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The Classical Limit of Entropic Quantum Dynamics

An honors thesis presented to the
Department of Physics,
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in partial fulfillment of the requirements
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and
graduation from The Honors College.

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ABSTRACT

The framework of entropic dynamics (ED) allows one to derive quantum mechanics as an application of entropic inference. In this work we derive the classical limit of quantum mechanics in the context of ED. Our goal is to find conditions so that the center of mass (CM) of a system of N particles behaves as a classical particle. What is of interest is that \hbar remains finite at all steps in the calculation and that the classical motion is obtained as the result of a central limit theorem. More explicitly we show that if the system is sufficiently large, and if the CM is initially uncorrelated with other degrees of freedom, then the CM follows a smooth trajectory and obeys the classical Hamilton-Jacobi with a vanishing quantum potential.

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INTRODUCTION

The development of quantum theory during the early twentieth century is one of the largest scientific breakthroughs to have ever been achieved. From the concept of the photoelectric effect that won Albert Einstein his Nobel prize to the work done by Robert Oppenheimer during the development of the atomic bomb, quantum theory has revolutionized the way that scientists approach atomic and molecular physics. Unfortunately while quantum theory is an incredibly powerful tool that can be used to model many previously unsolvable phenomena, it is both an incomplete framework for the world and conflicts with thoroughly tested theories such as relativity.

A common misconception about quantum mechanics is that it is a single theory that describes atomic physics, while it is in fact a collection of theories which all describe different atomic processes. From Quantum Chromodynamics which accurately describes the strong interaction in an atomic nucleus, to Quantum Field Theory which is used in high energy particle physics, the different theories that comprise quantum mechanics thoroughly span a significant amount of known atomic phenomena. These theories are not always congruent, however, and while one theory can model a given phenomena more accurately than any other theory, it may not be easily applicable to another atomic phenomena. It is because of this that there exist many theories which all jockey to be considered the most accurate representation of the quantum world.

The most common interpretation of quantum mechanics is the Copenhagen interpretation. This was one of the first strong attempts at the creation of a quantum theory. Among its other postulates is that particles have no definite position, but instead are described by a wave function which can be manipulated into probability distributions that define observable values such as position or momentum. This concept is that which led to the famous Schrödinger's cat thought experiment. In this thought experiment a cat is put into a box with a vial of poison which is attached to an apparatus with a radioactive atom which breaks the vial when the atom exists in a given energy state. Since the wave function of the atom does not give the actual state of the atom, but instead a probability distribution of the atom over all possible states, the atom exists in all states at once before it is measured, and thus simultaneously both breaks and does not break the vial. Thus until the box is opened and the state of the atom observed, the cat is both alive and dead. The Copenhagen interpretation of quantum mechanics illustrates both the unusual probabilistic nature of the framework built to model atomic processes, but also that it is clearly unknown how the atomic world truly behaves. The concepts put forth by quantum mechanics are just interpretations of abstract mathematics used to model the system, and not what actually occurs in a given system.

The discrepancies between the numerous quantum theories is often overshadowed by the classical physics. Due to classical physics being modeled generally by Hamilton-Jacobi mechanics and quantum mechanics is modeled through the use of a complex wave function, systems generally cannot be modeled by both quantum and classical physics. This has led to both an enormous problem for the scientific community and an interesting subsection of study. The problem that has arisen for the community is that the theories of quantum mechanics and general relativity cannot be reconciled and a functioning theory of quantum gravity has not yet been produced to reconcile these two fundamental theories of physics. A subsection of physics that must be studied is that of semi-classical physics.

Semi-classical physics is the study of systems that due to size, such as macromolecules, exhibit both quantum and classical effects. This transition is generally modeled by the branch of statistical mechanics. Conveniently, several paradigms of quantum mechanics are based upon the methods of statistical mechanics. Many of these theories originate from famed theoretical physicist David Bohm, and as a result are described as Bohmian theories. In this paper we will use one such theory, Entropic Dynamics (ED), to explore the transition from the quantum world to classical world.

Entropic Dynamics is a framework that allows the formulation of dynamical theories as applications of entropic methods of inference.¹ Its main application so far has been the derivation of the Schrödinger equation [2]-[5]. At this stage in its development, the value of ED has been clarifying conceptual issues such as the uncertainty relations, the quantum measurement problem, the connection to Bohmian mechanics, etc. (see [5] and references therein). Indeed, given the experimental success of non-relativistic quantum mechanics one should not expect any predictions that deviate from those of the standard quantum theory — at least not in the non-relativistic regime discussed here. This situation will probably change once ED is extended to realms, such as gravity, cosmology, or very high energies, where the status of quantum theory is less well tested.

Whenever we reason on the basis of incomplete information it is inevitable that the inference of most quantities will be afflicted by uncertainty. There are however situations where for some very specially chosen variables one can, despite the lack of information, achieve complete predictability. One prominent example is thermodynamics where on the basis of just a few pieces of information one can make precise predictions about the macroscopic behavior of systems with huge numbers of particles. The catch, of course, is that the answers to the vast majority of the questions one might conceivably ask about the system (such as what is

¹The principle of maximum entropy as a method for inference can be traced to the pioneering work of E. T. Jaynes. For a pedagogical overview of Bayesian and entropic inference and for further references see [1].

the position of a particular molecule) will remain completely unanswered. However, for some very special variables — macrovariables such as temperature, pressure, heat capacity, etc. — the incomplete information happens to be sufficient to attain remarkable levels of certainty. It is this phenomenon, which relies on the very careful choice of special variables, that accounts for the emergence of determinism from theories such as statistical mechanics or entropic dynamics that are intrinsically indeterministic.

In this paper we are concerned with studying a similar phenomenon, the emergence of a deterministic classical mechanics from an intrinsically indeterministic ED. The classical limit is usually taken by letting $\hbar \rightarrow 0$ [6]. This is mathematically correct, but does not address the physical fact that any realistic macroscopic object is comprised of a large number of particles whose motion is described by a quantum mechanics in which \hbar is non zero. We consider the entropic dynamics of a system of N quantum particles and show that for large N the center of mass (CM) behaves as a classical particle. The result is not in itself surprising but some features of the derivation might however be of interest. One is that \hbar remains finite at all steps in the derivation which means that a mesoscopic or macroscopic object will behave classically while all its component particles remain explicitly quantum mechanical.

Two other interesting features are more directly related to the formalism of ED. In ED particles follow Brownian trajectories. For large N we find, as a consequence of the central limit theorem, that the CM follows a smooth trajectory. This condition is clearly necessary for a fully classical motion but it is definitely not sufficient. (Particles in Bohmian mechanics, for example, also follow smooth trajectories. [7][8])

In ED the motion is described through two coupled equations, one is a Fokker-Planck equation for the probability density and the other is an equation for the phase of the wave function. The latter is a Hamilton-Jacobi modified by the addition of a quantum potential the form of which is suggested by considerations of information geometry [4]. It is the presence of this quantum potential that is ultimately responsible for quantum behavior.

We find that under rather general conditions the CM motion decouples, and for a sufficiently large system the quantum potential for the CM motion vanishes. Thus, for $N \rightarrow \infty$, the CM follows smooth trajectories described by a classical Hamilton-Jacobi equation. In conclusion, the CM motion of a system with a large number of particles follows a classical trajectory.

We begin with a brief overview of ED [4]. In section 3 we use the central limit theorem to discuss the ED of the CM following the discussion in [1][9]. The Hamilton-Jacobi equation for the CM motion, which is the new contribution of this paper, is derived in section 4. We conclude in section 5 with some final remarks.

ENTROPIC DYNAMICS – A BRIEF REVIEW

Our system consists of N particles living in a flat Euclidean space \mathbf{X} with metric δ_{ab} . The particles have definite positions x_n^a . (The index $n = 1 \dots N$ denotes the particle and $a = 1, 2, 3$ the spatial coordinate.) The position of the system in configuration space $\mathbf{X}_N = \mathbf{X} \times \dots \times \mathbf{X}$ — what one might call a microstate — is denoted x^A where $A = (n, a)$.

The goal is to predict the particle positions and their motion on the basis of some limited information. The main piece of information is that particles follow continuous trajectories which means that the motion can be analyzed as a sequence of short steps. Thus, the first goal is to find the probability $P(x'|x)$ that the system take an infinitesimally short step from x^A to $x'^A = x^A + \Delta x^A$. The tool to assess this probability is the method of maximum entropy and the relevant information is introduced through constraints.

The information that particles take short steps from x_n^a to $x_n'^a = x_n^a + \Delta x_n^a$ is expressed through N independent constraints,

$$\langle \Delta x_n^a \Delta x_n^b \rangle \delta_{ab} = \kappa_n, \quad (n = 1 \dots N). \quad (1)$$

where we shall eventually take the limit $\kappa_n \rightarrow 0$. But particles do not move independently of each other. In order to introduce correlations among them one imposes one additional constraint,

$$\langle \Delta x^A \rangle \partial_A \phi = \sum_{n=1}^N \langle \Delta x_n^a \rangle \frac{\partial \phi}{\partial x_n^a} = \kappa', \quad (2)$$

where ϕ is called the drift potential and $\partial_A = \partial / \partial x^A = \partial / \partial x_n^a$. κ' is another small but for now unspecified position-independent constant. Eq.(2) is a single constraint; it acts on the $3N$ -dimensional configuration space and is ultimately responsible for such quantum effects as interference and entanglement. The physical nature of the ϕ potential need not be discussed at this point — it is sufficient to postulate its existence and to note that it will eventually be transformed into the phase of the wave function.²

The result of maximizing entropy leads to

$$P(x'|x) = \frac{1}{\zeta} \exp\left[-\sum_n \left(\frac{1}{2} \alpha_n \Delta x_n^a \Delta x_n^b \delta_{ab} - \alpha' \Delta x_n^a \frac{\partial \phi}{\partial x_n^a}\right)\right], \quad (3)$$

²Since ϕ affects the motion of particles it plays a role analogous to that of a pilot wave or an electromagnetic field. Indeed, ϕ is as real as the vector potential A^a and the intimate relation between the two manifests itself through a gauge symmetry (see [2][5]). In [2] ϕ was interpreted as being itself of entropic origin and, in the context of particles with spin, it is possible to interpret ϕ as an angular variable. Clearly much remains to be clarified here.

where ζ is a normalization constant and α_n and α' are Lagrange multipliers. The specification of α' is relatively simple: one can rescale $\alpha'\phi \rightarrow \phi$ which amounts to choosing $\alpha' = 1$ without affecting the quantum dynamics [11]. The specification of α_n is considerably more involved.

Once the probability $P(x'|x)$ for an infinitesimal step is found one proceeds by iteration to derive a Fokker-Planck equation for the probability distribution $\rho(x, t)$. This requires the introduction of an “entropic” time t as a book keeping device to keep track of the accumulation of changes. The “clock” that measures this entropic time is provided by the particle fluctuations which leads to

$$\alpha_n = \frac{m_n}{\eta\Delta t}. \quad (4)$$

The m_n 's are particle-specific constants that will be called “masses” and η is a constant that fixes the units of time relative to those of length and mass [2]-[5].

As discussed in [4], up to an arbitrary scale factor C the geometry of the configuration space \mathbf{X}_N is uniquely determined by the information metric,³

$$m_{AB} = C \int dx' P(x'|x) \frac{\partial \log P(x'|x)}{\partial x^A} \frac{\partial \log P(x'|x)}{\partial x^B}. \quad (5)$$

With an appropriate choice of C , the metric m_{AB} is recognized as the “mass” tensor while its inverse m^{AB} is the “diffusion” tensor,

$$m_{AB} = m_n \delta_{AB} \quad \text{and} \quad m^{AB} = \frac{1}{m_n} \delta^{AB}. \quad (6)$$

The choice of α_n in (4) leads to a simple dynamics: $P(x'|x)$ in eq.(3) is a Wiener process. A generic displacement Δx^A is expressed as an expected drift plus a fluctuation,

$$\Delta x^A = b^A \Delta t + \Delta w^A, \quad (7)$$

where $b^A(x)$ is the drift velocity,

$$\langle \Delta x^A \rangle = b^A \Delta t \quad \text{with} \quad b^A = \eta m^{AB} \partial_B \phi, \quad (8)$$

³For an introduction to information geometry with references to the literature see *e.g.*, [1].

and the fluctuations Δw^A satisfy,

$$\langle \Delta w^A \rangle = 0 \quad \text{and} \quad \langle \Delta w^A \Delta w^B \rangle = \frac{\eta}{m_n} \delta^{AB} \Delta t = \eta m^{AB} \Delta t . \quad (9)$$

Having introduced a convenient notion of time through (4), the accumulation of many changes leads to a Fokker-Planck equation for the probability distribution $\rho(x, t)$, [2][3]

$$\partial_t \rho = -\partial_A (\rho v^A) , \quad (10)$$

where v^A is the velocity of the probability flow in configuration space or *current velocity*,

$$v^A = b^A + u^A \quad \text{and} \quad u^A = -\eta m^{AB} \partial_B \log \rho^{1/2} \quad (11)$$

is the *osmotic velocity*. Since both b^A and u^A are gradients, the current velocity is a gradient too,

$$v^A = m^{AB} \partial_B \Phi \quad \text{where} \quad \Phi = \eta \phi - \eta \log \rho^{1/2} . \quad (12)$$

The FP equation (10) describes a standard diffusion. In order to obtain a “non-dissipative” dynamics [12] one must revise the constraint 2 after each step Δt . The result is that the drift potential ϕ (or equivalently Φ) becomes a dynamical degree of freedom instead of an externally prescribed field. The required updating $\Phi \rightarrow \Phi + \delta\Phi$ is such that a certain functional $\tilde{H}[\rho, \Phi]$ is conserved,

$$\tilde{H}[\rho + \delta\rho, \Phi + \delta\Phi] = \tilde{H}[\rho, \Phi] . \quad (13)$$

The requirement that \tilde{H} be conserved for arbitrary choices of ρ and Φ implies that the coupled evolution of ρ and Φ is given by a conjugate pair of Hamilton’s equations,

$$\partial_t \rho = \frac{\delta \tilde{H}}{\delta \Phi} \quad \text{and} \quad \partial_t \Phi = -\frac{\delta \tilde{H}}{\delta \rho} . \quad (14)$$

The form of the “ensemble” Hamiltonian \tilde{H} is chosen so that the first equation reproduces the FP equation (10). Then, the second equation in (10) becomes a Hamilton-Jacobi equation. A more complete specification

of \tilde{H} is suggested by information geometry. The natural choice is

$$\tilde{H}[\rho, \Phi] = \int dx \left[\frac{1}{2} \rho m^{AB} \partial_A \Phi \partial_B \Phi + \rho V + \xi m^{AB} \frac{1}{\rho} \partial_A \rho \partial_B \rho \right], \quad (15)$$

where the first term in the integrand is the “kinetic” term that reproduces (10). The second term includes the standard potential $V(x)$, and the third term, which is motivated by information geometry, is called the “quantum” potential. The parameter $\xi = \hbar^2/8$ defines the value of what we call Planck’s constant \hbar [4].

The formulation of ED is now complete. Its equivalence to quantum mechanics is verified by combining ρ and Φ into a single complex function,

$$\Psi = \rho^{1/2} \exp(i\Phi/\hbar) \quad \text{where} \quad \hbar = (8\xi)^{1/2}. \quad (16)$$

Then Hamilton’s equations (14) can be written as a single complex linear Schrödinger equation,

$$i\hbar \partial_t \Psi = -\frac{\hbar^2}{2} m^{AB} \partial_A \partial_B \Psi + V \Psi. \quad (17)$$

A FOKKER-PLANCK EQUATION FOR THE CENTER OF MASS

As seen in eqs. (7) and (9) the particle trajectories are afflicted by fluctuations; ED is intrinsically indeterministic. To achieve a classical limit it is necessary to suppress these fluctuations which is conveniently accomplished by making η (or \hbar) sufficiently small or by making m sufficiently large. A more realistic approach [9] is to consider the motion of a very special macro-variable, the center of mass of a large body composed of N particles. The goal is to show that while the positions and motions of all the component particles remain uncertain, the CM motion is fully predictable.

From (3) with (4) the transition probability for a short step is

$$P(x'|x) = \frac{1}{Z_N} \exp \left[-\sum_{n=1}^N \frac{m_n}{2\eta \Delta t} \delta_{ab} (\Delta x_n^a - \Delta \bar{x}_n^a) (\Delta x_n^b - \Delta \bar{x}_n^b) \right]. \quad (18)$$

The CM coordinates are

$$X^a = \frac{1}{M} \sum_{n=1}^N m_n x^a \quad \text{where} \quad M = N\bar{m} = \sum_{n=1}^N m_n. \quad (19)$$

The probability that the center of mass moves from X to $X' = X + \Delta X$ is found from (18),

$$P(X'|X) = \int d^{3N} x' P(x'|x) \delta \left(\Delta X - \frac{1}{M} \sum_n m_n \Delta x \right). \quad (20)$$

The evaluation of $P(X'|X)$ is a straightforward consequence of the central limit theorem (see e.g. [1]). The result is

$$P(X'|X) \propto \exp \left[-\frac{M}{2\eta\Delta t} \delta_{ab} (\Delta X^a - \Delta \bar{X}^a) (\Delta X^b - \Delta \bar{X}^b) \right]. \quad (21)$$

Therefore, a short step of the CM is given by

$$\Delta X^a = \Delta \bar{X}^a + \Delta W^a, \quad (22)$$

where, using eq.(7) and (8), the expected step $\Delta \bar{X}^a$ is

$$\Delta \bar{X}^a = \frac{1}{M} \sum_{n=1}^N m_n \Delta \bar{x}^a = \frac{\eta\Delta t}{N\bar{m}} \sum_{n=1}^N \frac{\partial \phi}{\partial x_n^a}, \quad (23)$$

and the fluctuations ΔW^a such that

$$\langle \Delta W^a \Delta W^b \rangle = \frac{\eta}{N\bar{m}} \Delta t \delta^{ab}. \quad (24)$$

Thus, we see that the expected drift $\Delta \bar{X}^a$ is of order N^0 (because the N terms in the sum offset the N in the denominator) whereas the fluctuations are of order $N^{-1/2}$. For large N the fluctuations become negligible and the CM follows a smooth trajectory.

Equation (23) can be simplified considerably by introducing the internal coordinates \hat{x}_n^a relative to the CM,

$$x_n^a = X^a + \hat{x}_n^a \quad \text{so that} \quad \sum_{n=1}^N m_n \hat{x}_n^a = 0. \quad (25)$$

Differentiate

$$\phi(x_1^a \dots x_N^a) = \phi(X^a + \hat{x}_1^a, \dots, X^a + \hat{x}_N^a) \quad (26)$$

keeping \hat{x} constant to get

$$\frac{\partial \phi}{\partial X^a} = \sum_{n=1}^N \frac{\partial \phi}{\partial x_n^a} \frac{\partial x_n^a}{\partial X^a} = \sum_{n=1}^N \frac{\partial \phi}{\partial x_n^a}. \quad (27)$$

Then eqs. (23) and (24) become

$$\Delta \bar{X}^a = \frac{\eta}{M} \frac{\partial \phi}{\partial X^a} \Delta t = B^a \Delta t \quad \text{and} \quad \langle \Delta W^a \Delta W^b \rangle = \frac{\eta}{M} \Delta t \delta^{ab}. \quad (28)$$

Now, in exactly the same way [1] that the FP eq.(10) can be derived from eqs.(8) and (9), we can use (28) to derive a FP equation for the probability $\rho_{CM}(X, t)$ of the CM. The result is

$$\partial_t \rho_{CM} = -\partial_a (\rho_{CM} V^a), \quad (29)$$

where V^a is the *current* velocity,

$$V^a = B^a + U^a \quad \text{and} \quad U^a = -\frac{\eta}{M} \partial_B \log \rho^{1/2} \quad (30)$$

is the *osmotic* velocity. Notice however that for large N or M the osmotic contribution vanishes, there is no diffusion and the probability distribution ρ_{CM} flows along the drift velocity. This is exactly what one expects in classical mechanics. For large N (28) shows that the CM trajectory follows the gradient of the scalar function,

$$M \frac{dX^a}{dt} = M \frac{d\bar{X}^a}{dt} = \eta \frac{\partial \phi}{\partial X^a}. \quad (31)$$

which is the classical equation of motion of a particle of mass M in the Hamilton-Jacobi formalism.

As mentioned earlier this condition is necessary to obtain the classical limit but it is not sufficient. We must now show that the scalar function $\eta\phi$ obeys the classical HJ equation.

THE HAMILTON-JACOBI EQUATION FOR THE CENTER OF MASS

We start from eqs.(14) and (15) to get the HJ equation for the N particles,

$$-\partial_t \Phi = \frac{\delta \tilde{H}}{\delta \rho} = \frac{1}{2} m^{AB} \partial_A \Phi \partial_B \Phi + V + V^Q. \quad (32)$$

The quantum potential V^Q is given by

$$V^Q = \frac{\delta}{\delta \rho} \xi m^{AB} I_{AB}[\rho] \quad \text{where} \quad I_{AB}[\rho] = \int dx \frac{1}{\rho} \partial_A \rho \partial_B \rho, \quad (33)$$

which leads to

$$V_Q = -4\xi m^{AB} \frac{\partial_A \partial_B \rho^{1/2}}{\rho^{1/2}} . \quad (34)$$

Next we change from x_n^a to the coordinates (X^a, \hat{x}_n^a) which requires some identities. Use (19) and (25), to write

$$\frac{\partial \Phi}{\partial x_n^a} = \frac{m_n}{M} \frac{\partial \Phi}{\partial X^a} + \frac{\partial \Phi}{\partial \hat{x}_n^a} , \quad (35)$$

which, using the analogue of (27),

$$\frac{\partial \Phi}{\partial X^a} = \sum_{n=1}^N \frac{\partial \Phi}{\partial x_n^a} , \quad (36)$$

leads to

$$\sum_{n=1}^N \frac{\partial \Phi}{\partial \hat{x}_n^a} = 0 . \quad (37)$$

In the HJ formalism these equations have a straightforward interpretation: eq.(36) expresses the CM momentum in terms of the momentum of the component particles and eq.(37) states that the total momentum relative to the CM vanishes.

Using (35) with (37) in the kinetic term in (32),

$$\frac{1}{2} m^{AB} \partial_A \Phi \partial_B \Phi = \frac{1}{2M} \left(\frac{\partial \Phi}{\partial X^a} \right)^2 + \sum_{n=1}^N \frac{1}{2m_n} \left(\frac{\partial \Phi}{\partial \hat{x}_n^a} \right)^2 , \quad (38)$$

shows that the contributions of the CM and the internal coordinates decouple. It remains to study whether this decoupling survives the effects of the potential V and quantum potential V_Q terms in (32).

The situation with V is straightforward: there exist broad families of potentials of practical interest for which the CM and the internal coordinates decouple. For example,

$$V(x) = V_{\text{ext}}(X) + \sum_{n,\ell} V_{\text{int}}(\hat{x}_n - \hat{x}_\ell) , \quad (39)$$

where V_{ext} and V_{int} represent an external potential and the interparticle interactions respectively.

The situation with the quantum potential (33) is subtler. A generic probability distribution

$$\rho(x) = \rho(X) \rho(\hat{x}|X) \quad (40)$$

leads to quantum correlations between the CM and the internal coordinates. In order for the CM motion to

decouple it is necessary to invoke some decoherence mechanism that leads to initial conditions where X and \hat{x} are independent,

$$\rho(x) = \rho_{CM}(X)\hat{\rho}(\hat{x}) . \quad (41)$$

Then the relevant term in (33) decouples,

$$\begin{aligned} m^{AB}I_{AB}[\rho] &= \sum_n \frac{1}{m_n} \int dx \rho \left(\frac{\partial \log \rho}{\partial x_n^a} \right)^2 \\ &= \sum_n \frac{1}{m_n} \int dXd\hat{x} \rho_{CM}\hat{\rho} \left(\frac{m_n}{M} \frac{\partial \log \rho_{CM}}{\partial X^a} + \frac{\partial \log \hat{\rho}}{\partial \hat{x}_n^a} \right)^2 \\ &= \frac{1}{M} \int dX \rho_{CM} \left(\frac{\partial \log \rho_{CM}}{\partial X^a} \right)^2 + \sum_n \frac{1}{m_n} \int d\hat{x} \hat{\rho} \left(\frac{\partial \log \hat{\rho}}{\partial \hat{x}_n^a} \right)^2 . \end{aligned} \quad (42)$$

Therefore, the quantum potential becomes,

$$V^Q(x) = V_{CM}^Q(X) + \hat{V}_{CM}^Q(\hat{x}) \quad (43)$$

where

$$V_{CM}^Q(X) = -\frac{4\xi}{M\rho_{CM}^{1/2}} \delta^{ab} \frac{\partial^2 \rho_{CM}^{1/2}}{\partial X^a \partial X^b} = -\frac{4\xi}{M\rho_{CM}^{1/2}} \nabla^2 \rho_{CM}^{1/2} , \quad (44)$$

and

$$\hat{V}_{CM}^Q(\hat{x}) = -4\xi \sum_n \frac{1}{m_n} \frac{1}{\hat{\rho}^{1/2}} \delta^{ab} \frac{\partial^2 \hat{\rho}^{1/2}}{\partial \hat{x}_n^a \partial \hat{x}_n^b} . \quad (45)$$

Collecting these results, (38), (39), and (42) into (32), the HJ becomes

$$\begin{aligned} -\partial_t \Phi &= \frac{1}{2M} \left(\frac{\partial \Phi}{\partial X^a} \right)^2 + V_{\text{ext}}(X) + V_{CM}^Q(X) \\ &+ \sum_{n=1}^N \frac{1}{2m_n} \left(\frac{\partial \Phi}{\partial \hat{x}_n^a} \right)^2 + \sum_{n,\ell} V_{\text{int}}(\hat{x}_n - \hat{x}_\ell) + \hat{V}_{CM}^Q(\hat{x}) . \end{aligned} \quad (46)$$

Thus, under the conditions (39) and (41) the HJ equation can be solved by separation of variables,

$$\Phi(x) = \Phi_{CM}(X) + \hat{\Phi}(\hat{x}) . \quad (47)$$

Direct substitution leads to the HJ equation for the CM,

$$-\partial_t \Phi_{CM} = \frac{1}{2M} \left(\frac{\partial \Phi_{CM}}{\partial X^a} \right)^2 + V_{\text{ext}}(X) + V_{CM}^Q(X). \quad (48)$$

Inspection of the FP equation (29) and the HJ equation (48) shows that the CM motion is described by a standard Schrödinger equation. If the system is sufficiently large, $M \rightarrow \infty$, we see from (44) that V_{CM}^Q is suppressed. In conjunction with (28) which implies smooth trajectories we see that the CM motion is classical. This concludes our derivation.

FINAL REMARKS

We have derived the classical limit of entropic quantum dynamics. The derivation hinges on two features that are of an intrinsically probabilistic or inferential nature: one is the central limit theorem, the other is the specific form of the quantum potential as dictated by information geometry.

The ED framework appears to be ideally suited for the study the transition between classical and quantum mechanics. In future work we intend to study the effect of large but still finite values of N . The diffraction of macro-molecules, for example, might offer an interesting test case where N might be small enough that the quantum potential is not yet negligible, but large enough that a discussion in terms of a smooth CM trajectory might be useful [13]. In addition, being an inference theory, entropic dynamics provides the natural framework to account for loss of information through decoherence [14][15]. The results of this paper provide a first step in that direction.

REFERENCES

- [1] For a pedagogical review see A. Caticha, *Entropic Inference and the Foundations of Physics* (monograph commissioned by the 11th Brazilian Meeting on Bayesian Statistics – EBEB-2012); <http://www.albany.edu/physics/ACaticha-EIFP-book.pdf>.
- [2] A. Caticha, *J. Phys. A: Math. Theor.* **44**, 225303 (2011); arXiv.org:1005.2357.
- [3] A. Caticha, *J. Phys.: Conf. Ser.* **504**, 012009 (2014); arXiv.org:1403.3822.
- [4] A. Caticha, D. Bartolomeo, M. Reginatto, in *Bayesian Inference and Maximum Entropy Methods in Science and Engineering*, ed. by Mohammad-Djafari, A. and Barbaresco, F. *AIP Conf. Proc.* **1641**, 155 (2015); arXiv.org:1412.5629.

- [5] A. Caticha, *Entropy* **17**, 6110 (2015); arXiv.org:1509.03222.
- [6] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory* (Pergamon, New York 1965)
- [7] D. Bohm and B. J. Hiley, *The undivided universe – An ontological interpretation of quantum theory* (Routledge, New York, 1993).
- [8] P. R. Holland, *The Quantum Theory of Motion* (Cambridge UP, 1993).
- [9] D. T. Johnson, “Generalized Galilean Transformations and the Measurement Problem in the Entropic Dynamics Approach to Quantum Theory”, Ph.D. thesis, University at Albany (2011) (arXiv:1105.1384).
- [10] D. Bartolomeo, A. Caticha, “Entropic Dynamics: the Schrödinger equation and its Bohmian limit,” to appear in *Bayesian Inference and Maximum Entropy Methods in Science and Engineering*, ed. by Giffin, A., AIP Conf. Proc. (2016); (arXiv:1512.09084).
- [11] D. Bartolomeo, A. Caticha, *J. Phys.: Conf. Ser.* **701**, 012009 (2016); (arXiv:1603.08469).
- [12] E. Nelson, “Connection between Brownian motion and quantum mechanics,” in *Einstein Symposium Berlin*, Lect. Notes Phys. **100**, 168 (Springer-Verlag, Berlin 1979).
- [13] M. Arndt, K. Hornberger, and A. Zeilinger, *Phys. World* **18**, 35 (2005); M. Arndt and K. Hornberger, *Nature Physics* **10**, 271 (2014).
- [14] W. H. Zurek, *Rev. Mod. Phys.* **75**, 715 (2003).
- [15] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Statamescu, and H. D. Zeh, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer, Berlin 2003).
- [16] A. Demme, A. Caticha, “The Classical Limit of Entropic Quantum Dynamics,” Paper presented at MaxEnt 2016, the 36th International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering (July 10-15, 2016, Ghent, Belgium), AIP Conf. Proc. (2016); (arXiv:1612.01905)