Mathematical and Software Engineering, Vol. 3, No. 2 (2017), 209-216.
Varεpsilon Ltd, varepsilon.com

Determination of the Minimum Antenna Mast Height for Microwave Links with Nonzero Path Inclination: Method I

Swinton C. Nwokonko[1], **Vital K. Onwuzuruike**[1], and

Chibuzo Promise Nkwocha[2*]

*Corresponding Author Chibuzo Promise Nkwocha: chibuzorpromise525@yahoo.com
[1]Department of Electrical/Electronic Engineering, Imo State University, Owerri, Nigeria.
[2]Department of Chemical Engineering, Federal University of Technology, Owerri (FUTO), Owerri, Nigeria

Abstract

In this paper, a method that can be used to determine the minimum antenna mast height when the path inclination is not equal to zero is presented. In this method, none of the antenna height is known. In this case, the two antenna mast heights are determined from the knowledge of the location and height of the maximum obstruction in the communication link path. The mathematical models and the algorithm pertaining to the method are presented in this paper along with sample numerical example using path profile data for a line of sight 4 GHz microwave communication link with path length of 38887.6 m. From the results, the receiver antenna height is 176.07 m and transmitter antenna height is 127.09 m. With respective to the elevation height, this gives the transmitter antenna mast height of 37.25m and the receiver antenna mast height of 127.2 m. In effect, the transmitter antenna is lower than the receiver antenna. The transmitter is also below the maximum height of the tip of the obstruction which is 144.21 m high. The path inclination is 1.26. *The ideas presented in this study are useful for installation of entirely new line of sight microwave communication link.*

Keywords: Microwave Communication Link; Line Of Sight, Earth Bulge; Elevation Profile; Path Inclination; Antenna Mast Height; Elevation Height

1. Introduction

Line Of Sight (LOS) communication links are very useful for terrestrial and satellite communications [1-6]. Particularly, during the planning stage of terrestrial point to point LOS communication link, it is important to determine the minimum antenna heights that will ensure clear line of sight between the transmitter antenna and the receiver antenna [7-11]. From the antenna heights, the required antenna mast heights can be established. The method to be used in determining the minimum antenna heights depends on the available data. Superficially, when the installation is an extension of existing LOS link, in that case, the height of one of the antennas is known, the method to be employed will take into consideration the know height of one of the antennas. This constraint helps the designer to narrow the possible values that can be obtained for the minimum height of the second antenna.

However, when the installation is a fresh LOS link, no antenna height is specified, the method to be employed must be able to use the limited information on the obstruction heights and their locations to determine the possible minimum antenna heights for the link. In this paper, this second case is addressed, a situation where both antenna heights are to be determined based on the elevation profile , earth bulge and obstruction dataset for the link. Particularly, the method utilizes the location and height of the maximum obstruction in the signal path to determine the minimum antenna mast heights. Sample microwave link is used to demonstrate the applicability of the method.

2. Theoretical Background

2.1 Fresnel Geometry Parameters For Line Of Sight (LOS) Link

Elevation Profile : Let n_e be the number of elevation points taken from the transmitter location to the receiver location . Also, let $H_{el(x)}$ be the elevation at point x, where x = 1,2,3,…, n_e; let $d_{t(x)}$ be the distance of location x from the transmitter ; let $d_{r(x)}$ be the distance of location x from the receiver, where x = 1,2,3,…, n_e and let d be the distance (in meters) between the transmitter and the receiver. Then,

$$d = d_{r(x)} + d_{t(x)} \qquad (1)$$

$$d_{r(x)} = d - d_{t(x)} \qquad (2)$$

H_{elt} is the elevation at the transmitter location where x = 0, hence, $H_{elt} = H_{el(0)}$

H_{elr} is the elevation at the receiver location where x = n_e, hence, $H_{elr} = H_{el(n_e)}$

Earth bulge :Earth bulge is the height an obstruction is raised higher in elevation (into the path) owing to earth curvature. Earth bulge is given as [13];

$$H_{eb(x)} = \frac{(d_{t(x)})(d_{r(x)})}{12.75 * K} \qquad (5)$$

Where $H_{eb(x)}$ is the height (in meters) of the earth bulge at location x between the transmitter and the receiver; H_{ebt} is the height (in meters) of the earth bulge at the transmitter mast location; H_{ebr} is the height (in meters) of the earth bulge at the receiver mast location; $d_{t(x)}$ and $d_{r(x)}$ are as defined earlier.

Radius of the nth Fresnel : The radius of the nth Fresnel zone ($\mathbf{r}_{(n,x)}$) at location x is given as [14 – 18];

$$\mathbf{r}_{(n,x)} = \sqrt{\frac{n\left(\lambda(d_{t(x)})(d_{r(x)})\right)}{(d_{t(x)} + d_{r(x)})}} \text{ ; for n =1,2,3,… and } d_{t(x)} >> \mathbf{r}_{(n,x)} \text{ and } d_{r(x)} >> \mathbf{r}_{(n,x)} \qquad (8)$$

λ in metres is given as;

$$\lambda = \frac{c}{f} \qquad (9)$$

Obstruction height: Let $h_{ob(x)}$ be the height of the obstruction at point x, where $h_{ob(x)}$ is measured from the ground level (where the ground level is at the top of the elevation point at point x) and it does not include the elevation and earth bulge at point x. The elevation point is measured from the sea level. Let $H_{ob(x)}$ be the overall height of the obstruction at point x, where $H_{ob(x)}$ is measured from the reference line. $H_{ob(x)}$ includes the elevation at point x and also include the earth budge at point x, Then ;

$$H_{ob(x)} = h_{ob(x)} + H_{eb(x)} + H_{el(x)} \qquad (10)$$

Percentage Clearance: Let $P_{C(n)}$ be the percentage clearance allowed for the Fresnel zone n, given in % where $P_{C(n)}$ positive if the obstacle tip is below the line of sight and $P_{C(n)}$is negative if the obstacle tip is above the line of sight. It must be noted that $P_{C(n)} = 0\ \%$ when the tip of the obstruction is on the LOS. Let X_{MAX} be the distance from the transmitter to the point at which the tip of the obstruction attains its maximum height, H_{MAX} and let H_{MAX} be the maximum height attained by the tip of the obstruction at location X_{MAX} and at distance, d_{MAX} from the transmitter. Then,

$$H_{MAX} = \text{Maximum}\left(H_{eb(x)} + H_{el(x)} + h_{ob(x)} + \left\{ \left(\frac{P_{C(n)}}{100}\right) \sqrt{\frac{n\left(\lambda\left(d_{t(x)}\right)\left(d_{r(x)}\right)\right)}{\left(d_{t(x)} + d_{r(x)}\right)}} \right\} \right)$$

(11)

2.2 Development Of The Mathematical Models For Determining The Transmitter And Receiver Antenna Mast Heights

This paper resents a method of determining the transmitter and receiver antenna mast when the transmitter and receiver antenna heights are not. In this case, the path inclination is not equal to zero. Particularly, for the method titled method I, analysis is based on the knowledge of the location (X_{MAX}) and height (H_{MAX}) of the maximum obstruction in the signal path. In this analysis, a communication link (figure 1) with the transmitter (T) and the receiver (R) at distance d apart is considered. It is assumed that the higher antenna is the receiver antenna ($H_t \leq H_r$) but the actual heights of both the transmitter and receiver are not. If however, $H_t > H_r$, then the notation r for transmitter and t for receiver will have to be swapped, whereby the transmitter becomes the receiver and vice versa.

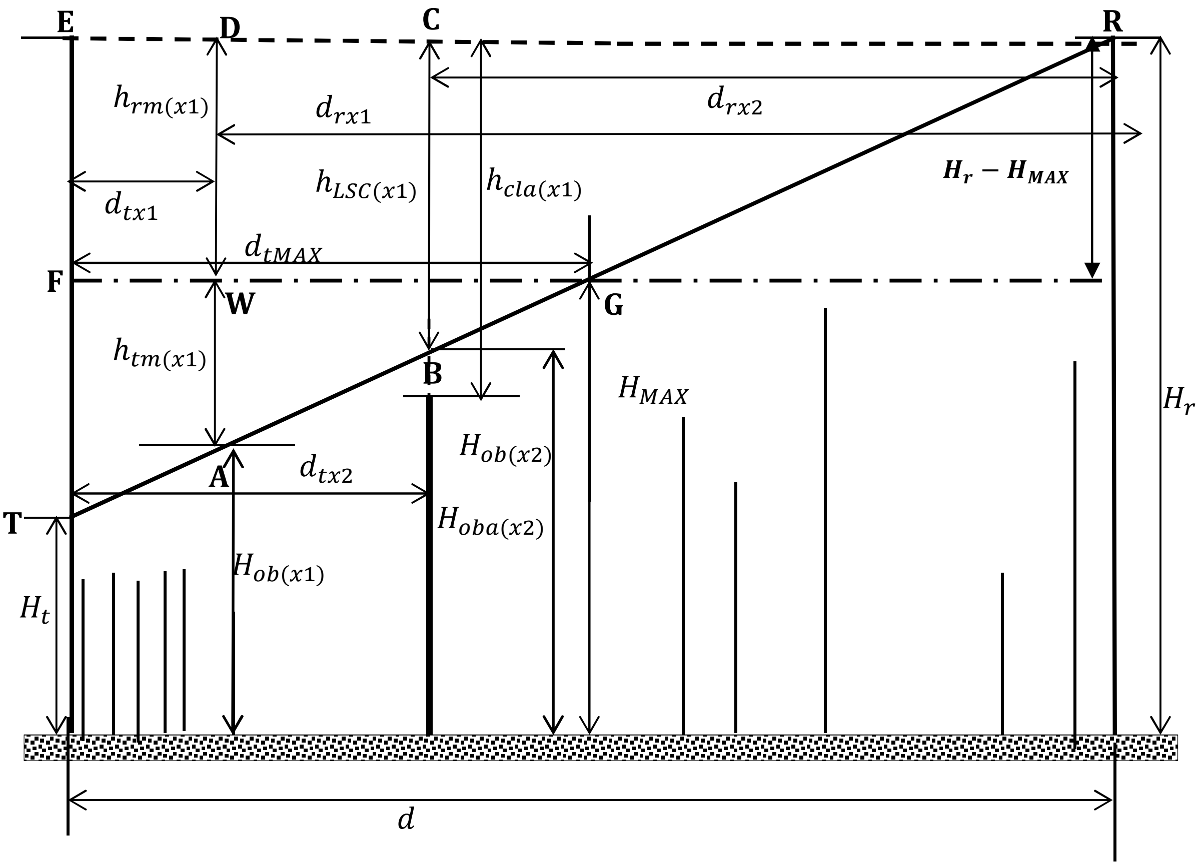

Figure 1 Model for determining the antenna mast height when the path inclination is not equal to zero

The path length is d . The locations between the transmitter and receiver are represented as x where x = 0,1,2,3,…,ne. Distance d_{tx} are measure from the transmitter and distance d_{rx} measured from the receiver. At the transmitter, x = 0 , $d_{t(0)}$ = 0 and x = ne , $d_{t(ne)}$ = d .

$$d_{rx} = d - d_{tx} \quad (12)$$

Let d_{rmx} bee defined as $d_{rmx} = d_{tMAX} - d_{tx}$

Let $H_{oba(x)}$ be defined as the actual obstruction height at location x, where

$$H_{oba(x)} = H_r - \left(h_{ob(x)} + H_{eb(x)} + H_{el(x)}\right) \quad (13)$$

Let $H_{ob(x)}$ be defined as the expected line of sight obstruction height at location x. Simply, $H_{ob(x)}$ is the maximum obstruction height that can be accommodated at location x without violating the LOS percentage clearance specified for the link. If $H_{oba(x)} > H_{ob(x)}$, then the LOS percentage clearance specified for the link will be violated. The equation for determining $H_{ob(x)}$ is derived by considering two locations x1 and x2 at point A and B respectively in figure 1. By applying similar triangle relationships on triangle AWG and triangle ADR then,

$$\frac{(H_r - H_{MAX}) - (H_{MAX} - H_{ob(x1)})}{d - d_{tx1}} = \frac{H_{MAX} - H_{ob(x1)}}{d_{tMAX} - d_{tx1}} \quad (14)$$

$$\frac{H_r - H_{ob(x1)}}{d_{rx1}} = \frac{H_{MAX} - H_{ob(x1)}}{d_{rmx1}} \quad (15)$$

$$H_r = H_{ob(x1)} + \left[\left(\frac{d_{rx1}}{d_{rmx1}}\right)\left(H_{MAX} - H_{ob(x1)}\right)\right] \quad (16)$$

Also, by applying similar triangle relationships on triangle ACR and triangle ADR then,

$$\frac{H_r - H_{ob(x2)}}{d_{rx2}} = \frac{H_r - H_{ob(x1)}}{d_{rx1}} \quad (17)$$

$$H_{ob(x2)} = H_r - \left[\left(\frac{d_{rx2}}{d_{rx1}}\right)\left(H_r - H_{ob(x1)}\right)\right] \quad (18)$$

$H_{ob(x2)}$ is the maximum obstruction height that can be accommodated at location x2 without violating the LOS percentage clearance specified for the link. If $H_{oba(x2)} > H_{ob(x2)}$, then the LOS percentage clearance specified for the link will be violated at location x2. In order to satisfy the line of sight clearance requirement at point x the following condition must be met:

$$H_{ob(x2)} \le H_{oba(x2)} \quad \text{for all x = 0,1,2,3}\ldots n_e. \quad (19)$$

Initially, $x1 = x = 0$, and $H_{ob(x1)} = H_{ob(0)} = H_{oba(0)}$. Also, at x1 = x = 0 , $d_x = 0$ and $d_{r0} = d - 0 = d$. In essence, with $H_{ob(x1)} = H_{ob(0)} = H_{oba(0)}$, the line of sight clearance requirement is satisfied at x1 = x = 0. Then, $H_{ob(x2)}$ is computed for x2 = x1+1 , x1+2, x1+3,…, n_e. At each point of x2, the line of sight clearance requirement conduction $H_{ob(x2)} \le H_{oba(x2)}$ is evaluated. If the condition is not satisfied, then , the current x2 becomes the x1 (that is, x1 = x2) and the current $H_{ob(x1)}$ becomes $H_{oba(x2)}$(that is, $H_{ob(x1)} = H_{oba(x2)}$). Next, $H_{ob(x2)}$ is computed for x2 = x1+1 , x1+2, x1+3,…, n_e. When all the points from x= 0 to x = n_e are considered, the transmitter height is adjusted based on the last value of $H_{ob(x1)}$ which is at a distance of d_{rx1} from the receiver. The adjustment is done as follows;

$$H_{ob(0)} = \left(\frac{d_{r0}}{d_{rx1}}\right) H_{ob(x1)} \quad (20)$$

$$H_t = H_r - H_{ob(0)} \quad (21)$$

The height (in meters) of the transmitter antenna mast measured from the ground is given as $h_{t(mast)}$ where;

$$h_{t(mast)} = H_t - H_{elt} = H_r - H_{ob(0)} - H_{elt} \quad (22)$$

Where H_{elt} is the elevation at the transmitter. The height (in meters) of the receiver antenna mast measured from the ground is denoted as $h_{r(mast)}$;

$$h_{r(mast)} = H_r - H_{elr} \quad (23)$$

2.3 The Procedure For Determining The Minimum Transmitter and Receiver Antenna Mast Height When The Path Inclination Is Nonzero

The following algorithm states the procedure for determining the minimum transmitter and receiver antenna mast height when the path inclination is nonzero.

Step 1: $Input\ H_{elr}, H_{elt}, h_{ob(0)}, d, n_e$
Step : $H_t = H_{elt};\ c = H_{elt};\ H_{eb(0)} = 0$
Step 2: $H_{ob(0)} = H_{oba(0)} = h_{ob(0)} + h_{el(0)}$
Step 3: $d_{r0} = d$
Step 4: x1 = 0
Step : Input $d_{tx1},\ H_{MAX},\ d_{MAX}$
Step : $d_{rx1} = d - d_{x1}$
Step : $d_{mx1} = d_{MAX} - d_{x1}$
Step : $H_r = \left(\frac{d_{rx1}}{d_{mx1}}\right)\left(H_{MAX} - H_{ob(x1)}\right)$
Step 5: For x2 = x1 to n_e Increment 1
Step : Input $d_{tx2,} h_{ob(x2)}, H_{eb(x2)}, H_{el(x2)}$
Step : $d_{rx2} = d - d_{x2}$
Step 6: $H_{ob(x2)} = \left(\frac{d_{rx2}}{d_{rx1}}\right) H_{ob(x1)}$
Step 7: $H_{oba(x2)} = H_r - H_{ob(x2)} = H_r - \left(h_{ob(x2)} + H_{eb(x2)} + H_{el(x2)}\right)$
Step 8: if ($H_{ob(x2)} < H_{oba(x2)}$ **) then**
Step 9: $H_{ob(x1)} = H_{oba(x2)}$
Step 10: x1 = x2
Step : $d_{mx1} = d_{MAX} - d_{x1}$
Step : $d_{rx1} = d - d_{x1}$
Step : $H_r = \left(\frac{d_{rx1}}{d_{mx1}}\right)\left(H_{MAX} - H_{ob(x1)}\right)$
Step 11: Endif
Step 12: Next x2
Step 13: $H_{ob(0)} = \left(\frac{d_{r0}}{d_{rx1}}\right) H_{ob(x1)}$
Step 14: $H_t = H_{ob(0)}$
Step 15: $h_{t(mast)} = H_t - H_{elt} = H_{ob(0)} - H_{elt}$
Step 16: $h_{r(mast)} = H_r - H_{elr}$
Step 17:End

3. Results and Discussions

The parameters of the microwave link used in the study are; path length = 38887.6 m, frequency = 4 GHz, k-factor = 1.33333 and obstruction height (hob) = 10 m. The specified minimum LOS percentage clearance with respect to Fresnel zone 1 is 60%. The result show that the maximum height of the tip of the obstruction (Hmax) is 144.21 m and it occurred at a distance of 14306.98 m from the transmitter. The receiver antenna height is obtained as 176.07 m while the transmitter is 127.09 m high.

In Table 1, the minimum percentage clearance of 60% with respect to Fresnel zone 1 occurred at a distance of 11415.14 m from the transmitter. The radius of the first Fresnel zone at that point is 24.59317 m and the LOS clearance height at that point is -14.76 m which gives the percentage clearance of 60% at that point. The 60% percentage clearance tallies with the 60% clearance specified at the link design stage.

Table 1 The Antenna Mast Computation Result

x, Elevation Point	dx, Distance (m)	hel(x) Elevation (m)	Heb(x) Earth Bulge (m)	Hx, LOS Height (m)	hLsc(x), LOS Clearance height (m)	r1, radius Of The First Fresnel Zone (m)	P(x,1) , Percentage Clearance Of The First Fresnel Zone %
1	0	89.8	0	127.1	-27.25	0	#DIV/0!
24	1750.32	78	3.82	129.3	-37.48	11.19667	334.7813
48	3576.74	96	7.43	131.6	-18.14	15.60713	116.2299
72	5403.17	92.3	10.64	133.9	-20.93	18.67974	112.058
96	7229.59	97	13.46	136.2	-15.73	21.00989	74.87233
120	9056.01	94.4	15.89	138.5	-18.24	22.8261	79.92583
144	10882.44	94.2	17.93	140.8	-18.67	24.24416	77.02571
151	11415.14	98.3	18.45	141.5	-14.76	24.59317	60
168	12708.86	90.5	19.57	143.1	-23.03	25.33102	90.92611
192	14535.28	94	20.82	145.4	-20.58	26.12804	78.76856
216	16361.71	93	21.68	147.7	-22.98	26.66124	86.18608
240	18188.13	92.3	22.15	150	-25.54	26.94627	94.76332
256	19405.74	92.2	22.24	151.5	-27.05	27.00258	100.1825
264	20014.55	89.4	22.22	152.3	-30.68	26.991	113.6627
288	21840.97	30.8	21.9	154.6	-91.87	26.79663	342.8401
312	23667.4	23	21.19	156.9	-102.74	26.35788	389.7966
336	25493.82	29.8	20.09	159.2	-99.3	25.66221	386.9364
360	27320.24	26.8	18.59	161.5	-106.1	24.68791	429.7789
384	29146.67	49.9	16.7	163.8	-87.15	23.4002	372.4393
408	30973.09	20	14.42	166.1	-121.68	21.74348	559.6235
432	32799.51	22.4	11.75	168.4	-124.27	19.62451	633.2387
456	34625.93	18.3	8.68	170.7	-133.71	16.86999	792.6158
480	36452.36	36.7	5.22	173	-121.13	13.08453	925.7446
504	38278.78	29	1.37	175.3	-134.97	6.704158	2013.261
512	38887.59	48.9	0	176.1	-117.16	0	#DIV/0!

Also, the transmitter antenna mast height is 37.25m while the receiver antenna mast height is 127.2 m. This gives receiver antenna height of 176.07 m and transmitter antenna height of 127.09 m. In effect, the transmitter antenna is lower than the receiver antenna. The transmitter is also below the maximum height of the tip of the obstruction which is 144.21 m high. The path inclination is $\frac{|H_r - H_t|}{d} = \frac{|176.07 - 127.09|}{38.88759} = 1.26$, where d is in km and H_r and H_t are in m.

4. Conclusion

In this paper, a first method that can be used to determine the minimum antenna mast height when the path inclination is not equal to zero is presented. In the second method,(not presented here), one of the antenna height is known. In this first method, none of the antenna height is known. In this case, the two antenna mast heights are determined from the knowledge of the location and height of the maximum obstruction in the communication link path. The mathematical models and the algorithm pertaining to the method are presented in this paper along with sample numerical example using path profile data for a line of sight microwave communication link.

References

[1] Kerczewski, R. J., Wilson, J. D., & Bishop, W. D. (2016, March). UAS CNPC satellite link performance—Sharing spectrum with terrestrial systems. In *Aerospace Conference, 2016 IEEE* (pp. 1-9).

[2] Kerczewski, R. J., Wilson, J. D., & Bishop, W. D. (2015, April). Parameter impact on sharing studies between UAS CNPC satellite transmitters and terrestrial systems. In *2015 Integrated Communication, Navigation and Surveillance Conference (ICNS)* (pp. X2-1). IEEE.

[3] Thompson, P., & Evans, B. (2015, June). Analysis of interference between terrestrial and satellite systems in the band 17.7 to 19.7 GHz. In *2015 IEEE International Conference on Communication Workshop (ICCW)* (pp. 1669-1674).

[4] Al Mahmud, M. R. (2009). *Analysis and planning microwave link to established efficient wireless communications* (Doctoral dissertation, Blekinge Institute of Technology).

[5] Wang, G., Lee, B. S., & Ahn, J. Y. (2016, October). Authentication and Key Management in an LTE-Based Unmanned Aerial System Control and Non-payload Communication Network. In *Future Internet of Things and Cloud Workshops (FiCloudW), IEEE International Conference on* (pp. 355-360).

[6] Zeng, Y., Zhang, R., & Lim, T. J. (2016). Wireless communications with unmanned aerial vehicles: opportunities and challenges. *IEEE Communications Magazine*, *54*(5), 36-42.

[7] Bao, L., Rydström, M., Rhodin, A., & Ligander, P. (2016). *U.S. Patent No. 20,160,127,059*. Washington, DC: U.S. Patent and Trademark Office.

[8] Agbinya, J. I. (2013). Radio Wave Propagation Models for Cellular Communications. *4G Wireless Communication Networks: Design Planning and Applications*, Chapter 2, 37-66.

[9] Ghasemi, A., Abedi, A., & Ghasemi, F. (2013). *Propagation engineering in radio links design*. Springer Science & Business Media.

[10] Munilla Diez, M. (2016). Derive a methodology for the design of a broadband (over 1 Gbps) microwave backhaul link in E-band. http://hdl.handle.net/10902/8446.

[11] Osman, M. A. A. (2015). *A Compartive Study of Microwave versus WiMAX Link* (Doctoral dissertation, Sudan University of Science and Technology).

[12] Dalbakk, L. E. (2014). Antenna System for Tracking of Unmanned Aerial Vehicle. http://hdl.handle.net/11250/2371136.

[13] Ma, Y., & Wang, Y. (2012, November). Study of characteristic prediction of radio wave propagation loss on complex irregular terrain of wide-range distance. In *Environmental Electromagnetics (CEEM), 2012 6th Asia-Pacific Conference on* (pp. 67-71).

[14] Guo, Y. J., & Barton, S. K. (2013). *Fresnel zone antennas*. Springer Science & Business Media.

[15] Kapusuz, K. Y., & Kara, A. (2014). Determination of scattering center of multipath signals using geometric optics and Fresnel zone concepts.*Engineering*

Science and Technology, an International Journal, *17*(2), 50-57.
[16] Freeman, R. L. (2006). *Radio system design for telecommunication* (Vol. 98). John Wiley & Sons.
[17] Ratnayake, N. L., Ziri-Castro, K., Suzuki, H., & Jayalath, D. (2011, January). Deterministic diffraction loss modelling for novel broadband communication in rural environments. In *Communications Theory Workshop (AusCTW), 2011 Australian* (pp. 49-54).
[18] Mazar, H. (1991, March). LOS radio links, clearance above tall buildings. In*Electrical and Electronics Engineers in Israel, 1991. Proceedings., 17th Convention of* (pp. 145-148).