

Early Childhood Educators' and Teachers' Early Mathematics Education Knowledge, Beliefs, and Pedagogy

Alexandra Youmans
Queen's University

Andrew Coombs
Queen's University

Lynda Colgan
Queen's University

Abstract

In the spring of 2017, an educational needs assessment of early mathematics education in Ontario was conducted using an online survey for early childhood educators, kindergarten teachers, and Grade 1 teachers. The purpose of the needs assessment was to acquire a brief account of the state of early mathematics education in the province and to identify any

potential gaps between current and desired early mathematics education practices. Educators' responses ($n = 130$) about their early mathematics knowledge, beliefs, and pedagogy revealed four critical areas that need to be addressed to improve the quality of early mathematics education.

Keywords: early childhood education, kindergarten, mathematics, teacher knowledge and beliefs

Résumé

Au printemps 2017, une évaluation des besoins éducatifs de l'éducation mathématique précoce en Ontario a été réalisée en utilisant un sondage en ligne pour les éducateurs de la petite enfance, les enseignants de la maternelle et les enseignants de première année. Le but de l'évaluation des besoins avait pour but d'obtenir un bref compte rendu de l'état de l'éducation précoce en mathématiques dans la province et d'identifier tout écart potentiel entre les pratiques actuelles et futures d'éducation en mathématiques. Les réponses des éducateurs ($n = 130$) au sujet de leurs connaissances, de leurs croyances et de leur pédagogie en mathématiques ont révélé quatre domaines critiques qui doivent.

Mots-clés : l'éducation de la petite enfance, jardin d'enfants, mathématiques, connaissances et croyances des enseignants

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Introduction

There is a growing awareness of the importance of mathematics in Canadian society due to an increased dependence on mathematical methods in science, technology, commerce, and government (Lancaster, 2013). Despite this awareness, less than 50% of secondary school graduates have pre-requisite Grade 11 and Grade 12 mathematics courses needed for a growing number of post-secondary pathways (Let's Talk Science & Amgen Canada Inc., 2012). This discrepancy may exist because many students are not developing foundational mathematics knowledge, skills, and confidence needed for success in upper-secondary mathematics courses during elementary schooling. Moreover, research suggests that students' struggles in elementary mathematics are related to weaknesses in early number competence, a fundamental early mathematics concept (Gersten, Jordan, & Flojo, 2005; National Mathematics Advisory Panel, 2008). Unfortunately, elementary students who lack number competence may experience diminishing mathematics achievement as they progress through school. For example, Ontario's Education Quality and Accountability Office (EQAO) reports that mathematics scores in English-language elementary schools have experienced an eight-year steady decline, with only 63% of Grade 3 students and 50% of Grade 6 students meeting provincial standards in 2017 (EQAO, 2017).

Given that mathematics knowledge is cumulative, it is critical for young children to develop strong early number competence. Upon kindergarten entry, children vary widely in their mathematical knowledge, and those with less knowledge tend to lag behind their peers as they advance through school, resulting in a widening achievement gap in mathematics (Duncan et al., 2007; Jordan, Kaplen, Ramineni, & Locuniak, 2009; Romano, Babchishin, Pagani, & Kohen, 2010). Accordingly, the Critical Transitions in Early Mathematical Development Community of Practice (Critical Transitions CoP) was established in the spring of 2017, as part of the Mathematics Knowledge Network (MKN), to identify and support best practices that foster successful early mathematics transitions (e.g., into formal schooling, kindergarten to Grade 1) and align with the Renewed Mathematics Strategy (RMS) in Ontario. The RMS was initiated in September 2016 as a series of school supports to promote mathematics achievement across the province (Ontario Ministry of Education, 2016). At the elementary school level, supports include a play-based model for kindergarten mathematics instruction, 60 minutes a day of protected mathematics learning for students in Grades 1 to 8, up to three mathematics

teacher leads in all elementary schools, and increased math-related professional learning opportunities for teachers and principals. With respect to the Early Years, the RMS's reference to a play-based model for kindergarten instruction reinforces the play-based approach taken in the province's full-day kindergarten (FDK) program, first introduced in September 2010 (Ontario Ministry of Education, 2010). In addition to focusing on play, the FDK program in Ontario utilizes a team teaching approach that combines the child development expertise of an early childhood educator (ECE) with a certified teacher in classrooms with 26 students or more. Given that the majority of kindergarten classrooms in Ontario include both an ECE and a teacher, the term "educator" will be used to refer to both ECEs and teachers. As a starting point for their work, the Critical Transitions CoP designed and conducted an educational needs assessment of early mathematics educators (i.e., early childhood educators, kindergarten teachers, and Grade 1 teachers) in Ontario to acquire a brief account of the current state of early mathematics education in Ontario and identify whether any potential gaps between current and desired early mathematics education practices existed. The method used for the educational needs assessment was an online survey that asked educators about their early mathematics knowledge, beliefs, and pedagogy.

Literature Review

The Importance of Early Number Competence

Early number competence refers to a young child's ability to know the value of small quantities instantly, grasp the magnitude of numbers, comprehend relationships between numbers, understand counting principles, and perform simple addition and subtraction (Jordan et al., 2009). Research suggests that early number competence is an important predictor of future mathematics and school success. For example, Jordan et al.'s (2009) study of early mathematics suggests that early number competence in kindergarten predicts rate of growth in mathematics achievement (between Grade 1 and Grade 3) and achievement levels in Grade 3. Similarly, in a meta-analysis of six longitudinal studies relating school entry skills to later academic achievement, early number competence (i.e., number knowledge and ordinality) was the most powerful predictor of later school

success, even more powerful than early literacy and attention skills, with an effect size of .34 (Duncan et al., 2007). A similar study examining the influence of kindergarten mathematics and literacy skills on Grade 3 academic outcomes, using a Canada-wide dataset, replicated Duncan et al.'s finding that early number competence is the strongest predictor of later school success (Romano et al., 2010).

Early Mathematics Educators' Knowledge, Beliefs, and Pedagogy

Teaching is a complex task that requires three types of teacher knowledge: (a) subject matter knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge (Shulman, 1986). Subject matter knowledge is an understanding of how mathematics works (i.e., its extent and depth, structure and unifying concepts, procedures and strategies, connections with other subjects), while pedagogical content knowledge refers to how to teach mathematics effectively and curricular knowledge refers to what to teach (Ernest, 1989). Teachers' subject matter knowledge of mathematics has been linked to student achievement. For example, one study exploring the relationship between teachers' mathematics knowledge and their students' mathematics achievement found that students of high-achieving teachers exhibited the highest mathematical achievement (Schofield, 1981). Similarly, an evaluation of teacher candidates' (TCs) ability to teach numbers during teaching placements (using a 3-point scale of weak/capable/strong), revealed that TCs who received high (or even middle) scores on a mathematics subject knowledge test were more likely to be assessed as strong teachers (Goulding, Rowland, & Barber, 2002). In a study examining the mathematics content knowledge and attitudes of elementary teachers, it was found that upper elementary teachers (Grades 3–5) had greater mathematics content knowledge and more positive attitudes toward mathematics than primary teachers (K–2; Wilkins, 2008). This is concerning because primary teachers' lack of mathematics content knowledge and less positive attitudes toward mathematics may undermine their confidence and effectiveness as early mathematics educators. Appropriate mathematics education training is needed for teachers to feel confident about their instructional abilities. Mathematics educators who are confident model confidence for their students: teachers' self-confidence as mathematics educators is significantly correlated with students' perceptions of their own competence as mathematics learners (Stipek, Givvin, Salmon, & MacGyvers, 2001).

Educators' beliefs about the nature of education influence their pedagogy through content selection and emphasis, teaching style, and the type of learning opportunities provided for students (Ernest, 1989). With respect to early mathematics education, the type and depth of mathematical concepts taught is influenced by educator beliefs about young children's mathematics learning. In light of growing evidence that young children are more capable of engaging with complex mathematical concepts than previously thought (Fisher, Hirsh-Pasek, Newcombe, & Golinkoff, 2013; Presser, Clements, Ginsburg, & Ertle, 2015), attempts are being made to help educators understand children's early mathematics development and build on young children's prior mathematical understanding (Clements, 2018; Hachey, 2013; Knaus, 2017; Stipek, 2013). In addition, there has been a recent shift in early mathematical education toward intentional teaching characterized by the communication of key mathematical concepts in a supportive play or inquiry environment (Hachey, 2013; Knaus, 2017; Stipek, 2013). In accordance with this, Moss, Bruce, and Bobis (2016) identify four components of an effective early mathematics education program: (a) it builds on children's prior knowledge and incorporates children's interests, (b) mathematical ideas are introduced and extended in planned ways (i.e., play alone is not enough to develop mathematical thinking), (c) it is based on child developmental theory, and (d) responsive adult guidance is needed. Beliefs about what young children are capable of learning and how they learn has shifted the landscape of early mathematics education. As a result, early mathematics educators require specialized training in early mathematical development and play-/inquiry-based approaches to mathematical instruction.

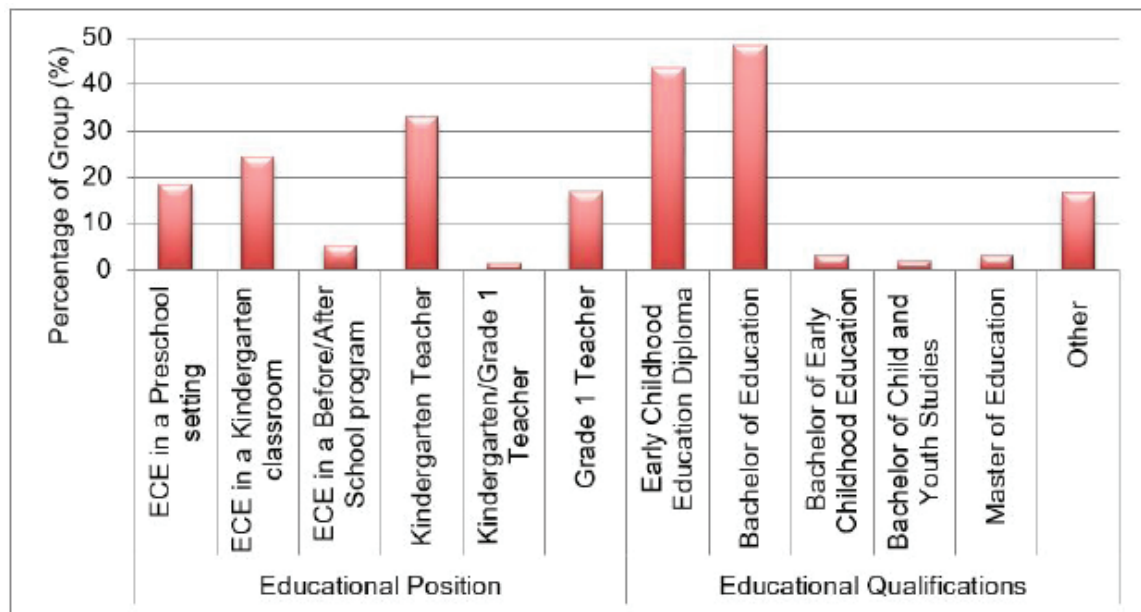
Method

The Critical Transitions CoP conducted an educational needs assessment targeted at early mathematics educators (i.e., ECEs, kindergarten teachers, Grade 1 teachers) in Ontario. The purpose of the needs assessment was to obtain a brief account of the state of early mathematics education and determine if any gaps between current and desired early mathematics education practices existed. For the needs assessment, the CoP developed the Early Mathematics Education Survey to collect information about Ontario educators' early mathematics education knowledge, beliefs, pedagogy, and professional

development needs. In total, 178 educators completed the Early Mathematics Education Survey.

Sample

Participants were recruited through electronic survey distribution to LISTSERVs of a large school board in southern Ontario and one professional development organization with province-wide membership. In addition, CoP-related social media outlets (e.g., the Mathematics Knowledge Network's website, Twitter accounts) were used to recruit survey participants. The overall sample consisted of 178 educators who completed the survey over a two-month period (spring 2017). Listwise deletion was used to remove missing data, resulting in a final sample of 130 educators. Participants reported a range of years of experience as a full-time educator ($M = 14.30$, $SD = 8.15$) and years as an educator of young children ($M = 12.10$, $SD = 7.52$). Participants were fairly evenly split



between early childhood educators (ECE; 48.5%) and kindergarten or Grade 1 teachers (K/1 teachers; 51.5%), with both types of educators primarily associated with a kindergarten classroom (59.2%; see Figure 1).

Figure 1. All participants' type of current position and educational qualifications by group percentage

Instrument

The Early Mathematics Education Survey contained a combination of closed-ended and open-ended questions, with 25 questions in total, categorized into five sections: (a) Educator Demographic Data (e.g., educational position, educational qualifications, educator training), (b) Educator Knowledge of Early Mathematics Education, (c) Beliefs About Early Mathematics Development and Education, (d) Early Mathematics Education in Practice, and (e) Early Mathematics Education and Professional Development. Approval from the General Research Ethics Board at Queen's University, the Greater Essex County District School Board, and the Association of Early Childhood Educators Ontario was granted to administer the Early Mathematics Education survey via an online platform.

Data Analysis

For questions with categorical responses (e.g., *Please indicate the types of assessment you use in your early mathematics program*), frequency counts, contingency tables, and Pearson's chi-square tests were used to determine if there were statistical differences between ECE and K/1 teachers. For questions with continuous responses (e.g., *Please rate your level of agreement with the following statements*), means and standard deviations were calculated and independent t-tests were used to determine if there were statistical differences between ECE and K/1 teachers. All data analysis was completed using Statistical Program for the Social Studies version 22 (SPSS v. 22). Interaction effects (e.g., differences between ECEs and classroom teachers by their level of mathematics training) were not analyzed due to the small sample size of the study.

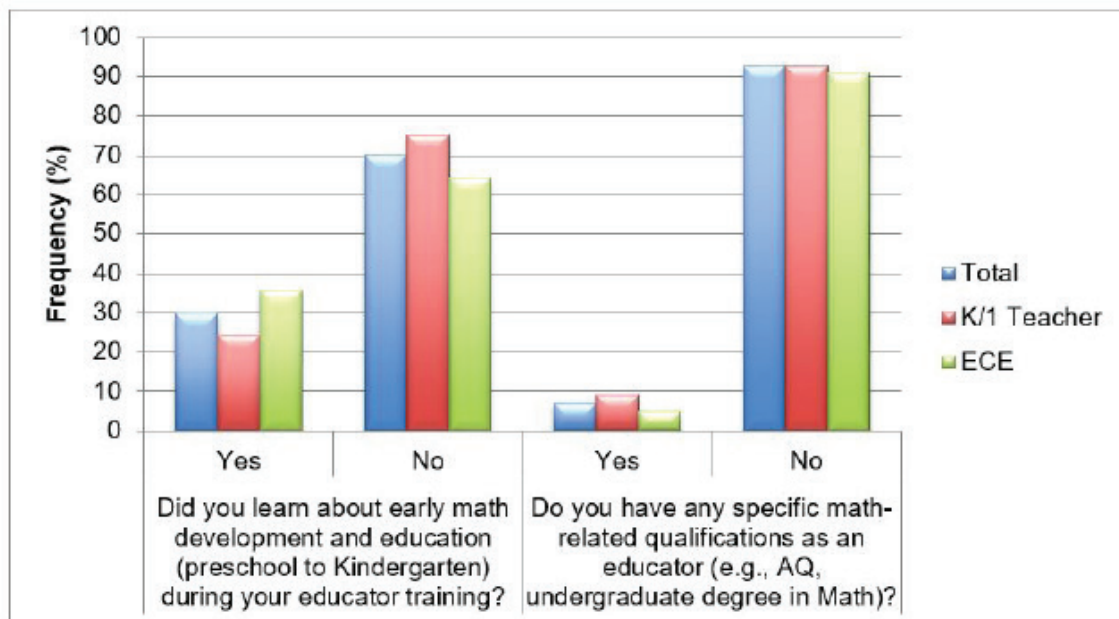
Open-ended survey responses were analyzed thematically for patterns within questions. Themes that emerged related to mathematics strands (e.g., Number Sense, Geometry and Spatial Sense) and teaching and learning topics (e.g., Instructional Strategies, Mathematics Training, etc.). For the purposes of this article, the three most common themes for each question are reported. Where questions had two parts, themes for each part are identified.

Results

The results of the Early Mathematics Education Survey are presented using three topic areas: Early Mathematics Educator Knowledge, Early Mathematics Educator Beliefs, and Early Mathematics Pedagogy. Each topic area begins with a description of quantitative responses and is followed with qualitative responses. A summary of results is outlined at the end of the section.

Early Mathematics Educators' Knowledge

To better understand educator knowledge, the Early Mathematics Education Survey asked educators to identify their early mathematics education training, describe the areas of early mathematics that they were most and least comfortable teaching, and communicate their early mathematics education professional development needs. As seen in Figure 2, the majority of ECE and K/1 teachers did not learn about early mathematics development and education during their educator training programs (64.4% and 75.4%, respectively)



and did not possess specific math-related qualifications (90.9% and 92.9%, respectively).

Figure 2. ECE and K/1 teachers' experience with early mathematics development and education with educators training and specific math-related qualifications

When asked which areas of early mathematics educators felt most comfortable teaching/ facilitating, they identified Number Sense and Numeration ($n = 42$), Patterning ($n = 27$), and Geometry and Spatial Sense ($n = 20$). Eleven educators indicated that they felt comfortable teaching/facilitating all strands. The most frequent explanations for comfort with teaching included the following examples: it was an area of teacher strength ($n = 13$), it was fun and engaging ($n = 8$), and it could be taught with manipulatives/concrete connections ($n = 8$). For example, an ECE communicated that she felt comfortable teaching particular areas of mathematics because they were her strengths: “Number sense and numeration, geometry and patterning...these are my strengths so I feel comfortable talking about ideas around these concepts.” Educators also felt comfortable teaching areas of early mathematics that were fun and engaging. A teacher described how she felt comfortable teaching shapes because it was fun for students and there were engaging resources available: “I feel most comfortable in teaching geometry, such as 2D and 3D shapes. Personally, I feel that it is the most fun to teach and there are so many play-based resources online for 2D and 3D shapes.” Another reason shared as to why educators felt comfortable teaching mathematics involved the use of manipulatives: an ECE working in a Montessori environment indicated “our school uses manipulatives for all mathematics areas...these materials give the child a solid foundation and understanding of mathematics in their very early years...all mathematics areas are shown with these hands-on, concrete representations.”

When asked which areas of early mathematics educators feel least comfortable teaching/facilitating, educators identified Geometry and Spatial Sense ($n = 13$), Instructional Strategies ($n = 12$), and Data Management and Probability ($n = 11$). Eight educators indicated that there were not any areas of mathematics education they felt least comfortable teaching. The most frequent explanations for least comfort teaching included the following examples: being unsure of what to teach conceptually/lack of expertise ($n = 8$), the mathematics area(s) is less engaging/interesting ($n = 4$), and a lack of teaching ideas ($n = 4$). An ECE described feeling least comfortable teaching Geometry because “I haven’t spent much time getting in-depth in the knowledge of this area of math.” Similarly, a teacher identified feeling least comfortable teaching Geometry and Spatial Sense “because these are the areas I struggle with most as an adult!” Educators also identified feeling least comfortable with particular teaching strategies. For example, five ECEs

indicated feeling least comfortable with instructional strategies, which included rote number facts, mathematics games, worksheets, large group mathematics instruction, and one-on-one teacher directed instruction. In regard to mathematics games, one ECE explained, “I have never been properly taught how to play [math] games (other than BINGO). If I can’t play them, how can I facilitate the students while they play?” Another ECE felt least comfortable with large groups for mathematics “because most of the children do not seem interested or they seem lost (not play-based).” Seven teachers identified feeling least comfortable with the following early mathematics instructional strategies: play-based learning ($n = 3$), whole group lessons ($n = 2$), use of manipulatives (1 educator), and time for one-on-one instruction ($n = 1$). One teacher explained her lack of comfort with play-based learning, “It’s difficult to reach all learners through play and to give them the practice they need due to time.” Similarly, another teacher shared the challenges she experiences with play-based learning: “It is a lot of activities going on at the same time and [it] requires more anecdotal notes, on top of trying to interact with the students.” A teacher explained she felt least comfortable with whole group instruction “because I feel the children won’t be able to apply the concept I am teaching as good [well] as if they were in a small group.” Some educators felt least comfortable teaching areas of early mathematics because they did not have a lot of ideas to teach with. For example, one ECE felt the least comfortable with Data Management because she “do[es] not have a lot of lot of ideas other than surveys and charts.”

Educators identified a variety of professional development (PD) opportunities and topics they would like to help improve their teaching and facilitating of early math. The PD opportunity most frequently suggested was early math-based training/workshops ($n = 36$). There were many suggested topics for this type of PD: mathematics games, early mathematics trajectories, development of mathematical concepts, provocations, teaching mathematics to young children, teaching mathematics through play, number talks, number sense activities, Rekenrek materials, engaging group mathematics activities, problem solving, use of technology for assessment in FDK, Smartboard activities, using manipulatives, and inquiry-based mathematics in Grade 1. The second most frequently suggested PD was ideas and resources for mathematics centres/play-based learning/inquiry ($n = 15$). The third most frequent PD item identified was read-aloud mathematics books ($n = 3$).

Early Mathematics Educators' Beliefs

The Early Mathematics Education Survey asked educators to identify their early mathematics education beliefs by rating their agreement with statements about early mathematics development and education and rating the importance of mathematics curriculum strands and problem solving. Educators were also asked to provide open-text responses to questions about areas of mathematics their students excelled in, areas of mathematics their students struggled with, what they believed students needed to be successful in Grade 1 math, and the effectiveness of full-day kindergarten in promoting early mathematics knowledge and skills. Early childhood educators and teachers were asked about their beliefs about early mathematics development and education by their level of agreement (4 = strongly agree, 3 = agree, 2 = disagree, 1 = strongly disagree) across 12 items. Educators endorsed (i.e., strongly agree or agree) all but two of the statements regarding beliefs of early mathematical education. The two items that were the most strongly endorsed were using manipulatives in mathematics helps young children develop conceptual understanding ($M = 3.91$, $SD = .34$) and it is important to develop positive mathematics attitudes and dispositions at an early age ($M = 3.89$, $SD = .31$). The two items that were not supported (i.e., disagree) were: *teaching early mathematics in a whole group setting is developmentally appropriate* ($M = 2.80$, $SD = .76$) and *rote memorization in mathematics helps young children develop automatic responses* ($M = 2.75$, $SD = .78$).

ECE and K/1 teachers had statistically different levels of endorsement within three items: (c) *Using manipulatives in mathematics helps young children develop conceptual understanding*; (f) *A solid foundation in early mathematical development leads to greater school success*; and (h) *mathematics is a subject that should be taught daily*. In all cases K/1 teachers indicated a greater level of endorsement. However, the effect sizes for these differences were moderate (Cohen's $d = .38$, $.41$, & $.60$, respectively). Please see Table 1 for further details.

Table 1. Item level descriptive statistics for beliefs about early mathematics development and education

Please rate your level of agreement with the following statements:	Mean (SD)		
	ECE	K/1 Teacher	Total
a) It is important to develop mathematical knowledge skills when children are young.	3.76(.47)	3.89(.31)	3.82(.40)

Please rate your level of agreement with the following statements:	Mean (SD)		
	ECE	K/1 Teacher	Total
b) It is important to develop positive mathematics attitudes and dispositions at an early age.	3.84(.36)	3.94(.24)	3.89(.31)
c) Using manipulatives in mathematics helps young children develop conceptual understanding.	3.84(.45)*	3.97(.18)*	3.91(.34)
d) Rote memorization in mathematics helps young children develop automatic responses.	2.89(.80)	2.62(.75)	2.75(.78)
e) Using games in mathematics reinforces early mathematical knowledge and understanding.	3.81(.40)	3.91(.29)	3.85(.36)
f) A solid foundation in early mathematical development leads to greater school success.	3.63(.60)*	3.84(.41)*	3.73(.53)
g) Teaching early mathematics in a whole group setting is developmentally appropriate.	2.85(.77)	2.73(.77)	2.80(.76)
h) Mathematics is a subject that should be taught daily.	3.42(.69)*	3.77(.46)*	3.60(.61)
i) Early mathematics education should be largely play-based.	3.46(.69)	3.48(.69)	3.47(.69)
j) Early mathematics education should be largely taught through learning centres.	3.08(.73)	3.09(.71)	3.01(.72)
k) I have a good grasp of early mathematics education and development.	3.19(.59)	3.27(.57)	3.23(.58)
l) I feel well-equipped to teach mathematics to young children.	3.25(.62)	3.36(.60)	3.30(.62)

Note. * denotes significance at $\alpha = 0.05$; Strongly Agree = 4; Agree = 3; Disagree = 2; Strongly Disagree = 1.

ECE and K/1 teachers were also asked to rate the importance (4 = very important, 3 = important, 2 = somewhat important, 1 = not important) they place upon the strands of mathematics curriculum (e.g., numbers, geometry, measurement, patterning, data management) and problem solving (see Figure 3). *Numbers* ($M = 3.26$, $SD = 1.43$) and *problem solving* ($M = 3.18$, $SD = 1.44$) were reported as important, while *patterning* ($M = 2.94$, $SD = 1.39$), *measurement* ($M = 2.78$, $SD = 1.36$), *geometry* ($M = 2.74$, $SD = 1.36$), and *data management* ($M = 2.68$, $SD = 1.34$) were reported only as somewhat important. No statistical differences between ECE and K/1 teachers were detected.

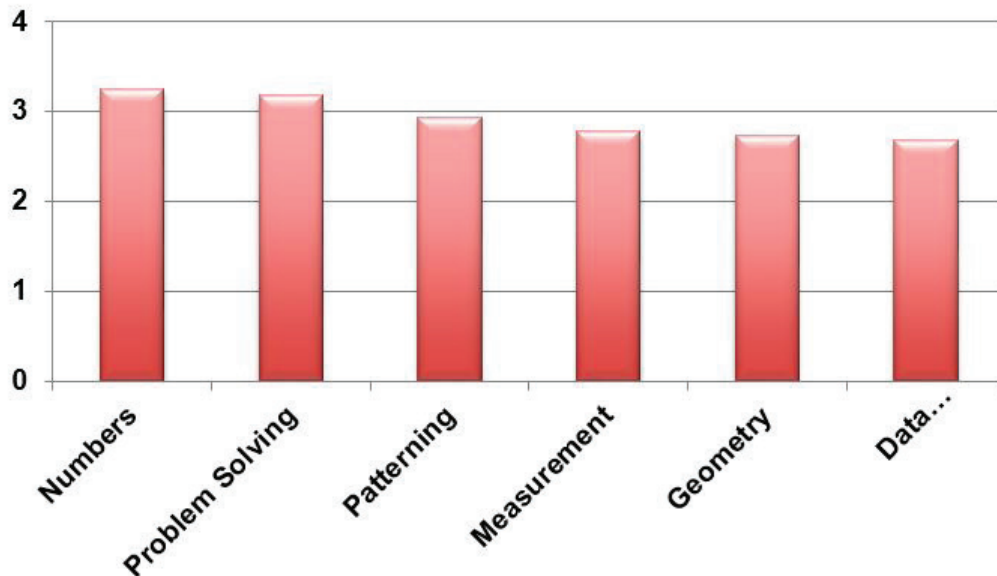


Figure 3. All participants' perceived importance of mathematics strands in early mathematics education programs using a 4-point scale

Qualitative analysis of open-ended responses concerning the type of mathematical knowledge, thinking, and skills students excel at, indicated that educators believed their students excelled most at Number Sense and Numeration ($n = 49$), Patterning ($n = 20$), and Geometry and Spatial Sense ($n = 13$). The most frequent explanations for why students excelled in these areas were the frequency of instruction/exposure ($n = 24$), use of manipulatives ($n = 11$), and the area of mathematics is interesting/enjoyable ($n = 9$). Educators described a connection between students excelling at Number Sense and Numeration and the frequency of exposure to that content area. One ECE noted that Number Sense “is something that is started at the beginning of the year and we have continued with it for a few minutes at the beginning of mathematics large group.” Similarly, a teacher communicated that students excel at Number Sense “because we infuse it into

all our other subject areas.” With respect to patterning, a teacher believed her students excelled at it because it involved “hands-on activities” and the “interest level is greater.” Similarly, another teacher indicated that her students excelled at Spatial Learning because it involved “visual[s] and manipulative[s] [that] represent the ideas easily and effectively. Students can see and move items to show their thinking.”

When asked what type of mathematical knowledge, thinking, and skills their students struggle with, educators indicated that students struggled most with Number Sense and Numeration ($n = 21$), Problem Solving ($n = 11$), and Representational Thinking/Communication of Thinking ($n = 10$). The most frequent explanations for why students struggled in these areas included the following: lack of developmental readiness/difficult concept ($n = 24$), lack of interest/motivation ($n = 4$), lack of time spent on the area ($n = 4$), and lack of foundational skills needed to build upon ($n = 4$). For example, one ECE reported, “Students struggle with subtracting by more than one and counting on from a given number because conservation of number hasn’t developed yet.” Similarly, a teacher noted that some students struggle with number sense and attributed this to their developmental readiness: “representing and manipulating numbers and really understanding what numbers represent. It can be a bit abstract for some of them and they are all at different stages in their development. Some are ready and some are not.” When students struggle with number sense and numeration, this impacts their ability to understand related mathematical concepts. A teacher explained, “Some of my students struggle with reading and understanding what numbers really are and how they relate to each other, impacting many mathematical concepts such as patterns [and] relationships of more and less in data and measurement.” With respect to students’ struggle with problem solving, two teachers indicated that some of their students were not developmentally ready for it, while another teacher believed her student had a “lack of motivation to try.” Educators reported that some students struggled with representational thinking/communication of thinking. An ECE believed her students found it difficult to share their reasoning because there was not enough time and teacher support to facilitate the process: “[there is] not enough time, due to high student: teacher ratio, to encourage students to share their thought processes with educators and engage one-to-one in these deep and rich discussions.” In contrast, a teacher who indicated that her students struggled with explaining their thinking stated, “I am beginning to wonder if it’s developmentally inappropriate.”

When asked what type of mathematical knowledge, thinking, and skills students should develop in kindergarten to be successful in Grade 1, educators indicated that kindergarten students should develop mathematical knowledge, skills, and thinking associated with Number Sense and Numeration ($n = 47$), Problem Solving ($n = 29$), and Geometry and Spatial Sense ($n = 20$). Despite the fact that most educators indicated that students should develop mathematical knowledge, skills, and thinking in Number Sense and Numeration to be successful in Grade 1, there was variability in what this entailed. For example, some educators thought students should focus on numbers 1 to 10, while other educators identified numbers 1 to 20 as the foci, and yet others suggested numbers 1 to 100. An ECE succinctly described the mathematics knowledge, skills, and thinking students need to be successful in Grade 1: “Students need to have a good understanding of numbers and their relationships. They need to be comfortable with exploring a wide variety of ways to solve problems that involve numbers... They need to be allowed to explore a variety of concepts around math, develop the language of mathematics and see it as an integral part of their lives.”

Educators held mixed opinions about the effectiveness of the full-day kindergarten program in equipping students to be successful in Grade 1 math. Educators who believed the program was successfully equipping students in Grade 1 mathematics believed the FDK program was successful because it used hands-on activities with mathematics manipulatives that lead to better preparation ($n = 8$), gave students the ability to develop strong Number Sense ($n = 5$), and developed students’ ability to see different ways to solve a problem ($n = 4$). Educators who believed the FDK program was less effective in equipping students to be successful in Grade 1 mathematics identified these challenges: a solely pedagogical play approach to mathematics instruction is not effective ($n = 11$), large class sizes and not enough support for individual students ($n = 8$), and a big mathematics curriculum gap between kindergarten and Grade 1 ($n = 4$).

Early Mathematics Pedagogy

To facilitate the examination of early mathematics pedagogy, the Early Mathematics Education Survey asked educators to rate the frequency of their instructional practices, identify which assessment practices they used, and provide information about their level of involvement. ECE and K/1 teachers reported the frequency (4 = 4–5 times a week, 3 =

3 times a week, 2 = 1–2 times a week, 1 = never) in which they employed various instructional strategies during their mathematics lessons (see Table 2). Common instructional strategies included *mathematics activities with manipulatives* ($M = 3.61$, $SD = .74$), *play-based learning* ($M = 3.56$, $SD = .80$), and *learning centres* ($M = 3.30$, $SD = .98$), while *mathematics worksheets* ($M = 1.58$, $SD = .84$) and *mathematics journals* ($M = 1.45$, $SD = .76$) were less commonly used. Statistical differences between ECE and K/1 teachers' frequency of instructional strategies were detected within four items: *whole group mathematics lessons*, *small group mathematics lessons*, *mathematics activities with manipulatives*, and *mathematics worksheets*. In all cases K/1 teachers indicated a greater level of frequency of instructional strategies. The effect sizes were moderate (Cohen's $d = .81$, $.45$, $.44$, and $.52$).

Table 2. Average frequency of instructional strategies within early mathematics program

Instructional strategies	Mean (SD)		
	ECE	K/1 Teacher	Total
Whole-group mathematics lessons	2.55(1.18)*	3.40(.88)*	2.98(1.12)
Small-group mathematics lessons	2.61(1.05)*	3.05(.86)*	2.83(.98)
One-on-one instruction	2.75(1.09)	2.77(.89)	2.76(.99)
Learning centres	3.18(1.06)	3.42(.88)	3.30(.98)
Mathematics activities with manipulatives	3.45(.87)*	3.77(.53)*	3.61(.74)
Mathematics worksheets	1.36(.61)*	1.79(.98)*	1.58(.84)
Play-based learning	3.67(.74)	3.47(.84)	3.56(.80)
Mathematics games	3.01(.96)	3.26(.81)	3.14(.89)
Mathematics journals	1.48(.92)	1.45(.64)	1.45(.76)
Read alouds	2.61(1.16)	2.77(.90)	2.69(1.04)

Note. * denotes significance at $\alpha = 0.05$; 4–5 times a week = 4; 3 times a week = 3; 1–2 times a week = 2; never = 1.

Educators were asked to select which of six assessment strategies were used in their early mathematics lessons. *Observation/anecdotal records* (91.5% of participants) and *photo-video documentation of activities/play* (65.4% of participants) were commonly used, with few educators utilizing either *worksheets* (24.6%) or *mathematics journals* (16.9%). Statistical differences between ECE and K/1 teachers were noted within four assessment practices. The chi-square tests showed a significant relationship between assessment practices and educational position for one-on-one conferencing ($\chi^2 [1, N = 130]$

= 14.136, $p < .001$); skills-based checklist ($X^2 [1, N = 130] = 9.450, p = .002$); worksheets ($X^2 [1, N = 130] = 12.013, p < .001$); and mathematics journals ($X^2 [1, N = 130] = 4.760, p = .029$). Within the four assessment practices, K/1 teachers indicated greater frequency of use. Please see Table 3 for more details.

Table 3. Frequency of assessment practices within the early mathematics program

Assessment Practices	Frequency (%)		
	ECE	K/1 Teacher	Total
Observation/Anecdotal records	90.5	92.5	91.5
One-on-one conferencing	49.2 ^a	80.6 ^a	65.4
Skills-based checklists	28.6 ^b	55.2 ^b	42.3
Photo/video documentation of activities/play	71.4	82.1	76.9
Worksheets	11.1 ^c	37.3 ^c	24.6
Mathematics journals	9.5 ^d	23.9 ^d	16.9

Note. ^{a, b, c, d} denotes significance at $\alpha = 0.05$.

Recognizing that some educators are less involved in mathematics education than desired, educators were asked about their level of involvement. Educators who desired greater involvement indicated they desired greater teacher knowledge (e.g., mathematics teaching ideas, understanding of mathematics development, and resources; $n = 14$), time for teaching/a greater teaching role ($n = 9$), and professional development ($n = 9$). With respect to a better conceptual understanding of mathematics development, one such teacher stated: “I would like to learn more about how young children learn math.” Three ECEs indicated that they would like similar mathematics PD opportunities as those made available to teachers. One ECE explained, “[I would like to] be a part of education programs in mathematics that the classroom teachers are involved in. They know what to do because they are in-serviced.” Another ECE indicated that optimal early mathematics PD for full-day early learning kindergarten teams would involve both teaching partners (i.e., the teacher and ECE) so they “can go together and learn.”

Summary of Results

In summary, the majority of ECE and K/1 teachers did not receive early mathematics education training and did not have specific math-related qualifications. Early mathematics educators felt most comfortable teaching Number Sense and Numeration, Patterning, and Geometry and Spatial Sense because these strands: were an area of teacher strength, were fun and engaging, and could be taught with manipulatives/concrete connections. Areas educators felt least comfortable with were Geometry and Spatial Sense, Instructional Strategies, and Data Management because they were unsure of what to teach conceptually/lacked expertise, the mathematics area(s) was less engaging/interesting, and they lacked teaching ideas. While some educators identified feeling comfortable teaching Geometry and Spatial Sense, others did not. Recent interest and professional development resources for Geometry and Spatial Sense may account for some teachers' comfort with teaching this strand (see Moss, Bruce, Caswell, Fynn, & Hawes, 2016). With respect to professional development, educators identified a need for early math-based training (e.g., how young children learn mathematics developmentally and how to teach young children mathematics effectively), ideas and resources for mathematics centres/play-based learning/inquiry, and mathematics read-aloud books.

When asked about early mathematics education beliefs, educators strongly endorsed the use of manipulatives to help develop conceptual understanding and believed it was important to help develop positive attitudes and dispositions to mathematics at an early age. However, educators believed less strongly in teaching early mathematics in a whole group setting and using rote memorization in mathematics to help young children develop automatic responses. Educators rated numbers and problem solving as important areas of math, while patterning, measurement, geometry, and data management were rated only as somewhat important. Educators believed their students excelled in the areas of Number Sense and Numeration, Patterning, and Geometry and Spatial Sense because they experienced a greater frequency of instruction/exposure, used manipulatives, and the area of mathematics was interesting/enjoyable. According to educators, their students struggled most with Number Sense and Numeration, Problem Solving, and Representational Thinking/Communication of Thinking because they lacked developmental readiness/it was a difficult concept, lacked interest/motivation, and lacked instructional time in the area(s). It is interesting to note that educators identified Number Sense and

Numeration as both an area of strength and an area of weakness for students. Given that Number Sense and Numeration represents foundational mathematical skills, it is likely that some students are developing these skills well, while others are falling behind early on. Educators believed that kindergarten students needed strong Number Sense and Numeration, Problem Solving, and Geometry and Spatial Sense abilities to be successful in Grade 1. However, there was variability in what strong Number Sense and Numeration skills meant: some educators thought students should focus on numbers 1 to 10, others identified numbers 1 to 20 as the foci, and yet others suggested numbers 1 to 100. There were mixed educator opinions about the effectiveness of the FDK program in equipping students to be successful in Grade 1 math. Some believed it was effective because it used hands-on activities with mathematics manipulatives, gave students the ability to develop strong Number Sense, and developed students' ability to see different ways to solve a problem. Other educators believed it was less effective because solely using a pedagogical play approach for mathematics instruction was ineffective, large class sizes made it a challenge to support the individual needs of students, and there is a big mathematics curriculum gap between kindergarten and Grade 1.

In terms of early mathematics education pedagogy, educators reported frequently (i.e., more than three times a week) using mathematics activities with manipulatives, play-based learning, and learning centres. Conversely, mathematics worksheets and mathematics journals were less commonly (e.g., less than once or twice a week) used. K/1 teachers reported using whole group mathematics lessons, small group mathematics lessons, mathematics activities with manipulatives, and mathematics worksheets more frequently than ECEs. The assessment practices commonly used by early mathematics educators included observation/ anecdotal records and photo-video documentation of activities/play. Few educators utilized worksheets or mathematics journals. K/1 teachers reported using one-on-one conferencing, skills-based checklist, worksheets, and mathematics journals more frequently than ECEs. Educators who desired greater involvement in early mathematics education identified a need for greater teacher knowledge (e.g. how young children learn mathematics and how to teach early math), more time to work with students, and professional development opportunities.

Discussion

The Early Mathematics Education Survey was an educational needs assessment of early mathematics education in Ontario. Survey responses from ECEs, kindergarten teachers, and Grade 1 teachers provided valuable insights about gaps that exist between current and desired early mathematics practices. This section of the article discusses potential ways to improve the quality of early mathematics education in Ontario and across Canada in regard to educator knowledge, educator beliefs, and pedagogy.

Educator Knowledge

One of the most important findings from the Early Mathematics Education Survey is that the majority of ECEs and teachers lacked training in early mathematics education. Given the importance of early mathematics education (Duncan et al., 2007; Jordan et al., 2009), a recent shift toward using intentional mathematics teaching in the context of play or inquiry (Hachey, 2013; Knaus, 2017; Stipek, 2013), and that educators identified areas of early mathematics they are less comfortable with (e.g., Geometry and Spatial Sense, Instructional Strategies, and Data Management), there is a need for pre-service and in-service early mathematics training. While the pre-service model for teachers in Ontario has been extended to two years and the RMS provides increased opportunities for mathematics professional development for educators, specialized training in early mathematics education is needed. Educators require a solid understanding of how young children learn mathematics and how to teach challenging mathematical concepts to young children. In addition, they need opportunities to learn, try, and collaboratively reflect on new instructional strategies with their colleagues (Borko, Mayfield, Marion, Flexer, & Cumbo, 1997). Building teachers' understanding and confidence in mathematics may help move them toward a more inquiry-based approach to mathematics education (Stipek et al., 2001).

Educators' Beliefs

Educators in this survey tended to ascribe to the notion of “developmentally appropriate practice” in early math. For example, educators indicated that students struggled with particular areas of mathematics (e.g., Number Sense and Numeration, Problem Solving,

and Representational Thinking/Communication of Thinking) because they lacked “developmental readiness” and they were less in favour of whole group instruction and rote memorization. Educators’ notion of “developmental readiness” in early mathematics may be a cause for concern because evidence suggests that young children are more capable of engaging with complex mathematical concepts than previously thought (Fisher et al., 2013; Presser et al., 2015). As a result, educators may need to learn more about young children’s mathematics capability based on mathematical learning trajectories (see Clements, 2018), and early mathematics curriculum and instructional practices may need to be aligned with such learning trajectories. With respect to educators being less in favour of whole group instruction, it may be that this instructional strategy has traditionally been used inappropriately in the past (e.g., educators have required young children to sit for long periods), but may still be effective if used for shorter periods of time. Although the idea of rote memorization is also considered developmentally inappropriate by some educators because it is often associated with “drill and kill strategies,” there are fun and engaging ways to help young children develop automaticity of numbers and number facts needed to engage with some challenging mathematical concepts. While some educators reported that a play-based approach to FDK is promoting mathematics because it provides young children with hands-on opportunities to learn math, other educators believed that solely a play-based approach to mathematics instruction was ineffective. With respect to the value of play-based learning in the context of early mathematics education, it is important for educators to be made aware that a continuum of play exists (see Pyle & Danniels, 2017), with only one specific type of play, namely guided play facilitated by a knowledgeable and responsive adult, being linked to early mathematics learning (Fisher et al., 2013; Moss, Bruce, & Bobis, 2016; Pyle, DeLuca, & Danniels, 2017).

Pedagogy

Survey responses from educators indicated that many are implementing an inquiry-/play-based approach to early mathematics instruction and assessment. This is in line with the recent shift toward intentional mathematics teaching in the context of play or inquiry (Hachey, 2013; Knaus, 2017; Stipek, 2013). The intentional teaching approach is complex and requires educators to have specialized knowledge about mathematics curriculum content and how young children’s mathematical thinking develops. With this in

mind, educators require pre-service and in-service training to help them skilfully implement an inquiry-/play-based approach to early math. Teachers and ECEs who indicated that they would like to be more involved in early mathematics education communicated that they needed a greater understanding of mathematics to facilitate their involvement. Survey responses from ECEs suggested that they may not be as involved in early mathematics education as their teacher partners. In addition, ECEs indicated that they required the same in-service early mathematics opportunities as teachers so that they would be equipped to teach and facilitate math. Ideally, in-service opportunities for teachers and ECEs should occur jointly to reinforce the team teaching approach in FDK.

Conclusion

The Early Mathematics Education Survey responses from ECEs, kindergarten teachers, and Grade 1 teachers provided a brief account of the current state of early mathematics education in Ontario and identified four major gaps that need to be addressed to improve the quality of early mathematics instruction across the province. First, the majority of ECEs, kindergarten, and Grade 1 teachers do not have training in early math. Given the foundational nature of early number competence for later mathematical understanding and success (Duncan et al., 2007; Jordan et al., 2009; Romano et al., 2010), educators working with young children require specialized pre-service and in-service early mathematics training. Second, educators' beliefs about developmental readiness and instruction in early mathematics should be informed by learning trajectories (see Clements, 2018) that outline what mathematical concepts young children should be taught and how they are learned. Early mathematics curriculum may also need to be reviewed to ensure it aligns with these learning trajectories. Third, educators need to be made aware that only one particular type of play-based learning is associated with early mathematics achievement: guided play facilitated by a knowledgeable and responsive adult (Fisher et al., 2013; Moss, Bruce, & Bobis, 2016; Pyle, DeLuca, & Danniels, 2017). Fourth, while the use of an inquiry-/play-based approach to early mathematics education is beneficial, it is complex and requires educator training and mentoring to be done effectively. Ideally, early mathematics training for ECE and teacher teams working in FDK should be administered jointly to promote team teaching.

References

- Borko, H., Mayfield, V., Marion, S., Flexer, R., & Cumbo, K. (1997). Teachers' developing ideas and practices about mathematics performance assessment: successes, stumbling blocks, and implications for professional development. *Teaching and Teacher Education, 13*, 259–278.
- Clements, D. (2018). Learning and teaching with *Learning Trajectories* [LT]². Retrieved from <https://www.learningtrajectories.org/>
- Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., . . . Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*(6), 1428–1446.
- Education Quality and Accountability Office. (2017). *The assessments*. Retrieved from <http://www.eqao.com/en/assessments>
- Ernest, P. (1989). The knowledge, beliefs, and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching, 15*, 13–33.
- Fisher, K. R., Hirsh-Pasek, K., Newcombe, N., & Golinkoff, R. M. (2013). Taking shape: Supporting pre-schoolers' acquisition of geometric knowledge through guided play. *Child Development, 84*(6), 1872–1878.
- Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38*(4), 293–304.
- Goulding, M., Rowland, T., & Barber, P. (2002). Does it matter? Primary teacher trainees' subject knowledge in mathematics. *British Educational Research Journal, 28*(5), 689–704.
- Hachey, A. (2013). The early childhood mathematics education revolution. *Early Education and Development, 24*(4), 419–430.
- Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early mathematics matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45*(3), 850–867. doi:10.1037/a0014939.

- Knaus, M. (2017). Supporting early mathematics learning in early childhood settings. *Australasian Journal of Early Childhood*, 42(3), 4–13.
- Lancaster, P. (2013, December 15). Mathematics and society. In *The Canadian Encyclopedia*. Retrieved from <http://www.thecanadianencyclopedia.ca/en/article/mathematics-and-society/>
- Let's Talk Science and Amgen Canada Inc. (2012). *Spotlight on science leaning: A benchmark of Canadian talent*. Retrieved from http://letstalkscience.ca/Portals/0/Documents/RPS/Spotlight/Science_Learning_Booklet_web_version.pdf
- Moss, J., Bruce, C., & Bobis, J. (2016). *Young children's access to powerful mathematical ideas: A review of current challenges and new developments in early years*. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (3rd ed., pp. 153–190). New York, NY: Taylor & Francis.
- Moss, J., Bruce, C., Caswell, B., Flynn, T., & Hawes, Z. (2016). *Taking shape: Activities to develop geometric and spatial thinking*. New York, NY: Pearson.
- National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.
- Ontario Ministry of Education (OME). (2010). *Full-day early learning kindergarten program, draft version*. Toronto, ON: Ontario Ministry of Education.
- Ontario Ministry of Education (OME). (2016). *Ontario's renewed mathematics strategy*. Retrieved from <https://news.ontario.ca/edu/en/2016/04/ontario-dedicating-60-million-for-renewed-math-strategy.html>
- Presser, A. L., Clements, M., Ginsburg H., & Ertle, B. (2015). Big maths for little kids: The effectiveness of a pre-school and kindergarten mathematics curriculum. *Early Education and Development*, 26(3), 399–426.
- Pyle, A., & Danniels, E. (2017). A continuum of play-based learning: The role of the teacher in a play-based pedagogy and the fear of hijacking play. *Early Education & Development*, 28(3), 274–289. doi:10.1080/10409289.2016.1220771

- Pyle, A., De Luca, C., & Danniels, E. (2017). Context and implications Document for: A scoping review of research on play-based pedagogies in kindergarten education. *Review of Education*. doi:10.1002/rev3.3098
- Romano, E., Babchishin, L., Pagani, L. S., & Kohen, D. (2010). School readiness and later achievement: Replication and extension using a nationwide Canadian survey. *Developmental Psychology*, *46*(5), 995–1007. doi:10.1037/a0018880
- Schofield, H. L. (1981). Teacher effects on cognitive and affective pupil outcomes in elementary school mathematics. *Journal of Educational Psychology*, *73*, 462–471.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4–14.
- Stipek, D. (2013). Mathematics in early childhood education: Revolution or evolution? *Early Education & Development*, *24*(4), 431–435.
- Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teacher's beliefs and practices related to mathematics instruction. *Teaching and Teacher Education*, *17*, 213–226.
- Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education*, *11*(2), 139–164.