# Analysis of Material Flow for Shoe Production Process using GERT

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***Abstract***

The Graphical Evaluation and Review Technique (GERT) is an approach used for analyzing a class of networks with the characteristics of probability that a branch of the network is part of a realization of the network duration time or time interval associated with the branch .

In this study, we deal with the network model from raw material to the finished footwear in Ethiopia's production factory. Such network is referred to as stochastic network and consists of a set of branches and nodes. Expected completion time, variance and probability distribution of the completion time of the material are evaluated. By utilizing the results, we try to propose an improvement plan on the material flow of the production process for the shoe manufacturing factory. We can plan to encounter the bottleneck process of lasting and stitching operations so as to improve the productivity and quality of the overall shoe production process along with the Kaizen activity.

***Keywords: GERT, Expected completion time, Variance, Probability distribution, Shoe production process, Material flow***

## 1. Introduction

Nowadays Ethiopia's footwear industry is becoming a promising manufacturing sector. The industry is supporting the growing economy of the country especially by exporting finished footwear to the global market. Even if the industry is becoming promising it is facing many straggles of being competitive in the international market. Almost all footwear factories in Ethiopia are operating the production intensively manual. This traditional manual operation has negative impact not only on the production output but also on the material flow of the shoe production process. Graphical Evaluation and Review Technique, commonly known as GERT [1)~7)] is a network analysis technique that allows probabilistic treatment both network logic and estimation of activity duration. GERT is an approach used for analyzing a class of networks with the characteristics of a probability that a branch of the network is part of a realization of the network and a duration time or time interval associated with the branch if the branch is part of the realization of the network. Such networks are referred to as stochastic networks and consist of a set of branches and nodes.

In this paper, we try to analysis the manufacturing process of the shoe production company in Ethiopia using GERT. After modeling the network from raw material to the finished footwear, the expected completion time, variance and probability distribution of the completion time are evaluated [8)~11)]. By utilizing the result, we try to propose an improvement plan of the manufacturing process for the shoe production.

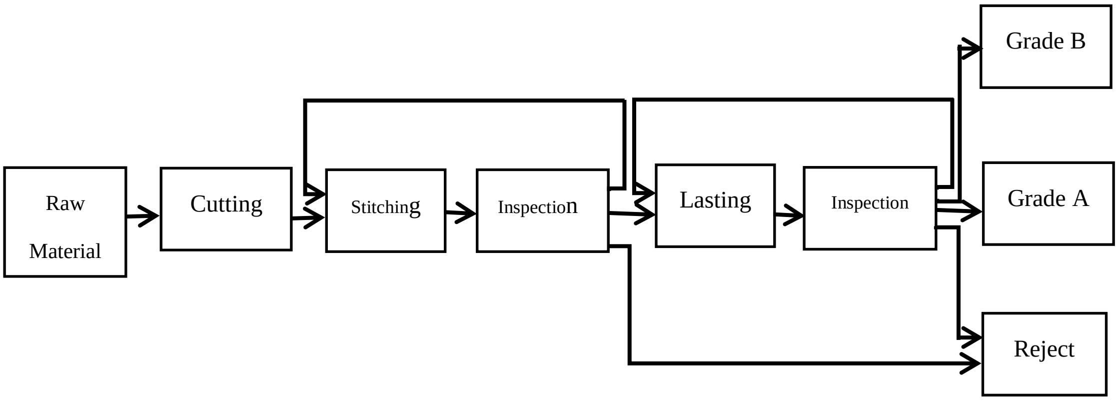



Fig.1 The material flow of shoe production process

## 2. Material flow for shoe production process

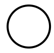
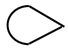
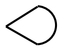
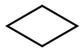
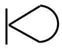
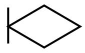
Figure 1 shows the flow and sequence of the shoe production process. The production process has mainly three sections. These are cutting, stitching and lasting sections. The first step in the shoe production process is cutting of the loaded raw materials into different components in cutting section, then the components sewn to be the upper part of the shoe in stitching section. The last step will be finalizing the shoe in lasting section. In the inspection sections the upper and the shoe will be given different grades according to the set standards and the rejected ones will be identified.

## 3. Analysis by using GERT

### 3.1 GERT model

The stochastic networks are composed of directed branches (arcs, edges, and transmittances) and logical nodes (vertices). A directed branch is associated with nodes at which it terminates. Two parameters are associated with a branch, which are, the probability that a branch is taken and the time required to accomplish the activity that the branch represents. A node in a stochastic network consists of an input (receiving and contributive) side and an output (emitting and distributive) side. The node types used in the network model graph for the shoe production process are several types of nodes types as shown in Table 1.

Table 1 Symbols of logical node

| Input \ Output | Deterministic | Probabilistic |
|---|---|---|
| “And” |  |  |
| Inclusive”OR ” |  |  |
| Exclusive”OR ” |  |  |

Signal flow graphs (SFGs), sometimes referred to simply flow graphs, are graphic representation used for the modeling and analysis of linear systems. The SFG representation of Fig.2 shows the transmittance from state $i$ to $j$ is dependent on the products $p_{ij}h_{ij}(z)$, where $p_{ij}$ and $h_{ij}(z)$ are the probability and the z-function of constant, respectively.

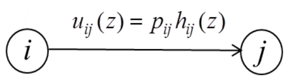



Fig.2 Basic element of SFG

### 3.2 Analysis of the shoe production process

We analyze the GERT network model of the shoe production process shown in Fig.3. Table 2 describes explicitly the activity codes of each branch, the time required to accomplish the activity that represents the branch and the probability that a branch is taken. We got the data of Table 2 from a shoe factory found in Ethiopia. From Table 2, we obtain Table 3 and Table 4.

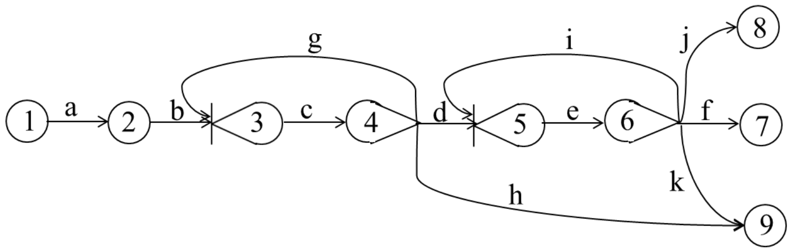



Fig.3 GERT network model graph for the shoe production process

Table 2 Activity code and its description

| Activity | Operation | Branch | Duration time(sec) | Probability |
|---|---|---|---|---|
| a | Cutting | 1 - 2 | 50 | 1 |
| b | Stitching | 2 - 3 | 110 | 1 |
| c | Stitching inspection | 3 - 4 | 50 | 1 |
| d | Lasting | 4 - 5 | 120 | 0.99 |
| e | Lasting inspection | 5 - 6 | 90 | 1 |
| f | Grade A product | 6 - 7 | 0 | 0.97 |
| g | Stitching rework | 4 - 3 | 124 | 0.008 |
| h | Reject | 4 - 9 | 0 | 0.002 |
| i | Lasting rework | 6 - 5 | 148 | 0.015 |
| j | Grade B product | 6 - 8 | 0 | 0.01 |
| k | Reject | 6 - 9 | 0 | 0.005 |

Table 3 Parameter values for each activity of Fig.3

$a=(1, 50),\ b=(1,\ 110),\ c=(1, 50),\ \ d=(0.990, 120),\ e=(1, 90)$
$f=(0.970,\ 0),\ g=(0.008, 124),\ \ h=(0.002, 0),\ i=(0.015, 148)$
$j=(0.010,\ 0), k=(0.005,\ 0)$

Table 4 z-function (constant)

$u_a=z^{50},\ u_b=z^{110},\ u_c=z^{50},\ u_d=0.990z^{120}, u_e=z^{90},$
$u_f=0.970z^0, u_g=0.008z^{124}, u_h=0.002z^0, u_i=0.015z^{148},$
$u_j=0.010z^0, u_k=0.005z^0$

Put the variable of node to $x_1 \sim x_{11}$ and transmittance on the arcs to z-transform $u_a(z) \sim u_k(z)$. The variables $u_a(z) \sim u_k(z)$ are represented as $u_a \sim u_k$ for simplification and the parentheses $z$ in can be omitted. By using the rules of signal flow graph, the following equation of $x_2$ is obtained.

$$x_2 = u_a x_1 \tag{1}$$

By the same way, the following equations for $x_3 \sim x_8$ are obtained.

$$x_3 = u_b x_2 + u_g x_4 \tag{2}$$

$$x_4 = u_c x_3 \tag{3}$$

$$x_5 = u_d x_4 + u_i x_6 \tag{4}$$

$$x_6 = u_e x_5 \tag{5}$$

$$x_7 = u_f x_6 \tag{6}$$

$$x_8 = u_j x_6 \tag{7}$$

By solving eqs.(1)~(7) for $x_7 \sim x_9$, we obtain the following equations.

$$x_7 = \frac{x_1 u_a u_b u_c u_d u_e u_f}{(1-u_c u_g)(1-u_e u_i)}, \text{ (Grade A products)} \tag{8}$$

$$x_8 = \frac{x_1 u_a u_b u_c u_d u_e u_j}{(1-u_c u_g)(1-u_e u_i)}$$ ,( Grade B products) (9)

$$x_8 = \frac{x_1 u_a u_b u_c u_d u_e u_j}{(1-u_c u_g)(1-u_e u_i)}$$ ,( Rejects) (10)

By substituting $u_i$ of Table 2 into eqs.(8)~(10) and $x_1 = 1$, we obtain the following equations of $x_7 \sim x_9$.

$$x_7 = \frac{0.9603z^{420}}{(1-0.008z^{174})(1-0.015z^{238})} \quad (11)$$

$$x_8 = \frac{0.0099z^{420}}{(1-0.008z^{174})(1-0.015z^{238})} \quad (12)$$

$$x_9 = \frac{z^{210}(0.002+0.00495z^{210}-0.00003z^{238})}{(1-0.008z^{174})(1-0.015z^{238})} \quad (13)$$

By using the equations (11), (12) and (13), we are able to calculate the expected completion time, the variance and the probability distribution of grade A shoes, grade B shoes and rejects.

### 3.3 Probability of each path

In this section, we evaluate the probability that the completion time of path $(1 \to i)(i = 7.8.9)$ is realized.

$$p(1 \to 7) = x_7\big|_{z=1} = 0.9828 \quad (14)$$

$$p(1 \to 8) = x_8\big|_{z=1} = 0.0101 \quad (15)$$

$$p(1 \to 9) = x_9\big|_{z=1} = 0.0071 \quad (16)$$

### 3.4 Expected completion time of each path

In this section, the expected completion time of path $(1 \to i)$ $(i = 7.8.9)$ are evaluated.

$$E(t_{1\to7}) = \frac{\partial}{\partial z}\left(\frac{x_7}{0.9828}\right)\Big|_{z=1} = 425[\mathrm{sec}] \quad (17)$$

$$E(t_{1\to8}) = \frac{\partial}{\partial z}\left(\frac{x_8}{0.0101}\right)\Big|_{z=1} = 425[\mathrm{sec}] \quad (18)$$

$$E(t_{1\to9}) = \frac{\partial}{\partial z}\left(\frac{x_9}{0.0071}\right)\Big|_{z=1} = 364[\mathrm{sec}] \quad (19)$$

### 3.5 Variance of completion time of each path

In this section, the variance of completion time of path $(1 \to i)$ $(i = 7.8.9)$ is evaluated.

$$V(t_{1\to7}) = \frac{\partial^2}{\partial z^2}\left(\frac{x_7}{0.9828}\right)\Big|_{z=1} + \frac{\partial}{\partial z}\left(\frac{x_7}{0.9828}\right)\Big|_{z=1} - \left\{\frac{\partial}{\partial z}\left(\frac{x_7}{0.9828}\right)\Big|_{z=1}\right\}^2 = 1122 \quad (20)$$

$$V(t_{1\to8}) = \frac{\partial^2}{\partial z^2}\left(\frac{x_8}{0.0101}\right)\Big|_{z=1} + \frac{\partial}{\partial z}\left(\frac{x_8}{0.0101}\right)\Big|_{z=1} - \left\{\frac{\partial}{\partial z}\left(\frac{x_8}{0.0101}\right)\Big|_{z=1}\right\}^2 = 1122 \quad (21)$$

$$V(t_{1\to9}) = \frac{\partial^2}{\partial z^2}\left(\frac{x_9}{0.0071}\right)\Big|_{z=1} + \frac{\partial}{\partial z}\left(\frac{x_9}{0.0071}\right)\Big|_{z=1} - \left\{\frac{\partial}{\partial z}\left(\frac{x_9}{0.0071}\right)\Big|_{z=1}\right\}^2 = 10166 \quad (22)$$

### 3.6 Probability distribution

By using the series expansion on the right side of eqs.(11)~(13), we obtain probability distribution of the completion time path $(1 \to i)(i = 7.8.9)$ (Table 5 (a)~(c)).

Table 5 Probability distribution of the completion time

(a) Path $(1 \to 7)$

| $t_{1\to7}$ | $P(t_{1\to7})$ |
|---|---|
| 420 | 0.9771 |
| 594 | 0.0078 |
| 658 | 0.0147 |
| 768 | 0.0001 |
| 832 | 0.0001 |
| 896 | 0.0002 |
| … | … |

(b) Path $(1 \to 8)$

| $t_{1\to8}$ | $P(t_{1\to8})$ |
|---|---|
| 420 | 0.9771 |
| 594 | 0.0078 |
| 658 | 0.0147 |
| 768 | 0.0001 |
| 832 | 0.0001 |
| 896 | 0.0002 |
| … | … |

(c) Path $(1 \to 9)$

| $t_{1\to9}$ | $P(t_{1\to9})$ |
|---|---|
| 210 | 0.2824 |
| 384 | 0.0023 |
| 420 | 0.699 |
| 594 | 0.0056 |
| 658 | 0.0105 |
| 832 | 0.0001 |
| 896 | 0.0002 |
| … | … |

## 4. Conclusion

In this paper, we evaluated the probability of completion time of each path, the expected completion time, the variance and the probability distribution of grade A shoes, grade B shoes and rejects of the production process. In the inspection section there are list of check points to identify the grade of the shoe. These requirements are related with the nature of the raw material, the correct alignment of the shoe, the scratch occurred on the surface of the finished shoe and so on. The shoe which passes all the inspection requirements will be graded as grade A shoe and it will be exported to the international market. The shoe which fall only requirements related to upper surface is graded as grade B shoe and sold only in local market. The remains will be rejected one. By utilizing the results, the improvement plan of the shoe production process will be proposed to enhance the material flow of shoe production process along with the Kaizen method. The result gained up to now is that the bottleneck processes are identified. The bottleneck processes are lasting and stitching operations. The future work is improving this bottleneck so as to improve the material flow of shoe production process and the productivity and quality of the overall shoe production process.

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# GERT を用いた靴製造工程における物の流れの解析

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概要

GERT(Graphical Evaluation and Review Technique）は確率を伴うある種類のネットワークを解析するための 1 つのアプローチであり，ネットワークのブランチはブランチと関連したネットワーク経過時間または時間間隔の実現の一部である。

本研究では，エチオピアにある靴の生産工場における原料から最終製品の靴となるネットワークモデルを扱う．そのようなネットワークは確率的ネットワークと呼ばれ，ブランチとノードの組から構成される．靴の期待完成時間，完成時間の分散および確率分布が求められる．結果を利用することによって，靴の生産工場における製造工程の製品の流れに対して改良計画を提案する．本研究では，作業のカイゼン活動とともに全体の靴製造工程の生産性と品質を改良するために，釣り込み（lasting）と縫いあげ作業(Stitching)のボトルネック工程に直面して計画できる．