Mathematical and Software Engineering, Vol. 3, No. 2 (2017), 173-182.
Varεpsilon Ltd, varepsilon.com

Modelling and Forecasting of Residential Electricity Consumption in Nigeria Using Multiple Linear Regression Model and Quadratic Regression Model with Interactions

Isaac A. Ezenugu[1], Swinton C. Nwokonko[1], and Idorenyin Markson[2]

[1] Department of Electrical/Electronic Engineering, Imo State University, Owerri, Nigeria
[2] Department of Mechanical Engineering, University of Uyo, Uyo, Nigeria

Abstract

In this paper statistical analysis of residential electricity demand in Nigeria is presented. Multiple linear regression model and quadratic regression model with interactions are applied to estimate residential electricity consumption and to forecast long-term residential electricity demand in Nigeria. Population and temperature are used as explanatory variables. The results show the Quadratic Regression with interaction has RMSE of 52.77 and r-square value of 0.9389 which indicates that 93.89% of the variation in residential electricity consumption is explained by the model. On the other hand , the multiple linear regression model has RMSE of 69.97 and r-square value of 0.873 which indicates that 87.3% of the variation in residential electricity consumption is explained by the model. Essentially, the quadratic regression model with interaction with lower RMSE and higher r-square value is selected and then used to forecast the residential electricity demand in Nigeria from 2015 to 2029. From the results, the Residential Electricity Consumption in Nigeria will reach 6521.09 MW/h in the year 2029. Furthermore, the results show that population has a positive sign and it is significant in the short run and in the long run forecasting. On the other hand, the result also revealed insignificant moderately weak relationship between residential electricity consumption and temperature.

Keywords: Electricity; Multiple Linear Regression Model; Quadratic Regression Model; Regression Model with Interactions; Statistical Analysis; Forecasting of Residential Electricity Demand

1 Introduction

Over the years , especially since the 1990s, Nigerian electricity industry has been bedevilled by so much inefficiency [1-7]. Equally, the demand for electricity in Nigeria has continued to outstrip its capacity, the end result has been the delivery of poor and shoddy services, evidenced by frequent power failures [8-12]. Available studies show that electricity consumption in Nigeria has been growing at a very rapid rate over the decades. Between 1970-2004, consumption of electricity in the country increased from 752 million kWh to 8576.3 million kWh [1], [13]. Given the current trends in population growth, industrialization, urbanization, modernization and income growth, electricity consumption is expected to increase substantially in the coming decades as well. All these require matching supply of infrastructure and public service to ensure sustainability.

Adequate supply and distribution of electricity constitute a central development issue which cannot be over-emphasized. The continuous electricity supply problems in Nigeria has been extensively linked to the inability of energy and urban planners to accurately forecast the effect of the various socio-economic and climatic factors that influence the electricity consumption rate across the various geopolitical regions of the country [1], [14]. This study therefore is aimed at statically analysing the residential electricity consumption pattern in Nigerian as well as determines the influence of the socio-economic and climatic parameters on residential electricity consumption. Data for the study was collected through national Bureau of Statistics and CBN annual Bulletin. Two socio-economic and one climatic.

2 Methodology

In this paper, modelling of the yearly average residential electricity consumptions in Nigeria is done using multiple linear regression model and quadratic regression with interaction model. Performance evaluation of the model is also performed using prediction accuracy evaluation measures, particularly, the Root Mean Square Error (RMSE) and coefficient of determination (R^2). Data on residential electricity consumption (MW/h), between 2006 -2014 were obtained from Central Bank of Nigeria Statistical Bulletin [15] while data on temperature (^{0}C) and population were obtained from the Bulletin of the National Bureau of Statistics[16].

2.1 The Multiple Linear Regression Model

Multiple linear regression model expresses electricity consumption (E_t) as a linear function of Population (P) and temperature (T_t) as follows:

$$EC_t = f\left(P_t, T_t\right) \quad (1)$$

$$E_t = \alpha_0 + \alpha_1 P_t + \alpha_2 T_t + \varepsilon_t \quad (2)$$

where

E_t is the yearly average residential electricity consumption (in MW/h) estimated at time, t;

P_t is Population at time, t and T_t is Temperature at time, t.

α_0 is the intercept

α_1 and α_2 are the regression coefficients that are used to quantify the contributions of Population (P_t) and Temperature (T_t) respectively.

Econometric Views (EViews) statistical package is used to perform the regression from which the values of the intercept α_0 and the regression coefficients α_1 and α_2 are obtained for the multiple regression model.

2.2 The Quadratic Regression Model with Interaction Terms.

The quadratic regression model with interaction terms expresses electricity consumption (E_t) as a quadratic function of Population (P_t) and temperature (T_t) as follows:

$$E_t = \alpha_0 + \alpha_1 P_t + \alpha_2 T_t + \alpha_3 P_t^2 + \alpha_4 T_t^2 + \alpha_5 P_t T_t + \varepsilon_t \quad (4)$$

where E_t is the yearly average residential electricity consumption (in MW/h) estimated at time, t;

P_t is Population at time, t and T_t is Temperature at time, t.

α_0 is the intercept

α_1 , α_2 α_3 , α_4 and α_5 are the regression coefficients.

Again, Econometric Views (EViews) statistical package is used to perform the regression from which the values of the intercept α_0 and the regression coefficients α_1 , α_2 α_3 , α_4 and α_5 are obtained for the quadratic regression model with interaction.

2.3 Models for the Explanatory Variables

In this study, data for the explanatory variables, namely, P_t and T_t are available for the years 2006 to 2014. Mathematical expression for each of the independent variables P_t and T_t are themselves obtained from models applied to the data sets of each of the variables over time (t). Particularly, the population for the years beyond 2014 are projected using the mathematical expression;

$$P_t = P_{t-1}(1+r)^n \quad (5)$$

where n is the number of years, P_{t-1} is previous year population, P_t is the population of the year to be estimated and r is the population growth rate of Nigeria which is given as 3.2% according to 2006 population census [17].

Also, temperature is predicted using simple linear regression model as follows;

$$T_t = a_0 + a_1 t \quad . \quad (6)$$

Where β_0 and β_1 are the simple linear regression coefficients and t is time. The model parameters β_0 and β_1 are estimated using Microsoft excel and the following values are obtained; $\beta_0 = 34.602$ and $\beta_1 = 0.0613$. Therefore,

$$T = 34.602 + 0.0613t \, . \quad (7)$$

Where t = time, t= 10,11,…,20.

2.4 Validation and Diagnostic Criteria

2.4.1 F- Test

To check the goodness of fit of the model, F test use where :

$$F = \frac{MSR}{MSE} \quad (8)$$

Where

$$MSR = \frac{SSR}{P-1} \quad (9)$$

P is the number of parameters

$$MSE = \frac{SSE}{n - P} \quad (10)$$

n = number of observation

$$\text{SST (Sum of square Total)} = E_t^1 E_t - \left(\frac{1}{n}\right) E_t^1 J E_t \text{ -(11)}$$

Sum of square regression, SSR, can be calculated for each model as follows:

$$\text{SSR (model 1)} = \alpha_a^1 U_a - \left(\frac{1}{n}\right) E_t^1 J E_t \quad - \quad (12)$$

$$\text{SSR (Model 2)} \quad = \quad \alpha_b^1 U_b - \left(\frac{1}{n}\right) E_t^1 J E_t \quad (13)$$

$$\text{SSR (Model 3)} \quad = \quad \alpha_c^1 U_c - \left(\frac{1}{n}\right) E_t^1 J E_t \quad (14)$$

2.4.2 Coefficient of Determination.

The coefficient of determination shows the percentage of the variation in the dependent variable explained by the independent variables. It is given as

$$r^2 = \frac{SSR}{SST} \quad (15)$$

2.4.3 Root Mean Square Error (RMSE).

The square of the sum of square of differences between the predicted and observed value divided by the number of observation: It can be expressed mathematically as:

$$RME = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(E_t - \hat{E}_t\right)^2} \quad (16)$$

2.4.4 Absolute Mean Error (MAE)

This is the mean of the absolute deviation of the predicted values from the observed value.

$$AME = \sum_{t=1}^{n} \frac{\left|E_t - \hat{E}_t\right|}{n} \quad (17)$$

Where, E_t and $\hat{E}_t$ are the observed and predicted residential electricity consumption.

Also, the error analysis is also conducted to determine the goodness of fit for each of the two models. The results of the analysis with Econometric Views (EViews) statistical package are then presented.

3 Results and Discussion

From the result in Table 1 the multiple linear regression model is given as:

$$E_t = 290.35 + 13.52 P_t - 34.32 T_t \quad (18)$$

Table 1 The model parameters and goodness of fit measures for the two models

	Regression Coefficients Values For The Model Parameters		Goodness Of Fit Measures		
	Multiple Linear Regression Model	Quadratic Regression Model with Interaction Terms		Multiple Linear Regression Model	Quadratic Regression Model with Interaction Terms
α_0	**290.35**	**152014.9**			
α_1	**13.52**	**386.57**	r^2 (%)	**0.873**	**0.9389**
α_2	**34.32**	**-10855.92**	RMSE	**69.97**	**52.77**
α_3		**-0.4**			
α_4		**177.38**			
α_5		**-7.05**			

Result in Table 1 shows that for the multiple linear regression model, population (P_t) and temperature accounted for 87.3% of the total variation in residential electricity consumption while the remaining 12.7% was accounted for by other variables not in the model. The contribution of population (α_1 = 13.52) was found to be positive. This means that as the population increases, residential consumption of electricity also increases. The implication of this result is that as population increases, there is a corresponding increase in the quantity of residential electricity consumption. The actual and predicted electricity consumption for the multiple linear regression model is presented in the Table 2.

Table 2: Values of actual and predicted residential electricity consumption based on Model 1.

Year	Actual(MW/h)	Predicted(MW/h)
2006	894.11	1000.67
2007	1151.94	1066.51
2008	1165.72	1123.65
2009	1104.54	1206.68
2010	1365.50	1269.13
2011	1401.01	1374.28
2012	1437.43	1409.97
2013	1474.81	1504.51
2014	1513.15	1552.79

Again , from the result in Table 1, the Quadratic Regression with interaction is given as:

$$E_t = 152014.90 + 386.57P_t - 10855.92T_t + 0.40P_t^2 + 177.38T_t^2 - 7.05P_t\,T_t \qquad (19)$$

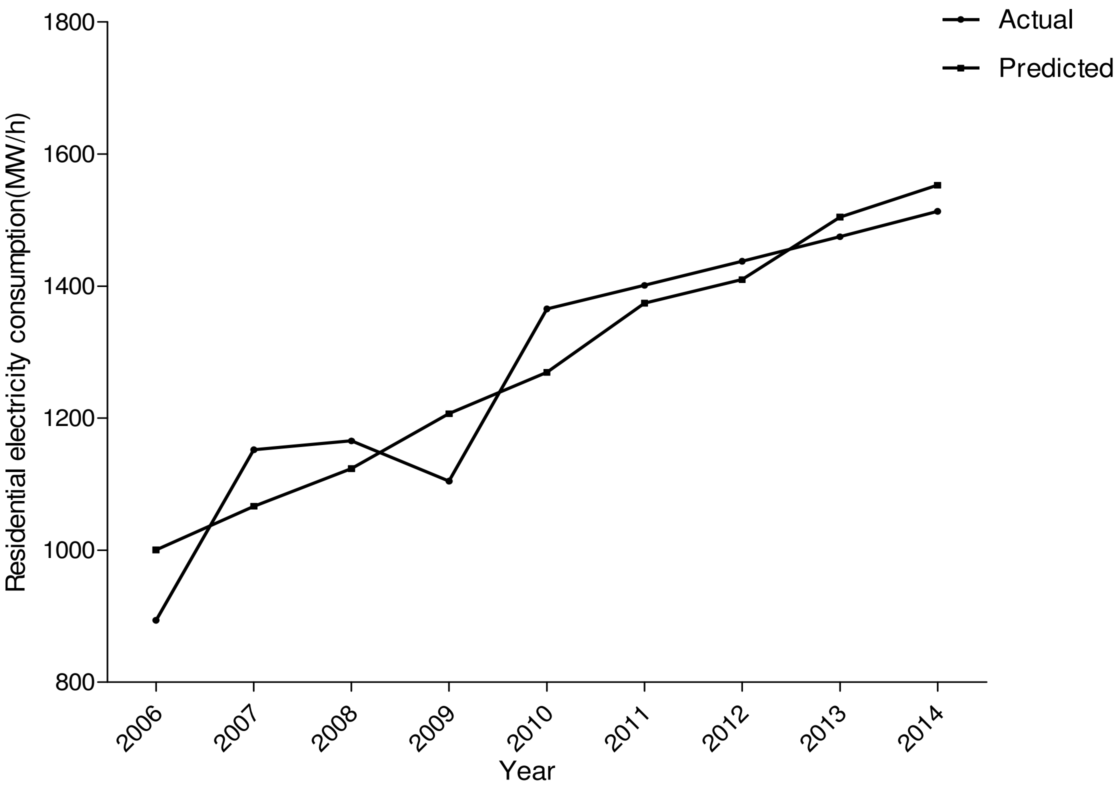

Figure1: Graph of the Actual and Predicted Residential Electricity Consumption for the multiple linear regression model.

Result in Table 1 shows that for the Quadratic Regression with interaction has r-square value of 0.9389 which indicates that 93.89% of the variation in residential electricity consumption was explained by the model (r^2 = 93.89%). The model also reveals that the linear term of population (α_1=386.57) and the quadratic term for temperature (α_2= 177.38) have positive contribution to residential electricity consumption. This result indicates that as these variables increases in value, residential electricity consumption also increases. Summary results of the actual and predicted residential electricity for the Quadratic Regression with interaction are as shown in Table 3.

Table 3: Values of actual and predicted electricity consumption using for the Quadratic Regression with interaction

Year	Actual(MW/h)	Predicted(MW/h)
2006	894.11	948.39
2007	1151.94	1045.49
2008	1165.72	1214.45
2009	1104.54	1179.20
2010	1365.50	1317.55
2011	1401.01	1400.75
2012	1437.43	1459.71
2013	1474.81	1480.52
2014	1513.15	1515.73

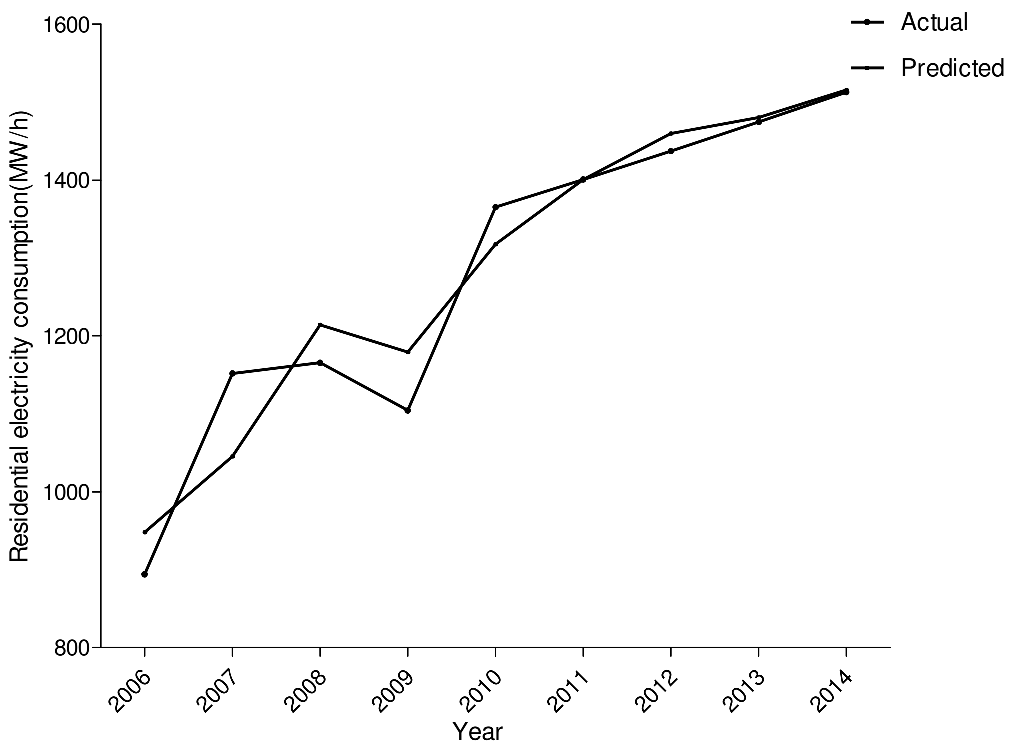

Figure 2: Graph of the Actual and Predicted Residential Electricity Consumption for the Quadratic Regression with interaction.

Bolded values is the r- square and least Root Mean Square Error

Table 4 shows performance evaluation of the two competing models. Model 2 (the Quadratic Regression with interaction) gave the highest value of coefficient of determination ($r^{2)}$ compared with model 1 (the Multiple Linear Regression Model). Also, Model 2 gave the least value of the Root Mean Square Error. Hence, the Quadratic Regression with interaction is recommended for forecasting residential electricity consumption. The population for 2015 and 2029 were projected using eq 5, $P_t = P_{t-1}(1+r)^n$, where n= number of years, P_{t-1}= previous year population, P_t= the population of the year to be estimated and r is the population growth rate of Nigeria which is given to 3.2% according to 2006 population census.

Table 4: Comparison of the Performance of the two Models

Models	r^2 (%)	RMSE
Model 1: for the Multiple Linear Regression Model	87.30	69.97
Model 2: the Quadratic Regression with interaction.	**93.87**	**52.77**

Also, the temperature was predicted using eq 6, $T = a_0 + a_1 t$. The temperature model parameters a_0 and a_1 were estimated using Microsoft excel programme which gave $a_0 = 34.602$, $a_1 = 0.0613$. Hence :

$$T = 34.602 + 0.0613t + \varepsilon_t \text{ , where t = time, } t = 10, \cdots, 24 \qquad (20)$$

Table 5 and Figure 3 show the forecasted residential electricity consumption in Nigeria from 2015-2029 using the quadratic regression with interaction. From the results in Table 5, the Residential Electricity Consumption in Nigeria will reach 6521.09 MW/h in the year 2029.

Table 5: The Forecast values for Residential Electricity Consumption (2015-2029) Using the Quadratic Regression with interaction

S/N	Year	Population (Million)	Temp.(^{0}C)	Forecasted Residential Electricity Consumption (MW/h) Using the Quadratic Regression with interaction
1	2015	186.84	33.99	2937.90
2	2016	192.82	33.93	3116.81
3	2017	198.99	33.87	3305.97
4	2018	205.36	33.81	3504.57
5	2019	211.93	33.74	3714.71
6	2020	218.71	33.68	3934.14
7	2021	225.71	33.62	4166.97
8	2022	232.94	33.56	4410.53
9	2023	240.39	33.5	4667.68
10	2024	248.08	33.44	4937.62
11	2025	256.02	33.38	5222.63
12	2026	264.21	33.31	5522.81
13	2027	272.67	33.25	5838.59
14	2028	281.39	33.19	6171.24
15	2029	290.4	33.13	6521.09

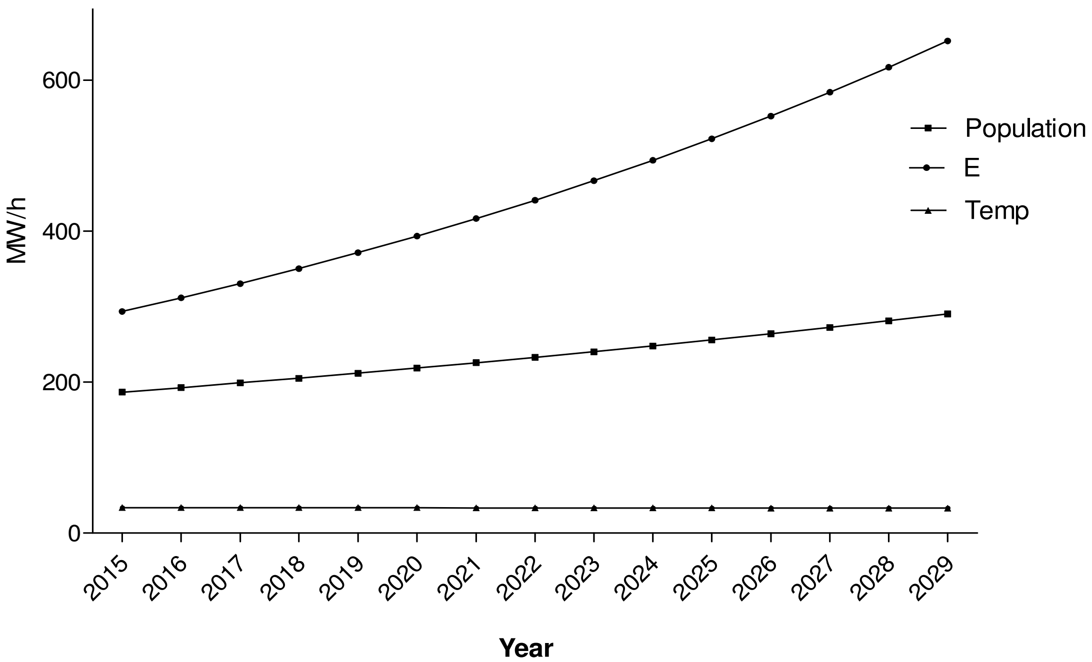

Figure 3: Graph of Forecasted Residential Electricity Consumption (2015-2029) Using the Quadratic Regression with interaction

Result in Table 6 shows highly positive significant relationship between residential electricity consumption and population ($r = 0.932$, $p< 0.01$). The result also reveal insignificant moderately weak relationship between residential electricity consumption and temperature ($r = -0.406$, $p> 0.02$). Based on this result, the only independent variable found to significantly correlated with residential electricity consumption is population. This result implies that as the population increases, residential consumption of electricity also increases significantly.

Table 6: Correlation matrix between the variables

	E	P	T
E	1		
P	0.932**	1	
T	-0.406	-0.369	1

E = Electricity consumption, P = Population, T = Temperature

4 Conclusion

The paper presented statistical analysis of residential demand for electricity in Nigeria, employing annual data over the period 2006–2014. Multiple linear model and quadratic regression with interactions were applied to estimate residential electricity consumption and to forecast long-term residential demand for electricity. Also , population and temperature were used as explanatory variables. The results shows that the quadratic regression model with interaction was more accurate due to the fact that it has the highest coefficient of determinant and the least value of root mean square error as compared to the linear regression model. Also, population of demand has a positive sign and it is significant in the short run and in the long run forecasting. The result also revealed insignificant moderately weak relationship between residential electricity consumption and temperature.

References

[1] Ubani, O. J., Umeh, L., & Ugwu, L. N. (2013). Analysis of the electricity consumption in the south-east geopolitical region of Nigeria. *Journal of Energy Technologies and Policy*, *3*(1), 20-31.

[2] Mojeed, M. (2014). The Epileptic Nature of Electricity Supply and its Consequences on Industrial and Economic Performance in Nigeria (Error Correction Modelapproach). *Global Journal of Research In Engineering*, *14*(4).

[3] Awosope, C. A. (2014). Nigeria Electricity Industry: Issues, Challenges and Solutions, Covenant University Public Lecture Series. Vol. 3, No. 2.

[4] Alila, P. O., & Atieno, R. (2006). Agricultural policy in Kenya: Issues and processes. *Nairobi: Institute of Development Studies*.

[5] Adenikinju, A. (2008). Efficiency of the Energy Sector and its Impact on the Competitiveness of the Nigerian Economy. *International Association for Energy Economics*, 27-31.

[6] Agbo, C.O. (2011). A critical evaluation of motor vehicle manufacturing in Nigeria. *Nigerian Journal of Technology*, *30*(1), 8-16.

[7] Orji, J. I. (2005). An Assessment of Impacts of Poverty Reduction Programmes in Nigeria as a Development Strategy, 1970-2005. *Unpublished Ph. D Dissertation, St. Clements University, Turks and Caicos Island*.

[8] Sambo, A. S. (2008). Matching electricity supply with demand in Nigeria. *International Association of Energy Economics*, *4*, 32-36.

[9] Essien, A. U., & Igweonu, E. I. (2014). Coal based generation: a solution to Nigeria electricity problem. *Int Archive Appl Sci Technol*, *5*(1), 74-80

[10] Ogunleye, E. K. (2016). *Political economy of Nigerian power sector reform* (No. UNU-WIDER Research Paper wp2016-009). World Institute for Development Economic Research (UNU-WIDER).

[11] Oyedepo, S. O. (2012). On energy for sustainable development in Nigeria. *Renewable and Sustainable Energy Reviews*, *16*(5), 2583-2598.

[12] Ubani, O. J., Umeh, L., & Ugwu, L. N. (2013). Analysis of the electricity consumption in the south-east geopolitical region of Nigeria. *Journal of Energy Technologies and Policy*, *3*(1), 20-31.

[13] Central Bank of Nigeria Statistical Bulletin (2006), Vol. 20, December 2006.

[14] Edenhofer, O. (2011). The IPCC special report on renewable energy sources and climate change mitigation.

[15] Annual Reports - Central Bank of Nigeria, 2015.

[16] National Bureau of Statistics, 2015.

[17] Agboola, B.M., & Adeyemi, J.K. (2013). Projecting Enrollment for Effective Academic Staff Planning in Nigerian Universities. *Educational Planning*, *21*(1).