $u'(0)$ , at the interface are later approximated to be compared with the simulation results of the non linear shooting method.

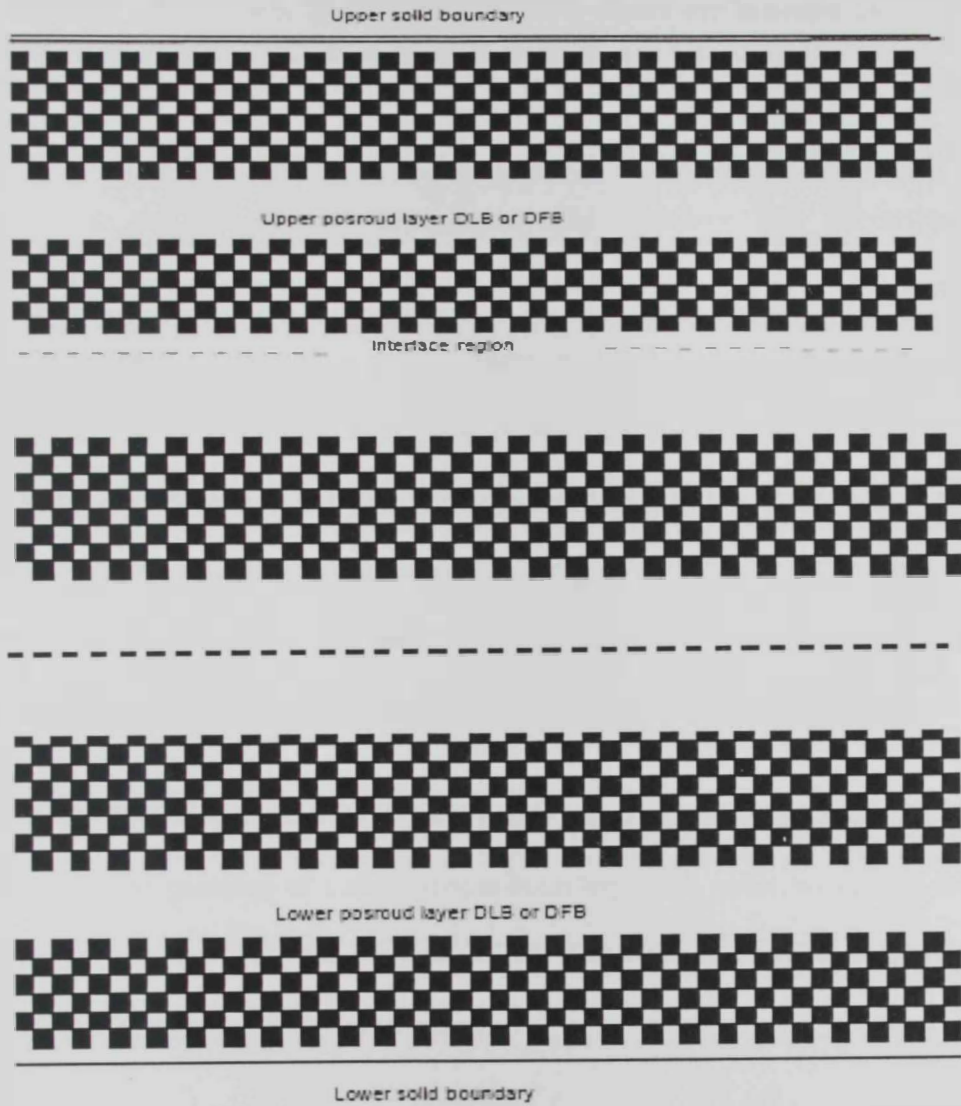


Figure 10: Configuration of the three layer problem.

## 7.0 RESULTS AND DISCUSSION

For our problem, we assume that the flow is planar, fully developed, and driven by a constant pressure gradient. We consider the flow through a channel which is composed of three different porous layers. The flow through the layers is governed by two different flow models. The upper and lower layers are bounded by solid, impermeable walls on which a no slip condition is valid. Near the solid walls, strong viscous shear effects are present, and hence one must choose a flow model that is compatible with the presence of macroscopic solid boundary. The interactions between the fluids in the three layers of the channel take place at the interface regions between the porous layers. As a result, there will be a discontinuity in the permeability at the interface due to the assumption that the interface between the layers is sharp. In addition, across the interface, the momentum transfer and shear stress effects are transmitted from the faster flow region to the slower flow region. However, it is assumed that the velocity and shear stress are continuous along the interface which makes it possible to determine the fluid velocity at the interface.

The DFB (Darcy-Lapwood-Forchheimer-Brinkman) model is known to be compatible with the presence of a macroscopic boundary. As a result, we choose the DFB flow model to govern the top and bottom layer. Then, the governing equation for the DFB model for the top and bottom layer is:

$$\frac{d^2u}{d^2y} = \text{Re}C + \frac{u}{k} + \frac{\text{Re}C_d}{\sqrt{k}}u^2, \quad (7.0)$$

Where  $u(y)$ ,  $-2 \leq y \leq -1$  and  $0 \leq y \leq 1$ , is the velocity of the fluid and the various parameters in (7.0) are defined as follows:

- $Re = \frac{\rho U_{\infty} L}{\mu}$  is the Reynolds number with  $\rho$  is the fluid density,  $U_{\infty}$  is the free stream characteristic velocity,  $\mu$  is the fluid viscosity, and  $L$  is the channel characteristic length.
- $\kappa$  is the permeability of the porous channel.
- $C_d$  is the form drag coefficient.
- $C < 0$  is a dimensionless pressure gradient.

The flow in the middle layer is governed by the DLB (Darcy-Lapwood-Brinkman) model which has the following governing equation:

$$u'' = Re C + \frac{u}{k_m}, \quad -1 \leq y \leq 0, \quad (7.1)$$

where  $k_m$  is the permeability associated with the middle layer. The permeabilities are denoted by  $k_t$  and  $k_b$  for the upper and lower layers respectively. The boundary conditions associated with the configuration above are as follows:

- Conditions along the solid walls: the no slip condition is employed at the solid walls, thus

$$u(-2) = 0, \text{ and } u(1) = 0$$

- Conditions at the interface,  $y = 0$  and  $y = -1$  : the velocity and shear stress are continuous at the interface between the layers, and thus

$$u(0^+) = u(0^-)$$

$$u'(0^+) = u'(0^-)$$

$$u(-1^+) = u(-1^-)$$

$$u'(-1^+) = u'(-1^-)$$

The solutions have been obtained for the flow through the top and bottom layers which are governed by the DFB model and through the middle layer which is governed by the DLB model. They have been obtained after applying the non linear shooting method and the finite difference method for comparison purposes. We ran the simulation for 3 different cases, where in every case we had different values for the permeabilities of the three layers. The results of the simulation to the three cases by both methods the shooting method and the finite difference method are shown in figures 5 through 10 below. Tables 3, 4, and 5, list the fluid interface velocity obtained by the non linear shooting method and by the finite difference method scheme.

**Table IV : Case I of the comparison between two methods:** The interface velocities for the values:  $K_b = 0.1$ ,  $K_c = 0.2$ ,  $K_t = 10$ ,  $a = -2$ ,  $b = 1$ ,  $c_1 = -1$ ,  $c_2 = 0$ ,  $R = 1$ ,  $C = -1$ ,  $C_d = 0.1$

	Finite Difference	Shooting Method	Difference
U(m)	<b>0.143945</b>	<b>0.143091</b>	<b>0.000854</b>
U(2m)	<b>0.278743</b>	<b>0.277124</b>	<b>0.001619</b>

**Table V : Case II of the comparison between two methods :** The

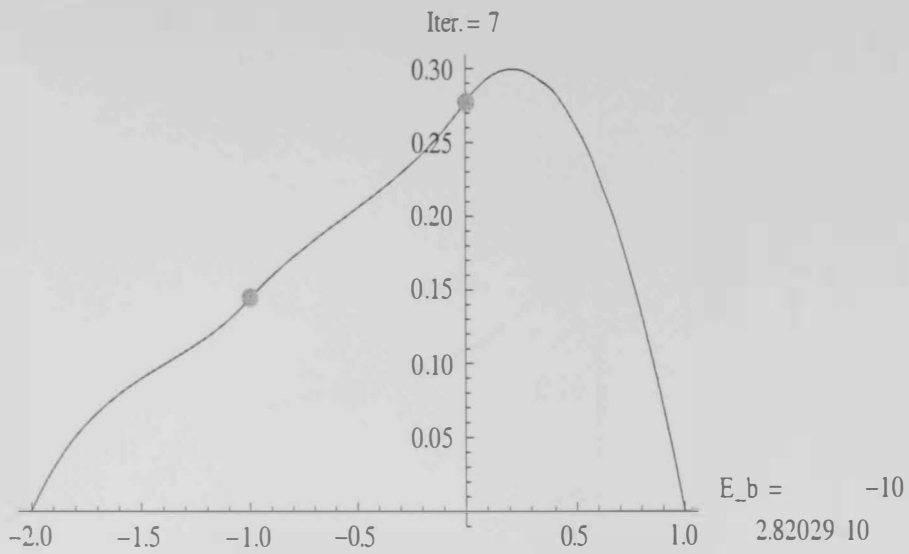
interface velocities for the values:  $K_b = 0.1$ ,  $K_c = 0.10$ ,  $K_t = 10$ ,  $a = -2$ ,  $b = 1$ ,  $c_1 = -1$ ,  $c_2 = 0$ ,  $R = 1$ ,  $C = -1$ ,  $C_d = 0.1$

	Finite Difference	Shooting Method	Difference
U(m)	<b>0.0994792</b>	<b>0.099396</b>	<b>0.0000832</b>
U(2m)	<b>0.194545</b>	<b>0.19261</b>	<b>0.001935</b>

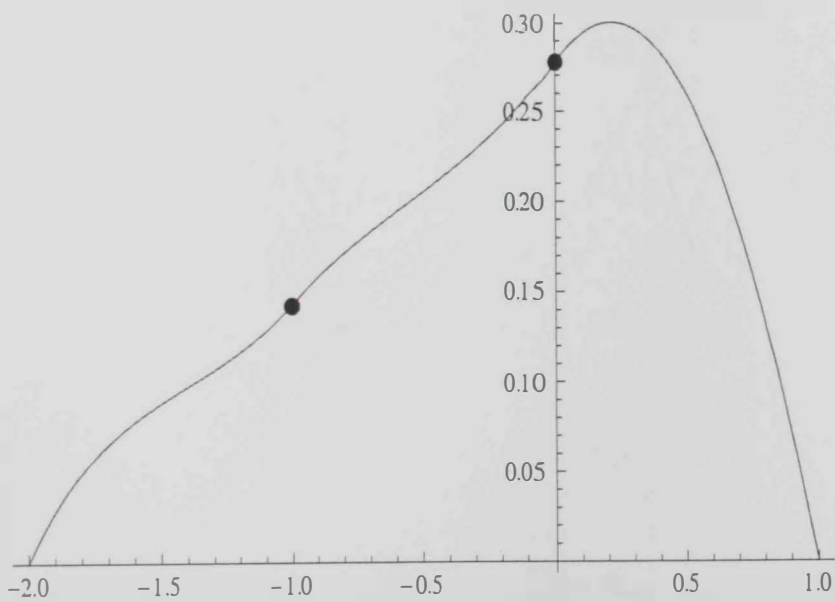


**Table VI : Case III of the comparison between two methods :** The interface velocities for the values:  $K_b = 0.5$ ,  $K_c = 10$ ,  $K_t = 0.1$ ,  $a = -2$ ,  $b = 1$ ,  $c_1 = -1$ ,  $c_2 = 0$ ,  $R = 1$ ,  $C = -1$ ,  $C_d = 0.1$

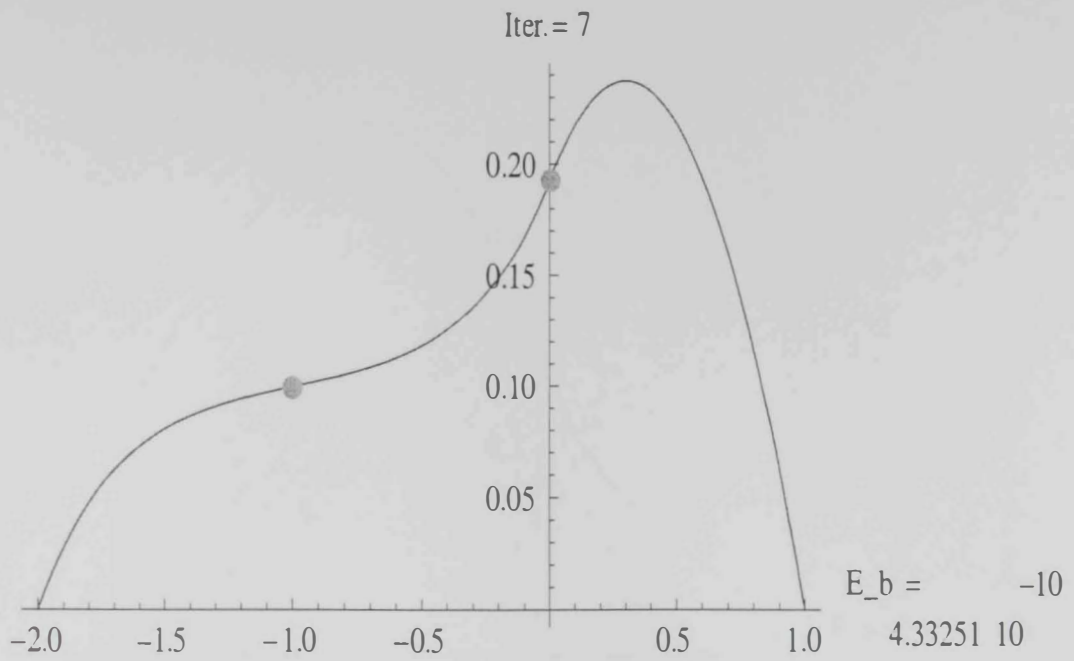
	Finite Difference	Shooting Method	Difference
U(m)	<b>0.458824</b>	<b>0.458096</b>	<b>0.000728</b>
U(2m)	<b>0.286518</b>	<b>0.292454</b>	<b>0.005936</b>



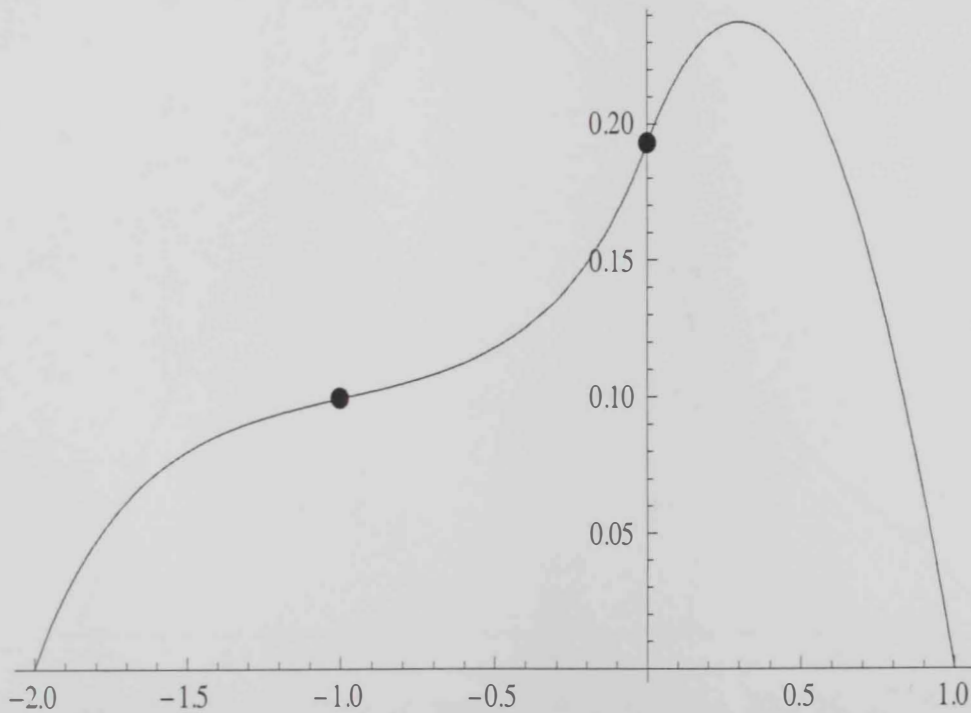
**Figure 11:** shooting method: With  $K_b = 0.1$ ,  $K_c = 0.2$ ,  $K_t = 10$ ,  $a = -2$ ,  $b = 1$ ,  $c_1 = a + (b-a)/3$ ,  $c_2 = a + 2(b-a)/3$



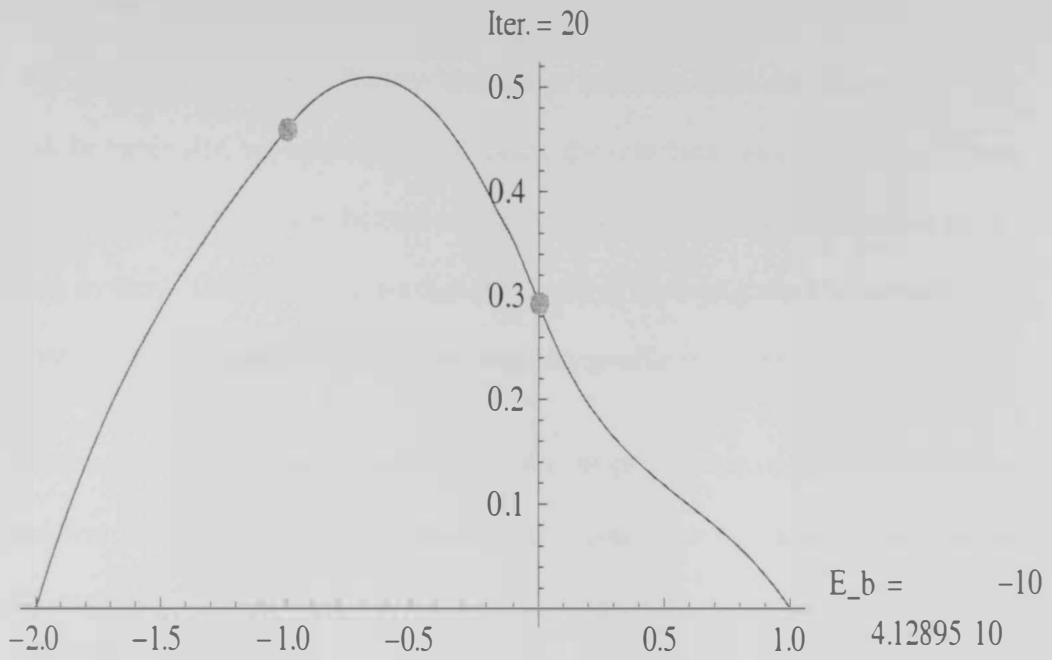
**Figure 12:** finite difference method: With  $K_b = 0.1$ ,  $K_c = 0.2$ ,  $K_t = 10$ ,  $a = -2$ ,  $b = 1$ ,  $c_1 = a + (b-a)/3$ ,  $c_2 = a + 2(b-a)/3$



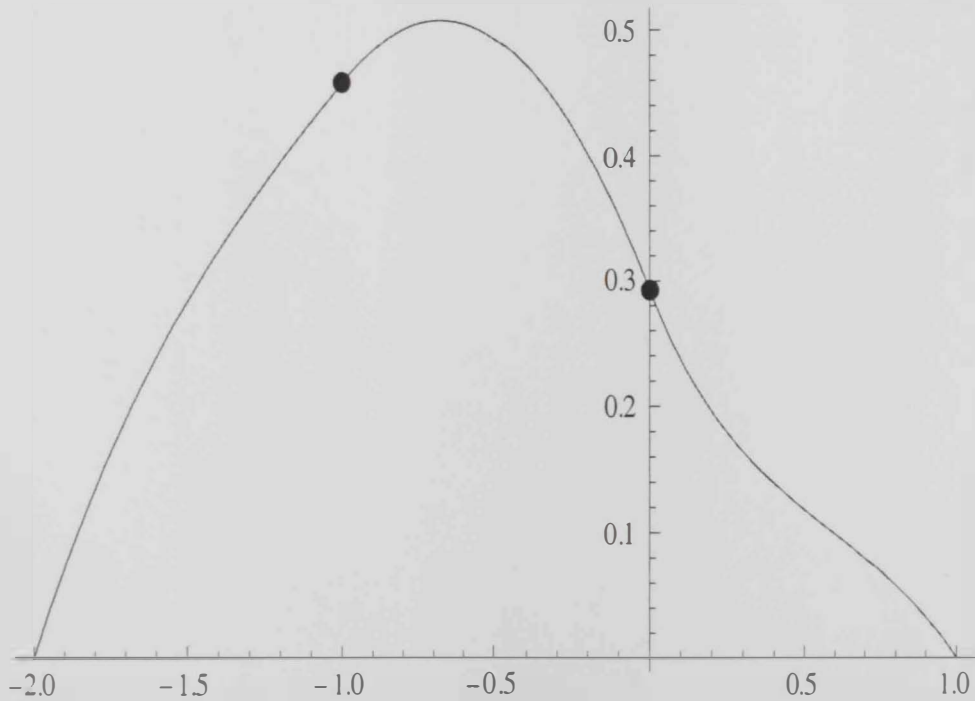
**Figure 13:** shooting method: With  $K_b = 0.1$ ,  $K_c = 0.10$ ,  $K_t = 10$ ,  $a = -2$ ,  $b = 1$ ,  $c_1 = a + (b-a)/3$ ,  $c_2 = a + 2(b-a)/3$ ,  $R = 1$ ,  $C = -1$ ,  $C_d = 0.1$



**Figure 14:** Finite difference method:  $K_b = 0.1$ ,  $K_c = 0.10$ ,  $K_t = 10$ ,  $a = -2$ ,  $b = 1$ ,  $c_1 = a + (b-a)/3$ ,  $c_2 = a + 2(b-a)/3$ ,  $R = 1$ ,  $C = -1$ ,  $C_d = 0.1$



**Figure 15:** shooting method: With  $K_b=0.5$ ,  $K_c=10$ ,  $K_t=0.1$ ,  $a=-2$ ,  $b=1$ ,  $c_1 = a + (b-a)/3$ ,  $c_2 = a + 2(b-a)/3$



**Figure 16:** Finite difference method: With  $K_b=0.5$ ,  $K_c=10$ ,  $K_t=0.1$ ,  $a=-2$ ,  $b=1$ ,  $c_1 = a + (b-a)/3$ ,  $c_2 = a + 2(b-a)/3$

The results of applying the shooting method to the three layer flow problem show that the algorithm is more efficient and more accurate than the finite difference method. In particular, we note that, in all cases, the interface velocity and shear stress values obtained by the finite difference approach tend to the values obtained by the shooting method. This demonstrates that the shooting method gives a better resolution at the interface. The results show that the velocity profile is similar in all three cases.

For the parameter values considered in this work, the convergence of Newton's method was very fast. In all cases considered, convergence was achieved in first and second case in 7 iterations and for the third case in 20 iterations. On the other hand, the finite difference method was time consuming as it has to handle a large system.

## CHAPTER EIGHT

### 8.0 ENVIRONMENTAL IMPACT

Porous materials are encountered literally everywhere in everyday life, in technology, and in nature. Many natural substances such as rocks, and biological tissues such as lungs and bones, and man made materials such as cements, foams and ceramics can be considered as porous media. More examples of porous material include soil, building materials such as bricks, concrete, limestone, and sandstone. The concept of porous media is used in many areas of applied science and engineering. It is used in mechanics such as geo-mechanics, soil and rock mechanics, in engineering such as petroleum and construction engineering, in geosciences such as hydrogeology and geophysics, in biology and biophysics, in material sciences, and many more fields of science. However, the most important areas of technology that depend significantly on the properties of porous media are hydrology, which relates to water movement in earth and sand structures, petroleum engineering which is mainly concerned with petroleum, and natural gas production, exploration, well drilling, and logging. Additionally, the flow of blood and other body fluids, and electro-osmosis are few examples where porous media plays a critical role in medicine and biological engineering.

Consequently, flow through porous media has become an important environmental problem that had attracted the attention of many scientists. It has many environmental implications in several areas such as the study of pollution, fate of contaminants, contaminations issues related to agriculture, civil constructions, coastal management, and flow of ground water, oil and gas in the ground layers are also

typical problems. In addition, flow through porous media finds agricultural applications in irrigation processes and the movement of nutrients, fertilizers, and pollutants into plants. Fluid flow through porous media has emerged as a separate field of study because it is a subject of most common interest.

Differential equations play an important role in many fields of science and engineering. They model many important physical phenomena. In fact, fluid mechanics of the interface region of multilayer flows has gained interest over the past three decades due to its applications in various physical settings. These applications include packed bed heat exchangers, heat pipes, thermal insulation, petroleum reservoirs, nuclear waste repositories, and geothermal engineering [1].

In general, petroleum products represent a dangerous potential source of groundwater pollution and sediment contamination due to the toxicity of a number of the oil components, such as benzene, toluene, ethylene and xylene. These can reach the water table through various pathways, including highway runoff, direct oil spills resulting from road accidents, wave washing of oil spilled in coastal waters, and the improper disposal of hydrocarbon products through urban sewer systems.

Due to the costly clean up processes of oil contaminated sediments, it is imperative to predict the outreach of oil components, their motion in the sediment, the physical and chemical behavior of the redistributed oil and how the oil components reach the water table. A large volume of work, both theoretical and experimental, has been carried out in this field in recent years and has been centered around a single and multiphase flow through porous media [2].

For example, organic tissues are porous media permeated by organic liquid. They are made of packed cells which constantly absorb organic material and nutrients from the liquid environment outside them to feed, grow, and duplicate. On the other hand, they constantly produce waste products, mainly water, and a multitude of chemical factors. At the end, when cells die, their membrane ruptures releasing their content and in particular re-usable organic materials. One can then model these bodies as growing and deformable porous media and study the filtration of organic liquids through them. Of course, this is not the only field which involves porous media with mass exchanges. Other examples can be found in composite material manufacturing, crystallization, petroleum and chemical engineering, glaciology, sedimentation and erosion, contamination and decontamination of soils.



## REFERENCES

1. Nield, D., Bejan, A. 1999. Convection in Porous Media. Springer: New York, 2<sup>nd</sup> edition.
2. Porous Media. (n. d.) Retrieved on November 26<sup>th</sup>, 2007, from: <http://www.fys.uio.no/~eaker/thesis/node5.html#SECTION00410000000000000000>
3. Dullien, F. (Feb 10, 1997). Porous Media: Fluid Transport and Pore Structure. Academic Press: Porous Media. Retrieved on November 26<sup>th</sup>, 2007, from: <http://www.fys.uio.no/~eaker/thesis/node5.html#SECTION00410000000000000003>
4. Hansen, V.E., Israelsen, O.W. (1962). Irrigation Principles and Practices. John Wiley and Sons: New York, 3<sup>rd</sup> edition.
5. Tiab, D., Donaldson, E. (1999). Theory and Practice of Measuring Reservoir Rock and Fluid Transport Properties. Butterworth-Heinemann published by the Gulf Publishing Company, Houston, Tx.
6. Verraijt, A. Groundwater Flow. The Macmillan Press: Hong kong, 2<sup>nd</sup> edition.
7. Dullen, F. (1979). Porous Media: Fluid Transport and Pore Structure. Academic Press: New York.
8. Fredlund, D.G., XING, A. (March 18, 1994). Predicting the permeability function for unsaturated soils using the soil-water. Retrieved on November 12, 2007, from: <http://www.soilvision.com/downloads/docs/pdf/research/cangeopredict.pdf>

9. Walter, T., Gao, B., Parlange, J.Y. (September 17, 2007). Modeling Soil: Solute Release into Runoff with Infiltration. Science direct: Elsevier, Journal of Hydrology, volume 347. issues 3-4.
10. Saturated Hydraulic Conductivity. (n.d.) Retrieved on November 12, 2007 from: <http://soils.usda.gov/technical/technotes/note6.html>
11. Water in Soil. (1997). Retrieved on November 12, 2007 from: <http://www.css.cornell.edu/faculty/hmv1/watrsoil/wtsl.htm>
12. Bouma, J., Brown, R.B. (n.d.). Soil as a Porous Medium. Retrieved on November 15, 2007, from: [http://edis.ifas.ufl.edu/scripts/htmlgen.exe?DOCUMENT\\_SS108](http://edis.ifas.ufl.edu/scripts/htmlgen.exe?DOCUMENT_SS108)
13. Israelsen, O., Hansen, V. (1962). Irrigation Principles and Practices. John Wiley and sons: New York, 3<sup>rd</sup> edition.
14. Nofziger, D.L. (March 18, 2003). Water Movement in Soils. Retrieved on November 12, 2007, from: <http://soilphysics.okstate.edu/software/water/infil.html>
15. Soil Structure. (November 5, 2007). Wikipedia: the free encyclopedia. Retrieved on November 12, 2007 from: [http://en.wikipedia.org/wiki/Soil\\_structure](http://en.wikipedia.org/wiki/Soil_structure)
16. Water Movement in Soil. (n.d.). Retrieved on November 12, 2007, from: [http://weather.nmsu.edu/teaching\\_Material/soil252/Chap10.htm](http://weather.nmsu.edu/teaching_Material/soil252/Chap10.htm)
17. Trautmann, N., Porter, K. (n.d.). Water and the Soil. Retrieved on November 15, 2007, from: <http://pmep.cce.cornell.edu/facts-slides-self/facts/wat-so-grw85.html>

18. Watson, J., Hardy, L. (et al). (September 1995). How Water Moves Through Soil. Retrieved on November 15<sup>th</sup>, 2007 from:  
<http://cals.arizona.edu/pubs/water/az9516.pdf>
19. Physics of Porous Media. (November 21, 1995). Retrieved on November 26<sup>th</sup>, 2007, from: <http://www.fys.uio.no/faststoff/coop/cop92/node29.html>
20. Beavers, G.S., Joseph, D.D. (1967). Boundary conditions at a naturally permeable wall. *Journal of Fluid Mechanics*, volume 30, issue 1, pp. 197.
21. Beavers, G.S., Sparrow, E.M. and Masha, B.A. (1974). Experiments of coupled parallel flows in a channel and a bounding porous medium. *Applied Mathematics*, volume 20, pp. 596.
22. Vafai, K., Thiyagaja, R. (1987). Analysis of flow and heat transfer at the interface region of a porous medium. *International journal of heat Mass and Transfer*, Volume 30, pp 1391.
23. Vafai, K., Kim, S.J. (1990). Fluid mechanics of the interface region between a porous medium and a fluid layer: an exact solution. *International journal of heat mass and transfer*, Volume 11, issue 3, pp. 334.
24. Allan, F., Hamdan, M.H. (2002). Fluid mechanics at the interface region between two porous layers. *Applied Mathematics Computations*, Volume 128, Issue 1, pp. 37.

25. Solbakken, S., Andersson, H. (2005). Direct numerical simulation of lubricated channel flow. *Fluid Dynamic Research*, Volume 37, Issue 3, pp 203-230.
26. Lemos, M., Renato, S. (2006). Turbulent over a layer of a highly permeable medium simulated with a diffusion jump model for the interface. *International journal of heat mass and transfer*, volume 49, issues 3-4, pp. 546-556.
27. Allan, F., Abu Ghniem, S., Hasan, S. (2007). *Simple Mathematical Models for the Fluid Flow through Channels*. Preprints.
28. Allan, F., Hajji, M.A., (2008). *Mathematical Manipulation of the Interface Region of Multilayer Flow*. *International Journal of Porous Media* (accepted).
29. Burden, R.L. and Faires, J.D., (2001). *Numerical Analysis*. 7<sup>th</sup> ed., Brooks/Cole.
30. Deeba, E., Khuri, S.A., and Xie, S. (2000). An Algorithm for Solving Boundary Value Problems, *Journal of Computational Physics*. vol. 559, pp. 125-138.
31. Ford, R.A., and Hamdan, M.H., (1998). Coupled Parallel Flow Through Composite Porous Layers. *Applied Mathematics and Computations*, vol. 97, pp. 261-271.
32. Charbeneau, R. (2000). *Groundwater Hydraulics and Pollutant Transport*. Prentice Hall: New Jersey.

33. Hwang, N., Hita, C. (1987). *Fundamentals of Hydraulic Engineering Systems*. Prentice Hall: New Jersey.
34. Verruijt, A. (1982). *Theory of Groundwater Flow*. The Macmillan Press: London, 2<sup>nd</sup> edition.
35. Schwab, G., Fangmeier, D., Elliot, W. (1996). *Soil and Water Management Systems*. John Wiley and sons: Toronto, 4<sup>th</sup> edition.
36. Allan, F., Hamdan, M.H. (2002). Fluid Mechanics of The Interface Region Between Two porous Layers. *Applied Mathematics Computation*, vol. 128, no. 1, pp. 37-43.
37. Rudraiah, N. (1986). Flow Past Porous Layers and Their Stability. *Encyclopedia of Fluid Mechanics, Slurry Flow Technology*, ch. 14.
38. Assher, U.M., Matthij, R.M.M., and Russel R.D. (1995). Numerical solution of boundary value problems for ordinary differential equations. Society for industrial and Applied Mathematics, Philadelphia, PA.
39. B.C. Chandra Sekhara, K. Rajani, N. Rudraiah. (1987). Effect of slip on porous walled squeeze films in the presence of a magnetic field. *Appl. Sci. Res. Volume 34*, pp. 393.
40. Verwoerd, W.S. (February 2009). New stochastic model for dispersion in heterogeneous porous media: Application to unbounded domains. *Applied Mathematical Modelling*, Volume 33, Issue 2, Pages 605-625.

41. Karim, M., Tanimoto, K., and Hieu, P.D. (January 2009). Modelling and simulation of wave transformation in porous structures using VOF based two-phase flow model. *Applied Mathematical Modelling*, Volume 33, Issue 1, January 2009, Pages 343-360.
42. Röckner, M., Wang, F.Y. (December 15, 2008). Non-monotone stochastic generalized porous media equations. *Journal of Differential Equations*, Volume 245, Issue 12, Pages 3898-3935.
43. A. Ramírez, A. Romero, F. Chavez, F. Carrillo, S. Lopez. (November 2008). Mathematical simulation of oil reservoir properties. *Chaos, Solitons & Fractals*, Volume 38, Issue 3, Pages 778-788.
44. Polubarinova, K. (1960). *Theory of Groundwater Movement*, Princeton University Press, Princeton, NJ.
45. Harr, M. (1962). *Groundwater and Seepage*. McGraw Hill, New York.
46. Awartani, M.M., Hamdan, M.H. (2005). Fully developed flow through a porous channel bounded by flat plates. *Applied Mathematics*, volume 169, pp. 749.
47. M. Khan, Ehnber Naheed, C. Fetecau, T. Hayat. (November 2008). Exact solutions of starting flows for second grade fluid in a porous medium. *International Journal of Non-Linear Mechanics*, Volume 43, Issue 9, Pages 868-879

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## المخلص

لقد نالت المسائل المتعلقة بالسوائل المتحركة في الأوساط المسامية الكثير من الاهتمام في الآونة الأخيرة وذلك لأهميتها في العديد من المجالات البيئية والصناعية, على سبيل المثال عملية الفلترة والتخلص من النفايات, واستخراج والتنقيب عن البترول. وكذلك المسائل المتعلقة بالبيئة مثل التلوث ومصير الملوثات الأخرى المتعلقة بالزراعة والبناء وغيرها.

في هذه الرسالة نلقي الضوء على حركة السوائل في الأوساط المسامية المتعددة الطبقات حيث تم إعداد المعادلات الرياضية المتعلقة بهذه الحركة وكذلك الشروط الابتدائية والشروط الحدودية عند كل طبقة. وقد تم التركيز على سرعة السائل عند الأوساط الحدودية بين هذه الطبقات. وقد تم استخدام طريقتين مختلفتين (طريقة التصويب والخطأ وطريقة الفروقات المنتهية) لحل المسألة والمقارنة بين النتائج في كلتا الحالتين.





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عمادة الدراسات العليا  
برنامج ماجستير علوم البيئة

عنوان الرسالة

حركة الموانع في الأوساط المسامية وتأثيراتها البيئية

رسالة مقدمة من الطالبة

نجود محمد جوهر

الى

جامعة الإمارات العربية المتحدة

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